ONLINE APPENDIX:
A MEASURE OF RATIONALITY AND WELFARE

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Appendix D. Discussion

D.1. Preference Restrictions. The swaps index aims to evaluate the distance between individual choices and the preference maximization model. Let us note at this point that exactly the same logic can be applied to measure the distance between choices and stronger notions of rationalizability. In this section, we show how this is done by measuring how far an agent is from being an expected utility agent, but the same logic could be followed to incorporate other types of properties, such as time stationarity, quantity monotonicity, etc. Let X be a finite set of lotteries and denote by $\mathcal{P}^{EU} \subset \mathcal{P}$ the set of all linear orders over X having an expected utility representation.[1] We define the EU-swaps index by

$$I_{EU-S}(f) = \min_{P \in \mathcal{P}^{EU}} \sum_{(A,a)} f(A,a)|\{x \in A : xPa\}|.$$

The EU-swaps index minimizes the number of swaps needed to accommodate all the observations considering only the set of expected utility preferences.[2]

Example 1. Consider $X = \{x, y, z, w\}$ and let independence impose that x is above y if and only if z is above w. Let $f(\{x, y\}, x) = \frac{2}{5}$ and $f(\{x, y, w\}, y) = f(\{z, w\}, w) = \frac{3}{10}$. The swaps index computed in the space of all linear orders identifies the preference $xPyPwPz$ with an associated inconsistency of $\frac{3}{10}$. Since $P \notin \mathcal{P}^{EU}$, the EU-swaps index establishes a greater inconsistency, $\frac{2}{5}$, with associated preference $yP^{EU}xP^{EU}wP^{EU}z$. Notice that the comparison between alternatives x and y is now influenced not only by the observations $(\{x, y\}, x)$ and $(\{x, y, w\}, y)$, but also, through independence, by $(\{z, w\}, w)$.

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[1]Notice that standard expected utility representations usually involve infinite domains and indifferences. Here, by setting a finite domain of lotteries, we can assume that these preferences have no indifferences. We study the infinite case and the presence of indifferences in the next sections.

[2]The axiomatic characterization of I_{EU-S} follows the same structure as that of I_S with minor modifications.

D.2. Indifferences. So far we have considered linear orders and, hence, have ruled out the possibility of indifferences. Clearly, allowing for unrestricted indifferences makes the entire exercise vacuous, since data can always be rationalized by total indifference.[3] We can, however, introduce restricted indifference in a meaningful way. Let X be a finite set of alternatives, described by vectors of quantities of goods or attributes and hence, is partially ordered by a strictly monotone binary relation $\succ$.[4] Under these conditions, the swaps index allowing for indifferences again adopts the functional form of I_S, but minimizing over the set of weak orders that extend $\succ$, which we denote generically by $\succeq^*$, with strict part $\succ^*$. Namely,

$$I_{I-S}(f) = \min_{\succeq^*} \sum_{(A,a)} f(A,a)|\{x \in A : x \succ^* a\}|.$$

Example 2. Consider $X = \{(0,3),(0,6),(1,3),(2,0),(3,2),(6,1),(9,0)\}$ where $x = (x_1, x_2)$ describes the quantities of goods 1 and 2. Monotonicity forces $(0,6)$ and $(1,3)$ to be preferred to $(0,3)$. It also makes $(3,2),(6,1)$ and $(9,0)$ to be preferred to $(2,0)$. Otherwise, indifferences are allowed. Consider the collection $f(\{(0,6),(1,3),(2,0)\},(2,0)) = \frac{1}{2} = f(\{(0,3),(3,2),(6,1),(9,0)\},(0,3))$. The individual is directly revealing that $(0,3)$ is weakly preferred to $(3,2),(6,1)$ and $(9,0)$. Monotonicity implies that any of the latter is strictly better than $(2,0)$. But the individual is also directly revealing that $(2,0)$ is weakly preferred to $(0,6)$ and $(1,3)$, which dominate the chosen option $(0,3)$ in terms of monotonicity. Hence, the data cannot be rationalized by any monotonic weak order.

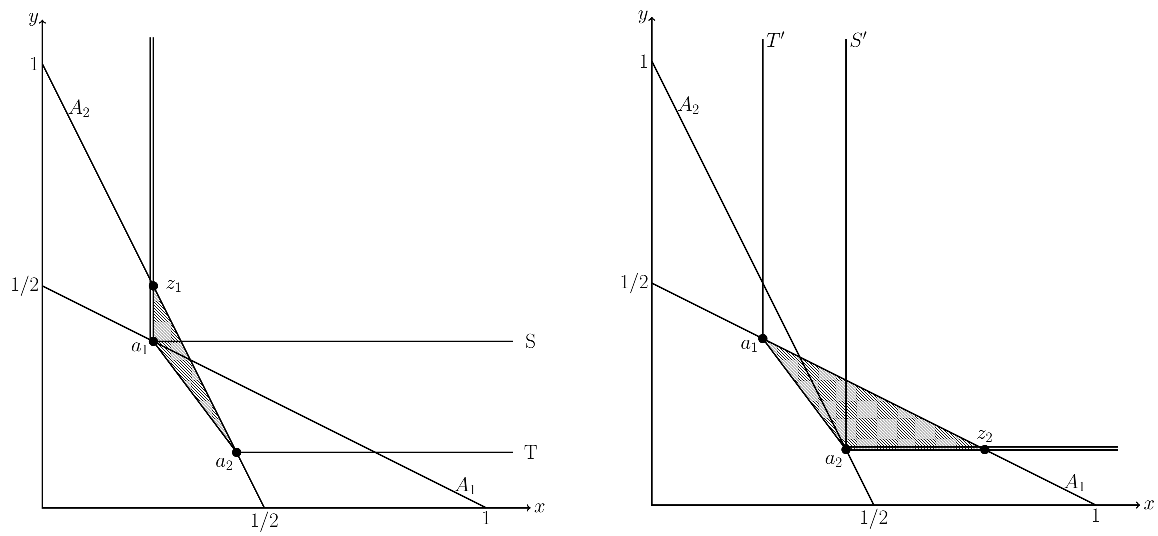
The revised version of the swaps index would work as follows. If $(2,0)$ is placed strictly above $(0,3)$, then the mass of required swaps is equal to $\frac{3}{2}$, since monotonicity requires that $(3,2),(6,1)$ and $(9,0)$ must be placed strictly above $(2,0)$ and hence, strictly above $(0,3)$. If on the contrary, $(0,3)$ is placed weakly above $(2,0)$, then the mass of required swaps is 1, since monotonicity implies that $(0,6)$ and $(1,3)$ must be placed strictly above $(0,3)$ and, hence, strictly above $(2,0)$. Then, the optimal weak order ranks $(0,3)$ weakly above $(2,0)$.

D.3. Infinite Sets of Alternatives. Economic models sometimes involve infinite sets of alternatives which are, typically, subsets of the Euclidean real space. We now show how the swaps index can be extended to these settings. Consider the standard consumer setting, where the set of all possible bundles is $X = \mathbb{R}^n_+$, and preferences are continuous, strictly monotonic and convex weak orders, that we denote by $\succeq$ (where the strict part is denoted by $\succ$). Menus are defined by $A = \{x : px \leq 1\}$, where $p \in \mathbb{R}^n_{++}$ describes the price vector,

[3] Formally, we would say that f is rationalizable by the weak order $\succeq$ if for every (A,a) with $f(A,a) > 0$, $a \succeq x$ for every $x \in A$. Again, small modifications of our axioms can be presented to characterize the index that follows.

[4] That is, $x \geq x'$ with $x \neq x'$ implies that $x \succ x'$.

Figure 1.



a: $a_1 \succ a_2$.　　b: $a_2 \succ a_1$.

and the data comprise a finite number of observations with positive mass. We define the consumer setting swaps index by

$$I_{CS-S}(f) = \inf_{\succeq} \sum_{(A,a)} f(A,a)\mu(\{x \in A : x \succ a\}),$$

where μ is the Lebesgue measure. That is, μ measures the volume of the upper contour set of the chosen element a in menu A according to the preference $\succeq$. Given the infinite number of weak orders over which I_{CS-S}, the infimum is used.

Example 3. Consider the set $X = \mathbb{R}^2_+$ and the following two observations, $(A_1, a_1) = (\{(x,y) : x + 2y \leq 1\}, (\frac{1}{4}, \frac{3}{8}))$ and $(A_2, a_2) = (\{(x,y) : 2x + y \leq 1\}, (\frac{7}{16}, \frac{1}{8}))$. Let f be the collection that assigns mass $1/2$ to each of these two observations. Clearly, f cannot be rationalized by any continuous, strictly monotonic and convex weak order. To see this, simply note that in observation 1 the individual has revealed that $a_1 \succeq a_2$. By strict monotonicity, $z_1 = (\frac{1}{4}, \frac{1}{2})$ must be strictly preferred to a_1 and hence to a_2. However, the individual reveals in menu 2 that a_2 is weakly preferred to z_1.

In order to describe $I_{CS-S}(f)$, let us divide the set of weak orders into those that place a_1 strictly above a_2, those that place a_2 strictly above a_1 and those that make them indifferent. Considering the first case, let $S = \{(x,y) : (x,y) \geq a_1\}$ and $T = \{(1/4, y) : y \geq$

$3/8\} \cup \{(x, y) : 4x + 3y = 17/8, 1/4 \leq x \leq 7/16\} \cup \{(x, 1/8) : x \geq 7/16\}$. In order to respect continuity, strict monotonicity and convexity, the smallest volume to be swapped in observation 2 can be achieved by considering the indifference curve of a_1 to be S and the limit of the indifference curve of a_2 to be T (see Figure 1a). Since these assumptions lead to no swap in menu 1, they provide the infimum swap for the case in which a_1 is strictly above a_2. The volume of the upper contour set would be exactly the area of the triangle formed by bundles a_1, a_2 and z_1, which is $3/256$. A similar analysis for the case of a_2 strictly above a_1 would require us to measure the area of the triangle formed by bundles a_1, a_2 and $z_2 = (3/4, 1/8)$, which is $10/256$ (see Figure 1b). Ranking a_1 and a_2 as indifferent would require the sum of these two volumes. Hence, it is optimal to place a_1 strictly above a_2 and $I_{CS-S}(f) = \frac{1}{2}\frac{3}{256}$.

Appendix E. Application: Data

Table 2: Inconsistency values

Individual	I_S	I_{HM}	I_W	Individual	I_S	I_{HM}	I_W
1	0.18	0.18	0.18	48	0.09	0.09	0.18
4	0.09	0.09	0.18	51	0.36	0.18	0.45
6	0.27	0.18	0.36	53	0.18	0.18	0.18
9	0.09	0.09	0.27	58	0.09	0.09	0.09
10	0.18	0.18	0.18	60	0.27	0.27	0.45
11	0.09	0.09	0.18	62	0.18	0.18	0.36
12	0.55	0.27	0.64	67	0.18	0.18	0.27
13	0.09	0.09	0.18	70	0.09	0.09	0.09
15	0.18	0.18	0.27	71	0.55	0.27	0.82
16	0.09	0.09	0.18	73	0.09	0.09	0.09
17	0.55	0.36	1	76	0.09	0.09	0.18
18	0.09	0.09	0.18	79	0.09	0.09	0.09
20	0.09	0.09	0.09	82	0.09	0.09	0.09
22	0.45	0.27	0.64	83	0.09	0.09	0.09
23	0.09	0.09	0.18	86	0.18	0.18	0.36
24	0.09	0.09	0.18	88	1.36	0.55	2.27
25	0.09	0.09	0.09	89	0.73	0.36	1.36
26	0.09	0.09	0.09	91	0.27	0.18	0.36
27	0.18	0.18	0.27	94	0.09	0.09	0.18
28	0.18	0.18	0.55	98	0.09	0.09	0.09
29	0.09	0.09	0.18	104	0.09	0.09	0.09
30	0.18	0.18	0.18	108	0.09	0.09	0.27
31	0.09	0.09	0.09	112	0.09	0.09	0.27
33	0.27	0.18	0.45	115	0.09	0.09	0.18
34	0.09	0.09	0.09	116	0.09	0.09	0.09
36	0.09	0.09	0.09	119	0.45	0.18	0.45
38	0.09	0.09	0.09	121	0.09	0.09	0.27
44	0.18	0.09	0.27	126	0.09	0.09	0.09
46	0.09	0.09	0.09	128	0.45	0.27	0.45

Summary For All Individuals

Group	I_S	I_{HM}	I_W
Avg. All	0.09	0.07	0.14
Avg. 7-years	0.13	0.11	0.21
Avg. 11-years	0.07	0.05	0.1
Avg. 21-years	0.08	0.05	0.13

NOTE. – Every subject that is not in the table is rationalizable. Participants 1 to 31 are 7−year old. Participants 32 to 73 are 11−year old. Participants 74 to 128 are 21−year old.