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Comparing cointegrating regression estimators: Some additional Monte Carlo results

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Abstract

This paper compares the finite sample performance of the canonical correlation regression estimator (CCR) and Stock and Watson's (A simple estimator of cointegration vectors in higher order integrated systems, *Econometrica*, 1993, 61(4), 783-820) dynamic ordinary least squares estimator (DOLS) using the models proposed by Inder (*Journal of Econometrics*, 1993, 57, 53-68). The CCR estimator shows smaller bias than the OLS and the fully modified. The DOLS estimator performs systematically better than the CCR estimator.

Keywords: Cointegration; Small sample; Simulation

JEL classification: C15; C22

1. Introduction

The estimation of a long-run relationship involving cointegrated variables has been the focus of a lot of recent papers. Many studies have reported alternative cointegrating vector estimators and their asymptotic properties (e.g. Phillips and Loretan, 1991). The general result is that those asymptotic properties are not affected by endogeneity or serial correlation if the estimators are properly corrected. However, the applied researcher does not usually have enough data to justify the application of asymptotic theory. For this reason it is important to consider the small-sample performance of alternative cointegrating vector estimators. On the one hand, the general result points to a large bias in small samples for any estimator that ignores short-run dynamics. On the other, the error correction mechanism (ECM) estimator, which considers explicitly knowledge of the short-run dynamics, has problems in terms of t-statistics far from their theoretical distributions.

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This paper compares, using a common model, the finite sample performance of two recently proposed cointegrating vector estimators: the canonical cointegration regression estimator (CCR) (Park, 1992) and Stock and Watson's (1993) dynamic OLS estimator (DOLS).

2. The estimators

2.1. The specification of the model

Let $y_t = (y_{1t}, y_{2t})$ be an m-dimensional I(1) process. The generating mechanism for y_t is the cointegrated system in its triangular form

$$y_{1t} = \beta' y_{2t} + u_{1t} , \tag{1}$$

$$\Delta y_{2t} = u_{2t} , \tag{2}$$

where $u_t = (u_{1t}', u_{2t}')$ is, in the general case, strictly stationary with zero mean and finite covariance matrix Σ. The benchmark case can be defined by u_t being IIDN$(0, \Sigma)$ and Σ block-diagonal. In this situation, Δy_{2t} is strictly exogenous and the OLS estimator of β in (1) is the MLE. In the general case, whenever Σ is not block-diagonal and/or the u_t process is weakly dependent, the OLS estimator is not efficient.

2.2. The CCR estimator

The CCR estimator is based on a transformation of the variables in the cointegrating regression that removes the second-order bias of the OLS estimator in the general case mentioned in Subsection 2.1.

The long-run covariance matrix corresponding to (1) and (2) can be written as

$$\Omega = \lim_{n\to\infty} \frac{1}{n} E\left(\sum_{t=1}^{n} u_t\right)\left(\sum_{t=1}^{n} u_t\right)' = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} . \tag{3}$$

The matrix Ω can be represented as the following sum:

$$\Omega = \Sigma + \Gamma + \Gamma' , \tag{4}$$

where

$$\Sigma = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E(u_t u_t') , \tag{5}$$

$$\Gamma = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n-1} \sum_{t=k+1}^{n} E(u_t u_{t-k}') , \tag{6}$$

$$\Lambda = \Sigma + \Gamma = (\Lambda_1, \Lambda_2) = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} . \tag{7}$$

The transformed series is obtained as[1]

$$y_{2t}^* = y_{2t} - (\Sigma^{-1}\Lambda_2)'u_t\,, \tag{8}$$

$$y_{1t}^* = y_{1t} - (\Sigma^{-1}\Lambda_2\beta + (0, \Omega_{12}\Omega_{22}^{-1})')'u_t\,. \tag{9}$$

The canonical cointegration regression takes the following form:

$$y_{1t}^* = \beta' y_{2t}^* + u_{1t}^*\,, \tag{10}$$

where

$$u_{1t}^* = u_{1t} - \Omega_{12}\Omega_{22}^{-1}u_{2t}. \tag{11}$$

Therefore, in this context the OLS estimator of (10) is asymptotically equivalent to the ML estimator. The reason is that the transformation of the variables eliminates asymptotically the endogeneity caused by the long-run correlation of y_{1t} and y_{2t}. In addition (11) shows how the transformation of the variables eradicates the asymptotic bias due to the possible cross correlation between u_{1t} and u_{2t}.

2.3. Stock and Watson's approach

Stock and Watson (1993) have proposed to estimate β running the following regression:

$$y_{1t} = \beta' y_{2t} + d(L)\,\Delta y_{2t} + v_t\,, \tag{12}$$

where $d(L)$ is two-sided.

This approach is motivated as an MLE for the triangular representation in (1) and (2) assuming that u_t is a Gaussian linearly regular stationary stochastic process. The leads and lags of Δy_{2t} eliminate asymptotically any possible bias due to endogeneity or serial correlation.

3. The Monte Carlo results

This paper adopts the models in Inder (1993) to compare the finite sample properties of the CCR and the DOLS estimators. Inder (1993) points out the bias problems of the FM estimator and 'blames' the particular generating mechanism used by Phillips and Hansen (1990) as the reason for the good performance of the fully modified estimator in their Monte Carlo experiment.[2]

The generating process is

[1] The fully modified estimator (FM) is similar to the CCR. The difference is that the FM transforms only the dependent variable and, in the second step, corrects the OLS estimate in the regression of the modified y_{1t}.

[2] Inder (1993) reports that the bias of the FM estimator is as large as the bias of the OLS estimator. Stock and Watson (1993) also indicate that, for their generating processes, the FM estimator tends to have biases comparable with the OLS estimator.

$$y_{1t} = \mu + \beta_0 y_{2t} + \beta_1 y_{2,t-1} + \alpha_1 y_{1,t-1} + u_{1t} , \tag{13}$$

$$y_{2t} = y_{2,t-1} + u_{2t} , \tag{14}$$

$$u_{1t} = \rho_{11}\eta_{1t} , \tag{15}$$

$$u_{2t} = \rho_{21}\eta_{1t} + \rho_{22}\eta_{2t} + \rho_{23}\eta_{1,t-1} , \tag{16}$$

where η_{1t} and η_{2t} are independently and identically distributed standard normal variables. The value of ρ_{11} is equal to 0.2 for all the experiments. The set of parameter values is included in Table 1. The benchmark case considered in Section 2 is presented in the first panel of Table 1 [$(\beta_0 = 1, \beta_1 = 0, \alpha_1 = 0)$ and $\rho = (\rho_{21} = 0, \rho_{22} = 1, \rho_{23} = 0)$].

Table 1 compares the bias and the root mean squared error (RMSE) of four estimators:

- OLS: ordinary least squares estimator.
- CCR: canonical cointegration regression estimator. A consistent estimator of Σ is

$$\hat{\Sigma} = \frac{1}{n}\sum_{t=1}^{n} \hat{u}_t\hat{u}_t' . \tag{17}$$

The CCR results in Table 1 are obtained using the non-parametric estimator of Γ:

$$\hat{\Gamma} = \frac{1}{n}\sum_{k\geq 1} c(k) \sum_{t=k+1}^{n} \hat{u}_t\hat{u}_{t-k}' , \tag{18}$$

where $c(k)$ has been chosen to be a QS kernel with an automatic bandwidth.

- CCRPW: CCR estimator using a VAR prewhitened kernel estimator of the long-run covariance matrix.[3]
- DOLS: dynamic OLS estimator. Among all the estimators reported by Stock and Watson (1993), the DOLS has the smallest bias in their Monte Carlo results excluding, of course, Johansen's MLE. The set of regressors contains one lead and one lag of the first difference of y_{2t}.

4. Conclusions

The results in Table 1 show the following facts:

1. The CCR estimator performs much better than the OLS estimator for all the models. The efficiency improvement of the CCR estimator over the fully modified, measured as the ratio of the root mean squared error, ranges from a 20% improvement to a 200% improvement. The smallest improvement corresponds to high values of α_1.
2. The CCRPW estimator does not improve over the performance of the standard CCR for the models considered in Table 1.
3. The DOLS estimator has substantial bias when $\alpha_1 = 0.8$. However, it has smaller bias and root mean squared error than the other estimators presented in Table 1.

[3] Ogaki and Park (1993). We thank Masao Ogaki for providing the GAUSS code to run this estimator.

Table 1

$T = 50$	$\rho = (0, 1, 0)$		$\rho = (0.5, 0.866, 0)$		$\rho = (0.5, 0.707, 0.5)$	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE
$\beta_0 = 1$ $\beta_1 = 0$ $\alpha_1 = 0$						
OLS	−0.0000	0.013	0.0092	0.016	−0.0002	0.008
CCR	−0.0007	0.015	0.0020	0.013	−0.0000	0.007
CCRPW	−0.0006	0.014	0.0025	0.013	−0.0000	0.007
SW	−0.0000	0.015	0.0001	0.013	0.0018	0.008
$\beta_0 = 0.6$ $\beta_1 = 0$ $\alpha_1 = 0.4$						
OLS	−0.0648	0.081	−0.0482	0.061	−0.0425	0.054
CCR	−0.0293	0.049	−0.0209	0.036	−0.0169	0.027
CCRPW	−0.0273	0.047	−0.0209	0.036	−0.0173	0.027
SW	−0.0112	0.030	−0.0092	0.024	−0.0079	0.015
$\beta_0 = 0.2$ $\beta_1 = 0$ $\alpha_1 = 0.8$						
OLS	−0.2880	0.332	−0.2518	0.291	−0.2282	0.266
CCR	−0.2228	0.281	−0.1935	0.246	−0.1729	0.210
CCRPW	−0.2323	0.293	−0.1969	0.248	−0.1734	0.219
SW	−0.2008	0.245	−0.1743	0.214	−0.1627	0.201
$\beta_0 = 0.6$ $\beta_1 = 0.4$ $\alpha_1 = 0$						
OLS	−0.0405	0.052	−0.0300	0.040	−0.0274	0.036
CCR	−0.0069	0.019	−0.0051	0.015	−0.0058	0.012
CCRPW	−0.0073	0.019	−0.0065	0.017	−0.0058	0.012
SW	−0.0003	0.016	0.0001	0.014	0.0020	0.008
$\beta_0 = 0.4$ $\beta_1 = 0.2$ $\alpha_1 = 0.4$						
OLS	−0.0952	0.118	−0.0788	0.099	−0.0670	0.086
CCR	−0.0502	0.076	−0.0436	0.065	−0.0321	0.048
CCRPW	−0.0506	0.075	−0.0439	0.066	−0.0320	0.048
SW	−0.0150	0.032	−0.0135	0.027	−0.0118	0.019
$\beta_0 = 0.1$ $\beta_1 = 0.1$ $\alpha_1 = 0.8$						
OLS	−0.3244	0.373	−0.2875	0.332	−0.2610	0.306
CCR	−0.2519	0.317	−0.2281	0.286	−0.2002	0.252
CCRPW	−0.2629	0.327	−0.2341	0.291	−0.2082	0.259
SW	−0.2292	0.280	−0.2035	0.246	−0.1862	0.226

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