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Volume versus GARCH effects reconsidered: an application to the Spanish Government Bond Futures Market

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The mixture distribution model is one of the benchmarks for modelling the relationship between volume and return. A basic variable in that theoretical construction is the number of intraday equilibria, which is empirically unobservable. This paper re-examines the finding in Lamoureux and Lastrapes (*Journal of Finance*, **45**, 1990) using alternative proxies for the number of intraday equilibria, which are included in the conditional variance equation of a GARCH model. The results show, using data of the Spanish Government Bond Futures Market for the 1992–94 period, that the number of transaction clusters and the average volume have a positive effect on conditional volatility.

I. INTRODUCTION

Recent research on volatility of financial variables has been dominated by the ARCH model and derived processes. However, these is no consensus in the answer to the question of why we observe these autoregressive effects in the conditional variance of many variables. There are several possible explanations for the existence of ARCH effects. One possibility is the existence of autocorrelation in the news arrival process (Diebold and Nervole, 1989). If information reaches the market in clusters then we should observe volatility serial correlation. Another explanation for volatility clustering in financial variables is related with the time it takes market participants to fully process information. Therefore, following this explanation, it is agents' slow adaptation to news what generates ARCH-GARCH effects (Brock and LeBaron, 1996).[1]

There is an additional explanation for ARCH-GARCH type of effects based on the notion of time deformation. Conditional heteroscedasticity may appear in situations where economic and calendar time move at different speeds. Typical examples of this approach, also called the mixture of distributions model, are Clark (1973) and Tauchen and Pitts (1983). In this model the number of trades per unit of calendar time is a random variable and the price change per unit of calendar time is the sum of the price changes occurring in the transactions that take place during that period. Therefore, if the number of trades per unit of calendar time is serially correlated then the conditional variance of returns, in calendar time, will display a GARCH-type of behaviour.

Following the last explanation several authors have proposed alternative definitions of the mixing variable considered in the mixture of distributions model. Lamoureux and Lastrapes (1990)[2] show, using daily trading volume as a proxy for the mixing variable, that the introduction of volume as an explanatory variable in the conditional variance equation eliminates the Garch effects. Glosten *et al.* (1991) point out that using the interest rate as variable into the conditional variance equation leads to a decrease in the persistence as measured by the sum of the GARCH parameters.

In this article we construct alternative proxies for the mixing variable and estimate GARCH models, with those

[1]There are some others explanations like the possibility of parameter instability (Tsay, 1987) or market microstructure effects (Bollerslev and Domowitz, 1991).
[2]Thereafter LL (1990).

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proxies included in the conditional variance equation, for the 10-years Government Bonds Spanish Futures Market. The results show the importance of the average size of operations and the number of intraday transaction clusters in the explanation of the conditional variance of the returns.

II. THE MIXTURE DISTRIBUTION MODEL

The mixture distribution model (e.g., Clark, 1973; Tauchen and Pitts, 1983) and its extensions, assumes that the number of trades occurring per unit of calendar time is a random variable and the price change, or return, per unit of calendar time (ε_t) is the sum of the I_t intraday equilibria price increments in day t, δ_{it},

$$\varepsilon_t = \sum_{i=1}^{I_t} \delta_{it} \qquad \delta_{it} \sim \text{iid N}(0, \sigma^2)$$

Conditional on I_t the daily price change can be written as

$$\varepsilon_t = \sigma\sqrt{I_t}z_t \qquad z_t \sim N(0, 1)$$

or as

$$\varepsilon_t | I_t \sim N(0, \sigma^2 I_t)$$

The mixing variable, I_t measures the rate at which information arrives to the market. There are several ways to deal with the fact that the number of intraday equilibria are generally unknown. One possibility is to make assumptions about the marginal distribution of the mixing variable. Clark (1973) and Tauchen and Pitts (1983) use the Poisson and the lognormal distribution. Foster and Viswanathan (1993) fit a lognormal distribution for the information flow deriving implications for the conditional moments of these variables. Lamoureux and Lastrapes (1993) use a model of signal extraction to obtain the unobserved flow of information. Andersen (1996) has combined the noise and the informed components of trading where the distribution of the daily informed volume is given by a Poisson distribution. Richardson and Smith (1994) propose to test the mixture of distributions hypothesis using a GMM estimator. Foster and Viswanathan (1995) have used the simulated method of moments to match simulated and observed values without using assumptions on the distribution of information arrival. Recently, Liesenfeld (1998) has approached the problem through a simulated maximum likelihood estimator.

However, other authors have looked for proxies of the mixing variable. The already mentioned paper by LL (1990) belongs to this group. LL (1990) choose daily trading volume as the measure of daily information flows into the market. The GARCH effect in the returns is explained as the consequence of the serial correlation in the daily number of information arrivals. We can write I_t as a correlated process

$$I_t = \alpha + \Theta(L)\, I_{t-1} + u_t$$

and define the conditional variance of ε_t as

$$\sigma^2_{\varepsilon_t|I_t} = E(\varepsilon_t^2|I_t) = \sigma^2 I_t$$

Then the MA structure of the innovation to the mixing variable are translated into the conditional variance of ε_t

$$\sigma^2_{\varepsilon_t|I_t} = \sigma^2\alpha + \Theta(L)\sigma^2_{\varepsilon_{t-1}|I_{t-1}} + \sigma^2 u_t$$

generating the typical GARCH structure.

Lamoureux and Lastrapes (1990) estimate a generalized GARCH model where volume is included as an explanatory variable in the variance equation. They find that, when volume is included, the other coefficients in the conditional variance equation become statistically insignificant for most of the 20 stocks in their sample. However, trading volume is a very imperfect proxy for the mixing variable. In fact, if we consider the volume per unit of calendar time as the sum of the volume traded during the I_t within day equilibria, volume will have a heteroscedastic error term that will be 0 only when the variance of the within day traded volume is 0.

III. ALTERNATIVE MEASURES FOR THE MIXING VARIABLE

Besides volume there are some other variables that could account for the autocorrelation observed in conditional volatility and approximate the process of information arrival. We discuss several of them in this section.

Duration between trades

The recent literature on ultrahigh frequency data (Engle, 1998) and the Autoregressive Conditional Duration Model (ACD) (Engle and Russell, 1998; Ghysels and Jasiak, 1997) have stressed the importance of modelling duration when dealing with transaction by transaction data. Given that we have aggregated the data in daily observations, one possible variable related to duration could be the average time between trades or, its reciprocal, the number of transactions or daily frequency (FREQ). Jones *et al.* (1994) point out that the number of transactions has more explanatory power than volume to explain the volatility of daily returns.

There are at least two alternative explanations of the relationship between duration and volatility. Admati and Pfleiderer (1988) argue that short durations are the result of the bunching of liquidity traders. In contrast, slow trading means that liquidity traders are out of the market because the probability of trading with an informed agent is very high. In this case slow trading means high volatility. Easley and O'Hara (1992) reach the opposite conclusion. Their model presents a situation where there is uncertainty whether there is information. However, informed traders know if there are news or not. If these news are good they will buy and if the news are bad they will sell. Therefore, the specialist will keep the price stable if the frequency of transactions is low. In this case slow trading means low volatility.

Information arrival and transactions clusters

One problem with a summary statistic like the average time between trades is that it does not take into account one phenomenon that is the most probable reason for volatility clustering: the arrival of new information and the cluster of trades that it generates. The average time between trades assumes that during one day the arrival of new information happens at the same rate, which could be different from one day to another. However, it does not give information on intraday clusters of transactions over one day. In order to deal with this situation we construct an index based on the number of transactions that have a duration respect to the last trade less than 60 seconds[3] (*NLT60*). This is an index of the number of transactions in trade clusters and, therefore, a proxy for the rate of information arrival.

The index *NLT60* is not the only alternative measure for the mixing variable based on the transactions clusters. There is also the possibility of constructing the ratio of *NLT60* over *FREQ*, *RLT60*, which measures the proportion of transactions that take place without a duration longer than 60 seconds. An additional index, *NTC*, measures the number of transaction clusters during one day. In fact this variable could be interpreted as the number of news arrivals, which is the exact content of the mixing variable in the mixture distribution model. The trading process assumed by this interpretation is the following. When new information reaches the market or there are differential interpretations of public signals (Kandel and Pearson, 1995) traders start trading. Trading stops when all the traders get their desired level of assets given the information known until that point and the interpretations (expectations) of other agents in the market. Trading resumes when new pieces of information arrive in the market.

In practical terms the variable *NTC* is constructed as the number of transaction clusters during one day. A cluster is defined as a sequence of transactions that do not have trade durations over 60 seconds. Obviously, the amount of information released during those periods will be associated with the average size of each trade (*ASIZE*).

The proposed measures are related to the variance ratio constructed in Jones *et al.* (1994) who compare the return variance of nontrading and trading periods. Jones *et al.* (1994) define the nontrading periods as days when exchanges are open but traders endogenously choose not to trade, opposite to the usual approach where the nontrading periods are weekends or nights (exogenous). In this way, trading and nontrading periods are comparable because both are nonpredictable and the activities which generate public information are unchanged.

IV. AN APPLICATION TO THE SPANISH BOND FUTURES MARKET

In this article we apply the approach proposed by LL (1990), with alternative indicators to capture the information flow into the Spanish Government Bond Futures Market. The data are real time observations on prices and trading volume for the 10-years government bonds futures contracts, the most actively traded contract in the Spanish futures market, traded at the Barcelona Financial Futures and Options Exchange. Actively traded futures are most likely to have a large number of intraday information arrivals per day. The contracts call for the delivery of a 10 million pesetas face value government bond. The sample period covers contracts with maturity September 1992 until contracts with maturity September 1994. The 10-years government bond contract was presented in March of 1992. The first delivery date was June 1992 but, being the initial one, was scarcely traded and has not been considered in the sample. Originally prices for different delivery dates were aggregated into a price for a single composite contract. However, trades of futures with delivery dates different from the first one are very scarce, practically nonexistent. This means that the results using the single composite contract are practically identical to the results that consider the switching of the contract at the date of the roll-over.[4] For simplicity we have adopted this second approach. Summarizing, the original database contains 338 374 transaction by transaction records. Daily aggregation results in 503 observations.[5]

The variables are constructed in the following way. Daily returns (*RET*) have been constructed as the log difference between the closing price and the opening price, $100 \cdot \ln(P_c - P_o)$. Therefore, overnight and weekend returns are not included. This is an important consideration given that we are trying to measure the effects of information clustering on volatility. During the overnight period there could be many pieces of new information that do not have any effect until the opening of the market next day. For this reason, the opening price could be very different from the closing price of the last day but we cannot trace the effect on prices of the release of information during the overnight period. The volume variable (*VOL*) measures the number of contracts traded while the variable frequency (*FREQ*) counts the number of transactions that took place every day. Average volume (*ASIZE*) is the size, in contracts, of the

[3]Other time periods, like 30 seconds, were tried with very similar results.
[4]The day of the roll-over is not included in the sample.
[5]This sample size is larger than the average sample size in LL (1990).

Table 1. *Daily descriptive statistics, 1992–1994*

Variables	T	Mean	Std.
RET	503	− 0.03	0.57
VOL	503	16 753.28	18 874.23
FREQ	503	672.71	679.27
ASIZE	503	23.75	4.61
NLT60	503	586.38	660.02
RLT60	503	0.75	0.14
NTC	503	86.32	35.75

Notes: *RET*: return; *VOL*: volume; *FREQ*: number of transactions; *ASIZE*: average size; *NLT60*: number of transactions with duration less than 60 seconds; *RLT60*: *NLT60/FREQ*; *NTC*: number of transactions clusters.

average transaction. *NLT60* and *NTC* have been defined in Section III.

Table 1 presents the descriptive statistics associated with these variables. An important consideration with respect to the returns to the returns is to check if they are normality distributed. The approach used for testing the normality assumption involves the Lagrange multiplier procedure applied on a normal distribution embedded as the null hypothesis on a more general family of distributions. The test statistic for skewness (*SKEW*) is calculated following the formula in Davison and McKinnon (1993),

$$SKEW = (6T)^{-1/2} \sum_{t=1}^{T} e_t^3$$

where

$$e_t \equiv (r - \mu)/\sigma$$

and r are the returns, μ is the sample mean of the returns, σ is the standard deviation of the returns and T is the sample size.

The test statistic of the excess kurtosis is

$$KUR = (24T)^{-1/2} \sum_{t=1}^{T} (e_t^4 - 3)$$

These two test statistics are distributed under the null as a $\chi^2(1)$.[6] In addition, it can be shown that these two statistics are independent. Therefore, the sum of their squares will be asymptotically a $\chi^2(2)$.[7]

$$NT = SKEW + KUR$$

The results show that we cannot reject the normality assumption from the point of view of the skewness test ($SKEW = 0.57$, p-value $= 0.45$) but, when considered together with the excess kurtosis test ($KUR = 10.56$, p-value $= 0.00$), the p-value of the sum shows a clear rejection of the normality assumption ($NT = 11.13$, p-value $= 0.00$).

Table 2 presents the correlations among the variables to be included as proxy for the information flow. Volume and frequency have a 0.98 correlation but the correlation of average volume with any of them is not larger than 0.36. *NLT60* has a high correlation with volume and frequency and a low correlation with average size. The ratio of *NLT60* over *FREQ* has also a high correlation with volume, frequency and *NLT60*. *RTL60* is positively correlated with volume, frequency and *NTL60*. Finally, the number of transactions clusters, *NTC*, has a positive correlation with volume and frequency, although not as high as *NLT60*, and a negative correlation with average size.

Table 3 shows the results of the GARCH models with and without exogenous explanatory variables. The specifications are based on the econometric mode

$$r_t = \beta_0 + \varepsilon_t$$

$$\varepsilon_t | I_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \sum_{k=1}^{K} \alpha_{k+2} I_{kt}$$

where K is the number of exogenous explanatory variables to be included in the conditional volatility equation.[8] The first specification shows that, when there are not exogenous explanatory variables in the conditional variance equations, the sum of the GARCH coefficients ($\alpha_1 + \alpha_2$), is close to 1. This implies a high degree of volatility persistence. Once volume is included as explanatory variable this persistence is reduced and volume has a positive effect on conditional volatility, findings that are consistent with the results in LL (1990).

We have argued that the number of transactions (*FREQ*) is also a suitable proxy for the flow of information into the market. The coefficient of *FREQ* is significantly different from 0 (Specification 3), based on the asymptotic t-statistic and has a positive impact on volatility. Specification 4 shows the results of the estimation when daily average volume is included as explanatory variable for volatility. The effect is also positive meaning that the larger is the average size of the trades the higher is volatility.

Using volume, the number of transactions and the average size of trades as explanatory variables in the conditional variance equation we observe that only average size has a significant effect on conditional variance. It is interesting to notice that volume and frequency turn out to be insignificant and that the coefficients α_1 and α_2 remain very close to the ones obtained using only average size as the

[6] These test statistics seem different from that proposed by Kiefer and Salmon (1983) but, in fact, they are numerically identical.
[7] Jarque and Bera (1980).
[8] Using the constraint $\beta_0 = 0$ does not modify the results.

Table 2. *Correlations*

Variables	*VOL*	*FREQ*	*ASIZE*	*NLT60*	*RLT60*	*NTC*
VOL	1.00					
FREQ	0.98	1.00				
ASIZE	0.35	0.24	1.00			
NLT60	0.98	0.99	0.25	1.00		
RLT60	0.75	0.79	0.07	0.77	1.00	
NTC	0.48	0.55	− 0.05	0.51	0.70	1.00

Notes: *RET*: return; *VOL*: volume; *FREQ*: number of transactions; *ASIZE*: average size; *NLT60*: number of transactions with duration less than 60 seconds; *RLT60*: *NLT60*/*FREQ*; *NTC*: number of transactions clusters.

Table 3. *Maximum likelihood estimates of GARCH(1, 1) models without day-of-the-week dummies*

Specifications	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6	Spec. 7	Spec. 8	Spec. 9
α_0	0.06	0.12	0.12	− 0.19	− 0.16	0.12	− 0.11	− 0.05	− 0.13
	(2.73)	(5.16)	(4.91)	(− 7.77)	(− 5.01)	(4.99)	(− 2.19)	(− 3.07)	(− 3.23)
α_1	0.04	0.31	0.28	0.24	0.21	0.28	0.37	0.01	0.22
	(3.92)	(3.99)	(3.85)	(3.56)	(2.77)	(3.82)	(4.43)	(1.21)	(3.05)
α_2	0.93	0.24	0.27	0.17	0.15	0.27	0.19	− 0.64	0.25
	(7.19)	(2.41)	(2.58)	(2.22)	(1.53)	(2.58)	(2.30)	(− 7.37)	(2.41)
VOL		0.02			− 0.01				
		(3.44)			(− 0.39)				
FREQ			0.53		0.95				
			(3.09)		(0.88)				
ASIZE				0.01	0.01			0.02	0.01
				(5.24)	(7.23)			(13.05)	(5.06)
NLT60						0.55			
						(3.03)			
RLT60							0.37	− 0.05	
							(4.93)	(− 0.66)	
NTC									0.05
									(2.30)

exogenous explanatory variable. This is an argument in favour of the robustness of the effect of average size on volatility.

Specifications 6 and 7 show the impact of two additional variables: the number of transactions with duration less than 60 seconds, *NLT60*, and the proportion of those transactions over the total number of transactions. The variable *NLT60* has a very high correlation with frequency, which in fact means that the estimated coefficients are very similar to the ones obtained using *FREQ* as explanatory variable. The same positive effect is observed when the conditional variance equation contains the ratio *RLT60* instead of *NTL60*. However, when the average size is included the coefficient of *NTL60* or, alternatively, *RLT60* turn out to be insignificant, as specification 8 shows for the second case.[9]

Specification 9 shows the estimation of the model when *NTC* and average size are included both as explanatory variables in the conditional variance equation. The effect of both variables is positive. Given that the *NTC* variable tries to measure the arrival of information this result implies that volatility is high when there are many news. The fact that the average size has also a positive effect on volatility implies that given a number of transactions clusters, each block of information has more informational content and, therefore, generates larger transactions and higher volatility.

Table 4 shows the results of a second model where the conditional mean is specified as

$$r_t = \beta_0 + \sum_{j=1}^{4} \beta_j D_{jt} + \varepsilon_t$$

where the *D*'s are four day-of-the-week dummies used to account for possible differences in expected returns (French, 1980). The results are essentially identical to the ones in Table 3.

[9]The first case is identical to the combination of frequency and average size, given the high degree of correlation between frequency and *NLT60*.

Table 4. *Maximum likelihood estimates of GARCH(1, 1) models with day-of-the-week dummies*

Specifications	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6	Spec. 7	Spec. 8	Spec. 9
α_0	0.06	0.11	0.11	− 0.14	− 0.13	0.12	− 0.09	− 0.03	− 0.13
	(2.83)	(5.26)	(5.03)	(− 3.50)	(− 2.20)	(5.14)	(− 2.01)	(− 0.48)	(− 3.54)
α_1	0.04	0.33	0.31	0.22	0.25	0.31	0.36	0.02	0.23
	(3.02)	(4.01)	(3.86)	(2.99)	(3.07)	(3.87)	(4.40)	(1.53)	(3.11)
α_2	0.91	0.20	0.23	0.21	0.18	0.23	0.21	− 0.65	0.25
	(6.55)	(2.13)	(2.25)	(1.87)	(1.79)	(2.25)	(2.30)	(− 7.25)	(2.33)
VOL		0.03			− 0.01				
		(3.43)			(− 0.26)				
FREQ			0.62		0.89				
			(3.10)		(0.69)				
ASIZE				0.01	0.01			0.02	0.01
				(5.55)	(4.44)			(7.77)	(5.08)
NLT60						0.64			
						(3.08)			
RLT60							0.33	− 0.04	
							(4.16)	(− 0.39)	
NTC									0.05
									(2.02)

IV. CONCLUSIONS

This paper has re-examined the empirical findings of Lamoureux and Lastrapes (1990) by using alternative variables to measure the effect of the information flow on volatility. Using data from the 10-years Spanish Government Bonds Futures Market we have found that the inclusion of several proxies for the rate of information flow in the conditional variance equation reduces persistence in volatility but do not eliminate completely GARCH effects.

Daily volume and frequency have a positive effect on volatility. However, when the average size of trades is included in the conditional variance equation, the coefficients of volume and frequency become insignificant.

This paper proposed some additional variables that could approximate the flow of information arrival into the market defined as the number of transactions clusters. It measures the number of daily periods where the duration between two consecutive transactions is less than 60 seconds. The positive effect of the average size and the number of trades clusters indicates that not only the number of news arriving to the market is important but also its informational content.

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