ANISOTROPY TEST FOR GAUSSIAN FIELDS

Different models, which are obtained as deformations of the Fractional Brownian field $B_H(t), t \in \mathbb{R}^2$, have been introduced to model rough anisotropic images. A curious phenomenon of some of them is related to the fact that anisotropy is better observed and measured on the Radon transform of the field than directly, when looking at increments in different directions. We will in particular consider Gaussian fields X with stationary increments and spectral density f. For f given in polar coordinates by $f(|\xi|, \theta) := |\xi|^{-h(\theta)-2}(1 + o(1))$ when $|\xi| \to \infty$, in a previous work of Bonami, Ayache and Estrade these authors used Radon transform to give an estimator of $h(\theta)$ for some fixed value of θ (under some technical assumptions). In a recent work, Biermé and Richard have proved that these estimators are asymptotically independent when considering two different directions θ_1 and θ_2, which allows them to propose a test of anisotropy. The same phenomenon of independence can be observed in other models, such as those where anisotropy is given in the constant, that is $f(|\xi|, \theta) := \frac{\Omega(\theta)}{|\xi|^{2H+2}}$. This last case is considered in a joint work in progress with Aline Bonami and Hermine Biermé.
Our framework is a Gaussian field $X = \{X(t) : t \in \mathbb{R}^2\}$, with zero mean and variogram

$$v(t) = \mathbb{E}(X(t) - X(0))^2 = \int_{\mathbb{R}^2} \sin(\frac{< t, \xi >}{2})f(\xi)d\xi.$$

With espectral density f. We have

$$X \text{ isotropic } \Leftrightarrow f \text{ radial}$$

A typical example is the Fractional Brownian field $B_H(t), t \in \mathbb{R}^2$. Here

$$f(\xi) = \frac{c_H}{|\xi|^{2H+2}}, \quad v(t) = a_H|t|^{2H}.$$

This process is auto-similar with Hurst parameter H. Two examples of anisotropy are

$$X(t) - X(0) = \int_{\mathbb{R}^2} (e^{i<t,\xi>} - 1)f^{1/2}(\xi)dW(\xi),$$

where

$$f(|\xi|, \theta) = \frac{1}{|\xi|^{2h(\theta)+2}}(1 + o(1)) \quad \text{when } |\xi| \to \infty, \text{ and } f(|\xi|, \theta) = \frac{\Omega(\theta)}{|\xi|^{2H+2}},$$

in the first case $h(\theta)$ a homogenous function of degree 0.
In this work by using the observation data $X(\frac{\mathbf{k}^j}{n})$ with $\mathbf{k}^j = (k_1^j, k_2^j)$ and $0 \leq k_i^j \leq N$.

- We propose a test of isotropy.
- We built an identification procedure of unknown function Ω.

In the first part we study the Istas-Lang type p-variations defined as

$$\frac{c_H N^{2H}}{\delta^{2H}} \sum_{1}^{N-1} |\Delta^2_{\delta/N,\mathbf{u}} X(\frac{k\delta}{N}\mathbf{u})|^p.$$

where

$$\Delta^2_{\delta/N,\mathbf{u}} X(t) = X(t+\delta\mathbf{u}) + X(t-\delta\mathbf{u}) - 2X(t).$$

We study also a more useful modification that is constructed by using the Random Transform.