Designing Dynamic Contests*

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Abstract

Innovation contests have emerged as a viable alternative to the standard research and development process. They are particularly suited for settings that feature a high degree of uncertainty regarding the actual feasibility of the end goal. Participants race towards completing an innovation project and learn about the underlying environment from their own efforts as well as from their competitors’ gradual progress. Learning about the status of competition can alleviate some of the uncertainty inherent in the contest, but it can also adversely affect effort provision from the laggards as they become discouraged about their likelihood of winning. Thus, the contest’s information provision mechanism is critical for its success. This paper explores the problem of designing the award structure of a contest as well as its information disclosure policy in a dynamic framework, and provides a number of guidelines with the objective of maximizing the designer’s expected payoff. In particular, we show that intermediate awards, apart from directly affecting the participants’ incentives to exert costly effort, may also be used as a way for the designer to appropriately disseminate information about the status of competition. Interestingly, our proposed design matches many features observed in real-world innovation contests.

Keywords: Contests, learning, dynamic competition, open innovation, information.

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1 Introduction

Innovation contests are fast becoming a tool that firms and institutions use to outsource their innovation tasks to the public. In these contests, an open call is placed for an innovation project that participants compete to finish, and the winners, if any, are awarded a prize.\(^1\) Recent successful contests include The NetFlix Prize and the Heritage Prize\(^2\), and a growing number of ventures like Innocentive, TopCoder, and Kaggle provide online platforms to organize and connect innovation seekers with potential innovators.

The objective of the contest designer is to maximize the probability of reaching the innovation goal while minimizing the time it takes to complete the project. Obviously, the success of an innovation contest crucially depends on the pool of participants and the amount of effort and experimentation they decide to provide, and a growing literature considers the question of how to best design these contests. Three key modeling features are crucial for our analysis and distinguish our work from prior literature. First, in our model, an agent's progress towards the goal is not a deterministic function of effort. As is typically the case in real-world settings, progress is positively correlated with effort but the mapping involves a stochastic component. Secondly and quite importantly, it is possible that the innovation in question is not attainable, either because the goal is actually infeasible or because it requires too much effort and resources that it makes little economic sense to pursue. We model such a scenario by having an underlying state of the world (whether the innovation is attainable or not) over which participants have some prior belief. Taken together, these two features imply that an agent's lack of progress can be attributed to either an undesirable underlying state (the innovation is not attainable) or simply to the fact that the agent was unlucky in how her effort was stochastically mapped to progress. Thirdly, we consider a dynamic framework to capture both how competition between the agents evolves over time as well as to incorporate the fact that they learn from each other's partial progress. In particular, we include well-defined intermediate milestones as part of the modeling setup which constitute partial progress towards the end goal. Apart from being an integral feature of most innovation contests, explicitly accounting for intermediate milestones allows us to study the complex role that information plays in this context.

The previous discussion implies that information about the progress of one of the participants has the following interesting dual role: it makes participants more optimistic about the state of the world, as the goal is more likely to be attainable and thus agents have a higher incentive to exert costly effort. Following the literature on strategic experimentation, we call this the encouragement effect. At the same time, such information implies that one of the participants is leading the contest, which might negatively affect effort provision from the rest of the agents as the likelihood of them

\(^1\)We use the terms 'participant', 'contestant', 'competitor', and 'agent' interchangeably throughout.

\(^2\)The NetFlix Prize offered a million dollars to anyone who succeeded in improving the company's movie recommendation algorithm by a certain margin, and was awarded in 2009. The Heritage Prize was a multi-year contest whose goal was to provide an algorithm that better predicts patient readmissions to hospitals. A successful breakthrough was obtained in June of 2013.
beating the leader is now considerably lower. We refer to this as the competition effect. These two effects interact with each other in subtle ways and understanding this interaction is of paramount importance to the design of an innovation contest.

The primary contribution of this paper is twofold. First, to the best of our knowledge, our framework is the first that explicitly models the three features of innovation contests described above, i.e., uncertainty regarding the feasibility of the end goal, stochastic mapping between effort and progress, and intermediate milestones. Doing so allows us to focus on the information disclosure policy of the contest designer and show how this policy depends on whether the competition or the encouragement effect dominates. In particular, we explicitly consider the question of whether and when should the contest designer disclose information regarding the competitors’ (partial) progress with the goal of maximizing her expected payoff. Interestingly, we show that the optimal design features a non-trivial information disclosure policy thus illustrating the active role information about the status of competition can have in incentivizing agents to participate in the contest.

We then identify the role of intermediate awards as a way for the designer to implement the desired information disclosure policy and thus increase the agents’ effort provision and consequently the chances of innovation taking place in the process. Intermediate awards are very common in innovation contests (the aforementioned NetFlix and Heritage prizes are examples of contests that have employed intermediate awards), but to the best of our knowledge, the exact role they play as information revelation devices has not yet been studied.

A simple illustration of the main ideas in this paper is the following. Consider an innovation contest that consists of well-defined milestones towards the end goal. In this case, reaching a milestone constitutes partial progress and we assume that agents and the designer are able to verifiably communicate this. Assume for now that the innovation is attainable with certainty and the agents are fully aware of that. The lack of progress towards the goal is then solely a result of the stochastic return on effort. If no information is disclosed about the agents’ progress, then they become progressively more pessimistic about the prospect of them winning as they believe that they may be lagging behind in the race towards the end goal. This may possibly lead them to abandon the contest, thus decreasing the aggregate level of effort and consequently increasing the time to complete the contest.

In contrast, when there is uncertainty on whether the end goal is attainable, agents that have made no or little progress towards the goal become pessimistic about whether it is even possible to complete the contest. If this persists, an agent may choose to drop out of the competition as she believes that it is not worth putting the effort for what is likely an unattainable goal, reducing the aggregate experimentation in the process and decreasing the chances of reaching a possibly feasible innovation.

This discussion highlights the complex role that information about the agents’ progress may play in this environment. In the first scenario, when the competition effect is dominant (since there is no uncertainty regarding the attainability of the end goal), disclosing that one of the participants
is ahead may deter future effort provision as it implies that the probability of winning is lower for the laggards. On the other hand, in the second case, when the encouragement effect dominates, news about an agent's progress are considered good news, since it reduces the uncertainty involved in the contest.

The information disclosure mechanism is only one of the levers that the designer has at her disposal to affect the agents' effort provision decisions. Another is obviously the compensation scheme that, in the context of an innovation contest, takes the form of an award structure. In a setting with potentially multiple milestones a design may involve compensating agents for reaching a milestone or having them compete for a grand prize given out for completing the entire contest. Our analysis sheds light on the interplay between information disclosure and the contest's award structure by comparing different mechanisms in terms of their expected payoff for the designer, assuming a fixed budget for the awards given out to the contestants. This way we essentially bring a contest's information provision mechanism to the forefront and interestingly we show that the probability of obtaining the innovation as well as the time it actually takes to complete the project are largely affected by when and what information the designer chooses to disclose.

Related Literature Several papers study the design of innovation contests mostly focusing on static models that resemble all-pay auctions. Typically, these models take the form of a one-shot game in which agents choose their effort levels incurring the associated cost of effort which can be thought of as their "bid". The agent that puts most effort typically wins the contest and claims an award. Several questions have been explored in this setup. For example, Taylor (1995) finds that a policy of free and open entry is not optimal because high participation may give rise to low levels of effort at equilibrium. Thus, the sponsor of the innovation contest restricts participation and extracts all expected surplus by taxing contestants through an entry fee. Moldovanu and Sela (2001) consider the case when the agents' cost of effort is their private information. They show that when the cost is linear or concave in effort, it is optimal to allocate the entire prize sum to the winner whereas when it is convex several prizes may be optimal. Che and Gale (2003) find that for a set of procurement settings it is optimal to restrict the number of competitors to two and, in the case that the two competitors are asymmetric, handicap the most efficient one. Moldovanu and Sela (2006) explore the performance of contest architectures that may involve splitting the participants among several sub-contests whose winners compete against each other. They identify conditions under which it is optimal to have a single grand contest or split the competitors into two divisions and have a final between the two winners. Siegel (2009) provides a general framework to study such static all-pay contests that allows for several features such as differing production technologies, attitudes toward risk, and conditional and unconditional investments. Terwiesch and Xu (2008) and Ales et al. (2014) explore static contests in which there is uncertainty regarding the value of an agent's contribution and explore the effect of the award structure and the number of competitors on the contest's expected performance. Finally, Boudreau et al. (2011) examine related questions empirically using
Unlike the papers mentioned above, a central feature in our model is the fact that there is uncertainty with respect to the attainability of the end goal. In addition, agents dynamically adjust their effort provision levels over time responding to the information they receive regarding the status of the competition and the state of the world, i.e., whether the contest can be completed. Early papers that consider the dynamics of costly effort provision in the presence of uncertainty are Choi (1991) and Malueg and Tsutsui (1997). They both assume that agents can perfectly observe the experimentation outcomes of their competitors and the value of the innovation is fixed thus abstracting away from the main focus of our study which is the role of the designer’s information disclosure mechanism and the contest’s award structure on the success of the innovation contest. There is also a recent stream of papers (Bolton and Harris (1999), Keller et al. (2005), Keller and Rady (2010), and Bonatti and Horner (2011)) that study the dynamics of collaboration within a team of agents that work towards completing a project. Since all agents receive the same payoff upon the project’s completion, the aforementioned papers focus on free-riding and how this affects the team’s aggregate experimentation level. Bimpikis and Drakopoulos (2014) also consider collaboration within a team and show that having agents work independently and then combining their efforts strictly increases the probability of having the project completed compared to the case when the entire team works together from the onset. Although our model builds on the exponential bandits framework that was introduced in Keller et al. (2005), our setup and focus are considerably different than prior literature in strategic experimentation. In particular, agents compete with one another for a set of awards that are set ex-ante by the designer. Furthermore, we allow for imperfect monitoring of the agents’ progress (experimentation outcomes). This, in combination with the fact that agents dynamically learn about the attainability of the end goal as well as the status of competition, significantly complicates the analysis as agents not only form beliefs about whether they can complete the contest but also they may need to form beliefs about their progress relative to their competitors. The latter is not an issue in the strategic experimentation literature since experimentation outcomes are typically assumed to be perfectly observable.

Our paper is also related to the literature on dynamic competition when there is uncertainty in the mapping between effort and progress. For example, Harris and Vickers (1987) show that in a one-dimensional model of a race between two competitors, the leader provides more effort than the follower and her effort increases as the gap between the competitors decreases. Building on this work (and very closely in spirit with our motivation), Moscarini and Smith (2007) study the design of optimal incentives in a two-player dynamic contest. Theirs is a model of perfect monitoring where the focus is on the design of a scoring function in which the leader is appropriately “taxed” whereas the laggard is “subsidized”. The goal of such a function is to increase overall effort provision by carefully trading off the ex-ante incentives of becoming a leader (and getting taxed) with the ex-

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3Several other papers, e.g., DiPalantino and Vojnovic (2009) and Chawla et al. (2012), study crowdsourcing contests as all-pay auctions and exploit this connection to provide design guidelines that maximize their performance. Slivkins and Vaughan (2013) provide a recent survey of the theoretical challenges related to crowdsourcing.
post incentives of the laggard giving up. Unlike both of these papers we allow the contest designer to choose what information and when to disclose it, thus putting more emphasis on how the designer can incentivize agents to take a certain set of actions by controlling the information they have access to.

A general premise of our work is that a principal may incentivize an agent (or agents) to take a given set of actions by controlling what information they have access to. Relatedly, Kamenica and Gentzkow (2011) consider the problem of a sender persuading a receiver to take an action by committing ex-ante to which of a potentially large pool of noisy signals to reveal (albeit in a static setting). Furthermore, in a social learning context, Che and Hörner (2013) and Kremer et al. (2014) study a framework in which a principal provides recommendations to a set of short-lived agents that decide sequentially on which of two actions to take. The principal is interested in maximizing the discounted sum of the agents' expected payoffs whereas the agents take the action that maximizes their own payoff based on the information they can infer from the principal’s recommendation.

Closest to our work are Lang et al. (2014) as well as the independent and concurrent contribution of Halac et al. (2014). Lang et al. (2014) study a two-player continuous time contest in which there is no uncertainty about the underlying environment but agents exert costly effort to complete as many milestones as they can before a predetermined deadline. They characterize equilibrium behavior and establish a close relation with the outcomes of (static) all-pay auctions. Halac et al. (2014) study an experimentation contest that ends after the occurrence of a single success, thus not incorporating the possibility of partial progress and effectively not allowing for the encouragement effect. They compare “winner-takes-all” contests that feature full information disclosure with “hidden-equal-sharing” contests in which agents are guaranteed a fraction of the total monetary prize if they complete the contest within a deadline. Although our work shares some features with theirs, particularly, the uncertainty regarding the attainability of the end goal, a major part of our study focuses on exploring the interplay between the contest’s award structure and the information disclosure policy that it implies in relation to the encouragement and competition effects. This becomes extremely relevant in the presence of partial progress towards the end goal and discounting, which are two features that are unique to our model and that, we believe, capture realistic aspects of contests where innovation seekers are interested in completing their projects as quickly as possible. As Halac et al. (2014) consider a contest with no intermediate milestones and assume that the designer and participants do not discount future outcomes, the time it takes to complete the contest is immaterial for their analysis.

Outline of the Paper The rest of the paper is organized as follows. Section 2 presents our basic model and assumptions. Section 3 provides the analysis for the scenarios discussed above and highlights the role of intermediate awards. Finally, Section 4 summarizes our findings and discusses directions for future research.
2 Model

Our benchmark model is an innovation contest with two sequential stages, Stage $A$ and Stage $B$, and two competitors, Agent 1 and Agent 2.\textsuperscript{4} Innovation happens if both stages are successfully completed by a competitor, at which time the contest also ends. Stage $j = A, B$, is associated with a binary state $\theta_j$ that describes whether that stage can be completed ($\theta_j = 1$) or not ($\theta_j = 0$). If $\theta_j = 0$, then the breakthrough required to complete stage $j$ is not feasible (and therefore innovation overall is not possible). If $\theta_j = 1$, then a breakthrough is feasible and has an arrival rate that is described by a Poisson process with parameter $\lambda_j$.\textsuperscript{5} We assume that agents have a common prior on $\theta_j$ and we denote that prior by $p_j = \mathbb{P}(\theta_j = 1)$. We also assume that the events that stages $A$ and $B$ can be completed are independent, i.e., $\mathbb{P}(\theta_B = 1|\theta_A = 1) = \mathbb{P}(\theta_B = 1)$.

Agents choose their effort levels continuously over time. Agent $i \in \{1, 2\}$ chooses effort level $x_{i,t} \in [0, 1]$ at time $t$ and incurs an instantaneous cost of effort equal to $c \cdot x_{i,t}$ for some $c > 0$. An agent in stage $j$ who puts effort $x_t$ at time $t$ obtains a breakthrough with instantaneous probability $\theta_j \lambda_j x_t$. Agent $i$ is endowed with an information set $\Gamma_{i,t} = (p^i_{j,t}, q_{i,t}, R(t))$ that summarizes her information about the tournament at time $t$, where $p^i_{j,t}$ is her belief about the feasibility of stage $j$, $q_{i,t}$ is her belief about whether the competing agent is in Stage $B$ (for example, $q_{i,t} = 1$ implies that agent $i$ believes with certainty that her competitor is in Stage $B$), and $R(t)$ is the award structure at time $t$. A strategy $x_i(t) = \{x_{i,t}\}_{t \geq 0}$ for agent $i$ maps the agent’s information set at time $t$ to an effort provision level in $[0, 1]$, i.e., $x_{i,t} : \Gamma_{i,t} \rightarrow [0, 1]$. Finally, let $\Pi_i(x_i(t), x_{-i}(t))$ denote the payoff to agent $i$ when she uses strategy $x_i(t) = \{x_{i,t}\}_{t \geq 0}$ and her competitor uses strategy $x_{-i}(t) = \{x_{-i,t}\}_{t \geq 0}$. The strategy profile $x^*(t) = (x^*_1(t), x^*_2(t))$ is an equilibrium if for $i \in \{1, 2\}$ the following holds

$$x^*_i(t) = \arg\max_{x_i(t)} \Pi_i(x_i(t), x^-_{-i}(t)).$$

One can see from this formulation that agents’ effort decisions are a direct function of their information sets (which in turn summarize the agents’ beliefs about the feasibility of the end goal and the status of the competition). Our goal in this paper is to explore how the designer’s information disclosure policy along with the contest’s award structure affect the agents’ information sets and consequently influence their effort provision decisions. Understanding these effects allows us to prescribe guidelines for maximizing the designer’s expected payoff.

Note that since in our setting the tournament is over once one of the agents completes Stage $B$, the information provision mechanism is centered around partial progress, i.e., the completion (or not) of Stage $A$.\textsuperscript{6} Formally, the designer’s information provision strategy takes the following form:

$$m_t : H^1_t \times H^2_t \rightarrow \{0, 1\} \cup \emptyset,$$

\textsuperscript{4}Section 4 discusses how our insights apply to multi-stage tournaments and a setting with multiple competitors.
\textsuperscript{5}It is possible that agents have access to different technologies and therefore have different progress rates. This introduces a new set of interesting questions especially when the agents’ skills, i.e., progress rates, are their private information. We further discuss this point in Section 4.
\textsuperscript{6}It should be clear by now that progress in this setting takes the form of discrete breakthroughs, and it is not a smooth function of the agents’ efforts.
where \( H^i_t \in \{0, 1\} \cup \emptyset \) for \( i = 1, 2 \) represents the designer’s information about whether agent \( i \) has completed Stage \( A \) by time \( t \) (0 implies that she has not, 1 that she has completed the stage, and \( \emptyset \) implies that the designer does not have any information). In turn, the designer can take one of three actions at every time \( t \): announce nothing (\( \emptyset \)), announce that a competitor has completed Stage \( A \) (1), or announce that none of the competitors has completed Stage \( A \) (0). We assume that 0 or 1 announcements are truthful, e.g., because partial progress is verifiable, but the designer can choose not to disclose the information she possesses. We do not allow for mixed strategies, i.e., probability distributions over the set of available actions at a given time instant \( t \). In addition to simplifying the analysis, the focus on pure strategies stems from the desire to keep the contest rules unambiguous and easy to interpret, with the implicit assumption that simple rules are more conducive to participation.

Intertwined with the information provision mechanism is an award structure that specifies the size and timing of the awards handed out to contestants. Apart from directly affecting the agents’ effort provision incentives (larger awards lead to higher effort levels from the agents), they can also be used as a lever to implement the designer’s information provision policy. In particular, there is no a priori reason for a contestant to reveal her partial progress to the designer unless this revelation leads to an increase in her expected utility. As we show in Section 3, the contest designer may incentivize agents to provide information about their progress by appropriately giving out intermediate awards. In particular, the problem we study is designing a mechanism by which awards are given out to contestants as a function of their progress towards the end goal and, quite importantly, time. Formally, the decision problem that the designer faces is determining the award policy \( R(t) \) that maximizes the expected payoff from running the contest. Denoting this payoff by \( \Pi_d \) and letting \( U \) be the utility that the designer gets from obtaining the innovation, we have

\[
\Pi_d = \mathbb{E}[e^{-r\tau_B}U],
\]

where \( \tau_B \) is the (random) time at which Stage \( B \) (and the tournament) is completed. Throughout the paper we assume that \( U \) is normalized to 1. In order to facilitate the comparison between different contest designs and illustrate the role of the designer’s information disclosure policy on the contest’s expected outcome, we assume that the designer’s budget for running a contest is fixed and equal to \( B \). In other words, the expected discounted sum of the awards given out by the designer has to satisfy the following constraint

\[
\mathbb{E} \left[ e^{-r\tau_A}R_A + e^{-r\tau_B}R_B \right] \leq B,
\]

where \( \tau_A \) is the random time at which the award for completing Stage \( A \) is given out, and \( R_A \) and \( R_B \) are the sizes of the awards for completing stages \( A \) and \( B \), respectively. In other words, the objective for the designer is to maximize her expected discounted utility subject to a budget constraint by choosing how to allocate this budget to the competitors in the form of awards (and implicitly designing an information provision mechanism in the process).
Finally, we note that we use this two-stage tournament as a way of providing a reasonable approximation of the dynamics of multi-stage contests. Naturally, the more progress being made, the less uncertain agents are about the feasibility of the end goal. Thus, at a high level a multi-stage contest can be thought of as having two distinct phases. First, during the early stages, uncertainty regarding the attainability of the end goal is the main driving force behind the competitors’ actions. Competition is of secondary importance as there is still plenty of time for the laggards to catch up. We capture this situation as a two-stage contest in which the feasibility of the discovery required to complete the first stage is uncertain, i.e., $p_A < 1$. On the other hand, the second stage – which models the remainder of the contest – takes on average a much longer time to complete, i.e., the arrival rate associated with Stage $B$ is much lower than that of Stage $A$, i.e., $\lambda_B < \lambda_A$.

As the contest draws to an end, the dynamics become quite different. Agents are more optimistic about the feasibility of the end goal, but the chances for the laggards to catch up with the leader are slimmer. Thus, the agents’ behavior is mainly prescribed by the competition effect. We capture this scenario by examining two successive stages that feature little or no uncertainty, i.e., $p_A, p_B \to 1$.

Studying these two extreme cases, namely, the behavior of contestants early and late in a tournament, allows us to abstract away from a multi-period setting and simplifies the analysis. This way we are able to provide sharp insights about the interplay of the competition and encouragement effects and discuss the role of information provision (and information provision devices such as intermediate awards) in the success of a contest.

### 3 Analysis

Our analysis focuses on the two cases discussed above that capture the dynamics near the beginning and near the end of a contest. The main goal is to study how the designer's information disclosure mechanism and the contest’s award structure affect the agents' effort provision and consequently the expected payoff for the designer. Having an understanding of this relationship allows us to provide concrete prescriptions for the design of dynamic contests. We start with the case when the contest is still at its early stages and there is significant uncertainty regarding the feasibility of the end goal.

#### 3.1 Early Stages: Uncertainty is Dominant

As discussed earlier, when there is uncertainty about whether the end goal is attainable or not, agents may become more optimistic about their chances of finishing the contest by observing the partial progress of others. This section explores how this “encouragement effect” can be utilized by the designer to improve the outcome of the contest. To simplify the analysis and exposition, we assume that $p_A < 1$ and that $P(\theta_B = 1|\theta_A = 1) = 1$, so that there is no uncertainty in the second stage provided that the first stage can be completed. Conditional on $\theta_A = 1$, the nominal arrival rate of a breakthrough in the first stage is $\lambda$ for both agents. In keeping with our focus on modeling the
situation near the beginning of a contest, we let the arrival rate in the second stage, which we denote by $\mu$, to be such that $\mu < \lambda$. This assumption together with the assumption that $p_A < 1$ provide a good approximation of the dynamics at the early stages of a contest, when there is both a significant amount of uncertainty as well as plenty of time before a competitor reaches the end goal.

Before we proceed with the analysis, we describe the belief update process under the two extremes of information disclosure. These are full disclosure, i.e., when information regarding an agent’s progress is immediately shared among all competitors, and no disclosure, i.e., when an agent observes only the outcomes of her own experimentation (as well as whether the contest has come to an end).

- Full disclosure: If agents can observe each others’ outcomes, then the (common) posterior belief at time $t$ regarding the feasibility of stage $A$ in the absence of any progress is given by Bayes rule as:

$$ p_{1,t} = p_{2,t} = \frac{p_A e^{-2\lambda t}}{p_A e^{-2\lambda t} + (1 - p_A)}, \quad (3) $$

when both competitors put full effort up to time $t$, i.e., $x_{1,\tau} = x_{2,\tau} = 1$ for $\tau \leq t$. Obviously, in the event that one of the competitors finishes stage $A$ then $p_{1,t}$ and $p_{2,t}$ jump to one as the uncertainty regarding stage $A$ is resolved. Finally, if agents do not put full effort up to $t$ but rather use effort provision strategies $\{x_{1,t}\}_{t \geq 0}$ and $\{x_{2,t}\}_{t \geq 0}$, then we replace term $e^{-2\lambda t}$ in Expression (3) by $e^{-\lambda \int_0^t (x_{1,\tau} + x_{2,\tau}) d\tau}$.

- No disclosure: In the absence of any partial progress (and assuming that her competitor has not claimed the final award), agent $i$’s belief at time $t$ is given by:

$$ p_{i,t} = \frac{p_A e^{-\lambda t} \left( \frac{\lambda e^{-\mu t} - \mu e^{-\lambda t}}{\lambda - \mu} \right) \left[ \frac{\lambda e^{-\mu t} - \mu e^{-\lambda t}}{\lambda - \mu} \right] + (1 - p_A)}{p_A e^{-\lambda t} \left( \frac{\lambda e^{-\mu t} - \mu e^{-\lambda t}}{\lambda - \mu} \right) + (1 - p_A)}, \quad (4) $$

when both competitors put full effort up to time $t$. Note that the term $\left( \frac{\lambda e^{-\mu t} - \mu e^{-\lambda t}}{\lambda - \mu} \right)$ captures the information agent $i$ infers from the fact that her competitor has not completed Stage $B$ up to time $t$. In particular,

$$ P(\text{contest has not ended until } t|\theta_A = 1) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda \tau} e^{-\mu (t-\tau)} d\tau \quad (5) $$

$$ = \frac{\lambda e^{-\mu t} - \mu e^{-\lambda t}}{\lambda - \mu}, $$

where the first term in Equation (5) is equal to the probability that the competitor has not completed Stage $A$ (conditional on $\theta_A = 1$) and the second term is equal to the probability that the competitor has completed stage $A$ but not $B$.

It is instructive to compare the expected outcomes of a contest under these two extreme information disclosure policies. Under full disclosure, an agent completing Stage $A$ is considered “good
news”, since it provides a positive signal about the feasibility of the end goal, and therefore instantly affects the competitors’ effort provision. On the flip side, absence of progress makes agents become pessimistic at a faster rate than when information is not public. Indeed, comparing Expressions (3) and (4) one can easily deduce that agents’ beliefs move downwards at a faster rate under full disclosure. This comparison clearly illustrates one of the designer’s main tradeoffs: on one hand, sharing progress between competitors allows for timely dissemination of good news. On the other hand, the absence of partial progress early on in the process can make agents pessimistic about the feasibility of the underlying project and, as we will see below, adversely affect their effort provision.

Our goal for the remainder of this section is to provide guidelines for the design of an innovation contest. Specifically, we consider the use of awards as a means of incentivizing agents to remain active in the contest as well as disclose their experimentation outcomes. The designer has to balance several conflicting trade-offs. First, the size of the awards directly affects the incentives of the agents to exert effort in the contest. Second, the timing of the awards implicitly induces an information disclosure policy. For example, the cases discussed above – full and no disclosure – can be implemented using a simple award structure. In the case of full disclosure, the designer can offer a large enough intermediate award \( R_A \) for the agent who first completes Stage \( A \), and no disclosure can be implemented by simply offering a single final award to the first agent who completes the entire contest.

Finally, the designer has to take the agents’ incentives to reveal information about their progress into account. Specifically, in the presence of an encouragement effect, agents may be reluctant to share their partial progress in the hope that their competitors become more pessimistic and drop out of the race. Thus, it is not clear whether and when agents will disclose information even when the designer allows them to do so.

As should be clear by the discussion above, the space of potential designs for an innovation contest is quite large. Throughout the paper, we restrict attention to the sub-space of designs that feature intermediate awards of constant size that may or may not be offered at a given time period, i.e., we assume that \( R_A(t) \in \{0, R_A\} \). We also assume that \( R_B(t) = R_B \), i.e., the award for completing the contest is always available to the agents. The second assumption is without loss of generality when the designer discounts future at a high enough rate. However, the designer may potentially benefit if she allows the size of the intermediate awards to vary over time. We chose not consider such a scenario mainly because of two reasons. First, optimizing the timing of awards even when their values are fixed is already quite challenging analytically. More importantly, we believe that in real-world tournaments simple award schemes are preferable and more likely to be implemented than awards whose sizes are a complex function of time. Indeed, most if not all real-world tournaments feature awards of constant size that are given out for reaching pre-specified milestones. Furthermore, this assumption allows us to clearly illustrate the interaction between the size of the awards and the time at which they are given out and how this interaction affects the agents’ beliefs about the underlying environment and consequently their effort provision.
As a natural starting point, Subsections 3.1.1 and 3.1.2 explore equilibrium behavior in contests that induce full and no information disclosure respectively. Our analysis clearly highlights the trade-offs involved and motivates Subsection 3.1.3 in which we show that more sophisticated information disclosure policies (that may be implemented by appropriately timing intermediate awards) lead to a higher expected payoff for the designer. These results provide a complete picture regarding the question of optimal contest design (at least in the space of deterministic disclosure policies) as a function of the parameters in the model, i.e., the designer’s budget, the discount rate, and the competitors’ skills (as captured by rates $\lambda$ and $\mu$).

### 3.1.1 Full Information Disclosure: Real-Time Leaderboard

In this subsection, we study equilibrium behavior in a contest where when an agent completes stage $i \in \{A, B\}$, she has the incentive to immediately report it (and claim the associated award). Before describing in detail the award structure that induces such behavior, we turn our attention to the subgame that results when one of the agents, the leader, completes Stage $A$. The leader’s optimal effort provision takes a very simple form for $t \geq \tau_A$, where $\tau_A$ is the random time at which Stage $A$ is completed. In particular, if we index the leader by $i$, we have

$$x_{i,t}^* = \begin{cases} 1 & \text{if } R_B \geq \frac{c}{\mu} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the designer should set $R_B$ to be at least as high as $\frac{c}{\mu}$ to ensure that the contest is finished (recall that Stage B can be finished with probability 1, so as soon as an agent breaks through to that stage, she will continue putting effort if the value of the award is high enough). Similarly, the laggard continues putting effort in the contest if her expected payoff is higher than the instantaneous cost of effort, i.e., $x_{j,t}^* = 1$ for $\tau_A \leq t \leq \tau_B$ if

$$\lambda \frac{\mu R_B - c}{2\mu + r} \geq c \Rightarrow R_B \geq \frac{c}{\mu} \left(1 + \frac{2\mu + r}{\lambda}\right).$$

where $j$ is the index of the laggard, and $\tau_B$ is the time at which Stage $B$ is finished (and the contest ends). In other words, upon completion of stage $A$, both the leader and the laggard remain in the contest and put full effort until one of them completes stage $B$ if the final award $R_B$ is at least

$$R_B \geq R_B^{\min} \equiv \frac{c}{\mu} \left(1 + \frac{2\mu + r}{\lambda}\right).$$

(6)

If, on the other hand, $\frac{c}{\mu} \leq R_B < R_B^{\min}$, the laggard drops out of the contest whereas the leader continues putting full effort until the end. If we let $V^i(k, \ell)$ denote the expected payoff of agent $i$
when she is in stage \( k \) and her competitor is in stage \( \ell \), we have

\[
V^i(A, B) = \begin{cases} 
\frac{\lambda}{\lambda + \mu + r} \frac{2\mu}{2\mu + r} R_B - \frac{2\mu + \lambda + r}{(\lambda + \mu + r)(2\mu + r)} c & \text{if } R_B \geq R_B^{\min} \\
0 & \text{otherwise}
\end{cases}
\]

and

\[
V^i(B, A) = \begin{cases} 
\left(\frac{\lambda}{\lambda + \mu + r} + \frac{\mu}{\lambda + \mu + r}\right) R_B - \frac{2\mu + \lambda + r}{(\lambda + \mu + r)(2\mu + r)} c & \text{if } R_B \geq R_B^{\min} \\
\frac{\mu R_B - c}{\mu + r} & \text{otherwise}.
\end{cases}
\]

We are now ready to describe the award structure that induces full information disclosure. Note that when the leader decides whether to claim an intermediate award and reveal her partial progress she is facing the following trade-off. On the one hand, she would be compensated by \( R_A \) for her revelation. On the other hand though, her progress encourages her competitor to stay in the contest and thus decreases her own likelihood of winning the final award of size \( R_B \). The increase in the leader’s expected payoff when her competitor quits providing effort is given by

\[
\Delta(R_B) = \frac{\mu R_B - c}{\mu + r} - \left(\frac{\lambda}{\lambda + \mu + r} + \frac{\mu}{\lambda + \mu + r}\right) R_B - \frac{2\mu + \lambda + r}{(\lambda + \mu + r)(2\mu + r)} c.
\]

Thus, if the intermediate award \( R_A \) is at least as high as \( \Delta(R_B) \) the leader will have an incentive to claim it and reveal her partial progress. This is formally stated in the next proposition (the proof is omitted as it follows from the arguments in the text).

**Proposition 1.** The following award schedule implements full information disclosure when \( R_B \geq R_B^{\min} \):

\[
R_A(t) = \begin{cases} 
R_A \geq \Delta(R_B) & \text{for } t \leq t_c \\
0 & \text{otherwise}
\end{cases}
\]

where \( t_c \) is a cutoff time described below. If \( \frac{c}{\mu} \leq R_B < R_B^{\min} \), then full information disclosure can be implemented for any \( R_A(t) \).

Following the discussion above agent \( i \)'s optimization problem can be written as follows when \( R_A \geq \Delta(R_B) \), i.e., information regarding partial progress is disclosed as soon as it occurs.

\[
\max_{\{x_{i,t}\}_{t \geq 0}} \int_0^\infty \left[ x_{i,t}(p_{i,t} \lambda (R_A + V^i(B, A)) - c) + x_{j,t} p_{i,t} \lambda V^i(A, B) \right] e^{-\int_0^t \left(p_{i,s}(\lambda(x_{1,s} + x_{2,s}) + r)ds \right) dt}.
\]

We are interested in characterizing the unique symmetric equilibrium in Markovian strategies in this setting. Proposition 2 below states that agents follow a cutoff experimentation policy in stage \( A \) and the aggregate amount of experimentation increases with the size of the intermediate award.

**Proposition 2.** When the award for completing stage \( A \) is such that \( R_A \geq \Delta(R_B) \), there exists a unique symmetric equilibrium in which agents experiment as follows:
(i) Agents follow a cutoff experimentation policy in stage A, i.e.,

\[ x_{i,t}^* = \begin{cases} 
1 & \text{for } t \leq t_c = \frac{1}{2\lambda} \ln \left( \frac{1-p_c}{p_c} \cdot \frac{p_A}{1-p_A} \right) \\
0 & \text{otherwise.} 
\end{cases} \]  

(10)

The cutoff belief \( p_c \) is given as follows

\[ p_c = \begin{cases} 
\frac{c}{\lambda \left( R_A + \frac{\mu R_B - c}{\mu + r} \right)} & \text{if } c \frac{\mu}{\mu + r} \leq R_B < R_B^{\text{min}} \\
\frac{c}{\lambda \left( R_A + \left( \frac{2\mu}{\lambda + r} + \frac{\mu}{\lambda + r} \right) R_B - \frac{2\mu + \lambda + r}{(\lambda + r)(2\mu + r)} c \right)} & \text{if } R_B \geq R_B^{\text{min}}.
\end{cases} \]  

(11)

(ii) If stage A is completed, experimentation continues as follows

(a) If \( R_B \geq R_B^{\text{min}} \): Both agents experiment with rate one until the end of the contest.

(b) If \( c \frac{\mu}{\mu + r} \leq R_B < R_B^{\text{min}} \): The laggard drops out of the contest whereas the leader experiments with rate one until the end.

Proposition 2 illustrates the tradeoff between setting \( R_B \) at a higher value (and thus providing an incentive for the laggard to stay active in the contest) and increasing the size of \( R_A \) (in which case agents experiment more in Stage A). In particular, the cutoff belief is decreasing in \( R_A \), implying that the aggregate amount of experimentation in Stage A is increasing with the size of the intermediate award. Note also that having both agents compete in the contest until its end adversely affects the aggregate amount of experimentation in Stage A, since the leader’s expected payoff for moving on to Stage B is smaller. Taken together, these facts imply that aggregate experimentation in Stage A – and thus the probability that the contest will be completed eventually – is maximized by setting \( R_B \) to the smallest possible value that ensures that an agent that completes Stage A has the incentive to complete Stage B as well, i.e., \( \frac{c}{\mu} \).

On the other hand, setting \( R_B \) to \( \frac{c}{\mu} \) implies that only one of the agents will put effort in stage B as opposed to when \( R_B \geq R_B^{\text{min}} \), in which case both the leader and the laggard continue to compete for the final award until the contest is over. Thus, when setting the sizes of the two awards the designer trades off a higher probability of obtaining the innovation (by setting \( R_B = c/\mu \)) with getting it sooner (by setting \( R_B \geq R_B^{\text{min}} \)).

To resolve the issue of whether it is optimal to have both agents compete until the end of the contest or set the intermediate award at a higher value requires that we obtain a characterization of the expected cost of running a contest since the designer is budget constrained (recall that \( R_A \) and \( R_B \) should be set such that the budget constraint in Equation 2 is not violated in expectation). Lemma 1 below provides an expression for the expected cost of a contest with awards \( R_A \) and \( R_B \).

**Lemma 1.** The cost of running a contest with awards \( R_A \) and \( R_B \) with \( R_A \geq \Delta(R_B) \) and \( \theta_A = 1 \) is given as follows
(a) If \( R_B \geq R_B^{\min} \), it is equal to
\[
\frac{2\lambda}{2\lambda + r} \left(1 - e^{-(2\lambda + r)t_c}\right) \left( R_A + R_B \left( \frac{\lambda}{\lambda + \mu + r} - \frac{2\mu}{\lambda + \mu + r} + \frac{\mu}{\lambda + \mu + r} \right) \right).
\]

(b) If \( \frac{\xi}{\mu} \leq R_B < R_B^{\min} \), it is equal to
\[
\frac{2\lambda}{2\lambda + r} \left(1 - e^{-(2\lambda + r)t_c}\right) \left( R_A + R_B \frac{\mu}{\mu + r} \right).
\]

The experimentation time cutoff \( t_c \) is given by expressions (10) and (11).

We conclude this subsection by providing conditions under which the optimal full information disclosure contest features a high intermediate award (and thus the design induces more experimentation in stage \( A \)) or a high final award (and thus the contest is completed sooner in expectation (conditional on it being completed)). In particular, Proposition 3 describes the optimal full information disclosure contest as a function of the discount rate \( r \) and the competitors’ skill \( \lambda \). It states that when the designer is sufficiently patient, i.e., she has a sufficiently low discount rate, it is optimal to set the final award \( R_B \) to its minimum value \( c/\mu \), and maximize the agents’ aggregate experimentation in stage \( A \) (recall that a higher intermediate award induces more experimentation in stage \( A \)). If, on the other hand, the designer discounts future at a sufficiently high rate, then it is optimal to allocate a larger fraction of the designer’s budget to the final award, and thus incentivize both agents to compete until the contest is over. Although this leads to lower aggregate experimentation in Stage \( A \) (and thus implies that the probability that the contest will be completed is lower), it also implies that conditional on the contest being completed the time it takes is lower since both agents compete until the end.

Let \( D^1 \) denote the design for which the final award is equal to \( R_B = \frac{c}{\mu} \) whereas let \( D^2 \) denote the design for which the final award is equal to \( R_B = R_B^{\min} \) and the rest of the designer’s budget is allocated to the award for partial progress \( R_A \) (here the superscript refers to the number of agents that continue putting effort if stage \( A \) is completed). Then,

**Proposition 3.** When the intermediate award \( R_A \geq \Delta(R_B) \), the two contest designs that maximize the designer’s expected payoff are \( D^1 \) and \( D^2 \). In particular,

(i) Contest design \( D^1 \) outperforms design \( D^2 \) when the designer is sufficiently patient, i.e., \( r \to 0 \).

(ii) When \( r > 0 \) there exists \( \tilde{\lambda}(r) \) such that for \( \lambda > \tilde{\lambda}(r) \), contest design \( D^2 \) outperforms design \( D^1 \).

Proposition 3 completes the discussion on optimal contest design under the constraint that the award structure is such that agents are incentivized to disclose their progress as soon as it occurs, i.e., \( R_A \geq \Delta(R_B) \). Inducing immediate dissemination of good news may be advantageous to the designer when the discount rate is high as we elaborate towards the end of the section, but it also implies that in the absence of progress agents become pessimistic at a fast rate. Subsection 3.1.2 considers the alternative of having just one final award and not disclosing any information about partial progress.
3.1.2 No Information Disclosure: Grand Prize

At the other extreme, we have contests that feature a single final award and no information disclosure in the interim. The single final award $R_B$ is set so that $\mathbb{E}[e^{-rT} R_B] = B$, i.e., the designer’s budget is entirely consumed conditional on the contest being completed. Agent $i$’s optimization problem can be then expressed as follows:

$$
\max_{\{x_{i,t}\}_{t \geq 0}} \int_0^\infty \left( x_{i,t} \lambda p_{i,t} \left( q_{i,t} V_i(B, B) + (1 - q_{i,t}) V_i(B, A, \{x_{i,t}\}) \right) - x_{i,t} c \right) e^{-r t} e^{-\int_{\tau=0}^t p_{i,\tau} (q_{i,\tau} \mu + \lambda x_{i,\tau}) d\tau} dt,
$$

where recall that $V_i(B, A)$ denotes the expected payoff for agent $i$ when she has completed stage $A$ whereas her competitor has not. Agent $i$ cannot observe agent $j$’s progress (as long as agent $j$ has not completed stage $B$). Therefore, she forms beliefs about whether or not her competitor has completed stage $A$ up to time $t$. In particular, $q_{i,t}$ denotes agent $i$’s belief that her competitor has advanced to stage $B$ by time $t$.

As we show in Proposition 4 equilibrium behavior takes the form of a threshold policy as in the case of full information disclosure. However, in this case the time threshold $t_c$ after which an agent stops experimenting in the absence of partial progress depends on both her own effort provision as well as her conjecture about her competitor’s strategy that collectively determine the speed at which the agent updates her beliefs about the attainability of the end goal as well as whether her competitor has already completed stage $A$. Note that as before as soon as the agent completes stage $A$ then it is optimal for her to put effort until the contest is over.

**Proposition 4.** When $R_B \geq R_B^{\text{min}}$ and agents’ progress is not observable, there exists a unique symmetric equilibrium in which agents experiment as follows:

(i) Agents follow a cutoff experimentation policy in stage $A$, i.e.,

$$
x^*_{i,t} = \begin{cases} 
1 & \text{for } t \leq t_c \\
0 & \text{otherwise},
\end{cases}
$$

where the cutoff time $t_c$ is given as the unique solution to the following equation

$$
\frac{p_A e^{-\lambda t}}{p_A e^{-\lambda t} \left( \frac{p_e^{-\mu t} - e^{-\lambda t}}{\lambda - \mu} \right) + (1 - p_A) \left( e^{-\lambda t} \frac{\mu R_B - c}{\mu + r} + \frac{e^{-\mu t} - e^{-\lambda t}}{\mu (\mu + r) \lambda - \mu} \right)} = \frac{c}{\lambda}.
$$

(ii) If the agent completes stage $A$, then she experiments with rate one until the end of the contest.

Proposition 4 is stated under the assumption that the single final award $R_B$ is such that $R_B \geq R_B^{\text{min}}$, i.e., completing Stage $A$ is seen as good news for both competitors, the leader and the laggard. In other words, such an award ensures that the encouragement effect dominates. Note that this assumption is in line with our focus on the dynamics at the beginning of the contest.
The juxtaposition of Propositions 2 and 4 clearly illustrates the main tradeoff that the designer faces. In particular, equilibrium experimentation takes the form of a threshold policy under both full and no information disclosure. The threshold under full disclosure satisfies the following equation

$$\frac{p_A e^{-2\lambda t_c}}{p_A e^{-2\lambda t_c} + (1 - p_A)} \left( R_A + \left( \frac{\lambda}{\lambda + \mu + r} \frac{2\mu}{2\mu + r} + \frac{\mu}{\lambda + \mu + r} \right) R_B - \frac{2\mu + \lambda + r}{(\lambda + \mu + r)(2\mu + r)} c \right) = \frac{c}{\lambda}, \quad (15)$$

when both agents compete until the end of the contest given that stage A is completed, i.e., when $R_B \geq R^\text{min}_B$. The threshold under no disclosure satisfies

$$\frac{p_A e^{-\lambda t_c}}{p_A e^{-\lambda t_c} \left( \frac{\lambda e^{-\mu t_c} - \mu e^{-\lambda t_c}}{\lambda - \mu} \right) + (1 - p_A)} \left( e^{-\lambda t_c} \mu R_B - c \frac{e^{-\mu t_c} - e^{-\lambda t_c}}{\mu} + \frac{\mu}{\lambda - \mu} \frac{e^{-\lambda t_c} - e^{-\mu t_c}}{2\mu + r} \right) = \frac{c}{\lambda}. \quad (16)$$

Comparing Equations (15) and (16) implies that agents stop experimenting earlier when information is fully disclosed and $\lambda > \mu$. Thus, the probability that the contest is completed is higher when the contest has only one final award. On the other hand though, there is a positive probability that in the case when information about partial progress is not disclosed one of the agents drops out even though the other has completed stage A. The latter never occurs under full information disclosure (when $R_B \geq R^\text{min}_B$). Thus, conditional on one agent completing stage A early enough, the contest is completed earlier in expectation when experimentation outcomes are publicly observable.

This tradeoff motivates the search for alternative information disclosure policies that could combine the benefits of these two extremes. The final part of this section shows that appropriately restricting the times at which an agent can disclose information about her progress leads to strictly better outcomes for the designer. Finally, we show that intermediate awards are crucial in implementing this more sophisticated information disclosure policy and we make a connection between our results and design features of real-world innovation contests.

### 3.1.3 Design with “Silent Periods”

As discussed in the beginning of the section, although full disclosure allows for the fast dissemination of good news, it may also adversely affect effort provision as agents become pessimistic regarding the feasibility of stage A in the absence of partial progress early in the process. The fact that “no news is negative news” motivates considering alternative designs that may feature silent periods, i.e., time intervals in which the designer does not disclose any information regarding the competitors’ progress (obviously, a special case is the design that features no information disclosure). In particular, for the remainder of this subsection we explore the performance of contest designs as they are defined by the timing of the designer’s announcement about the status of the competition between the agents as well as by the award structure. Specifically, we parameterize designs by $T \subseteq [0, \infty)$ and $R = (R_A, R_B)$, i.e., the set of times the designer discloses information about the competitors’ progress, i.e., whether one or both have completed stage A and the size of the awards for completing stage A and B respectively. For example, $T = [0, \infty)$ corresponds to full disclosure, $T = \emptyset$ to no
disclosure, whereas $T = [t_1, t_2]$ to a design in which information is disclosed only between $t_1$ and $t_2$. We say that the design features a “silent period” when $T \subset [0, \infty)$, i.e., the set of time periods that the designer does not disclose any information has non-zero measure.

As a first indication that silent periods may increase the designer’s expected payoff, Proposition 5 below provides a sufficient condition on the designer’s budget for such designs to outperform full information disclosure.

**Proposition 5.** Assume that the designer has budget $B > \frac{2\lambda}{2\lambda+r} \left( \frac{\lambda}{\lambda+\mu+r} \frac{2\mu}{2\mu+r} + \frac{\mu}{\lambda+\mu+r} \right) R_B^{\min}$. Then, a design with $T = [t_A, \infty)$, i.e., a design that features a silent period of length $t_A > 0$, always outperforms full information disclosure.

Proposition 5 shows that designs with $T = [t_A, \infty)$ and $t_A > 0$ outperform full disclosure. Essentially, agents may require encouragement (that takes the form of a positive announcement regarding their competitors’ progress) but only after some time has elapsed and their own experimentation has not generated any results. The lower bound on $B$, i.e., $B > \frac{2\lambda}{2\lambda+r} \left( \frac{\lambda}{\lambda+\mu+r} \frac{2\mu}{2\mu+r} + \frac{\mu}{\lambda+\mu+r} \right) R_B^{\min}$ ensures that the designer has enough budget to set the award for finishing stage $B$ at $R_B \geq R_B^{\min}$ which as we mentioned in the previous subsection it is necessary for the encouragement effect to be relevant in this setting (if $R_B < R_B^{\min}$ then being informed that a competitor has completed stage $A$ makes the laggard quit the contest).

Although this result establishes that full disclosure is never optimal, it does not preclude the optimality of a no disclosure information policy, i.e., $t_A = \infty$. On the one hand, when $t_A$ is finite positive news may be shared between the agents (albeit with some delay) but on the other in the absence of a positive announcement agents become pessimistic about the feasibility of the goal and drop out. No disclosure does not allow for sharing of information but agents’ beliefs are updated downwards at a slower rate.

Interestingly, we show (under some additional mild assumptions) that a design that features a silent period of finite length outperforms the no information disclosure policy.

**Theorem 1.** Let

$$B > \frac{2\lambda}{2\lambda+r} \left( \frac{\lambda}{\lambda+\mu+r} \frac{2\mu}{2\mu+r} + \frac{\mu}{\lambda+\mu+r} \right) R_B^{\min}$$

and $\lambda \geq \bar{\lambda}, r \geq \bar{r}$ for constants $\bar{\lambda}, \bar{r}$. Then, the design that features $T = [t_A, \infty)$ and $R_B = R_B^{\min}$ outperforms both the design that implements full information disclosure as well as the one that implements no information disclosure. Announcement time $t_A$ is given as the solution to Equation (17) below:

$$\frac{p_A e^{-\lambda t_A}}{p_A e^{-\lambda t_A} + \left( 1 - p_A \right)} \left( e^{-\lambda t_A} \left( R_B + V(B, A) \right) + \frac{\lambda e^{-\mu c}}{\lambda - \mu} \left( R_A + \frac{\mu R_B - c}{2\mu + r} \right) \right) = \frac{c}{\lambda}. \quad (17)$$

Theorem 1 states that the design in which information regarding the status of competition is first disclosed at time $t_A > 0$ outperforms both full information disclosure as well as no disclosure.
Note that such an information disclosure policy can be implemented by giving out an intermediate award for completing stage $A$ at time $t_A$. In other words, the proposed design that outperforms the extremes of a real-time leaderboard (full information disclosure) and a single final award (no information disclosure) takes the following form: the designer gives out an award equal to $R_B = R_B^{\min}$ to the agent that first completes the entire contest. This award is available at all times. In addition, the designer offers an intermediate award for completing stage $A$ by time $t_A$. If both agents completed stage $A$ by $t_A$ then they each receive half the award (or equivalently they receive the award with probability $\frac{1}{2}$). Importantly, the designer only announces the recipients of the intermediate award at time $t_A$, i.e., does not disclose any information before then.

Furthermore, it is straightforward to see that this design is incentive-compatible (assuming that the designer has enough budget and $R_A$ is large enough), i.e., the leader will have the incentive to disclose her progress at time $t_A$ and claim the intermediate award.

In fact, this design closely resembles the structure that many real-world tournaments follow. As an example, apart from the final grand prize, contestants in the Netflix prize competed for intermediate awards (progress prizes) that were given out at pre-specified times to the team that had the highest score given that the score was above a certain threshold. In particular, Netflix was offering a Progress Prize each year the contest continued to run to the team that showed the most improvement during the year as long as this improvement was above a given threshold. 

This mirrors the design with a silent period of length $t_A$ at which time the designer gives out an intermediate award to the team(s) that has completed stage $A$, i.e., has progressed above some threshold level. What is more, the Netflix design allowed participants to disclose their progress as it happened in a publicly observable real-time leaderboard. However, since the awards were given out only once a year, i.e., at pre-specified times, most of the teams posted their progress close to the deadline effectively implementing a silent period until the time the first award was to be given out.

The theorem confirms the role of intermediate awards as essential design levers for an innovation contest in the presence of uncertainty regarding the feasibility of the end goal. This result, i.e., that front-loading payments may lead to a higher expected payoff for the designer (principal), is in stark contrast with the standard principal-agent framework where it is typically optimal to backload the payments to the agent as a way to maximize her effort provision. In our setting, the contest designer exploits the fact that claiming an award generates information that increase the agents’ incentive to exert costly effort and thus finds it optimal to front load her payments to them.

We should note that the design that features one silent period of length $t_A > 0$ is actually the optimal design when there is uncertainty in the first stage and the designer is restricted to award structures and information disclosure mechanisms that do not depend on the history of outcomes.\textsuperscript{8}

\textsuperscript{7} Although it is hard to know exactly how much information regarding their progress teams were holding back before the deadline for each progress prize, much of the online discussions allow us to infer that teams were very strategic regarding to what information to post to the leaderboard and when. See for example the discussion in http://www.decompilinglife.com/post/575898924/the-netflix-prize-competition.

\textsuperscript{8} Considering designs that do not depend on the sample path of outcomes makes most sense in a setting where agents’
In particular, this implies that it would never be optimal to give out two intermediate awards for completing stage $A$ at different times separated with a silent period, i.e., the optimal design cannot features two silent periods. This is straightforward to see: as we show above it is always optimal for agents to follow a bang-bang experimentation strategy when the design features silent periods. They continue experimenting as long as their expected instantaneous payoff is non-negative. The latter depends only on the agents’ beliefs about the underlying state and not on the times at which awards are given out. This implies that one could replace the two silent periods with one and ensure the same aggregate amount of experimentation.

Finally, depending on the model’s primitives it may be optimal to set $t_A$ to infinity, i.e., consider giving out only one final award. For example, when the discount rate $r$ is sufficiently small ($r \to 0$) it is optimal for the designer not to give out any intermediate award (as this maximizes the aggregate amount of experimentation in Stage $A$).

In the next subsection we turn our attention to the dynamics towards the end of the contest when competition between the agents is fierce. We characterize equilibrium behavior under full and no information disclosure and illustrate that optimal designs feature silent periods as well (albeit for an entirely different reason that we discussed above).

### 3.2 Late Stages: Competition is Dominant

In this section we focus on the case where $p_A = p_B = 1$, i.e., given enough time and effort, innovation will occur with probability one. Since innovation is certain, the interest of the contest designer is in achieving that innovation as quickly as possible, as time is discounted at a rate $r > 0$. Having both agents actively participating in the contest, i.e., exerting effort towards its completion, naturally expedites innovation compared to the case when only one of them remains active in the race, and thus the focus of the designer is on providing the right informational incentives for agents to continue experimenting and not drop out of the contest. These incentives may involve signaling to the agents that, relative to the competition, they are not lagging behind.

For the remainder of the section and following the earlier discussion that this setting approximates the contest near its conclusion and after most of the uncertainty has been resolved, we assume that $\mu > \lambda$, so that on average the second stage (the remainder of the contest) requires less time to complete than the first.

As before, it is instructive to begin our analysis by comparing the performance of the following two design structures that can be thought as the two extremes as far as information disclosure is concerned. First, we analyze equilibrium behavior under full disclosure, i.e., when experimentation outcomes are publicly observable. Then, we study the performance of designs that feature no disclosure, i.e., all an agent knows about her competitor is that the latter has not completed the outcomes are their private information and the only way that the designer can elicit information disclosure is by giving out awards for reaching a pre-specified milestone (and thus the designer’s information set is always a subset of what individual agents know).
The tradeoff between the two designs can be summarized as follows: under full disclosure agents compete in stage $A$ by exerting effort with rate one but may drop out of the contest as soon as one of them advances to stage $B$. On the other hand, in the case of no disclosure, agents cannot observe each other’s progress towards the end goal. However, as time goes by and an agent has not completed stage $A$ yet, she becomes progressively more pessimistic about her probability of winning the contest, believing that it is more likely that her competitor is already in stage $B$ (which would imply that she should quit). As a response to this uncertainty, agents may drop their effort levels to strike a balance between quitting the competition early and persisting in an attempt to win, without losing too much if it turns out they are lagging behind.

In what follows, we provide a characterization of the unique symmetric equilibrium for both cases and then we show that a design that judiciously provides information to the agents about the status of competition outperforms both of these extremes.

### 3.2.1 Full Information Disclosure

The case of full disclosure, i.e., assuming that the experimentation outcomes are publicly observable, is straightforward to analyze. Indeed, as we show below equilibrium behavior features either no effort provision or full allocation of the agent’s experimentation efforts to the contest. Also, in the absence of any uncertainty regarding the feasibility of stage $A$, it is straightforward to see that it is optimal to have a single final award since this way the agents’ effort provision incentives are maximized throughout the contest (we will describe how to implement full disclosure when progress is an agent’s private information towards the end of the section).

Let $R_B$ denote as before the award for completing the contest. Then, the expected cost for running the contest is equal to

$$
\frac{2\lambda}{2\lambda + r} \frac{\mu}{\mu + \lambda + r} R_B,
$$

when the laggard drops out of the contest as soon as the leader completes stage $A$ and

$$
\frac{2\lambda}{2\lambda + r} \left( \frac{\mu}{\mu + \lambda + r} + \frac{\lambda}{\mu + \lambda + r} \frac{2\mu}{2\mu + r} \right) R_B,
$$

otherwise. Note that an agent will continue putting effort to the contest even after her competitor completes stage $A$ if her expected payoff by doing so is positive, i.e.,

$$
\lambda \frac{\mu R_B - c}{2\mu + r} \geq c \Rightarrow R_B \geq \frac{c}{\mu} \frac{2\mu + \lambda + r}{\lambda},
$$

where $\frac{\mu R_B - c}{2\mu + r}$ is the laggard’s continuation payoff for completing stage $A$. Similarly, no agent will put any effort in the contest if her expected payoff for doing so (even when the laggard drops out after the leader completes stage $A$) is negative. This implies that to ensure participation the designer should set $R_B$ so that

$$
\lambda \frac{\mu R_B - c}{\mu + r} \geq c \Rightarrow R_B \geq \frac{c}{\mu} \frac{\mu + \lambda + r}{\lambda}.
$$
The discussion above implies the following theorem that fully characterizes equilibrium behavior when outcomes are publicly observable.

**Theorem 2.** When agents’ progress is observable, there exists a unique symmetric equilibrium in which agents experiment as follows as a function of the final award $R_B$ (and consequently the designer’s budget):

(i) When $R_B < R_{\text{min}} \equiv \frac{\mu + \lambda + r}{\lambda}$: Agents never allocate any effort to the contest, i.e., $x_{1,t} = x_{2,t} = 0$ for all $t$.

(ii) When $R_{\text{min}} \leq R < R_{\text{max}} \equiv \frac{2\mu + \lambda + r}{\lambda}$: Agents put full effort until time $\tau_A$, i.e., the (random) time at which one of them completes stage $A$. After $\tau_A$ the leader continues putting full effort, whereas the laggard drops out of the race, i.e.,

$$x_{1,t} = x_{2,t} = 1, \text{ for } t \leq \tau_A,$$

and

$$x_{i,t} = 1 = 1 - x_{-i,t}, \text{ for } t > \tau_A \text{ if } i \text{ completes stage } A.$$

(iii) When $R \geq R_{\text{max}}$: Agents put full effort until the end of the contest, i.e., $x_{1,t} = x_{2,t} = 1$, for all $t$.

For the remainder of the section we will restrict attention to the case when the final award is $R_{\text{min}} \leq R_B < R_{\text{max}}$ so that when information is publicly disclosed agents compete by experimenting with rate one until stage $A$ is completed after which the leader continues in stage $B$ on her own. This is the most interesting case as when $R < R_{\text{min}}$ agents do not participate in the contest whereas when $R_B \geq R_{\text{max}}$ information disclosure about partial progress does not affect their effort provision.

Recall that in the interest of facilitating the comparison between different mechanisms we have assumed that the designer is budget constrained. This implies that the constraint on the size of the final award ($R_{\text{min}} \leq R_B < R_{\text{max}}$) corresponds to a constraint on the designer’s budget. For the case of full information disclosure the budget constraint can be written as follows

$$\frac{2\lambda}{2\lambda + r} \mu + r R_{\text{min}} \equiv B_{\text{min}} \leq B < B_{\text{max}} \equiv \frac{2\lambda}{2\lambda + r} \left( \frac{\mu}{\mu + \lambda + r} + \frac{\lambda}{\mu + \lambda + r} \frac{2\mu}{2\mu + r} \right) R_{\text{max}}.$$

We will state our subsequent results in terms of the sizes of the final awards as this eases the exposition, but we will also ensure that the associated budget constraint is satisfied.

Finally, note that unlike the case in which there is uncertainty in stage $A$, there is no need for the designer to incentivize the agents to announce their partial progress, i.e., completing stage $A$, when the latter is their private information. Instead, assuming that the designer provides them with a mechanism to do so, e.g., an online real-time leaderboard, an agent would broadcast a breakthrough as soon as it happens, since this report results in her competitor dropping out of the contest (when $R_B \in [R_{\text{min}}, R_{\text{max}}]$).
3.2.2 No Information Disclosure

When agents cannot observe their competitors’ experimentation outcomes, they form beliefs about their progress. As a first step in characterizing equilibrium behavior under no disclosure, we describe the evolution of these beliefs over time for an agent that has not experienced a breakthrough. Formally, let $q_{1,t}$ denote the probability that agent 1 assigns at time $t$ to the event that her competitor is already in stage B. Probability $q_{1,t}$ obviously depends on Agent 1’s belief about Agent 2’s effort provision policy. Assume that Agent 1 believes that if agent 2 is still in stage A, she allocates effort $x_{2,t}$ during time interval $[t, t + dt)$ to the contest (and allocates effort equal to one if she is in stage $B$). Then the rate of change of Agent 1’s belief at time $t$, denoted by $\dot{q}_{1,t}$, is given by

$$\dot{q}_{1,t} = (1 - q_{1,t})(x_{2,t}\lambda - q_{1,t}\mu),$$

(18)

with the boundary condition $q_{1,0} = 0$. This belief update process balances the following two facts: depending on Agent 2’s effort policy it is more likely over time that she has already completed stage A. On the other hand, if Agent 2 has already completed stage A, she should be putting full effort in completing stage $B$. Thus, the fact that the contest still has not ended makes it less likely that Agent 2 is actually in stage $B$. Note also that $q_{1,t}$ is always bounded above by $\frac{\lambda}{\mu}$.

Lemma 2 below uses Equation (18) to show that for every belief $q$ that agent 1 holds, there is an effort level $\psi(q)$ that Agent 2 can exert and that keeps that belief unchanged, perfectly balancing the opposing forces in the agent’s belief update process (the fact that $q_{1,t} \leq \frac{\lambda}{\mu}$ ensures that $\psi(q) \leq 1$).

**Lemma 2.** For every belief $q$ that agent $i$ holds, there is an effort level $\psi(q) = \frac{\mu q}{\lambda}$ that agent $j \neq i$ can exert such that the posterior belief of agent $i$ remains equal to $q$. The posterior belief of agent $i$ increases when $\psi(q) > \frac{\mu q}{\lambda}$ and decreases when $\psi(q) < \frac{\mu q}{\lambda}$.

Using Lemma 2, we show that there exists a unique symmetric equilibrium in Markovian strategies that takes a simple form: in the absence of any partial progress, agents put full effort up to a certain time $t_c$, after which they drop their effort level to $\frac{\mu q_{t_c}}{\lambda}$, where $q_{t_c}$ denotes the agent’s belief at time $t_c$. An agent continues to exert that effort level until she either experiences a breakthrough (in which case her effort level goes back up to 1 for stage $B$) or her competitor completes the contest. We explicitly characterize $t_c$ in the following result.

**Theorem 3.** When agents’ progress is not observable and $R_{\min} \leq R_B < R_{\max}$, there exists a unique symmetric equilibrium in which agents experiment as follows:

(i) In the absence of any partial progress, agent $i$ follows the experimentation strategy described

---

9Note that given prior belief $q_{1,t}$ and using Bayes’ rule, the posterior belief at time $t + dt$ is equal to:

$$q_{1,t+dt} = \frac{q_{1,t}(1 - \mu dt) + (1 - q_{1,t})x_{2,t}\lambda dt}{1 - q_{1,t} + q_{1,t}(1 - \mu dt)}.$$

Subtracting $q_1$ on both sides, dividing by $dt$, and taking the limit as $dt$ goes to zero gives us Equation (18).
below:

\[ x_{i,t}^* = \begin{cases} 
1 & \text{for } t \leq t_c = -\frac{\log \left( \frac{\Omega \mu - \mu + \lambda}{\mu - \lambda} \right)}{\mu - \lambda} \\
\frac{\mu q}{\lambda} & \text{otherwise,} 
\end{cases} \]  

(19)

where \( \Omega = \frac{c(2\mu + r + \lambda) - \lambda \mu R_B}{c\mu} \).

(ii) If the agent completes stage A, she experiments with rate one until the end of the contest.

We should note that due to discounting, it is not straightforward to compare the expected costs for running a contest under full information disclosure and under no disclosure, even when the final award has the same size for both designs. In particular, when information is disclosed then agents put full effort until Stage A is completed, after which only the leader remains active in the contest. This is not the case under no disclosure, where Stage A may take longer to complete on average (due to agents dropping their effort levels), but Stage B is completed at a faster rate in expectation. Our goal is to compare designs that allocate a fixed budget to awards, so the discussion above implies that the size of the final awards that correspond to full and no information disclosure under the same budget constraint may be different. Nevertheless, we are able to overcome this difficulty and compare the two extreme designs. Our results indicate that a full disclosure design dominates no disclosure for low budgets (but still such that the final award \( R_B \geq R_{\min} \)) whereas the situation is reserved for higher budgets (but still low enough so that \( R_B < R_{\max} \)).

Comparing the two extremes  Having characterized the equilibrium behavior under full and no disclosure, we turn our attention in the designer’s expected payoff from running the contest. Note that in both cases the contest will be eventually completed. Thus, the main concern for the designer is to minimize the expected time to completion.

We start by providing an expression for the designer’s payoff under full information disclosure. In particular, let \( \Pi_d^{\text{F}} \) denote the designer’s payoff under full disclosure and recall that \( U \), the designer’s utility from the innovation, is normalized to 1. Then, \( \Pi_d^{\text{F}} \) is given by:

\[
\Pi_d^{\text{F}} = U \cdot \int_{t=0}^{\infty} 2\lambda e^{-2\lambda t} dt \int_{\tau_A = t}^{\infty} \mu e^{-\mu(t-A-T)} e^{-r\tau_A} d\tau_A
\]

\[
= \frac{2\lambda \mu}{(2\lambda + r)(r + \mu)}. 
\]  

(20)

Furthermore, the budget that corresponds to a final award of value \( R_B \) (expressed in \( t = 0 \) terms) is given by

\[
R_B \cdot \frac{2\lambda \mu}{(2\lambda + r)(r + \mu)} = R_B \cdot \Pi_d^{\text{F}}. 
\]  

(21)
Although we cannot explicitly characterize the designer’s payoff under no disclosure, it should be clear that it is strictly increasing in the designer’s budget (unlike the case of full disclosure where the designer’s payoff is constant for all final awards in \([R_{\text{min}}, R_{\text{max}}])\). As we show below, this implies that for large enough final awards (but still within the interval \([R_{\text{min}}, R_{\text{max}}])\) no disclosure outperforms the full disclosure design.

**Proposition 6.** There exists a constant \(\bar{B}\) with \(R_{\text{min}} \cdot \Pi^F_d < \bar{B} < R_{\text{max}} \cdot \Pi^F_d\) such that if the designer’s budget \(B\) satisfies

(i) \(B \in [R_{\text{min}} \cdot \Pi^F_d, \bar{B}]\): Then, the designer’s payoff is higher under full disclosure.

(ii) \(B \in (\bar{B}, R_{\text{max}} \cdot \Pi^F_d]\): Then, the designer’s payoff is higher under no disclosure.

As we already noted above, when the budget is less than \(R_{\text{min}} \cdot \Pi^F_d\), agents do not participate in the contest under full disclosure (since the final award that the designer can give out is less than \(R_{\text{min}}\)). On the other hand, when the budget is higher \(R_{\text{max}} \cdot \Pi^F_d\) and the designer sets the size of the final award to be greater than \(R_{\text{max}}\), then the choice of contest design is irrelevant, since it is a dominant strategy for both agents to put full effort until the end of the contest. When the designer’s budget is in between these two extremes, the previous results suggest that in expectation, stage A is completed faster under full disclosure than under no disclosure. The reason is that both agents put full effort in stage A in the hope of being the leaders and ousting their competitors in full disclosure, whereas under no disclosure agents drop their effort levels as per Theorem 3 and hence progress towards completing stage A can be slow. However, because in full disclosure the progress of one agent immediately forces the other agent to quit, there is always a single agent in stage B. Under no disclosure, on the other hand, agents never exit the contest (but as mentioned, drop their effort levels), thus it is likely that both agents find themselves in stage B and put full effort from there on, finishing that stage faster than a single agent would. Note that the delay associated with the agents dropping their effort levels in stage A decreases as the final award increases which is the main reason why no disclosure outperforms full disclosure for a high enough final award (but still below \(R_{\text{max}}\)).

In the last part of this section, we present a design based again on silent periods that combines the desirable properties of these two designs.

### 3.2.3 Design with Silent Periods

Our mechanism for improving on both full and no information disclosure policies is based on having silent periods as in the previous section, whereby the designer announces the status of competition every \(T\) time periods and allows no disclosure of information in between. This announcement can be implemented through the introduction of an intermediate award that as discussed above can have zero monetary value as it is incentive compatible for the leader to publicly disclose her progress. Importantly, the reason why silent periods increase the designer’s payoff is very different than in the case when there is high uncertainty in stage A. Although before silent periods balanced
the tradeoff between agents becoming pessimistic at a slower rate under no information disclosure with sharing potentially positive news (an agent completing stage \(A\)), in this case the tradeoff is different. As time goes by and agents cannot observe their competitors’ progress, they believe that it is increasingly more likely that they are lagging behind (and thus have to quit the contest). Thus, an intermediate award (and in particular no agent claiming it) is good news in this case since it implies that their chances of winning are still high.

Equilibrium behavior when the design features silent periods is quite cumbersome to characterize explicitly for a general length \(T\). As in the case when there is no disclosure, agents form beliefs regarding the likelihood that their competitors advanced to stage B. Beliefs are reset at the time of an announcement if no progress is announced and the game essentially restarts. In the interval between two announcements, the belief that a competitor has completed stage \(A\) increases as time goes by and thus the agents’ incentives to compete weaken and their effort levels decrease.

As one would expect, the exact rate at which effort levels decrease, depends also on the value of \(T\) (and thus it is extremely hard to characterize explicitly for a general value of \(T\)). That said, we are able to show that there exists a finite value \(\bar{T} > 0\) that we explicitly characterize such that a design with silent periods of length \(\bar{T}\) strictly outperforms both full and no information disclosure.

More precisely, we let \(\bar{T}\) be such that agents put full effort in the contest as long as no progress has been announced. Clearly, this provides a lower bound on the designer’s expected payoff (as it may be optimal for the designer to set \(\bar{T}\) such that agents experiment with lower intensity but at a higher level on aggregate between two announcements). We show that the design with silent periods of length \(\bar{T}\) outperforms the two benchmark designs we analyzed previously.

**Proposition 7.** When the final award has value \(R_{\min} \leq R_B < R_{\max}\) and the design features silent periods of length \(\bar{T}\) given by

\[
\bar{T} = -\frac{\log \left( \frac{\Phi \mu + \mu - \lambda}{\Phi \lambda} \right)}{\mu - \lambda},
\]

with

\[
\Phi = \frac{\mu R_B - c}{\mu + r} - \frac{\mu R_B - c}{2\mu + r} - \frac{\lambda}{2\lambda + r}
\]

then in the unique symmetric equilibrium of the resulting game, both agents put full effort as long as no progress has been announced.

Note that Proposition 7 implies that if \(R_B \geq R_{\min}\), a design with silent periods of length \(\bar{T}\) defined as in Equation (22) will strictly outperform full disclosure since \(\bar{T}\) is strictly positive (and thus the probability that both agents will advance in the second stage is non-zero without sacrificing any of their experimentation efforts in stage \(A\)).

**Corollary 1.** When the final award is such that \(R_{\min} < R_B\), a design that features silent periods with length \(\bar{T}\) generates a strictly higher expected payoff for the designer than full disclosure.
Finally, we complete our discussion on designs that feature silent periods by proving the following result that identifies conditions under which silent periods of length $\bar{T}$ strictly outperform no disclosure as well.

**Proposition 8.** There exists constant $\tilde{B}$ with $\tilde{B} > \bar{B}$ such that if the designer's budget is in $[R_{\min} \cdot \Pi_d^F, \tilde{B}]$, then the design with silent periods of length $\bar{T}$ outperforms both full and no disclosure.

4 Concluding Remarks

This paper studies the role of information in innovation contests and how it is inextricably linked to the encouragement and competition effects present in these contests. In particular, we examine the role of intermediate awards as information revelation devices that can be used to improve the performance of contests both in terms of the probability of reaching the end goal as well as the time it takes to complete the project. Interestingly, the role of an intermediate award depends on which of the two effects dominates: for the competition effect, an intermediate award that is *not handed out* conveys positive information and increases the agents’ willingness to put in effort, since they believe they are still in the running and have a chance at winning the contest. When the encouragement effect dominates, an award that is handed out makes agents more optimistic about the feasibility of the project and hence provides an incentive for them to continue experimenting and putting effort. This implies that a budget-constrained designer has to trade-off a higher level of aggregate experimentation in the early stages of the contest with a larger number of participants in later stages (and hence, faster completion of the contest) when determining the sizes of the awards.

We show that a design that features silent periods – time intervals in which there is no information disclosure about the status of competition – as well as appropriately sized and timed intermediate awards for partial progress is optimal for this setting, and outperforms both the design when information about progress is not shared among competitors (implemented as a single grand prize for reaching the end goal) and the design that has a real-time leaderboard and gives out awards for partial progress as it happens (and thus agents are certain about the status of competition at all times). Silent periods have been implemented explicitly and implicitly as parts of real-world innovation contests. For example, although the Netflix Prize had an online real-time leaderboard most of the activity was recorded close to the deadline of the annual progress prizes (the intermediate awards for partial progress), effectively imposing a silent period between two consecutive such deadlines. Matlab programming contests organized by Mathworks explicitly feature a silent period early on in the contest, the so-called “Darkness Segment”, after which participants are allowed to share their progress with their competitors (in what are known as the “Twilight” and “Daylight” segments of the contest).

We believe that the modeling framework in the paper can be used as a foundation for subsequent work that investigates the role of information disclosure policies, as well as award structures, in dynamic competition settings. More generally, our work is applicable to settings that involve
mechanisms by which a designer or a social planner selectively provides feedback to the agents involved. We conclude with a list that summarizes our observations regarding a number of features in our setup.

**Uncertainty in Both Stages** The first part of the paper considers the early stages of an innovation contest. For the sake of tractability, we assume that Stage B can be completed conditional on Stage A being completed, so that the uncertainty is only regarding the feasibility of Stage A. Our analysis indicates that there is a trade-off between more experimentation in Stage A (and thus higher probability of having at least one agent move to Stage B) and the time it takes to complete the contest, which is critically important to the designer in the presence of discounting. Even in the absence of discounting, a similar trade-off exists if there is uncertainty regarding the feasibility of both stages in the contest. The designer may find it optimal to incentivize agents to remain active in the contest even after a competitor completes Stage A in order to increase the aggregate amount of experimentation in the second stage, and thus we expect that our main qualitative insights regarding the optimality of designs that feature silent periods will continue to hold. The analysis becomes quite challenging however, with the main difficulty being that in addition to the agents’ beliefs about the feasibility of Stage A and the status of competition, they also have to form beliefs about the feasibility of Stage B. Furthermore, the continuation payoff upon completing Stage A depends not only on whether a competitor has completed Stage A, but also on when exactly did this completion happen, since the time that a competitor has spent in Stage B is necessary for forming a belief about the feasibility of Stage B. Conditioning on the time that an agent has completed Stage A makes the belief update process analytically intractable.

**Skill Heterogeneity** We assume that agents are symmetric with respect to their skills as captured by rates $\lambda$ and $\mu$. An interesting direction for future research would be to relax this assumption and instead consider a setting in which agents are privately informed about their skills. In that case, giving out an intermediate award introduces an additional trade-off. The completion or not of stage A by a competitor provides a signal regarding her skills and may thus further affect effort provision. The choice of the timing and size of awards becomes even more involved as the designer has to take this additional signal into account.

**Contests with Many Stages** A typical contest may involve several milestones. As we have already argued, our analysis aims to capture the dynamics near the beginning and towards the end of the contest, where the encouragement and the competition effects, respectively, dominate. A contest consisting of a large number of stages may involve multiple intermediate awards. We conjecture that the interval between two consecutive awards increases at the beginning of the contest, thus reflecting the fact that as competitors progress uncertainty regarding the feasibility of the end goal is gradually resolved and the need for encouragement decreases. The situation is different after
enough time goes by and the competition effect becomes dominant, with agents becoming pessimistic regarding their progress relative to their competitors. Because of this, the interval between consecutive announcements by the designer, i.e., intermediate awards, decreases. At any given stage one of the two effects will be dominant and so our analysis would still apply. However, figuring out the optimal timing of giving out the intermediate awards can be quite challenging.

Multiple Competitors Our analysis focuses on the case when there are only two competitors. This is adequate for the purpose of bringing out the subtle role of the designer’s information disclosure policy and the contest’s award structure. Many of our results hold true for the case when there are \( N \) competitors. In particular, in the absence of discounting it is optimal to disclose no information (and thus maximize the aggregate amount of experimentation) whereas a design with a silent period in the case there is uncertainty in stage \( A \) works best when \( \lambda \geq \mu \). Allowing for multiple competitors introduces additional degrees of freedom for the designer. For example, the tradeoff between offering a higher intermediate award (and thus inducing more experimentation in Stage \( A \)) and reserving a higher fraction of the budget for the final award becomes more involved as the size of the final award can determine the maximum number of participants that are willing to put effort in Stage \( B \). For the setting with two competitors, if \( R_B < R_B^{\text{min}} \) the encouragement effect is absent (since the laggard will always find it optimal to quit the contest once her competitor has competed Stage \( A \)). When there are two or more competitors, the designer can incentivize any number of agents to compete in stage \( B \) by changing the size of \( R_B \) and thus agents may find it optimal to remain active in the contest unless more than a given number of their competitors advance to Stage \( B \). Despite this additional flexibility though the structure of the optimal design is qualitatively the same as it is still optimal to have a silent period and disclose information at a pre-determined time using an intermediate award.
Appendix

Preliminaries

Throughout the Appendix we use $V^i(k, \ell)$ to denote the utility that agent $i$ obtains when she is in stage $k$ and her competitor is in stage $\ell$ assuming that both agents exert full effort until the end of the contest. We also use $V^i(k, \ell, R)$ when we need to explicitly refer to the size of the final award $R$. Finally, we use $V^0(R)$ to denote the utility that a single agent obtains when she is in stage $B$ and exerts full effort until the end. We have

- The utility of an agent when both competitors are in stage $B$ is given by
  $$V^i(B, B) = \frac{\mu R_B - c}{2\mu + r}.$$

- The utility of agent $i$ when $i$ is in stage $A$ and $j$ is in stage $B$ and $R_B \geq R^\text{min}_B$ is given by
  $$V^i(A, B) = \frac{\lambda V^i(B, B) - c}{\lambda + \mu + r}.$$

- The utility of agent $i$ when $i$ is in stage $B$ and $j$ is in stage $A$ and $R_B \geq R^\text{min}_B$ is given by
  $$V^i(B, A) = \frac{\lambda V^i(B, B) - c}{\lambda + \mu + r} + \frac{\mu R_B}{\lambda + \mu + r}.$$

- Finally,
  $$V^0(R) = \frac{\mu R - c}{\mu + r}.$$

Proof of Proposition 2

We build on the proof of Theorem 1 in Bonatti and Horner (2011). In particular, we use Pontryagin’s maximum principle to show that there is a unique symmetric equilibrium which takes the form of a threshold experimentation policy. The proof assumes that an agent’s progress is observable. Moreover, let $R_i$ for $i \in \{A, B\}$ denote the award for completing stage $i$. Our focus is on effort provision before the intermediate award $R_A$ is claimed. Once, the first stage is completed both agents put full effort until the end of the contest and the expected payoff is equal to $R_A + V^1(B, A)$ for the leader and $V^2(B, A)$ for the laggard.

We begin the proof by noting that since progress is observable, the (common) posterior belief about the feasibility of stage $A$, $p_t$, at time $t$ is given by

$$\dot{p}_t = -(x_{1,t} + x_{2,t})\lambda p_t(1 - p_t),$$

when the agents’ rates of effort provision are $x_{1,t}$ and $x_{2,t}$ respectively and no breakthrough occurs in time interval $[t, t + dt]$. For simplicity of notation, in the remainder of proof we use $p_t$ instead
of \( p_{1,t} \). Agent 1’s expected payoff when she follows strategy \( \{x_{1,t}\}_{t \geq 0} \) is described by the following expression

\[
\int_{t=0}^{\infty} (x_{1,t} \lambda p_t (R_A + V^1(B, A)) + x_{2,t} \lambda p_t V^1(A, B) - cx_{1,t}) e^{-\int_{s=0}^{t} (p_r \lambda (\gamma_{x_{1,t}} + x_{2,t}) + r) ds} dt. \tag{24}
\]

Equation (23) implies that \((x_{1,t} + x_{2,t}) \lambda p_t = \frac{d \log(1 - p_t)}{dt}\). Given this, we can rewrite expression (24) as

\[
\int_{t=0}^{\infty} (x_{1,t} \lambda p_t (R_A + V^1(B, A)) + x_{2,t} \lambda p_t V^1(A, B) - cx_{1,t}) e^{-rt} \frac{1 - p_t}{1 - p_0} dt. \tag{25}
\]

For simplicity, let \( F \equiv R_A + V^1(B, A) \) and \( Z \equiv V^1(A, B) \). Also, note that since \((x_{1,t} + x_{2,t}) \lambda p_t = -\frac{\dot{p}_t}{1 - p_t}\) we have \( x_{1,t} = -\left(\frac{\dot{p}_t}{\lambda p_t(1 - p_t)} + x_{2,t}\right) \). Next, we simplify expression (25) by using integration by parts and ignoring terms that do not involve \( x_{1,t} \) and \( p_t \) (as they are irrelevant for agent 1’s optimization problem). Our objective is to derive an expression for agent 1’s objective function that depends on her actions only through \( p_t \). We have

\[
\int_{t=0}^{\infty} \left(-\frac{\dot{p}_t}{1 - p_t} + x_{2,t} \lambda p_t\right) F + x_{2,t} \lambda p_t Z + c(\frac{\dot{p}_t}{1 - p_t} \lambda p_t + x_{2,t}) e^{-rt} \frac{1}{1 - p_t} dt \Rightarrow
\]

\[
\int_{t=0}^{\infty} \frac{-\dot{p}_t}{(1 - p_t)^2} F e^{-rt} dt + c \frac{\dot{p}_t}{1 - p_t} e^{-rt} \frac{1}{1 - p_t} dt + \int_{0}^{\infty} \frac{(cx_{2,t} - x_{2,t} \lambda F p_t + x_{2,t} \lambda Z p_t)}{1 - p_t} e^{-rt} dt \Rightarrow
\]

\[
- \int_{0}^{\infty} \frac{rF}{1 - p_t} e^{-rt} dt + \frac{cr}{\lambda} \int_{0}^{\infty} (\frac{p_t}{1 - p_t} + \frac{1}{1 - p_t}) e^{-rt} dt + \int_{0}^{\infty} \frac{(cx_{2,t} - x_{2,t} \lambda F p_t + x_{2,t} \lambda Z p_t)}{1 - p_t} e^{-rt} dt \Rightarrow
\]

\[
\int_{0}^{\infty} \left(\frac{cr \ln(\frac{p_t}{1 - p_t})}{\lambda} + \frac{(cx_{2,t} - x_{2,t} \lambda F p_t + x_{2,t} \lambda Z p_t + \frac{cr}{\lambda} - rF)}{1 - p_t}\right) e^{-rt} dt \Rightarrow
\]

\[
\int_{0}^{\infty} \left(\frac{cr \ln(\frac{p_t}{1 - p_t})}{\lambda} + \frac{cx_{2,t} + \frac{cr}{\lambda} - rF}{1 - p_t} - (x_{2,t} \lambda F - x_{2,t} \lambda Z) \frac{p_t}{1 - p_t}\right) e^{-rt} dt. \tag{26}
\]

Letting \( \psi_t = \ln \frac{1 - p_t}{p_t} \) and ignoring terms that do not depend on \( \psi_t, p_t \), or \( x_{1,t} \), we obtain:

\[
\int_{0}^{\infty} \left(-\frac{cr}{\lambda} \psi_t + e^{-\psi_t} \left(cx_{2,t} + \frac{cr}{\lambda} - rF\right) - e^{-\psi_t}(x_{2,t} \lambda F - x_{2,t} \lambda Z)\right) e^{-rt},
\]

with \( \psi_t = x_{1,t} + x_{2,t} \). The Hamiltonian corresponding to agent i’s problem is given by

\[
H(x_{i,t}, \psi_t, \gamma_{i,t}) = e^{-rt} \left(-\frac{cr}{\lambda} \psi_t + e^{-\psi_t} \left(cx_{-i,t} + \frac{cr}{\lambda} - rF - x_{-i,t} \lambda F + x_{-i,t} \lambda Z\right)\right) + \gamma_{i,t}(x_{1,t} + x_{2,t}). \tag{27}
\]

According to the maximum principle the necessary conditions for an optimal solution \( \{x_{1,t}^*\}_{t \geq 0} \) are as follows:

(i) For every \( t \geq 0 \), \( x_{i,t}^* \) should be chosen to maximize \( \gamma_{i,t}(x_{1,t} + x_{2,t}) \), i.e.,

\[
x_{i,t}^* = \begin{cases} 1 & \text{if } \gamma_{i,t} > 0 \\ 0 & \text{if } \gamma_{i,t} = 0 \\ 0 & \text{if } \gamma_{i,t} < 0 \end{cases} \tag{28}
\]
(ii) The evolution of $\gamma_{i,t}$ is described by
\[ \dot{\gamma}_{i,t} = -H_{\psi}(x_{i,t}, \psi_t, \gamma_{i,t}) = e^{-r t} \left( e^{-\psi_t} \left( x_{-i,t} (\lambda Z + c - \lambda F) + \frac{cr}{\lambda} - r F \right) + \frac{cr}{\lambda} \right). \] (29)

(iii) Finally, the transversality condition requires that
\[ \lim_{t \to \infty} \gamma_{i,t}^*(\psi_t^* - \psi_t) \leq 0, \] (30)
for all $i$ and all feasible trajectories $\psi_t$.

The following strategy profile satisfies the necessary conditions (28)-(30)
\[ x_{i,t} = \begin{cases} 1 & \text{for } p_t \geq p_c = \frac{c}{\lambda F}, \\ 0 & \text{otherwise} \end{cases} \] (31)
where $p_t = \frac{p_A e^{-2t}}{p_A e^{-2t} + (1 - p_A)}$ as long as no agent has completed stage $A$. To see this consider a trajectory that starts at $(\gamma_{1,0}, \gamma_{2,0}, \psi_0) = (\bar{\gamma}_0, \bar{\gamma}_0, \ln \frac{1 - p_c}{p_c})$ where
\[ \bar{\gamma}_0 = \int_{p_t = p_A}^{p_t = \frac{c}{\lambda F}} \left( \frac{x(1 - p_A)}{(1 - x)p_A} \right) \frac{dp_t}{\left( \frac{p_t}{1 - p_t} \right) (\lambda Z + c - \lambda F + \frac{cr}{\lambda} - r F) + \frac{cr}{\lambda}}. \] (32)

Note that for this choice of $\bar{\gamma}_0$, if stage $A$ has not been completed by time $t_c$ such that $p_{t_c} = p_c = \frac{c}{\lambda F}$, then $\gamma_{i,t} = \gamma_{i,t} = 0$ for $t \geq t_c$ and thus $x_{1,t} = x_{2,t} = 0$ for $t \geq t_c$, which is consistent with the necessary conditions.

In order to establish that this profile is indeed an equilibrium we need to show that the Hamiltonian is concave in the state variable $\psi_t$, and thus the necessary conditions are sufficient. We have
\[ \frac{\partial^2}{\partial \psi_t^2} H(x_{i,t}, \psi_t, \gamma_{i,t}) = e^{-r t} e^{-\psi_t} \left( x_{2,t} (\lambda Z + c - \lambda F) + \frac{cr}{\lambda} - r F \right) < 0, \]
where the inequality follows from the fact that
\[ \lambda Z + c - \lambda F < 0 \text{ and } \frac{cr}{\lambda} - r F < 0, \] (33)
which we establish below. Note that since $Z \geq 0$, we only need to prove the first inequality and the second one follows directly. First consider the case where $R \geq R_{\text{min}}$ then the first inequality in (33) can be shown as follows:

\[ \lambda Z + c - \lambda F = \lambda V^1(A, B) + c - \lambda \left( R_A + V^1(B, A) \right) \]
\[ \leq \lambda V^1(A, B) + c - \lambda \left( V^0(R_B) - V^1(B, A) + V^1(B, A) \right) \quad \text{(since } R_A \geq V^0(R_B) - V^1(B, A)) \]
\[ = c - \lambda \left( V^0(R_B) - V^1(A, B) \right) \]
\[ \leq c - \lambda \left( V^0(R_B^{\text{min}}) - V^2(B, A, R_{B^{\text{min}}}) \right) \quad \text{(since } V^0(R_B) - V^1(A, B) \text{ is increasing in } R_B) \]
\[ = c - \lambda V^0(R_{B^{\text{min}}}) \quad \text{(since } V^2(B, A, R_{B^{\text{min}}}) = 0) \]
\[ = c - \lambda \left( \frac{c(2\mu + r)}{(\mu + r)\lambda} \right) < 0. \]
Similarly we prove inequalities in (33) for the case where \( R < R_{\min} \). Note that in this case, since the laggard will immediately drop out of the tournament, we have \( Z = V(1, B) = 0 \). So in this case, inequalities are the same so it is only enough to show that \( c - \lambda F < 0 \), which obviously should hold since \( F \) is the expected reward for the agent who finishes stage \( A \). Thus in order to have agents even start putting effort we must have \( \lambda F > c \).

Finally, to show that the equilibrium described above is the unique symmetric equilibrium we need to rule all other possible trajectories. The inequalities in (33) along with Equation (32) that describes the evolution of \( \gamma \) imply the following

(i) For any time \( t \), \( \dot{\gamma}_{i,t} \) is decreasing in \( x_{-i,t} \) for \( i \in \{1, 2\} \).

(ii) For any time \( t \) and \( i \in \{1, 2\} \) and any value \( x_{-i,t} \), \( \dot{\gamma}_{i,t} \) is increasing in \( \psi_t \).

(iii) There exists a value \( \bar{\psi} \) such that if \( \psi_t > \bar{\psi} \) then \( \dot{\gamma}_{i,\tau} > 0 \) for any \( i \in \{1, 2\} \) and \( \tau \geq t \).

In the remaining of the proof, we use our candidate equilibrium as a reference (feasible) trajectory to eliminate other trajectories by using the transversality condition. First, we consider trajectories that start with a positive \( \gamma_{i,0} \) for both agents and let \( t \geq 0 \) such that \( \dot{\gamma}_{i,t} > 0 \) for both agents. If

\[
e^{-\psi_t} \left( (\lambda Z + c - \lambda F) + \frac{cr}{\lambda} - rF \right) + \frac{cr}{\lambda} > 0,
\]

we obtain that \( \dot{\gamma}_{i,\tau} > 0 \) for \( \tau \geq t \) and thus \( \lim_{\tau \rightarrow \infty} \gamma_{i,\tau} = +\infty \). Moreover, \( \lim_{\tau \rightarrow \infty} \psi_{\tau} = +\infty \), and thus the transversality condition is violated if we consider as a reference trajectory the one that corresponds to the candidate equilibrium we described above.

On the other hand, when

\[
e^{-\psi_t} \left( x_{2,t} (\lambda Z + c - \lambda F) + \frac{cr}{\lambda} - rF \right) + \frac{cr}{\lambda} < 0,
\]

and while \( \gamma_{1,t}, \gamma_{2,t} > 0 \), we have \( x_{1,t} = x_{2,t} = 1 \) and consequently \( \psi_t \) increases. If at some point \( \psi_t \) becomes greater than \( \bar{\psi} \), then we have \( \dot{\gamma}_{i,\tau} > 0 \) for all \( \tau > t \) which results in \( \lim_{\tau \rightarrow \infty} \gamma_{i,\tau} \rightarrow +\infty \) and the transversality condition being violated.

So we are left to consider the case when \( \psi_t < \bar{\psi} \) for all \( t \) and thus \( \gamma_{i,t} < 0 \) while \( \gamma_{i,t} > 0 \). Let \( \tau \) be the first time such that \( \gamma_{1,\tau} = \gamma_{2,\tau} = 0 \) (the case where \( \gamma_{1,\tau} = 0 \) and \( \gamma_{2,\tau} > 0 \) can be eliminated as we focus on symmetric equilibria). We have following cases:

1. \( e^{-\psi_t} \left( \frac{cr}{\lambda} - rF \right) + \frac{cr}{\lambda} < 0 \). This implies that for any value of \( x_{i,\tau} \), \( \dot{\gamma}_{i,\tau} \) is negative and thus \( \gamma_{i,\tau} \) goes to \(-\infty\) which further implies that the transversality condition is violated.

2. \( e^{-\psi_t} \left( \frac{cr}{\lambda} - rF \right) + \frac{cr}{\lambda} > 0 \). In this case, we show that \( \psi_t \) increases over time and it eventually becomes larger than \( \bar{\psi} \). Recall that \( \dot{\gamma}_{i,t} \) is decreasing in \( x_{i,t} \), so if at time \( \tau \) agents set their effort levels \( x_{i,\tau} \) to small enough levels so that

\[
\dot{\gamma}_{i,\tau} = e^{-\psi_t} \left( x_{2,\tau} (\lambda Z + c - \lambda F) + \frac{cr}{\lambda} - rF \right) + \frac{cr}{\lambda} > 0,
\]
then \( \gamma_{i,\tau} \) eventually becomes positive and agents put full effort from then on and thus \( \psi_t \) increases. So consider the case where agents set their effort levels large enough so that

\[
\dot{\gamma}_{i,\tau} = e^{-\psi_t} \left( x_{2,\tau} (\lambda Z + c - \lambda F) + \frac{ct}{\lambda} - r F \right) + \frac{ct}{\lambda} \leq 0.
\]

In this case, we eventually reach time \( \tau' > \tau \) where \( \gamma_{i,\tau'} = 0 \). So through this cycle each time we reach the point \( \tau \) such that \( \gamma_{i,\tau} = 0, \psi_t \) increases and thus after some time we reach a point such that \( \psi_t > \bar{\psi} \).

3. So the only case left to consider is when \( e^{-\psi_t} \left( \frac{ct}{\lambda} - r F \right) + \frac{ct}{\lambda} = 0 \). At this point we must have

\[
e^{-\psi_t} = \frac{\frac{c}{F} - \frac{ct}{\lambda}}{\frac{ct}{\lambda}} \implies \psi_t = \ln \frac{1 - p_c}{p_c}.
\]

Note that the initial value of the state variable is \( \psi_0 = \ln \frac{1 - p_A}{p_A} \). Also, agents put full effort up to time \( \tau \), for which \( \psi_t = \ln \frac{1 - p_c}{p_c} \) and \( \dot{\gamma}_{i,\tau} = 0 \). Thus, we conclude that the only possible value for \( \gamma_{i,0} \) is \( \bar{\gamma} \).

Finally, we need to rule out the the trajectories for which \( \gamma_{i,0} \leq 0 \). Note that since \( \psi_0 = \ln \frac{1 - p_A}{p_A} \), we have that \( \dot{\gamma}_{i,t} > 0 \), which implies that eventually we have \( \dot{\gamma}_{i,t} > 0 \) and \( e^{-\psi_t} \left( \frac{ct}{\lambda} - r F \right) + \frac{ct}{\lambda} > 0 \), which is a case that we ruled out above.

**Proof of Lemma 1**

Let \( t_c \) denote the time at which agents stop experimenting in the absence of any partial progress, i.e., completing stage \( A \). Time \( t_c \) is characterized in Expressions (19) and (11). Then, the expected cost for running a design with awards \( R_A \) and \( R_B \) is given as follows (depending on whether \( R_B \geq R_B^{\min} \)):

(i) \( R_B \geq R_B^{\min} \). Then, the expected cost is given as follows\(^{11}\):

\[
\int_0^{t_c} 2\lambda e^{-2\lambda t} e^{-rr} \left( R_A + R_B \left( \frac{\lambda}{\lambda + \mu + r} 2\mu + \frac{\mu}{\lambda + \mu + r} \right) \right) dt
\]

\[
= \frac{2\lambda}{2\lambda + r} \left( 1 - e^{-(2\lambda + r)t_c} \right) \left( R_A + R_B \left( \frac{\lambda}{\lambda + \mu + r} 2\mu + \frac{\mu}{\lambda + \mu + r} \right) \right).
\]

\(^{10}\)Recall that since we have assumed \( e^{-\psi_t} \left( \frac{ct}{\lambda} - r F \right) + \frac{ct}{\lambda} > 0 \), as soon as \( \gamma_{i,t} \) becomes negative, \( \dot{\gamma}_{i,t} > 0 \), and thus we eventually reach the point when \( \gamma_{i,t} = 0 \).

\(^{11}\)In the case that agent \( i \) is in stage \( B \) and agent \( j \) is in stage \( A \) but continue to finish the tournament, the expected final reward that designer will handout is as follows:

\[
R_B \left( \int_{t=0}^{\infty} \lambda e^{-\lambda t} e^{-\mu t} e^{-rt} \left( \int_{t'=0}^{\infty} 2\mu e^{-2\mu t'} e^{-rt'} dt' \right) dt + \int_{t=0}^{\infty} e^{-\lambda t} e^{-\mu t} e^{-rt} dt \right) = R_B \left( \frac{\lambda}{\lambda + \mu + r} 2\mu + \frac{\mu}{\lambda + \mu + r} \right).
\]
and consequently $p R$ expected budget for the design with $\Pi$ Next consider the case when (b) optimal to set $R$ so it is enough to show that $\Pi$ implies that First, we show that when (a) Finally, we let $\delta$ denote the payoff for the designer. Lemma 1 implies that $d_i$ for sufficiently small discount rate $r$ that since $\lambda \mu \leq \mu + r$ then, the expected cost is given as follows:

$$
\int_0^{\lambda_c} 2\lambda e^{-2\lambda r} e^{-rr} \left( R_A + R_B \frac{\mu}{\mu + r} \right) d\tau
= \frac{2\lambda}{2\lambda + r} \left( 1 - e^{-2(\lambda + r)\tau} \right) \left( R_A + R_B \frac{\mu}{\mu + r} \right).
$$

This concludes the proof of Lemma 1. □

**Proof of Proposition 3**

Throughout the proof we use superscripts 1 and 2 to refer to the design for which only the leader and both the leader and laggard continue putting effort after stage $A$ is completed respectively. Note that

$$
R^1_B = \frac{c}{\mu} \quad \text{and} \quad R^2_B = R^\text{min}_B = \frac{c}{\mu} \left( 1 + \frac{2\mu + r}{\lambda} \right).
$$

Also,

$$
p^1_c = \frac{c}{\lambda R^1_A} \quad \text{and} \quad p^2_c = \frac{c}{\lambda} \left( R^2_A + \frac{\mu R^2_B - c}{\mu + \lambda} \right).
$$

Finally, we let $P^i_d$ denote the payoff for the designer. Lemma 1 implies that

$$
\frac{\Pi^2_d}{\Pi^1_d} = \frac{(1 - e^{-(2\lambda + r)t^2_0})}{(1 - e^{-(2\lambda + r)t^2_0})} \cdot \frac{\lambda}{\lambda + \mu + r} \cdot \frac{2\mu + r}{2\mu + r} + \frac{\mu}{\lambda + \mu + r}. \frac{\mu}{\mu + r}
$$

**(a)** First, we show that when $r = 0$, it is the case that $\Pi^2_d < \Pi^1_d$. Note that because of continuity this implies that $\Pi^2_d < \Pi^1_d$ for sufficiently small discount rate $r$. For $r = 0$, we have

$$
\frac{\lambda}{\lambda + \mu + r} \cdot \frac{2\mu + r}{2\mu + r} + \frac{\mu}{\lambda + \mu + r} \cdot \frac{\mu}{\mu + r}
= 1,
$$

so it is enough to show that $t^1_c > t^2_c$ (or equivalently $p^1_c < p^2_c$). The latter follows trivially from noting that since $r = 0$ and both designs consume same budget we have

$$
R^1_A + R^1_B = R^2_A + R^2_B \Rightarrow R^1_A = R^2_A + \frac{c}{\mu} \frac{2\mu + r}{\lambda},
$$

and consequently $p^1_c < p^2_c$. Thus, we conclude that when the discount rate $r$ is sufficiently small it is optimal to set $R_B = \frac{c}{\mu}$ and have only the leader continue in stage $B$.

**(b)** Next consider the case when $r > \bar{r} > 0$. In this case we show that for sufficiently large $\lambda$, we have $\Pi^2_d > \Pi^1_d$. Define $\delta = \frac{2\mu(\mu + r)}{(2\mu + r)\mu} > 1$ and consider setting the intermediate award $R^2_A$ so that the expected budget for the design with $R^2_B = R^\text{min}_B$ does not exceed $\mathcal{B}$. Then, we have

$$
\frac{\Pi^2_d}{\Pi^1_d} = \frac{(1 - e^{-(2\lambda + r)t^2_0})}{(1 - e^{-(2\lambda + r)t^2_0})} \cdot \frac{\lambda}{\lambda + \mu + r} \cdot \frac{2\mu + r}{2\mu + r} + \frac{\mu}{\lambda + \mu + r} \cdot \frac{2\mu + r}{\mu + r}
\geq \left( 1 - e^{-(2\lambda + r)t^2_0} \right) \frac{\lambda}{\lambda + \mu + r} \cdot \delta \geq \left( 1 - e^{-2\lambda t^2_0} \right) \frac{\lambda}{\lambda + \mu + r} \cdot \delta.
$$

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To complete the proof it is enough to have
\[
\left(1 - e^{-2 \lambda t_c^2}\right) \frac{\lambda}{\lambda + \mu + r} > \frac{1}{\delta}.
\] (34)

The left hand side of the expression above is continuous and strictly increasing in \(\lambda\). To see this note that \(t_c^2\) satisfies the following
\[
e^{-2 \lambda t_c^2} = \frac{p_c^2}{1 - p_c^2} \frac{1 - p_A}{p_A}.
\] (35)

The right hand side in Expression (35) above is decreasing in \(\lambda\) and thus \(1 - e^{-2 \lambda t_c^2}\) is increasing in \(\lambda\). Finally,
\[
\lim_{\lambda \to \infty} \left(1 - e^{-2 \lambda t_c^2}\right) \frac{\lambda}{\lambda + \mu + r} = 1,
\]
which implies that for sufficiently large \(\lambda\) inequality (34) holds. \qed

Proof of Proposition 4

We first show that the agents’ best response strategy takes the form of a threshold policy. To prove this we fix agent 2’s strategy to \(\{x_{2,t}\}\), and we show that agent 1’s best response strategy is to put full effort up to some point and then drop out. For simplicity of notation instead of \(p_{1,t}\) and \(q_{1,t}\), let \(p_t\) denotes the belief of agent 1 regarding the feasibility of stage \(A\) and \(q_t\) denotes the belief of agent 1 that agent 2 has proceeded to stage \(B\) conditional on stage \(A\) being feasible. The following expressions describe the evolution of beliefs:12
\[
\dot{p}_t = - (1 - p_t) p_t (q_t \mu + \lambda x_{1,t})
\] (37)
\[
\dot{q}_t = (1 - q_t) (x_{2,t} \lambda - q_t \mu).
\] (38)

**Lemma 3.** Let \(R \geq R_{\min}\) and fix the strategy of agent 2 to \(\{x_{2,t}\}\). Then, it is optimal for agent 1 to follow a threshold policy.

**Proof.** Agent 1 will choose strategy \(\{x_{1,t}\}\) to maximize:
\[
\Pi_1 = \int_{t=0}^{\infty} (x_{1,t} \lambda p_t (q_t V(B, B) + (1 - q_t) V(B, A, \{x_{2,t}\})) - x_{1,t} c) e^{-rt} e^{-\int_{t=0}^{t} p_r (q_r \mu + \lambda x_{1,t}) dr} dt.
\] (39)

Note that (37) implies that \(p_t = -\frac{\dot{p}_t}{(1 - p_t) (q_t \mu + \lambda x_{1,t})}\). By using this we can get the following alternative form for optimization problem (39):
\[
\Pi_1 = \int_{t=0}^{\infty} (x_{1,t} \lambda p_t (q_t V(B, B) + (1 - q_t) V(B, A, \{x_{2,t}\})) - x_{1,t} c) e^{-rt} \frac{1 - p_A}{1 - p_t} dt.
\] (40)

\[\]12Note that by using Bayes’ rule we have
\[
p_{t+dt} = \frac{((1 - q_t) + q_t (1 - \mu dt)) (1 - \lambda x_{1,t} dt) p_t}{(1 - p_t) + ((1 - q_t) + q_t (1 - \mu dt)) (1 - \lambda x_{1,t} dt) p_t}.
\] (36)

By subtracting \(p_t\) from both sides and diving by \(dt\), we get the desired result.
Now for sake of contradiction assume that strategy \( \{x_{1,t}\} \) that maximizes (40) is not a threshold policy, which implies that there is a time \( t \) such that \( x_{1,t} < 1 \) and \( x_{1,t+dt} > 0 \). Define

\[ \epsilon \equiv \min \{1 - x_{1,t}, x_{1,t+dt}\}. \]

We next show that decreasing \( x_{1,t+dt} \) by \( \epsilon \) and increasing \( x_{1,t} \) by the same amount leads to an increase in Expression (40) and thus we each a contradiction. Note that following this change, since the total effort of agent 1 in interval \([t, t + 2dt]\) remains the same, \( p_{t+2dt} \) does not change.

Let \( p_{t+dt} \) and \( p'_{t+dt} \) denote agent 1’s beliefs regarding the feasibility of stage A if the agent follows her original or the new strategy respectively. We will show that the difference in Expression (40) associated with moving \( \epsilon \) effort from \( x_{1,t+dt} \) to \( x_{1,t} \) is positive. First note that by increasing the effort level during interval \([t, t + dt]\) by \( \epsilon \), the expected utility increases by

\[ (\lambda p_{t} (q_{t} V(B, B) + (1 - q_{t}) V(B, A, \{x_{2,t}\})) - c_{x} e^{-r t} \frac{1 - p_{A}}{1 - p_{t}} dt. \] (41)

Also by decreasing effort level \( x_{1,t+dt} \) by \( \epsilon \), the expected continuation value for the agent changes as follows

\[
\Delta V = \frac{e^{-r t}(1 - p_{A})(1 - rdt)}{1 - p't_{t+dt}} (\lambda(x_{1,t+dt} - \epsilon)p'_{t+dt} (q_{t+dt} V(B, B) + (1 - q_{t+dt}) V(B, A, \{x_{2,t+dt}\})) - c(x_{1,t+dt} - \epsilon)) dt
- \frac{e^{-r t}(1 - p_{A})(1 - rdt)}{1 - p_{t+dt}} (\lambda x_{1,t+dt} p_{t+dt} (q_{t+dt} V(B, B) + (1 - q_{t+dt}) V(B, A, \{x_{2,t+dt}\})) - c x_{1,t+dt}) dt,
\] (42)

where we have

\[ p'_{t+dt} = p_{t+dt} - (1 - p_{t}) p_{t} \epsilon \lambda dt = p_{t} - (1 - p_{t}) p_{t} (q_{t} \mu + \lambda x_{1,t}) dt - (1 - p_{t}) p_{t} \epsilon \lambda dt, \] (43)

and

\[ V(B, A, \{x_{t+dt}\}) = V(B, A, \{x_{t}\}) + (c + (r + \lambda x_{2,t} + \mu) V(B, A, \{x_{t}\}) - R - \lambda x_{2,t} V(B, B)) dt. \] (44)

So our goal is to show that the sum of Expressions (41) and (42) is positive. To do this, by using Equations (37), (38), (43), (44), and factoring common positive terms \( 1 - p_{A} \) and \( e^{-r t} \), as well as ignoring second order terms, i.e., terms of the order of \( dt^3 \), one can see that we can write the summation of Expressions (41) and (42) as follows:

\[
\lambda p_{t} q_{t} V(B, B) (\lambda \epsilon + r + \mu + \lambda x_{1,t} - \lambda x_{1,t+dt}) dt^2 + \lambda p_{t} (1 - q_{t}) V(B, A, \{x_{2,t}\}) (\lambda \epsilon - \mu + \lambda x_{1,t} - \lambda x_{1,t+dt}) dt^2
- \lambda p_{t} + \lambda p_{t} \epsilon + \lambda p_{t} q_{t} + r + p_{t} q_{t} \mu + \lambda x_{1,t} - \lambda x_{1,t+dt}) dt + \lambda p_{t} \mu (1 - q_{t}) R dt^2.
\]

We want to show that the above expression is greater or equal to zero. By rearranging terms we reach the following inequality:

\[
\lambda p_{t} q_{t} V(B, B) (r + \mu + \lambda p_{t} (1 - q_{t}) \mu (R - V(B, A, \{x_{2,t}\})) dt^2
+ \lambda p_{t} (q_{t} V(B, B) + (1 - q_{t}) V(B, A, \{x_{2,t}\})) (\lambda \epsilon + \lambda x_{1,t} - \lambda x_{1,t+dt}) dt^2
- \lambda p_{t} + \lambda p_{t} \epsilon + \lambda p_{t} q_{t} + r + p_{t} q_{t} \mu + \lambda x_{1,t} - \lambda x_{1,t+dt}) dt^2 \geq 0. \] (45)
Note that since we put positive effort in interval $[t + dt, t + 2dt]$, we have

$$\lambda p_t (q_t V(B, B) + (1 - q_t) V(B, A, \{x_{2,t}\})) \geq c.$$  

Thus by using this inequality we can rewrite Expression (45) as follows:

$$\lambda p_t q_t V(B, B)(r + \mu) + \lambda p_t (1 - q_t) \mu (R - V(B, A, \{x_{2,t}\})) dt + c(1 - p_t)(\lambda e + \lambda x_{1,t} - \lambda x_{1,t+dt})$$

$$- c(\lambda p_t(1 - q_t) + r + p_t q_t \mu) \geq 0.$$  

Finally by canceling out terms we get:

$$\lambda p_t q_t V(B, B)(r + \mu) + \lambda p_t q_t \mu \left( R - \frac{c}{\mu} - V(B, A, \{x_{2,t}\}) \right) - c(r + p_t q_t \mu) \geq 0.$$  

Now by noting that $V(B, A, \{x_{2,t}\}) \leq V^0(R) = \frac{\mu R - e}{\mu + r}$ and $V(B, B) \geq \frac{c}{\lambda}$ (since $R \geq R_{\min}$), we can simplify the above as follows:

$$\lambda p_t q_t V(B, B)r + \lambda p_t (1 - q_t) \mu \left( \frac{r(R - \frac{c}{\mu})}{\mu + r} \right) - cr \geq \lambda p_t (q_t V(B, B) + (1 - q_t)V^0(R)) - c \geq 0.$$  

This completes the proof of the lemma.  

According to Lemma 3, the best response of an agent to any strategy by her competitor takes the form of a threshold policy: put full effort up to some point and then drop out. Thus, the unique symmetric features both agents putting full effort up to some time $t_c$ and then drop out\(^\text{13}\). Finally, we obtain from Expression (40) that time $t_c$ is given as the solution to:

$$\lambda p_{t_c} (q_{t_c} V(B, B) + (1 - q_{t_c})V^0(R)) - c = 0,$$

which completes the proof of the proposition.

**Proof of Proposition 5**

Consider a full information disclosure design and recall that $t_c$ denotes the time at which agents stop putting effort in stage $A$ if none of them has completed it. Recall that with intermediate reward $R_A$ and final reward $R_B^{\min}$, $t_c$ is the solution of the following equation:

$$\lambda p_{t_c} (R_A + V(B, A)) = c,$$  

where $p_{t_c} = \frac{p_A e^{-2\lambda t_c}}{p_A e^{-2\lambda t_c} + (1 - p_A)}$ is the agents’ common posterior belief about the feasibility of stage $A$ at time $t_c$. We show that there exists a design that has a silent period of appropriate length $t'_c > t_c$ and awards of the same size such that agents put full effort in time interval $[0, t'_c]$ (with $t'_c > t_c$). This

\(^{13}\)Agents will not stop sooner than $t_c$ as their instantaneous payoff is still positive. Also they will not continue to put effort after $t_c$ as their instantaneous cost will be negative.
further implies that the designer’s expected profit is higher in the presence of a silent period than under full information disclosure.

The proof relies on a similar argument as in the proof of Proposition 4. In particular, when \( R_B \geq R_B^{\min} \) then a design that features a silent period of length \( T \) induces agents to follow a threshold experimentation policy, i.e., experiment with rate one up to some time \( t \) and then quit the race (or stop putting effort until time \( T \) if \( t < T \)). This is fairly intuitive since when \( R_B \geq R_B^{\min} \) and the encouragement effect dominates, they do not have any incentive to postpone their effort provision. Formally showing this involves using similar arguments as those in Proposition 4.

To complete the proof, let \( t'_c \) be the solution to the following equation:

\[
\lambda \hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right) = c, \tag{47}
\]

where \( \hat{p}_t = \frac{p_A e^{-\lambda t} \left( \frac{e^{-\lambda t} - e^{-\lambda t'}}{\lambda - m} \right)}{p_A e^{-\lambda t'} \left( \frac{e^{-\lambda t} - e^{-\lambda t'}}{\lambda - m} \right) + (1-p_A)} \) is the agents’ belief about the feasibility of stage \( A \) at time \( t'_c \) if both agents put full effort up to that time. Note that since \( R_B = R_B^{\min} \), the payoff of an agent being a laggard is zero and, thus, agents do not have an incentive to free ride and they put effort as long as their instantaneous utility is non-negative, i.e., they put full effort during time interval \([0, t'_c]\). So the proof follows by showing that \( t'_c > t_c \) or equivalently

\[
\lambda p_t (R_A + V(B, A)) < \lambda \hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right) .
\]

This is shown as follows:

\[
\lambda \hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right) = c \Rightarrow \\
\lambda \hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right) < \frac{R_A}{2} + V(B, B) \Rightarrow \\
\hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right) \Rightarrow \\
\frac{1 - q_t}{1 - \hat{p}_t q_t} < q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \Rightarrow \\
\lambda \hat{p}_t \left( q_t \left( \frac{R_A}{2} + V(B, B) \right) + (1 - q_t) \left( R_A + V(B, A) \right) \right),
\]

where the first inequality comes from the fact that since \( R_B = R_B^{\min} \) we have \( c < \frac{R_A}{2} + V(B, B) \), and the last inequality comes from the fact that for any time \( t \) we have the following relation between \( p_t, \hat{p}_t, \) and \( q_t \)

\[
p_t = \frac{\hat{p}_t (1 - q_t)}{1 - \hat{p}_t q_t}.
\]

\[\square\]
Proof of Theorem 1

We show that a contest that features a silent period of appropriate length $t_A$ outperforms the design where agents progress is not observable. Recall, that in the no-disclosure design, at equilibrium an agent experiments up to time $t_c$ in stage $A$ and drops out of the contest in case she does not complete stage $A$, where $t_c$ is given as the solution to Equation (14).

The proof consists of several steps. First, we provide a lower and an upper bound on the designer’s expected payoff for the contest that features a silent period of length $t_A$ and the no disclosure design, in terms of $t_A$ and $t_c$ respectively. Then, the rest of proof provides expressions for $t_A$ and $t_c$ and bounds their difference. Given the bound, we are able to provide conditions under which the design with a silent period outperforms the design that features a single final award (and thus implements the no information disclosure policy).

Step 1 Consider a designer that features a silent period of length $t_A$ chosen so that agents put full effort in interval $[0, t_A]$ (we provide an expression for $t_A$ later in the proof). Then, the designer’s expected payoff is bounded below by:

$$
\Pi_S \geq \int_0^{t_A} 2\lambda e^{-2\lambda t} e^{-rt} \left( \frac{\lambda}{\lambda + r} \frac{2\mu}{2\mu + \rho} \right) = \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \frac{2\mu}{2\mu + r} \left( 1 - e^{-(2\lambda + r)t_A} \right) \geq \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \frac{2\mu}{2\mu + r} \left( 1 - e^{-2\lambda t_A} \right),
$$

Similarly, for the design with no information disclosure we have

$$
\Pi_N \leq \left( 1 - e^{-\lambda t_c} \right)^2 \frac{2\mu}{2\mu + r} + 2e^{-\lambda t_c} \left( 1 - e^{-\lambda t_c} \right) \frac{\mu}{\mu + r},
$$

where the inequality follows by noting that the expression in the right hand side is equal to the designer’s expected payoff when conditional that one or two agents complete stage $A$, they begin experimenting in stage $B$ at time $t = 0$. In other words, conditional on completing stage $A$, we assume that they do so at the beginning of the contest.

Let $\delta = \frac{2\mu}{2\mu + r} \cdot \frac{\mu + r}{\mu} > 1$. Then,

$$
\Pi_S - \Pi_N \geq \left[ \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \frac{2\mu}{2\mu + r} \left( 1 - e^{-2\lambda t_A} \right) \right] - \left[ \left( 1 - e^{-\lambda t_c} \right)^2 \frac{2\mu}{2\mu + r} + 2e^{-\lambda t_c} \left( 1 - e^{-\lambda t_c} \right) \frac{\mu}{\mu + r} \right] = \frac{\mu}{\mu + r} \left\{ \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \left( 1 - e^{-2\lambda t_A} \right) \cdot \delta - \left[ \left( 1 - e^{-\lambda t_c} \right)^2 \delta + 2e^{-\lambda t_c} \left( 1 - e^{-\lambda t_c} \right) \right] \right\}.
$$

The proof follows by showing that the difference between $t_A$ and $t_c$ is not large as we establish in the steps that follow.
**Step 2** Consider a no-disclosure design that features a final award equal to $R$. Note that according to Proposition 4 agents will put effort in stage $A$ up to time $t_c$ as long as their posterior belief $p_{t_c}$ satisfies the following inequality

$$\lambda p_{t_c} \left( q_{t_c} V(B, B, R) + (1 - q_{t_c}) V_0(R) \right) \geq c,$$

where $p_t = \frac{p_A e^{-\lambda t} (\frac{\alpha - \mu t - \lambda}{\lambda - \mu})}{p_A e^{-\lambda t} (\frac{\alpha - \mu t - \lambda}{\lambda - \mu}) + (1 - p_A)}$. So, we obtain that the belief $p_{t_c}$ that an agent stops putting effort in stage $A$ satisfies:

$$p_{t_c} = \frac{c}{\lambda (q_{t_c} V(B, B, R) + (1 - q_{t_c}) V_0(R))}.$$  \hfill \hspace{1cm} (51)

**Step 3** Next, we provide an expression for $e^{-\lambda A}$ when $R_B = R_B^{\text{min}}$ and $R_A = \frac{\mu}{\mu + r} \left( R - 2 \frac{\mu + r}{2 \mu + r} R_B^{\text{min}} \right)$. First, we establish that this is a feasible design for a budget-constrained designer, i.e., it uses no more than $B$ in expectation for the awards $R_A$ and $R_B$. To see this, first note that $t_c \geq t_A$, i.e., the probability that the designer has to spend her budget is smaller for the contest that has a silent period. Next, consider a sample path for which at least one of the agents completes stage $A$ at time $t \leq t_A$ (and thus the designer has to spend her budget in both cases). Then, note that the designer spends at least $e^{-r \tau} \frac{\mu}{\mu + r} R$ for the design with no-disclosure of information (this follows from the fact that this is the latest that an award $R$ will have to be handed out in expectation). On the other hand, the budget consumed for the design with a silent period is bounded above by

$$e^{-r \tau} \left( R_A + \frac{2 \mu}{2 \mu + r} R_B^{\text{min}} \right) = e^{-r \tau} \frac{\mu}{\mu + r} R.$$

So, for all sample paths the design with a silent period consumes less budget than what the designer spends for the the agent that completes stage $A$ first in the case of a single final award, which is assumed to be equal $B$.

Next we find the belief that agents stop putting effort in the design with a silent period, and compare it with $p_{t_c}$ which was derived in Equation (51). Consider $t_A$ to be equal to the solution of the following equation:

$$\lambda p_{t_A} \left( q_{t_A} \left( \frac{R_A}{2} + V(B, B, R_B) \right) + (1 - q_{t_A}) \left( R_A + V(B, A, R_B) \right) \right) = c,$$  \hfill \hspace{1cm} (52)

Then, it is optimal for agents to put full effort up to time $t_A$ which corresponds to belief about the feasibility of stage $A$ equal to

$$p_{t_A} = \frac{c}{\lambda \left( q_{t_A} \left( \frac{R_A}{2} + V(B, B, R_B) \right) + (1 - q_{t_A}) \left( R_A + V(B, A, R_B) \right) \right)}.$$  \hfill \hspace{1cm} (53)
Next note that \(q_A \leq q_e\), and thus we have:

\[
\frac{p_t}{p_t A} = \frac{\lambda(q_A \left(R_A + V(B, B, R_B)\right) + (1 - q_A)(R_A + V(B, A, R_B)))}{\lambda(q_e V(B, B, R) + (1 - q_e)V^0(R))}
\]

\[
\geq \left(q_t \left(R_A + V(B, B, R_B)\right) + (1 - q_t)(R_A + V(B, A, R_B)))\right)
\]

\[
\geq \frac{R_A + V(B, B, R)}{V(B, B, R)}.
\]

Finally if we assume that \(R \geq \frac{4c}{\mu}\), we get \(\frac{R_A + V(B, B, R_B)}{V(B, B, R)} \geq \frac{3\mu + r}{3\mu + 3r}\). So we have the following ratio between beliefs \(p_t A\) and \(p_t c\):

\[
\frac{p_t c}{p_t A} \geq \frac{3\mu + r}{3\mu + 3r} \geq \frac{1}{3}. \quad (54)
\]

**Step 4** Next we compute an upper bound for \(e^{-\lambda t A}\), and by using it we show that for an interval of values for \(\lambda\), \(\Pi_S - \Pi_N > 0\), i.e., the design with a silent period strictly outperforms the one that feature no information disclosure. Note that we have

\[
\frac{p_t c}{p_t A} = \frac{p_A e^{-\lambda t c} \left(\frac{\lambda t c - \mu t c - \mu e^{-\lambda t c}}{\lambda - \mu}\right)}{p_A e^{-\lambda t c} \left(\frac{\lambda t c - \mu t c - \mu e^{-\lambda t c}}{\lambda - \mu}\right) + (1 - p_A)} \implies e^{-\lambda t c} = \frac{p_t c}{1 - p_t c} = \frac{p_t c}{1 - p_t c} \frac{1 - p_A}{p_A} \frac{\lambda - \mu}{\lambda e^{-\mu t A} - \mu e^{-\lambda t c}}. \quad (55)
\]

Similarly we get the following expression for \(e^{-\lambda t A}\):

\[
e^{-\lambda t A} = \frac{p_t A}{1 - p_t A} \frac{1 - p_A}{p_A} \frac{\lambda - \mu}{\lambda e^{-\mu t A} - \mu e^{-\lambda t A}}. \quad (56)
\]

Finally, we obtain the following upper bound for \(e^{-\lambda t A}\):

\[
e^{-\lambda t A} = \frac{\frac{p_t A}{1 - p_t A}}{\frac{p_t c}{1 - p_t c}} \frac{\frac{p_t A}{1 - p_t A}}{\frac{p_t c}{1 - p_t c}} \frac{\frac{\lambda t c - \mu t c - \mu e^{-\lambda t c}}{\lambda - \mu}}{\frac{\lambda t c - \mu t c - \mu e^{-\lambda t c}}{\lambda - \mu}} \leq \frac{\frac{p_t A}{1 - p_t A}}{\frac{p_t c}{1 - p_t c}} \frac{\frac{p_t A}{1 - p_t A}}{\frac{p_t c}{1 - p_t c}} \frac{p_t A(1 - p_t A)}{p_A(1 - p_t A)} \frac{p_t A}{p_A} \frac{p_t c}{1 - p_t c} \leq \frac{p_A(1 - p_A)}{\lambda e^{-\mu t A} - \mu e^{-\lambda t A}}. \quad (55)
\]

where the first inequality comes from the fact that \(\frac{\lambda t c - \mu t c - \mu e^{-\lambda t c}}{\lambda - \mu} \leq 1\), and the second inequality comes from the ratio between beliefs \(p_t A\) and \(p_t c\) characterized in Step 3. Finally the last inequality follows from fact the \(\frac{p_A(1 - p_A)}{p_A(1 - p_t A)} \frac{p_t A}{p_A} \frac{p_t c}{1 - p_t c}\) is increasing in \(p_t A\) and we have \(p_t A \leq p_A\).

Before going to the final step and for simplicity of notation we define

\[
\beta \equiv \frac{p_A(1 - p_A)}{p_A(1 - p_t A)^{\frac{2}{3}}}.
\]
Step 5  Now we are ready to compare $\Pi_S$ and $\Pi_N$ and derive conditions under which $\Pi_S > \Pi_N$. By using inequality\(^{(50)}\), we have

$$\Pi_S - \Pi_N \geq \frac{\mu}{\mu + r} \left\{ \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \left( 1 - e^{-2\lambda t_A} \right) \cdot \delta - \left[ \left( 1 - e^{-\lambda t_B} \right)^2 \delta + 2e^{-\lambda t_B} \left( 1 - e^{-\lambda t_B} \right) \right] \right\}$$

$$= \frac{\mu}{\mu + r} \left\{ \frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \left( 1 - e^{-2\lambda t_A} \right) \cdot \delta - \left[ \left( 1 - e^{-\lambda t_B} \right)^2 \delta + 2e^{-\lambda t_B} \left( 1 - e^{-\lambda t_B} \right) \right] \right\}.$$ \(^{(57)}\)

Note that the above expression is quadratic in $e^{-\lambda t_B}$, and thus if

$$\frac{2\lambda}{2\lambda + r} \frac{\lambda}{\lambda + r} \geq \max \left\{ \frac{2 - \delta}{\beta^2 \delta}, \frac{2 - \delta + \beta^2 \delta}{2 \delta^2} \frac{\delta - 1}{\delta \beta + \lambda} - \frac{2 - \delta}{\beta^2 \delta} + F(\delta) \right\},$$

then $\Pi_S - \Pi_N > 0$ for $\lambda$ such that

$$e^{-\lambda t_B} \in \left[ \frac{\delta - 1}{\delta \beta + \lambda} - \frac{2 - \delta}{2 \lambda + r} \frac{\lambda}{\lambda + r} - \frac{2 - \delta}{\delta \beta + \lambda} + F(\delta), \frac{\delta - 1}{\delta \beta + \lambda} - \frac{2 - \delta}{2 \lambda + r} \frac{\lambda}{\lambda + r} + F(\delta) \right], \quad (58)$$

and

$$F(\delta) = \frac{1 - \delta}{\delta \beta + \lambda} - \frac{2 - \delta}{2 \lambda + r} \frac{\lambda}{\lambda + r} - \frac{2 - \delta}{\delta \beta + \lambda} + F(\delta).$$

Finally, since $e^{-\lambda t_B}$ is continuous in $\lambda$ the interval in Expression\(^{(58)}\) defines an interval of $\lambda$’s for which $\Pi_S > \Pi_N$. \(\square\)

Proof of Lemma 2

Recall that according to Equation\(^{(18)}\) we have $\dot{q}_{1,t} = (1 - q_{1,t})(x_{2,t} \lambda - q_{1,t} \mu)$. So if $x_{2,t} = \frac{q_{1,t} \mu}{\lambda}$, then $\dot{q}_{1,t} = 0$ which means agent 1 belief that agent 2 has already proceeded to stage $B$ does not change. Similarly if $x_{2,t} < \frac{q_{1,t} \mu}{\lambda}$ ($x_{2,t} > \frac{q_{1,t} \mu}{\lambda}$), then $\dot{q}_{1,t} < 0$ ($\dot{q}_{1,t} > 0$) and proof is complete. Obviously the same holds for agent 2 as well.

Proof of Theorem 3

In order to show that the strategy profile in the statement of the theorem is an equilibrium, we assume that the second agent plays this strategy and then show that the best response is for the first agent to play the same strategy as well. Let the strategy sequence $x_{2,t}$ of Agent 2 be given by $\{x_{2,t}\}$ where

$$x_{2,t} = \begin{cases} 1 & \text{if } t \leq T \\ \psi(q_{1,t}) & \text{if } t > T \end{cases} \quad (59)$$

Note that by fixing the strategy sequence $\{x_{2,t}\}$ of the second agent, the belief sequence of the first agent $\{q_{1,t}\}$ is also fixed, i.e. the belief sequence $\{q_{1,t}\}$ only depends on $\{x_{2,t}\}$. In particular, $\{q_{1,t}\}$ keeps increasing until time $T$, where it becomes equal to

$$q_{1,T} = \frac{\lambda(\mu R - c) - c(\mu + \lambda)}{c \mu},$$

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and remains at this value until the end of the contest.

For the purposes of this proof we will introduce the following notation. We denote by \( V^1(A, t, \{x_{2,t}\}) \) the value for Agent 1 of being in Stage A at time \( t \) when the second agent is following strategy \( \{x_{2,t}\} \) and Agent 1 puts full effort. Since we are focused on Agent 1, we drop the superscript and write \( V \) instead of \( V^1 \). At each point in time, Agent 1 chooses her effort \( x_{1,t} \) to maximize her expected payoff, given by

\[
(1 - \mu q_t dt)(1 - r dt) [\lambda x_{1,t} V(B, t + dt, \{x_{2,t}\}) + (1 - \lambda x_{1,t} dt) V(A, t + dt, \{x_{2,t}\})] - cx_{1,t} dt
\]

Because we are fixing Agent 2’s strategy to be that in 59, we will drop the argument \( \{x_{2,t}\} \) from the expression \( V(A, t, \{x_{2,t}\}) \) and write it instead as \( V(A, t) \) (and similarly, we write \( V(B, t, \{x_{2,t}\}) \) as \( V(B, t) \)). The above expectation is easily seen to be linear in \( x_{1,t} \) and hence has an optimal solution given by

\[
x^*_1(t) = \begin{cases} 
1 & \text{if } V(B, t + dt) - V(A, t + dt) > \frac{\xi}{\lambda} \\
0 & \text{if } V(B, t + dt) - V(A, t + dt) < \frac{\xi}{\lambda} \\
\in (0, 1) & \text{if } V(B, t + dt) - V(A, t + dt) = \frac{\xi}{\lambda} 
\end{cases}
\]

We first prove the following claim that – taken together with 60– implies that for every \( t \geq T \), Agent 1 is indifferent to putting effort if she is still in Stage A.

**Claim 1.** For every \( t \geq T \), we have \( V(B, t) - V(A, t) = \frac{\xi}{\lambda} \).

**Proof.** First note that for every \( t \geq T \) the second agent puts effort \( \psi(q_{1,T}) \) and thus her rate of success in stage A is equal to \( \psi(q_{1,T})\lambda \). If we denote by \( V(B, A, t) \) the value from being in stage B and putting full effort while Agent 2 is in Stage A at time \( t \), and recalling that \( V(B, B) \) is the value that either agent gets when they are both in Stage B, then

\[
V(B, A, t) = \frac{\mu R + \psi(q_{1,T})\lambda V(B, B) - c}{r + \mu + \psi(q_{1,T})\lambda}
\]

We can write \( V(B, t) \) as the weighted sum of two components, \( V(B, A, t) \) and \( V(B, B) \), to get

\[
V(B, t) = q_{1,T} V(B, B) + (1 - q_{1,T}) V(B, A, t) = \frac{c}{\lambda}.
\]

Also, we have

\[
V(A, t) = \int_{t_1=0}^{\infty} e^{-r(t_1-t)} x_{1,t+t_1} (\lambda V(B, t_1 + dt) - c) e^{-\int_{t_2=1}^{t_1} q_{2} \mu dt_2} e^{-\lambda \int_{t_2=1}^{t_1} x_{1,t} dt_2} dt_1 = 0.14
\]

and thus \( V(B, t) - V(A, t) = \frac{\xi}{\lambda} \). This proves the claim. \( \square \)

We now show that Agent 1 puts full effort during any time \( t' < T \). We do this by showing that it is optimal to put full effort during \( t' \) whenever the agent is indifferent or puts full effort in \( t \), where \( t' < t \leq T \). By taking \( t = T \) and noting that we have just proved in Claim 1 that Agent 1 is indifferent

\[14\text{Note that according to Equation (61) for any } t \geq T, \text{ we have } V(B, t) = \frac{\xi}{\lambda}, \text{ and thus } \lambda V(B, t + dt) - c = 0.\]
at $T$, we obtain the desired result. To this end, assume that Agent 1 is indifferent or puts full effort at $t$, then at time $t - dt$ we have\footnote{To show that Agent 1 puts full effort at $t - dt$, we need to show that $V(B, t) - V(A, t) > \frac{c}{\lambda}$.}

$$
V(B, t) - V(A, t) = (1 - r dt) \left( \mu dt R + (1 - \mu dt - q_t \mu dt) V(B, t + dt) \right) - c dt \\
- (1 - r dt) \left( (1 - q_t \mu dt) (\lambda dt V(B, t + dt) + (1 - \lambda dt) V(A, t + dt)) \right) + c dt \\
= (1 - r dt) \left( \mu dt R + (1 - q_t \mu dt - \lambda dt) (V(B, t + dt) - V(A, t + dt)) - \mu dt V(B, t + dt) \right) \\
\geq (1 - r dt) \left( \mu dt R + (1 - q_t \mu dt - \lambda dt - r dt) \left( \frac{c}{\lambda} - \mu dt - \frac{\mu R - c}{\mu + q_t \mu + r} \right) \right) \\
= \mu dt R + (1 - q_t \mu dt - \lambda dt - r dt) \left( \frac{c}{\lambda} - \mu dt - \frac{\mu R - c}{\mu + q_t \mu + r} \right) \\
\geq \frac{c}{\lambda},
$$

where the first inequality comes from the fact that the belief $q_{1,t}$ is increasing for $t < T$, and hence $V(B, t) < V(B, T) = \frac{\mu R - c}{\mu + q_t \mu + r}$. Therefore $V(B, t) - V(A, t) > \frac{c}{\lambda}$ and Agent 1 puts full effort at time $t$ according to (60).

The argument to show uniqueness is more intricate. The main idea is to show that if $\bar{\tau}$ is the first time where agents do not put full effort, then in all symmetric equilibria the belief from time $\bar{\tau}$ onward is constant and equal to $\bar{q}$. This means that agents put full effort up to time $\bar{\tau}$, where their belief becomes exactly equal to $\bar{q}$, and hence $\bar{\tau} = T$, implying that the equilibrium described above is the only valid symmetric equilibrium. In order to prove that the belief after time $\bar{\tau}$ is constant and equal to $\bar{q}$, we prove two lemmas. In the first lemma we show that for any symmetric equilibrium and for every time $t$, the belief $q_t$ is bounded by $\bar{q}$. In the second lemma we show that for any time $t \geq \bar{\tau}$ the belief cannot be less than $\bar{q}$. Putting the two lemmas together, we conclude that after time $\bar{\tau}$ the belief is equal to $\bar{q}$.

We begin by proving the first lemma:

**Lemma 4.** There is no unique symmetric equilibrium such that $\exists t, q_t > \bar{q}$.

**Proof.** The proof is by contradiction. Assume that there exists a time where the belief is strictly higher that $\bar{q}$ and let

$$M \equiv \sup\{q_t\}.$$

We will show that for any $\epsilon > 0$, there exists a time $t$ with three properties that cannot simultaneously hold if the belief is ever higher than $\bar{q}$. This existence implies that beliefs cannot increase beyond $\bar{q}$. We first assume existence of that time $t$ and show the incompatibility of the three properties below with $q_t > \bar{q}$, and then we show existence to conclude the proof of the lemma. With that in mind, assume that for any $\epsilon > 0$ there exists $t$ such that
1. \[ M - q_t \leq \epsilon \]

2. \[ V(B, t + dt) - V(A, t + dt) = \frac{\epsilon}{\lambda} \]

3. \[ V(B, t) - V(A, t) \geq \frac{\epsilon}{\lambda} \]

Define \( \epsilon' = M - \bar{q} \), and consider \( \epsilon < \frac{\lambda \epsilon'}{c} \left( \frac{\epsilon}{\lambda} - \frac{\mu R - c}{\mu (1 + q + \epsilon')} \right) \). Now compute

\[
V(B, t) - V(A, t) = (1 - rd) (\mu dt R + (1 - \mu dt - p \mu dt) V(B, t + dt)) - c dt - (1 - rd) (1 - q \mu dt) V(A, t + dt) \\
= (1 - rd) (\mu dt R + (1 - q \mu dt) (V(B, t + dt) - V(A, t + dt)) - \mu dt V(B, t + dt)) - c dt \\
< (1 - rd) \left( \mu dt R + (1 - q \mu dt) (V(B, t + dt) - V(A, t + dt)) - \mu \frac{\mu R - c}{\mu (1 + M) + r} dt \right) - c dt \\
= (\mu R - c) dt (\frac{\mu M + r}{\mu (1 + M) + r}) + (1 - q \mu dt - rd) (V(B, t + dt) - V(A, t + dt)) \\
= V(B, t + dt) - V(A, t + dt) + \left( \frac{(\mu M + r)(\mu R - c)}{\mu (1 + M) + r} - (\mu q_t + r) \frac{c}{\lambda} \right) \\
< V(B, t + dt) - V(A, t + dt)
\]

where the first equality comes from the fact that we have \( V(B, t + dt) - V(A, t + dt) = \frac{\epsilon}{\lambda} \) and so agents are indifferent in how much effort to put. This means that we can write \( V(A, t) \) with zero effort:

\[ V(A, t) = (1 - rd) (1 - q \mu dt) V(A, t + dt) \]

Additionally, the first inequality comes from the fact that \( V(B, t + dt) > \frac{\mu R - c}{\mu (1 + M) + r} \), and the second inequality is because the quantity inside the parenthesis is negative. However \( V(A, t) - V(A, t) < V(B, t + dt) - V(A, t + dt) \) contradicts properties 2 and 3 of chosen time \( t \).

Now, to complete the proof of the lemma, it suffices to show that for any \( \epsilon > 0 \) there exists a time \( t \) with those three properties. Note that for every \( \epsilon \) there is a time \( t' \) such that \( M - q_{t'} < \epsilon \). Consider the smallest \( t' \) such that \( M - q_{t'} < \epsilon \). We have the following cases:

- \( V(B, t' + dt) - V(A, t' + dt) > \frac{\epsilon}{\lambda} \): Since \( V(B, t' + dt) - V(A, t' + dt) > \frac{\epsilon}{\lambda} \), agents put full effort at time \( t' \) (and thus the belief path is increasing at \( t' \)). Consider the first time \( t'' > t' \) such that an agent does not put a full effort (this time exists because agents do not put full effort forever in any symmetric equilibrium with a final award less than \( R_{\text{max}} \)). We show that \( t'' \) has all the three properties mentioned above. First, it has Property 1 by definition. Second, since \( t'' \) is the first time after \( t' \) where agents do not put full effort, it means at time \( t'' - dt \) agents put full effort and therefore \( V(B, t'') - V(A, t'') > \frac{\epsilon}{\lambda} \), satisfying Property 3. Finally, since at time \( t'' \) agents do not put full effort, we have \( V(B, t'' + dt) - V(A, t'' + dt) \leq \frac{\epsilon}{\lambda} \), and because \( V(B, t) - V(A, t) \) is a continuous function of time, we cannot have \( V(B, t'' + dt) - V(A, t'' + dt) < \frac{\epsilon}{\lambda} \). Therefore \( V(B, t'' + dt) - V(A, t'' + dt) = \frac{\epsilon}{\lambda} \) and Property 2 is satisfied.

\[
^{16}\text{Because the belief that Agent 2 is in stage } B \text{ is equal to the supremum value } M \text{ in the worst case, we have } V(B, t) = \frac{\mu R - c}{\mu (1 + M) + r}.
\]
\[ V(B, t' + dt) - V(A, t' + dt) = \frac{c}{\lambda}: \] Note that because \( t' \) is the first time that beliefs become \( \epsilon \)-close to \( M \), agents put non-zero effort in the interval \([t' - dt, t']\), which implies that \( V(B, t') - V(A, t') \geq \frac{c}{\lambda} \). Thus time \( t' \) has all three properties.

\[ V(B, t' + dt) - V(A, t' + dt) < \frac{c}{\lambda}: \] The proof here utilizes the observations from the previous two cases. Consider \( \epsilon_1 < \epsilon \) and let time \( \tau \) be the first time that \( M - p_\tau < \epsilon_1 \). If \( V(B, \tau + dt) - V(A, \tau + dt) \geq \frac{c}{\lambda} \) then the proof is exactly the same as before, so assume instead that \( V(B, \tau + dt) - V(A, \tau + dt) < \frac{c}{\lambda} \). Again, since \( \tau \) is the first time that \( M - p_\tau < \epsilon_1 \), at time \( \tau_2 = \tau - dt \) (i.e. just before \( \tau \) and where \( p_{\tau_2} < p_\tau \)), agents would put non-zero effort at \( \tau_2 \) and because of the continuity of \( V(B, t) - V(A, t) \), we have \( V(B, \tau) - V(A, \tau) = \frac{c}{\lambda} \). Then, just before \( \tau_2 \) at time \( \tau_3 = \tau_2 - dt \), agents put non-zero effort since if not, their beliefs from \( \tau_3 \) to \( \tau_2 \) would decrease, which contradicts the assumption that \( \tau \) is the first time that \( M - p_\tau < \epsilon_1 \). Therefore at time \( \tau_2 \) we should have \( V(B, \tau_2) - V(A, \tau_2) \geq \frac{c}{\lambda} \) and \( \tau_2 \) satisfies all the three properties.

\[ \square \]

The above lemma shows that beliefs cannot go higher than \( \bar{q} \) in a symmetric equilibrium. Let \( \bar{\tau} \) be the first time in the effort path of the competitor where she does not put full effort. Define

\[ m = \inf \{ q_t | t \geq \bar{\tau} \}. \]

If we prove that \( m \) cannot be less than \( \bar{q} \) then the proof is complete, since this implies that agents put full effort until their beliefs reach \( \bar{q} \) and they remain at that level, exactly like the proposed equilibrium. The following lemma proves this to conclude our proof.

**Lemma 5.** In no symmetric equilibrium is \( m \) less than \( \bar{q} \).

**Proof.** Similar to the arguments of the previous lemma, we can show that there exists a time \( t > \bar{\tau} \) such that

1. \( q_t - m \leq \epsilon \)
2. \( V(B, t + dt) - V(A, t + dt) = \frac{c}{\lambda} \)
3. \( V(B, t) - V(A, t) \leq \frac{c}{\lambda} \)

Again we will show that these three properties cannot satisfy together if \( m > \bar{q} \). Define \( \epsilon' = \bar{q} - m \), and consider \( \epsilon < \frac{\lambda \mu R - c}{c} \left( \frac{c}{\lambda} \right. \left. - \frac{\mu R - c}{\mu (1 + q - \epsilon') \bar{q}} \right) \), and assume a time \( t \) with the above thee properties. We have
\[ V(B, t) - V(A, t) = (1 - rdt) (\mu dt R + (1 - \mu dt - q_t \mu dt) V(B, t + dt)) - cdt - (1 - rdt)(1 - q_t \mu dt)V(A, t + dt) \]
\[ = (1 - rdt) (\mu dt R + (1 - \mu dt)(V(B, t + dt) - V(A, t + dt)) - \mu dt V(B, t + dt)) - cdt \]
\[ > (1 - rdt) \left( \mu dt R + (1 - q_t \mu dt)(V(B, t + dt) - V(A, t + dt)) - \mu \frac{\mu R - c}{(1 + m) + r}dt \right) - cdt \]
\[ = (\mu R - c)dt \left( \frac{\mu M + r}{\mu (1 + m + r)} + (1 - q_t \mu dt - r dt)(V(B, t + dt) - V(A, t + dt)) \right) \]
\[ = V(B, t + dt) - V(A, t + dt) + \left( \frac{\mu m + r}{\mu (1 + m) + r} - (\mu q_t + r) \frac{c}{\lambda} \right) \]
\[ > V(B, t + dt) - V(A, t + dt) \]

Where the first inequality is because \( V(B, t + dt) < \frac{\mu R - c}{\mu (1 + m) + r} \) and the second inequality follows from the quantity inside the parenthesis being negative. However \( V(A, t) - V(A, t) > V(B, t + dt) - V(A, t + dt) \) contradicts properties 2 and 3 of chosen time \( t \).

\[ \square \]

**Lemma 6.** Consider two designs \( D_1 \) and \( D_2 \), each featuring a single final award and no intermediate award. Let the designer’s payoff be continuous in the amount of the final award in both designs. For the same final award \( R \), denote the designer’s payoff as \( \Pi_{D_1} \) under design \( D_1 \) and \( \Pi_{D_2} \) under design \( D_2 \), so that the costs of these designs are \( \Pi_{D_1} \cdot R \) and \( \Pi_{D_2} \cdot R \), respectively. Assume that \( \Pi_{D_1} > \Pi_{D_2} \), then for every budget \( \Pi_{D_2} R \leq B \leq \Pi_{D_1} R \) the designer payoff in \( D_1 \) is greater than her payoff in \( D_2 \).

**Proof of Lemma 6**

Consider a budget \( B \in [R \cdot \Pi_{D_2}, R \cdot \Pi_{D_1}] \): Under award \( R \), design \( D_1 \) consumes more budget than \( B \) and design \( D_2 \) consumes less budget than \( B \). Notice that for both designs to consume the same amount of budget \( B \), we should decrease the award in \( D_1 \) and increase it in \( D_2 \). Let \( R'_{D_1} \) and \( R'_{D_2} \) be the new awards for designs \( D_1 \) and \( D_2 \), respectively, such that these designs consume exactly budget \( B \). It then must be the case that
\[ R'_{D_1} \leq R \leq R'_{D_2}. \]

Let \( \Pi'_{D_1} \) and \( \Pi'_{D_2} \) be the designer’s payoffs under the new awards in designs \( D_1 \) and \( D_2 \), respectively. Since these designs consume the same budget \( B \) i.e. \( B = \Pi'_{D_1} \cdot R'_{D_1} = \Pi'_{D_2} \cdot R'_{D_2} \) thus we must have \( \Pi'_{D_1} > \Pi'_{D_2} \) and the proof is complete.

**Proof of Proposition 6**

We first note that for budget \( B_{min} = \Pi_{FD} R_{min} \), the designer’s payoff is higher in Full Disclosure than it is in No Disclosure. Indeed, for award \( R_{min} \) the designer’s payoff under Full Disclosure is \( \Pi_{FD} \) and the design uses budget \( R_{min} \Pi_{FD} = B_{min} \). However, for the same award \( R_{min} \) the payoff in No
Disclosure is 0, since for that award no agent puts any effort in the symmetric equilibrium. Using Lemma 6 with $R = R_{\min}$, $D_1$ being full disclosure, $D_2$ being No disclosure, $\Pi_{D_2} = 0$, and $\Pi_{D_1} = \Pi_{FD}$, we conclude that for any budget in $[0, B_{\min}]$, Full Disclosure is better than No Disclosure.

Let the maximum payoff be given by $\Pi_{max}$, which is achieved when both agents put full effort indefinitely. Denote by $R_{ND_{max}}^{max}$ the award value at which both agents put full effort indefinitely (and hence yield payoff $\Pi_{max}$ to the designer) under no disclosure. Recall from Corollary ?? that the value of this award is strictly less than $R_{max}$. Because of this, under award $R_{ND_{max}}^{max}$ no disclosure outperforms full disclosure. This is because $R_{max}$ is the minimum award value that leads to both agents putting full effort indefinitely under full disclosure, and any amount less than that means that one agent exits the tournament as soon as the other has obtained a breakthrough. This implies that the designer’s payoff under full disclosure with award $R_{ND_{max}}^{max}$ is the same as it is under $R_{min}$, and is equal to $\Pi_{FD}$. Let $B_{max} = \Pi_{FD} \cdot R_{max}^{NM}$, then applying Lemma 6 with $R = R_{max}^{NM}$, $D_2$ being full disclosure, $\Pi_{D_1} = \Pi_{max}$, and $\Pi_{D_2} = \Pi_{FD}$, we conclude that for any budget in $[B_{max}, \Pi_{d_{max}}^{max} R_{max}]$, no disclosure is better than full disclosure.

Putting all of the above together and noting that the payoff under no disclosure is a continuous function of the budget, we see that there exists a value $\bar{B}$, $B_{min} < \bar{B} < B_{max}$ that satisfies the statement of the proposition.

**Proof of Proposition 7**

We utilize the same notation employed in the proof of Theorem 3 to show that the unique symmetric equilibrium is for each agent to put full effort during time $[0, T]$. To show this consider any time $t < T$ and recall that the best response strategy in (60) is for an agent to put full effort at time $t$ if

$$V(B, t + dt) - V(A, t + dt) > \frac{c}{\lambda}.$$  

We first show that if an agent is indifferent or puts full effort at any time $t$ then she should put full effort in any time $t' < t$. We then show that at the time of announcement $T$, agents put full effort (therefore implying that they put full effort before $T$ as well). Define $q = \frac{\lambda - \lambda e^{-(\mu - \lambda)T}}{\mu - \lambda e^{-(\mu - \lambda)T}}$, i.e. $q$ is the maximum belief that an agent has about whether her competitor had proceeded to stage B by time $T$.

Claim 2. If an agent exerts full effort during $[t, T]$ then she exerts full effort at time $t + dt$ as well.

**Proof.** Recall that $V_M$ is the expected payoff for a monopolist, i.e. an agent who is by herself in the

\[ q_t = \frac{\lambda e^{-\lambda t} - \lambda e^{-\mu t}}{\mu e^{-\lambda t} - \lambda e^{-\mu t}}. \]

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contest. We have
\[
V(B,t) - V(A,t) = (1 - rdt)(\mu dt R + (1 - \mu dt - q_t \mu dt) V(B, t + dt)) - cd
t
-(1 - rdt)(1 - q_t \mu dt)(\lambda dt V(B, t + dt) + (1 - \lambda dt) V(A, t + dt)) + cd
\]
\[
= (1 - rdt)(\mu dt R + (1 - q_t \mu dt - \lambda dt)(V(B,t + dt) - V(A,t + dt)) - \mu dt V(B,t + dt))
\]
\[
> (1 - rdt)\left(\mu dt R + (1 - q_t \mu dt - \lambda dt)\frac{c}{\lambda} - \mu dt (q_t V(B,B) + (1 - q_t) V_M)\right)
\]
\[
> (1 - rdt)\left(\mu dt R + (1 - q_t \mu dt - \lambda dt)\frac{c}{\lambda} - \mu dt (q V(B,B) + (1 - q) V_M)\right)
\]
\[
\geq \frac{c}{\lambda},
\]
where the first inequality follows from the assumption that agents put full effort during \([t, T]\) and therefore \(V(B,t + dt) - V(A,t + dt) > \frac{c}{\lambda}\). Moreover, since beliefs \(q_t\) are increasing on \([t, T]\), we can bound \(V(B,t + dt)\) from above by assuming that beliefs remain at \(q_t\) during \([t, T]\), and therefore one can see that \(V(B,t + dt) < q_t V(B,B) + (1 - q_t) V_M\). The second inequality follows from the fact that \(q > q_t\) and since \(V_M - V(B,B) < \frac{c 18}{\lambda}\), one can see that replacing \(q_t\) with \(q\) can only decrease the value of that expression. Finally, it is straightforward to check that the last inequality is valid for any \(R \geq R_{\min}\). Thus \(V(B,t + dt) - V(A,t + dt) > \frac{c}{\lambda}\) and agents put full effort at time \(t - dt\) as well. \(\square\)

We use Claim 2 to show that the strategy profile where both agents exert full effort during \([0, T]\) is an equilibrium, and is indeed the unique symmetric equilibrium. Let us fix the strategy of the second agent so that she exerts full effort in the interval \([0, T]\). We show that the best response for the first agent is to do the same by first showing that she puts full effort just before the announcement. Consider time \(T - dt\). Agent 1 exerts full effort at this time if and only if
\[
V(B,T) - V(A,T) > \frac{c}{\lambda}
\]
Note that, since the strategy of the second agent is fixed to exert full effort, we have
\[
V(B,T) - V(A,T) = qV(B,B) + (1 - q)V_M - (1 - q)V_{T\text{design}}(A,A)
\]
where the first term indicates the expected payoff when both agents are in Stage B, the second is when Agent 1 is a monopolist in Stage B (since if the announcement at \(T\) indicates that Agent 1 has progressed to Stage B while Agent 2 has not, Agent 2 drops out), and \(V_{T\text{design}}(A,A)\) denotes the agents’ utility from the T-design when they are both in Stage A. Denote by \(V_{FD}(A,A)\) the utility of the agents under full disclosure when they are both in stage A. The following claim helps us bound the RHS of the above equation

\[\text{Since } V_M - V(B,B) = \frac{\mu R - c}{\mu + r} - \frac{\mu R - c}{2\mu + r} = \frac{\mu^2 R - c\mu}{(\mu + r)(2\mu + r)} < \frac{c\mu(2\mu + r)}{\lambda(\mu + r)(2\mu + r)} = \frac{c}{\lambda} \frac{\mu}{\mu + r},\]
where the inequality comes from our assumption that the laggard will drop out i.e. \(R < R_{\max} = \frac{c(2\mu + \lambda + r)}{\lambda \mu}\).
Claim 3. For every $T \geq 0$, we have $V_{T\text{design}}(A, A) \leq V_{FD}(A, A)$.

Proof. Note that as long as $t < T$ and there has been no breakthroughs, Agent 2 behaves the same under Full Disclosure and under the T-design: she puts full effort. The best response to this in Full Disclosure is for Agent 1 to put full effort and by symmetry this is also the best response under T-design and the payoffs under both regimes are the same. However, as soon as a breakthrough occurs, the situation is not symmetric anymore. Let us examine the payoff of Agent 1 under the following set of paths:

- **Paths in which Agent 1 is the laggard:** Assume that Agent 1 becomes the laggard at time $t < T$ (i.e. Agent 2 obtains a breakthrough at time $t$), then her continuation payoff is given by $V^1(A, B) < 0$ and it is optimal for her to drop out of the contest, which is indeed what will happen under full disclosure. In the T-design however, Agent 1 is not informed of that breakthrough until $T$, and may end up putting effort in all or part of the interval $[t, T]$, even though the expected payoff is negative, implying that the payoff under this scenario is higher in Full Disclosure compared to the T-design.

- **Paths in which Agent 1 is the leader:** Assume that Agent 1 experiences a breakthrough and becomes the leader at time $t < T$. Under Full Disclosure, Agent 2 will immediately drop out of the game once this happens, making Agent 1 a monopolist with expected payoff $V_M$. But in the T-design Agent 2 will not drop out of the game until time $T$ and thus she might complete stage $A$ and catch up with Agent 1 during the interval $[t, T]$, decreasing Agent 1’s expected payoff to $V(B, B) < V_M$ in the process.

From the discussion above, we see that Agent 1’s utility is higher under full disclosure compared to the T-design, and thus we have $V_{T\text{design}}(A, A) \leq V_{FD}(A, A)$.

Based on Claim 3, we can replace $V_{T\text{design}}(A, A)$ with $V_{FD}(A, A)$ in the above inequality to get

$$V(B, t) - V(A, t) = qV(B, B, R) + (1 - q)V_M(B, R) - (1 - q)V_{T\text{design}}(A, A) > qV(B, B, R) + (1 - q)V_M(B, R) - (1 - q)V_{FD}(A, A) \geq \frac{c}{\lambda},$$

Therefore, Agent 2 puts full effort just before $T$ and as a result of claim 2 she puts full effort during interval $[0, T]$. This proves the symmetric equilibrium.

Uniqueness follows from the proof of the previous claim since, in essence, we have proved the following stronger result that implies that the equilibrium highlighted above is the only symmetric equilibrium.

**Claim 4.** For any symmetric equilibrium under T-design, we have: $V_{T\text{design}}(A, A) < V_{FD}(A, A)$. 

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Proof. The proof is exactly the same as above. Consider a symmetric equilibrium path under T-design and compare the agents’ payoffs with the payoffs under Full Disclosure (where the symmetric path that maximizes agents' payoff is the full effort path) and the result follows.

Proof of Proposition 8

From Proposition 6, we know that the designer’s payoff under Full and No Disclosure is the same with budget $\bar{B}$. From Corollary 1, we have that T-design is strictly better than Full Disclosure at budget $\bar{B}$. Therefore, since the designer's payoff in No Disclosure is continuous in the budget (and final award), one can see that there is budget $\tilde{B} > B$ such that if No Disclosure consumes budget $\tilde{B}$ the designer's payoff will be equal to her payoff in T-design at budget $\bar{B}$. This implies that in the budget interval $[\bar{B}, \tilde{B}]$, T-design outperforms No Disclosure (and consequently, Full Disclosure) and the proof is complete.

References


