

Capacity Investment in Imperfect Capital Markets: The Interaction of Operational and Financial Decisions

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Abstract

This paper analyzes the effect of capital market imperfections on a firm's operational and financial decisions in a capacity investment setting. We model a monopolist who produces and sells two products in a single selling season. Product demands are price-dependent, random and correlated. The firm chooses between flexible and dedicated technologies, and determines the capacity level and the production quantities with the chosen technology. The firm's limited budget, which depends partly on a tradable asset, can be altered with financial risk management and can be increased by borrowing from a creditor. The borrowing terms are determined in a Stackelberg equilibrium by the creditor who is the leader. The creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The capital market imperfections, bankruptcy costs and underwriter fees, impose financing frictions on the firm. We derive the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm; and the creditor's optimal contracting decision in equilibrium. Our analysis contributes to the capacity investment literature by analyzing the effect of capital market imperfections on capacity investment and characterizing previously undocumented trade-offs that arise in imperfect capital markets; demonstrating that these trade-offs may change traditional insights concerning capacity investment derived under the perfect market assumption; and underlining the importance of the integration of operational and financial decisions.

Key Words: Capacity, Flexibility, Financing, Risk Management, Operational Hedging, Real Options.

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1 Introduction

Capacity investment is subject to internal or external financing frictions, especially in capital-intensive industries. However, as highlighted by Van Mieghem (2003, p. 275) “stochastic capacity models assume (often implicitly) either perfect capital markets, so that frictionless borrowing is possible, or that the investment size is relatively small, so that it can be internally financed without material impact on the overall valuation of the firm.” The objective of this paper is to increase our understanding of how capital market imperfections affect technology choice and capacity investment. Our main contributions are: (i) to analyze the effect of capital market imperfections on capacity investment and characterize previously undocumented trade-offs that arise in imperfect capital markets; (ii) to demonstrate that these trade-offs may change traditional insights concerning capacity investment derived under the perfect market assumption; and (iii) to underline the importance of the integration of operational and financial decisions.

To this end, we model a budget-constrained manufacturer who produces and sells two products. Product demands are price-dependent, random and correlated. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. The firm’s limited budget partially depends on a perfectly tradable asset. Thus, the firm is exposed both to product market (demand) and financial market (asset price) risk. The firm can relax its budget constraint by borrowing from a creditor. To capture capital market imperfections, we assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The firm can use forwards written on the asset price to alter its budget distribution so as to counterbalance the effect of external financing costs arising from capital market imperfections.

We derive the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm and the creditor’s optimal contracting decision in equilibrium. Based on these results, we show how capital market imperfections affect the operational decisions of the firm. This comparison is made possible by the existence of a natural perfect-market benchmark in our framework. In particular, we answer the following research questions:

1. How do capital market imperfections affect capacity investment and operational per-

formance?

2. For a given technology, what are the main drivers of capacity investment level and operational performance in imperfect capital markets?
3. What are the main drivers of technology choice in imperfect capital markets?
4. Do these drivers differ from those in perfect capital markets and if so, what explains the difference?
5. What is the value of financial risk management in the creditor-firm interaction?

We demonstrate that an increase in capital market imperfection costs decreases the operational performance and the optimal capacity investment of the firm. This is because higher imperfection costs lead to higher financing costs in equilibrium.

In perfect capital markets, for a firm selling a single product, or two products using dedicated technology, the firm's optimal capacity investment level and its operational performance only depend on the expectation of demand and not its variability. In imperfect capital markets, other factors also matter. In the single product setting, the firm's optimal capacity investment level and its operational performance *decrease* in demand variability. This is because higher variability increases the default risk of the firm and induces the creditor to charge a higher financing cost in equilibrium. In the two-product setting with dedicated technology, the firm's optimal capacity investment level and operational performance also *decrease* in demand correlation. This is because the two-market investment generates diversification benefits for the firm in reducing the default probability. An increase in correlation decreases the diversification benefit, increases the default risk of the firm and, in turn, increases the equilibrium level of financing costs.

Flexible technology incurs a higher investment cost and exhibits production-switching capability. For a given capacity investment level, production switching capability is beneficial. Production-switching capability also alters the optimal capacity investment level of the firm. In perfect capital markets, these two direct effects create positive (static) value (and are traded off against the higher cost of flexible technology in determining the optimal technology choice). In imperfect capital markets, production switching capability also has strategic value through the indirect effect on the equilibrium financing cost: With identical

technology costs, the expected value of production switching for a given capacity investment level acts to decrease the default risk of the firm and hence the equilibrium financing cost. However, the optimal adjustment of the capacity investment level acts to increase the expected borrowing level and the default risk of the firm, and may decrease or increase the equilibrium financing cost. The higher investment cost of flexible technology also has an effect on these static and strategic values. Putting these effects together, we show that the strategic value of production switching can be negative and this negative strategic value that only exists in imperfect capital markets can direct the firm towards dedicated technology in these markets.

Our analysis illustrates the value of financial risk management in creditor-firm interaction. In perfect capital markets, financial risk management does not have any value for the firm and does not affect the creditor's returns. In imperfect capital markets, financial risk management has both static and strategic values. For a given financing cost scheme, engaging in financial risk management has a positive value for the firm because it decreases the expected borrowing level of the firm. The strategic value comes from the effect on the creditor's expected returns. We demonstrate that the firm's engagement in financial risk management may have negative strategic value. The reduction in the firm's borrowing level decreases the expected returns of the creditor and the creditor increases the financing cost in equilibrium.

With these results, we contribute to the growing operations management literature that incorporates financial considerations in operational decision making. We provide managerial insights about technology and capacity investment for financially constrained firms that are exposed to capital market imperfection costs. In the next section, we provide more detail about how our work contributes to the existing literature. In §3, we describe the model and discuss the basis for our assumptions. §4 and §5 analyze the optimal strategy of the firm and the creditor, respectively. We provide a perfect market benchmark in §6. Our main results and contributions are provided in Sections 7 and 8 where we distill the effect of capital imperfections on the firm's decisions and performance. §7 examines this effect in a single-product setting. §8 extends this analysis to the two-product setting and investigates technology choice in imperfect capital markets. §9 concludes.

2 Literature Review

In this section, we review the streams of literature related to our paper and delineate our contributions to each stream. The stochastic capacity investment literature analyzes the value of resource flexibility in a variety of models. We refer readers to Van Mieghem (2003) for an excellent review. As highlighted in this review paper, the operations management literature (often implicitly) assumes that capital markets are perfect, in which case operational and financial decisions decouple (Modigliani and Miller 1958). In practice, capital market imperfections such as agency costs, taxes, underwriter fees and bankruptcy costs exist (Harris and Raviv 1991) and impose deadweight costs of external financing, leading operational and financial decisions to interact with each other. There is a growing body of work in operations and finance that analyze these interactions. Our paper's overall contribution to this literature is i) increasing our understanding of the effect of capital market imperfections on stochastic capacity investment; ii) demonstrating heretofore undocumented tradeoffs that arise in imperfect capital markets; iii) delineating the interaction between operational and financial decisions in capacity investment context.

In the Operations Management literature, Lederer and Singhal (1994), Buzacott and Zhang (2004), Xu and Birge (2004), Babich and Sobel (2004) and Babich et al. (2006) focus on the joint financing and operating decisions of the firm. We compare our results to two of these papers in particular. Lederer and Singhal (1994) study the joint financing (optimal mix of debt and equity) and capacity investment problem in a multi-period setting and show how the technology choice of the firm is related to its financing decision. In a numerical example, they show that the production switching value of flexible technology is even stronger with external financing because it decreases the firm's default risk by reducing the variability of cash flows. We analytically demonstrate that their result may not hold in general. Drivers of the production switching value create strategic effects on the equilibrium level of financing costs; these strategic effects can be negative and induce the firm to prefer dedicated technology.

Xu and Birge (2004) analyze the effect of taxes and bankruptcy costs on the firm's joint financing and operating decisions in a single-period single-product capacity investment setting. They demonstrate the value of integrated decision making and analyze the effect of demand variability and some other operational characteristics in imperfect capital

markets. Our work is complementary to theirs. We provide analytical proofs for some of their numerical observations and extend the interaction analysis to the two-product setting. In particular, we analyze the effect of technology choice and financial risk management on the creditor-firm interaction.

Several finance papers also investigate the interaction of financing and operating decisions. Dotan and Ravid (1985) and Dammon and Senbet (1988) are examples of early studies that demonstrate the effect of operational investments on the financing policy of the firm in a single-period setting. We refer the reader to Childs et al. (2005) for a recent review of papers in this stream. More recently, a number of papers (Mauer and Triantis 1994, Mello et al. 1995, and Mello and Parsons 2000) analyze the effect of various forms of operational flexibility (e.g. shutting down the production plant) on the joint operational and financing decisions of firms in the contingent claims framework. The focus of these papers is on the financing policy of the firm with strong modeling assumptions concerning the firm's operations. As highlighted in MacKay (2003), without agency cost concerns, operational flexibility has a positive strategic effect: Operational flexibility decreases the firm's default risk by generating higher returns due to its option value and this decreases the financing cost in equilibrium. We demonstrate that this argument may not hold in general with a stronger formalization of the firm's operations. Anticipating the option value of operational flexibility (flexible technology in our case), the firm optimally adjusts other operational decisions (capacity investment and production quantity). As a result, the firm's default risk may increase, yielding a net negative strategic effect.

Our work is also related to the recent stream of papers that analyze the interaction of operational and financial decisions from an integrated risk management point of view. Zhu and Kapuscinski (2004), Ding et al. (2005), Chod et al. (2006), Dong et al. (2006) are in this stream. We refer the reader to our companion paper, Boyabatlı and Toktay (2006) for a more detailed review of this literature. In Boyabatlı and Toktay (2006), we use a similar, more general model of the firm, but take external financing costs to be *identical and exogenous* for each technology. This paper formalizes the capital market imperfections and *endogenizes* the external financing costs in a creditor-firm interaction. We provide new insights and undocumented trade-offs that arise from this strategic interaction.

Several papers in the finance literature empirically (e.g. Gay and Nam 1998, Geczy et al. 1997, Haushalter 2000) and theoretically (e.g. Froot et al. 1993, Smith and Stulz 1985,

Leland 1998) analyze the value of financial risk management under borrowing frictions where the operating cash flows are correlated with a financially tradable index. The majority of these papers document the static value of financial risk management: With a given financing cost, the firm can use financial instruments to engineer its internal cash flows to reduce the dependence on external borrowing. A few papers (Smith and Stulz 1985, Leland 1998) demonstrate the positive strategic value of financial risk management with endogenous financing costs. Financial risk management enables the firm to reduce its default risk after the loan is taken and decreases the financing costs in equilibrium. We contribute to this literature by showing the negative strategic value of financial risk management. The main driver of this result is that in our setting financial risk management is effective before the loan is taken. Financial risk management reduces the expected borrowing level of the firm, and this may induce the creditor to charge higher financing costs in equilibrium to generate sufficient returns.

3 Model Description and Assumptions

We consider a creditor-firm strategic interaction where the creditor is the Stackelberg leader who determines the borrowing terms. The firm is a monopolist that sells two products in a single selling season under demand uncertainty. Differing from the majority of traditional stochastic technology and capacity investment problems, we model the firm as being budget constrained, where the budget partially depends on a hedgeable market risk. This firm can undertake financial risk management to hedge this risk. The firm chooses the technology (dedicated versus flexible), and the borrowing, financial risk management, capacity investment, and production levels so as to maximize expected shareholder wealth. After operating profits are realized, the firm pays back its debt; default occurs if it is unable to do so.

We model the firm's decisions as a three-stage stochastic recourse problem under financial market and demand risk. In stage 0, the firm decides its technology choice (flexible or dedicated), and the financial risk management level under demand and financial market risk. In stage 1, the financial market risk is resolved and the financial risk management contract (if any) is exercised; these two factors determine the internal cash level of the firm. The firm then determines the level of external borrowing and makes its capacity investment using its total budget (internal cash and borrowed funds). In stage 2, demand uncertainty

is resolved and the firm chooses the production quantities for each product. Subsequently, the firm either pays back its debt or defaults. In the remainder of this section, we define the creditor's and firm's objectives and discuss the assumptions concerning each decision epoch in detail.

Assumption 1 *The creditor is risk neutral and chooses the borrowing terms to guarantee a given expected return $U \geq 0$.*

The risk-neutrality of the creditor can be justified on the basis of the fact that banks have large diversified portfolios and they are approximately risk-neutral with respect to individual loans with small risks (Rochet and Frexias 1997, p.94). U represents the reservation utility of the creditor. In the financial economics literature, the common assumption is to have perfectly competitive loan markets such that the creditor makes zero-expected profit, i.e. $U = 0$ (see, for example, Melnik and Plaut 1986). We allow strictly positive values of U which can be interpreted as expected underwriter fees. $U > 0$ is one of the capital market imperfections in our model. We discuss the implications of $U > 0$ in §6.

Assumption 2 *The firm maximizes the expected shareholder wealth by maximizing the expected (stage 0) value of equity. The shareholders are assumed to be risk-neutral and the risk-free rate r_f is normalized to 0. Shareholders have limited liability.*

The main goal of corporations is to maximize shareholder wealth. The expected shareholder wealth is a function of the expected cash flows to equity of the firm and the required rate of return of the shareholders. By assuming the risk neutrality of shareholders, we focus on maximizing the expected equity value of the firm. The required rate of return is the risk-free rate, which is normalized to 0 by assumption.

3.1 Stage 0

In this stage, the creditor offers its borrowing terms. The firm then determines its technology choice $T \in \{D, F\}$, and the financial risk management level H_T under financial market and demand uncertainty. The flexible technology (F) has a single resource that is capable of producing two products. The dedicated technology (D) consists of two resources that can each produce a single product. Assumptions 3-6 summarize modelling choices related to these decisions.

Assumption 3 *The creditor offers separate secured loan commitment contracts for each technology and incurs fixed bankruptcy cost BC if the firm defaults on its loan. Technology choice is verifiable. The creditor has full information about the firm.*

Loan commitment is a promise to lend up to a pre-specified amount at a pre-specified terms. In practice, most short-term industrial and commercial loans in the US are made under loan commitment contracts (Melnik and Plaut 1986). When the firm agrees to a contract at stage 0, this means that it owns the right to a loan contract that can be exercised in stage 1. If the firm defaults on its loan, the creditor incurs the fixed bankruptcy cost BC . Bankruptcy cost is the other form of market imperfection in our model. This cost includes the administrative and legal fees (Altman 1980) of bankruptcy and is often used in the literature to represent default related capital market imperfections (e.g. Smith and Stulz 1985). We discuss the characteristics of the loan commitment contract in Assumption 7 of stage 1. The contracts are finalized before the firm commits to the technology and financial risk management level. It follows from the verifiability assumption that the creditor offers technology-specific contracts.

The information endowment of the creditor is an important determinant of external financing costs. The degree of information available to the creditor depends on several factors such as the source of borrowing (e.g. bank financing, vendor financing), the extent of interaction between the creditor and the firm (credit history), the available information about the firm's financial status (e.g. credit rating, stock price) and the industry characteristics (e.g. industry reports) in which the firm is operating. In this paper, we focus on one end of the spectrum where the creditor has full information about the firm. The full-information case may represent a vendor that finances its subsidiary or the financial partner of a firm (GM Acceptance Corporation) that provides financial services to the parent company (GM). In reality, most industrial and commercial firms occasionally lend money to their customers (Rochet and Frexias 1997). With full-information, the creditor can anticipate the optimal decisions of the firm.

Assumption 4 *Technology T has variable (c_T) capacity investment costs.*

Assumption 5 *At stage 0, the firm has rights to a known internal stage 1 endowment (ω_0, ω_1) . Here, ω_0 represents the cash holdings and ω_1 represents the asset holdings of the firm. The asset is a perfectly tradeable asset that has a known stage 0 price of α_0 and*

random stage 1 price of α_1 . The random variable α_1 has a continuous distribution with positive support and bounded expectation $\bar{\alpha}_1$.

With this assumption, in stage 0, the firm knows that the value of its endowment will be $\omega_0 + \alpha_1\omega_1$ in stage 1, where α_1 is random; this is the financial market risk in our model. This representation is consistent with practice: In general, firms hold both cash and tradable assets on their balance sheet, such as a multinational firm that has pre-determined contractual fixed payments denominated in both domestic and foreign currency, or a gold producer that produces a certain level of gold that is exposed to gold price risk. In these examples, the asset price α_1 represents the exchange rate and the gold price in stage 1, respectively. Although the cash and the asset holdings are certain, the price of the asset makes the stage 1 value of the internal endowment random. The firm can use financial risk management tools to alter the distribution of this quantity.

Assumption 6 *The firm uses forward contracts written on asset price α_1 to financially manage the market risk. Forward contracts are fairly priced. We restrict the number of forward contracts H_T such that the firm does not default on its financial transaction in stage 1.*

Forward contracts are the most prevalent type of financial derivatives used by non-financial firms (Bodnar et al. 1995). The fair-pricing assumption ensures that the firm can only affect the distribution of its budget in stage 1 – and not its expected value – by financial risk management. We restrict the feasible set of forwards to the range $\left[-\frac{\omega_0}{\bar{\alpha}_1}, \omega_1\right]$. Within this range of forwards the firm never defaults on its financial transaction in stage 1. This ensures that we can use default-free prices in forward transactions.

3.2 Stage 1

In stage 1, the market risk α_1 is resolved. The value of the firm's internal endowment and the exercise of the financial contract (if any) determine the firm's budget B . In this stage, the firm can raise external capital if the budget is not sufficient to finance the desired capacity investment. The firm determines the amount of external borrowing and the capacity investment level under demand uncertainty.

Assumption 7 *With the loan commitment contract, the firm can borrow up to credit limit E_T from a unit interest rate of $a_T \geq r_f = 0$ with each technology $T \in \{D, F\}$. The firm has physical assets of value P (e.g. real estate) that are pledged to the creditor as collateral. We assume that at a given unit cost a_T , the creditor chooses the credit limit that can be secured with the collateral value P , $E_T(a_T) = \frac{P}{1+a_T}$. The physical assets are illiquid; they can only be liquidated with a lead time. The value of the physical assets P is sufficient to finance the budget-unconstrained optimal capacity investment level of the firm. There is a fixed bankruptcy cost BC for the creditor if the firm defaults. The creditor incurs this cost as an out-of-pocket fee and does not deduct it from the seized assets of the firm.*

We assume that the loan commitment is fully collateralized by the firm's physical assets P , i.e. $E_T(1 + a_T) = P$, since most bank loans are secured by the company's assets (Weidner 1999) and modeled as such (Mello and Parsons 2000). Although the loan is fully collateralized, if the firm's final cash position is not sufficient to cover the face value of the debt, the firm cannot immediately liquidate the collateral assets to repay its debt since the physical assets are illiquid. Under limited shareholder liability, this leads to default, in which case the creditor can seize these physical assets, liquidate them and use their liquidation value to recover the loan. With our assumption that it is incurred by the creditor as out-of-pocket fees in the default states, this cost is effectively charged to the firm ex-ante in the equilibrium unit borrowing cost a_T . This assumption is made for analytical convenience. We can show that the equilibrium level of unit financing cost decreases if the creditor can deduct the fixed cost of the bankruptcy from the seized assets¹. This demonstrates that the firm is penalized ex-ante (before borrowing) through a more severe loan contract for not being responsible for the fixed cost of bankruptcy after default.

This external financing cost structure provides a parsimonious model that is consistent with real-life practices; allows us to capture capital market imperfections and enables us to preserve tractability.

3.3 Stage 2

In this stage, demand uncertainty is resolved. The firm then chooses the production quantities (equivalently, prices) to satisfy demand optimally. If the firm is able to repay its debt

¹The proof is available upon request

from its final cash position, it does so and terminates by liquidating its physical assets. Otherwise, default occurs. In this case, because of the limited liability of the shareholders, the firm goes to bankruptcy. The cash on hand and the ownership of the collateralized physical assets are transferred to the creditor. The firm receives the remaining cash after the creditor covers the face value of the debt from the seized assets of the firm.

Assumption 8 *Price-dependent demand for each product is represented by the iso-elastic inverse-demand function $p(q_i; \xi_1) = \xi_i q_i^{1/b}$ for $i = 1, 2$. Here, $b \in (-\infty, -1)$ is the constant elasticity of demand, and p and q denote price and quantity, respectively. ξ_i represents the idiosyncratic risk component. (ξ_1, ξ_2) are correlated random variables with continuous distributions that have positive support and bounded expectation $(\bar{\xi}, \bar{\xi})$ with covariance matrix Σ , where $\Sigma_{ii} = \sigma^2$ and $\Sigma_{ij} = \rho\sigma^2$ for $i \neq j$ and ρ denotes the correlation coefficient. (ξ_1, ξ_2) and α_1 have independent distributions. The marginal production costs of each product at stage 2 are 0.*

We make specific assumptions about the distribution of ξ throughout the text whenever necessary.

4 Analysis of the Firm's Problem

4.1 Stage 2: Production Decision

In this stage, the firm observes the demand realization $\tilde{\xi}$ and determines the production quantities $\mathbf{Q}_T' = (q_T^1, q_T^2)$ within the existing capacity limits to maximize the stage 2 equity value.

Proposition 1 *The optimal production quantity vector in stage 2 with technology $T \in \{D, F\}$ for given \mathbf{K}_T and $\tilde{\xi}$ is given by*

$$\mathbf{Q}_D^* = \mathbf{K}_D, \quad \mathbf{Q}_F^* = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}} \tilde{\xi}^{-b}.$$

Proof All proofs are relegated to a separate technical appendix. ■

Since the unit production cost is zero, the firm optimally utilizes the entire available capacity. With dedicated technology, the optimal individual production quantities are equal to the available capacity levels for each product. With flexible technology, the firm allocates

the available capacity K_F between each product in such a way that the marginal profits for each product are equal.

4.2 Stage 1: Capacity Choice and External Financing

In this stage, the firm exercises the forward contract H_T and observes the asset price $\tilde{\alpha}_1$. With fair pricing, the strike price of the forward is equal to $\bar{\alpha}_1$. The stage 1 budget is therefore $B(\tilde{\alpha}_1, H_T) \doteq \omega_0 + \tilde{\alpha}_1(\omega_1 - H_T) + \bar{\alpha}_1 H_T$. We henceforth suppress $\tilde{\alpha}_1$ and H_T and denote the available budget realization by $\tilde{B} \in [0, \infty)$. For given \tilde{B} and T , the firm determines the optimal capacity investment level $\mathbf{K}_T^*(\tilde{B})$ and the optimal external borrowing level $e_T^*(\tilde{B})$.

Proposition 2 *The optimal capacity investment vector $\mathbf{K}_T^*(\tilde{B})$ and the optimal external borrowing level $e_T^*(\tilde{B})$ for technology $T \in \{D, F\}$ with a given budget level \tilde{B} are*

$$\mathbf{K}_T^*(\tilde{B}) = \begin{cases} \mathbf{K}_T^0 & \text{if } \tilde{B} \in \Omega_T^0 \doteq \{\tilde{B} : \tilde{B} \geq c_T \mathbf{1}' \mathbf{K}_T^0\} \\ \bar{\mathbf{K}}_T & \text{if } \tilde{B} \in \Omega_T^1 \doteq \{\tilde{B} : c_T \mathbf{1}' \mathbf{K}_T^1 \leq \tilde{B} < c_T \mathbf{1}' \mathbf{K}_T^0\} \\ \mathbf{K}_T^1 & \text{if } \tilde{B} \in \Omega_T^2 \doteq \{\tilde{B} : c_T \mathbf{1}' \mathbf{K}_T^1 - E_T \leq \tilde{B} < c_T \mathbf{1}' \mathbf{K}_T^1\} \\ \bar{\bar{\mathbf{K}}}_T & \text{if } \tilde{B} \in \Omega_T^3 \doteq \{\tilde{B} : 0 \leq \tilde{B} < c_T \mathbf{1}' \mathbf{K}_T^1 - E_T\} \end{cases} \quad (1)$$

$$e_T^*(\tilde{B}) = \left(c_T \mathbf{1}' \mathbf{K}_T^*(\tilde{B}) - \tilde{B} \right)^+ . \quad (2)$$

The explicit expressions for the capacity vectors in the proposition are given in the proof. \mathbf{K}_T^0 is the optimal capacity investment in the absence of a budget constraint (the “budget-unconstrained optimal capacity”). If the budget realization is high enough to cover the corresponding cost $c_T \mathbf{1}' \mathbf{K}_T^0$ ($\tilde{B} \in \Omega_T^0$), then $\mathbf{K}_T^*(\tilde{B}) = \mathbf{K}_T^0$ with no borrowing. Otherwise, for each budget level $\tilde{B} \in \Omega_T^{123}$, the firm determines to borrow or not by comparing the marginal revenue from investing in an additional unit of capacity over its available budget with the marginal cost of that investment including the external financing cost, $(1 + a_T)c_T$. For $\tilde{B} \in \Omega_T^1$, the budget is insufficient to cover \mathbf{K}_T^0 , and the marginal revenue of capacity is lower than its marginal cost. Therefore, the firm optimally does not borrow, and only purchases the capacity level $\bar{\mathbf{K}}_T$ that fully utilizes its budget \tilde{B} . For $\tilde{B} \in \Omega_T^{23}$, the marginal revenue of capacity is higher than its marginal cost $(1 + a_T)c_T$. Therefore, the firm optimally borrows from external markets to invest in capacity. \mathbf{K}_T^1 is the optimal capacity investment with borrowing, in the absence of a credit limit (the “credit-unconstrained optimal

capacity”). If the budget realization and the credit limit can jointly cover its cost, \mathbf{K}_T^1 is the optimal capacity investment; otherwise, the firm purchases the capacity level $\overline{\mathbf{K}}_T$ that fully utilizes its budget and its credit limit.

The optimal external borrowing level $e_T^*(\tilde{B})$ is such that the firm borrows exactly what it needs to cover its capacity investment. Since production is costless, the firm does not incur any further costs beyond this stage. The firm only borrows for funding the capacity investment, which yields (2).

The optimal expected (stage 1) equity value of the firm with a given budget level \tilde{B} , $\pi_T(\tilde{B})$, can be obtained in closed form.

Corollary 1 $\pi_T(\tilde{B})$ is concave increasing in \tilde{B} on $[0, \infty)$.

4.3 Stage 0: Financial Risk Management Level and Technology Choice

In this stage, the firm decides on the technology choice $T \in \{D, F\}$, and the financial risk management level H_T , the number of forward contracts written on the stage 1 asset price α_1 . The optimal expected (stage 0) equity value $\Pi^*(\mathbf{W})$ as a function of the internal (stage 1) endowment $\mathbf{W}' = (\omega_0, \omega_1)$ is

$$\Pi^*(\mathbf{W}) = \max \{Z, \omega_0 + \bar{\alpha}_1 \omega_1 + P\}. \quad (3)$$

Here, Z denotes the expected (stage 0) equity value of the better technology calculated at the optimal risk management level H_T^* . In (3), the firm compares this equity value with $\omega_0 + \bar{\alpha}_1 \omega_1 + P$, the expected (stage 0) equity value of not investing in any technology. §4.3.1 derives H_T^* and §4.3.2 characterizes the optimal technology choice. This characterization is valid for any continuous α_1 and ξ distribution with positive support and bounded expectation.

4.3.1 Financial Risk Management

The expected direct gain from the financial contract is 0 due to the fair pricing assumption. At the same time, financial risk management affects the distribution of the stage 1 budget $B(\alpha_1, H_T)$, which is used to finance the firm’s capacity investment. In choosing H_T , the goal of the firm is to engineer its budget to maximize the expected gain from the technology commitment made in stage 0.

Proposition 3 *It is optimal for the firm to fully hedge: $H_T^* = \omega_1$.*

Full hedging is optimal because π_T is a concave function of the available budget \tilde{B} . This follows by Jensen's inequality: For concave π_T , $\mathbb{E}[\pi_T(B(\alpha_1, H_T))] \leq \pi_T(E[B(\alpha_1, H_T)]) = \pi_T(\omega_0 + \bar{\alpha}_1 \omega_1)$, the equity value under full hedging.

4.3.2 Technology Choice

We now turn to the technology selection problem. Since the credit limit E_T is uniquely determined by the unit financing cost a_T ($E_T = \frac{P}{1+a_T}$), we only use a_T to denote the loan contract terms. We define the vectors $\mathbf{a} \doteq (a_D, a_F)$ and $\mathbf{H}^* \doteq (H_D^*, H_F^*)$. The choice T^* between flexible versus dedicated technology is determined by a unit cost threshold that makes the firms indifferent between the two technologies.

Proposition 4 *For a given financing cost scheme a_T , and under the financial risk management level $H_T^* = \omega_1$ for each technology $T \in \{D, F\}$, there exists a unique variable cost threshold $\bar{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$ such that when $c_F < \bar{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$ it is more profitable to invest in flexible technology ($T^* = F$). Without financial risk management, there is a parallel threshold $\bar{c}_F(c_D, \mathbf{a}, \mathbf{0})$. With symmetric financing costs $a_F = a_D$,*

$$\bar{c}_F(c_D, \mathbf{a}, \mathbf{H}^*) = \bar{c}_F(c_D, \mathbf{a}, \mathbf{0}) = \bar{c}_F^S(c_D) = c_D \left(\frac{\mathbb{E}^{-b} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]}{2\bar{\xi}^{-b}} \right)^{-\frac{1}{b+1}} \geq c_D. \quad (4)$$

Investing in T^ dominates not making any technology investment.*

The cost threshold developed in Proposition 4 reveals the more profitable technology. Proposition 4 also concludes that investing in this technology is more profitable than not investing at all. This completes the characterization of the firm's optimal decisions. The next section analyzes the creditor's problem.

5 Analysis of the Creditor's Problem

As the Stackelberg leader, the creditor determines credit terms to ensure an expected return of U : $\mathbb{E}[\Lambda_T(a_T, E_T)] = U$. Recall from Assumption 7 that at a given unit cost a_T , the creditor offers the credit limit that can be secured with the collateral value P , $E_T(a_T) =$

$\frac{P}{1+a_T}$. Then we can write the stage 0 expected return of the creditor with a loan commitment contract $(a_T, E_T(a_T))$ as

$$\mathbb{E}[\Lambda_T(a_T)] = a_T \mathbb{E}[e_T] - BC \mathbb{E}[Pr\{\Gamma_T < e_T(1+a_T)\}], \quad (5)$$

where BC is the fixed cost of bankruptcy and Γ_T is the firm's optimal stage 2 operating profits under technology choice T . Since we focus on fully-secured loan commitment contracts ($e_T(1+a_T) \leq P$), the creditor always retrieves the face value of the debt and generates expected earnings of $a_T \mathbb{E}[e_T]$. However, since the physical assets are illiquid, default can occur because the firm is not able to immediately pay back the debt with the liquid assets, in which case the creditor incurs the bankruptcy cost BC .

By Proposition 3, the firm always fully hedges, so, we need only focus on the budget level \bar{B} to expand (5). The regions in Proposition 2 depend on a_T . If $\bar{B} \in \Omega_T^{01}(a_T)$, the firm does not borrow. It can also be shown that under our assumptions, the firm never borrows at the credit limit $E_T(a_T)$ for any a_T . Therefore, the firm only borrows from the creditor if $\bar{B} \in \Omega_T^2(a_T)$, where $e_T = c_T \mathbf{1}' \mathbf{K}_T^1 - \bar{B} < E_T$. For any a_T satisfying $\bar{B} \in \Omega_T^2$, we have

$$\mathbb{E}[\Lambda_T(a_T)] = (c_T \mathbf{1}' \mathbf{K}_T^1 - \bar{B}) a_T - BC Pr\left(\mathbf{N}'_T \mathbf{1}' \mathbf{K}_T^1^{1+\frac{1}{b}} < (c_T \mathbf{1}' \mathbf{K}_T^1 - \bar{B})(1+a_T)\right). \quad (6)$$

Let $EE_T \doteq (c_T \mathbf{1}' \mathbf{K}_T^1 - \bar{B}) a_T$ denote the expected earnings (without default) of the creditor, $P_T \doteq Pr\left(\mathbf{N}'_T \mathbf{1}' \mathbf{K}_T^1^{1+\frac{1}{b}} < (c_T \mathbf{1}' \mathbf{K}_T^1 - \bar{B})(1+a_T)\right)$ denote the default probability and $ED_T = BC P_T$ denote the expected default cost of the creditor. To build intuition into the creditor's problem, we analyze each of the two parts separately and demonstrate the effect of different parameters on each part. For the comparative statics analysis, we focus on local results where $\bar{B} \in \Omega_T^2$ holds.

Proposition 5 *EE_T decreases in the unit cost of the technology c_T and firm size \bar{B} . There exists a unique threshold \underline{a}_T such that EE_T increases in a_T for $a_T < \underline{a}_T$ and decreases in a_T for $a_T > \underline{a}_T$.*

A larger unit investment cost induces the firm to decrease its capacity investment level. As a result, the total investment cost decreases, and so does the borrowing level. As the budget of the firm increases, it's obvious that its borrowing level would decrease. As the financing cost a_T increases, the optimal investment level, and hence the borrowing level decreases. For small levels of a_T , the increase in the marginal return of increasing a_T dominates the

reduction in the borrowing level. As a_T becomes larger the percentage reduction in the borrowing level becomes more significant and dominates the positive effect of increasing marginal returns. Hence the result in Proposition 5 follows.

Proposition 6 *Let P_T denote the expected default probability with technology $T \in \{F, D\}$. For $\bar{B} \in \Omega_T^2$,*

$$P_T = Pr \left(H_T(\boldsymbol{\xi}) < M_T \left(1 + \frac{1}{b}\right) \left[1 - \frac{\bar{B}}{c_T \mathbf{1}' \mathbf{K}_T \mathbf{1}}\right] \right) \quad (7)$$

where $\bar{B} = \omega_0 + \bar{\alpha}_1 \omega_1$, $M_F = \mathbb{E} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$, $M_D = 2^{-1/b} \bar{\xi}$, $H_F(\boldsymbol{\xi}) = \left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}}$, and $H_D(\boldsymbol{\xi}) = \frac{\xi_1 + \xi_2}{2^{1+1/b}}$. P_T decreases in firm size (expected budget level \bar{B}) and in unit investment cost c_T for an arbitrary level of a_T . P_T decreases in unit financing cost a_T .

For an arbitrary level of a_T , an increase λ reduces the borrowing level while not affecting the other factors, so the default probability decreases. Hence large firms have lower default risk than small firms at the same level of a_T . One may expect a higher technology cost to result in a higher borrowing level and hence a higher default risk, but the firm's optimal capacity investment level decreases in c_T , leading to a decrease in the borrowing level. Therefore an increase in c_T leads to a lower default risk at a given level of a_T . As the unit financing cost a_T increases, the capacity investment level decreases. Both the operating cash flows decrease and the face value of the loan decrease, but the net effect is that the default probability decreases.

It is interesting to note that the default probability decreases in the unit financing cost. Melnik and Plaut (1986) derive several relations among the parameters of loan commitment contract after assuming that the borrowing level is independent of the unit financing cost, and that the default probability increases in the unit financing cost. Propositions 5 and 6 demonstrate that these assumptions may not be valid with a more formal representation of operations.

We have characterized the expected return of the creditor for a given a_T ; we now characterize equilibrium financing cost a_T^* for each technology. We focus on the the Pareto-Nash equilibrium that achieves a return of U for the creditor and the highest profit for the firm. This is consistent with the financial economics literature where the creditor makes zero expected return in equilibrium ($U = 0$) and the creditor chooses the best contract for the firm. In our model, the Pareto-optimality refinement guarantees the uniqueness of the equilibrium financing cost a_T^* , if such a cost exists:

Proposition 7 *If there exists a feasible $a_T \geq 0$ that satisfies $\mathbb{E}[\Lambda_T(a_T)] = U$ for the creditor, then in the Pareto-optimal equilibrium, the creditor offers a unique loan commitment contract for each technology T with parameters*

$$\begin{aligned} a_T^* &= \operatorname{argmin}_{a_T \geq 0} \mathbb{E}[\Lambda_T(a_T)] = U, \\ E_T^* &= \frac{P}{1 + a_T^*}. \end{aligned}$$

If such an $a_T \geq 0$ that satisfies $\mathbb{E}[\Lambda_T(a_T)] = U$ does not exist, then in equilibrium the creditor does not offer a contract. In this case we say $a_T^ \rightarrow \infty$ and $E_T^* \rightarrow 0$.*

The minimal a_T (which also corresponds to maximal credit limit E_T) is Pareto-optimal for the firm because the expected equity value of the firm (weakly) increases as more external capital becomes available at a lower unit cost; the creditor is indifferent between all a_T 's satisfying his reservation expected utility U . Since the firm's borrowing level depends on a_T , when the fixed bankruptcy cost or the reservation utility of the creditor is sufficiently large, then there may not exist a feasible a_T that satisfies the creditor's requirement of U . In this case, the creditor does not offer a contract and the firm cannot raise external capital.

Proposition 8 *The equilibrium level of unit financing cost a_T^* and credit limit E_T^* increase and decrease, respectively, in the fixed cost of bankruptcy and the underwriter fee. The optimal (stage 0) expected equity value of the firm and the expected capacity investment level decrease in the fixed cost of bankruptcy and the underwriter fee.*

It is intuitive that higher bankruptcy costs and underwriter fees induce the creditor to ask for a higher level of unit financing cost (which results in a lower credit limit). Higher imperfection costs in the financial markets are directly transferred to the firm's operations, resulting in a lower expected equity value.

6 The Perfect Capital Market Benchmark

The goal of this paper is to understand the effect of capital market imperfections on the firm's operational decisions and performance. As mentioned in the introduction, the capacity investment literature has implicitly assumed frictionless borrowing, from which follows a series of results on optimal capacity investment and technology choice. We will show that many of these conclusions do not hold once capital market imperfections are taken

into account. To this end, we first identify the natural perfect market benchmark in our modeling framework.

Proposition 9 *In our model, when capital markets are perfect, i.e. when there are no bankruptcy and underwriter fees, the equilibrium unit financing cost is the risk-free rate ($a_T^* = 0$) and the equilibrium credit limit is the value of the collateralized physical assets of the firm ($E_T^* = P$).*

The perfect market assumption prohibits any transaction costs (e.g. fixed cost of bankruptcy, underwriter fee) and requires the fair valuation of the debt obligation in terms of its underlying default exposure. Since we focus on collateralized debt, in the absence of transaction costs ($BC = 0, U = 0$), there is no risk for the creditor associated with default. Consequently, the fair price of any secured debt obligation is the risk-free rate ($a_T^* = 0$)² and the credit limit E_T is the value of the collateralized physical asset ($E_T^* = P$). If there are capital market imperfections, then $a_T^* > 0$ and $E_T^* < P$ in our framework.

Proposition 10 *If the capital markets are perfect, the firm's operational decisions are independent of the financing and financial risk management decisions. Financial risk management does not have any value. The firm invests in the budget-unconstrained capacity investment level for any budget realization, $\mathbf{K}_T^*(\tilde{B}) = \mathbf{K}_T^0$ and borrows to finance this capacity level, $e_T^*(\tilde{B}) = [c_T \mathbf{1}' \mathbf{K}_T^0 - B]^+$. Equilibrium technology choice T^* is determined by the variable cost threshold $\bar{c}_F^P(c_D) = \bar{c}_F^S(c_D)$ of Proposition 4 and the expected (stage 0) equity value in equilibrium is given by $\Pi^*(\mathbf{W}) = \bar{B} + \frac{c_T \mathbf{1}' \mathbf{K}_T^0}{-(b+1)} + P$.*

The optimal investment level \mathbf{K}_T^0 is identical to the one in traditional stochastic capacity models (Van Mieghem 2003). The firm simply chooses the optimal investment level without regard to the budget limit or financing costs, and implements it by borrowing if necessary. This is consistent with the decoupling of operational and financial decisions in perfect markets (Modigliani and Miller 1958). In the same line of reasoning, financial risk management has no value in perfect capital markets (Fite and Pflleiderer 1995) with which Proposition 10 is again consistent.

²If the debt were not fully secured, then the unit cost of financing would be larger than the risk-free rate even in a perfect market (see Xu and Birge 2004 for a detailed treatment of this with unsecured spot lending).

7 Effect of Capital Market Imperfections on the Firm's Operational Decisions and Performance - The Single Product Case

In this section, we characterize the creditor-firm equilibrium for specific demand distributions and a single product. We compare the results with the perfect market benchmark of Section 6 to demonstrate how conclusions arrived at under the (implicit) perfect market assumption may change when imperfections are taken into account. Our goal is not to undertake a complete characterization of the equilibrium, but to show the existence of certain heretofore unidentified effects. We focus on the firms that engages in financial risk management (except the last part where we analyze the value of financial risk management). All the analytical results of this section continue to hold for the firms that do not use financial risk management with minor modifications in the proofs.

In the single product setting, the firm uses a single resource and technology choice is not relevant so we eliminate the D and F subscripts. The optimal capacity investment, production quantity and financial risk management decisions of the firm follow from our analysis in §4 by setting the range of one of the product market uncertainties ξ_i to 0; we also eliminate the i subscript. Unless otherwise specified, this section assumes that ξ is uniformly distributed between $[0, 2\bar{\xi}]$. With this assumption, the expected return of the creditor for a given a satisfying $\bar{B} \in \Omega_2(a)$ can be written as

$$\mathbb{E}[\Lambda(a)] = \left(cK^0(1+a)^b - \bar{B} \right) \left(a - \frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b} \right).$$

The uniform distribution assumption leads to a nice decomposition of the creditor's expected return in product form: The first term is the expected amount of lending. The second term is the expected unit marginal profit of lending. For each unit of the loan, the creditor earns a and incurs an expected default cost. This decomposition holds because the default risk is a linear function of the expected amount of lending under the uniform distribution. The following proposition characterizes the equilibrium unit financing cost:

Proposition 11 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\bar{\xi}]$ and $U \geq 0$, there exists a unique bankruptcy cost threshold \widehat{BC} such that for $BC > \widehat{BC}$, the firm does not lend money in equilibrium. For $BC \leq \widehat{BC}$, if U and \bar{B} are sufficiently small, the*

creditor offers a rate $a^* < \left(\frac{cK^0}{B}\right)^{-1/b} - 1$ and the firm borrows; otherwise the creditor does not offer a contract or offers the rate $a^* = \left(\frac{cK^0}{B}\right)^{-1/b} - 1$ and the firm does not borrow.

If the bankruptcy cost is sufficiently high, then the expected marginal profit is negative for the creditor for all feasible a . In this case, the the creditor does not offer any loan. For a sufficiently low level of bankruptcy cost, the expected marginal profit can be positive for some range of a . In this case, if U and \bar{B} are not very high then the creditor offers a finite a^* in equilibrium and the firm borrows. If U is sufficiently high, since the expected loan amount is limited and decreasing in a , the creditor cannot generate sufficient expected returns to satisfy U . If \bar{B} is sufficiently high, since the expected amount of lending is low, similarly, the creditor cannot generate sufficient expected returns to satisfy U . In these cases, loan is not offered in equilibrium. In summary, borrowing only takes place if the bankruptcy cost and underwriter fee are sufficiently low.

We now investigate the effect of product market variability, firm size, unit capacity investment cost and engaging in financial risk management on the equilibrium level of financing costs and firm's decisions and performance. Our focus is to show differences from the perfect market case. For convenience, we summarize the results of the next four propositions in Table 1.

	Perfect Market	Imperfect Market
Product market variability	Does not impact capacity level and equity value	Decreases capacity level and equity value
Expected budget level	Does not impact capacity level and increases equity value	May decrease capacity level and equity value
Unit investment cost	Decreases the capacity level and equity value	Decreases the capacity level and equity value even more
Financial risk management	Does not have any value	Has positive static and possibly negative strategic value

Table 1: Differences between perfect and imperfect markets in single-product investments with uniform $[0, 2\bar{\xi}]$ product market uncertainty

The effect of product market variability. As suggested by traditional models, and as also follows from Proposition 10, in perfect capital markets, the firm's capacity decision and

the expected (stage 0) equity value depend on the expected value of product market uncertainty but not on product market variability. The following proposition demonstrates that independence from product market variability does not hold in imperfect capital markets.

Proposition 12 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\bar{\xi}]$, the expected optimal capacity investment level and (stage 0) equity value of the firm in equilibrium decrease in product market variability through an increase in the equilibrium financing cost level.*

This result also holds when the product market uncertainty ξ is normal or uniform with mean $\bar{\xi}$. For an arbitrary financing cost a , increasing variability alters neither the capacity investment level nor the expected (stage 0) equity value of the firm. The firm is only concerned with the mean of the product market uncertainty; this is in line with the perfect market benchmark. With normal and uniform distributions, higher variability corresponds to the mean-preserving spread of ξ – more probability mass is transferred to the tails. Since the creditor is only concerned with the downside risk of the firm’s operating cash flows; higher variability translates into higher downside risk. This leads to higher expected default risk for the firm and lower expected returns for the creditor. To compensate for this reduction, the creditor charges higher financing cost in equilibrium. In turn, the increase in unit financing cost decreases the capacity investment level and the operational performance of the firm in equilibrium. Proposition 12 provides an analytical proof for the numerical observation of Xu and Birge (2004) on the effect of product market variability in a single-product setting.

The effect of firm size. We proxy the firm size with the expected internal (stage 1) endowment (\bar{B}) of the firm. In perfect capital markets, an increase in the expected internal endowment does not alter the optimal capacity investment level, but increases the expected equity value due to the increase in the term \bar{B} . In imperfect capital markets, a change in \bar{B} also has an indirect effect through altering the equilibrium level of financing cost.

Proposition 13 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\bar{\xi}]$, the equilibrium unit financing cost increases in the expected budget level. The expected optimal capacity investment level decreases in the expected budget level if the firm borrows in equilibrium and increases otherwise. The expected (stage 0) equity value of the firm may decrease or increase in the expected budget level.*

At a given a , the expected marginal profit of the creditor is independent of the internal endowment of the firm; but a higher endowment level means a smaller loan as \bar{B} is larger. As a result, the creditor increases the unit financing cost in equilibrium to compensate for this reduction. Since the expected optimal capacity investment level depends on \bar{B} only when the firm does not borrow, increasing a^* decreases the capacity investment level when the borrowing takes place. The combined effect of increasing \bar{B} and increasing a^* on the expected (stage 0) equity value can be positive or negative.

In the literature, it has been argued qualitatively that larger firms can obtain lower financing costs than smaller firms, based on the premise that larger firms have more internal capital and borrow less; and hence have lower default risk. Proposition 13 highlights the importance of the expected borrowing level, and demonstrates that the lower external borrowing need of large firms may induce the creditor to charge a higher unit financing cost in equilibrium. Proposition 13 also demonstrates that larger firms may not perform better than smaller firms despite larger firms' higher internal endowment.

The effect of unit capacity investment cost. In perfect capital markets, an increase in the unit capacity investment cost decreases the expected equity value through reducing the optimal capacity investment level. In imperfect capital markets, an increase in c reduces these quantities even more due to an increase in the external financing cost.

Proposition 14 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\bar{\xi}]$, the equilibrium level of financing cost increases, the expected optimal capacity investment level and (stage 0) equity value of the firm decrease in the unit capacity investment cost.*

In §5, we demonstrated that in imperfect capital markets, an increase in c_D decreases the expected earnings but also decreases the expected bankruptcy cost of the creditor. The combined effect is indeterminate and depends on the product market uncertainty. With a uniform distribution, the first effect dominates the second effect. Therefore, in equilibrium, the creditor increases the unit financing cost to compensate for the reduction in expected returns.

The effect of engaging in financial risk management. As we demonstrated in Proposition 10, financial risk management does not have any value for the firm in perfect capital markets. It follows from Proposition 9 that the firm engaging in financial risk management does not have any effect on the creditor's returns in perfect markets. Under market

imperfections, the financial risk management policy of the firm affects both the firm and the creditor. We first focus on the value of engaging in financial risk management for the firm. Let Π^{FRM} and Π^{-FRM} denote the expected (stage 0) equity value of the firm with and without engaging in financial risk management respectively. The value of engaging in financial risk management for the firm, Δ , is given by

$$\begin{aligned}\Delta &= \Pi^{FRM}(a_{FRM}^*) - \Pi^{-FRM}(a_{-FRM}^*) \\ &= [\Pi^{FRM}(a_{FRM}^*) - \Pi^{FRM}(a_{-FRM}^*)] + [\Pi^{FRM}(a_{-FRM}^*) - \Pi^{-FRM}(a_{-FRM}^*)].\end{aligned}$$

The second term in brackets is the *static* value of engaging in financial risk management for the firm. At a given unit financing cost a_{-FRM}^* , the firm uses forward contracts to engineer its internal cash flow to avoid dependence on external borrowing. The static value is always non-negative because the firm can always choose not to engage in financial risk management.

The first term in brackets corresponds to the *strategic* value of engaging in financial risk management for the firm, obtained by altering the equilibrium level of financing cost by changing the expected returns of the creditor. The strategic value is negative if engaging in financial risk management increases the equilibrium level of unit financing cost; it is positive otherwise. For an arbitrary a , engaging in financial risk management decreases the expected earnings of the creditor, because the firm borrows less in expectation with financial risk management. The effect on the expected default cost is mainly determined by the product market uncertainty. The following proposition characterizes the equilibrium unit financing cost and the strategic value of financial risk management for the uniform case.

Proposition 15 *Let a_{FRM}^* (a_{-FRM}^*) denote the equilibrium financing cost with (without) financial risk management. Let the product market uncertainty ξ be uniformly distributed in $[0, 2\bar{\xi}]$. For $BC > \widehat{BC}$, the creditor does not offer financing with or without financial risk management. For $BC \leq \widehat{BC}$,*

1. *If $U = 0$, then $a_{FRM}^* \leq a_{-FRM}^*$ but there is no strategic value.*
2. *If $U > 0$ and \bar{B} are sufficiently small such that a finite a_{FRM}^* exists, then $a_{FRM}^* > a_{-FRM}^*$ and financial risk management has negative strategic value.*

3. *If $U > 0$ is sufficiently large and \bar{B} is sufficiently small such that the creditor would not offer financing to the firm engaging in financial risk management, then either a finite a_{-FRM}^* exists and financial risk management has negative strategic value, or the creditor does not finance the firm without risk management either and financial risk management has no strategic value.*
4. *If $U > 0$ and \bar{B} are sufficiently large such that the creditor would not offer financing to the firm engaging in financial risk management, then either a finite a_{-FRM}^* exists or the creditor does not finance the firm without risk management either and financial risk management has no strategic value.*

It follows from Proposition 15 that engaging in financial risk management may have a negative strategic value for the firm. In the uniform case, since the default risk is a linear function of the borrowing level, the strategic value is determined by the change in expected borrowing level. When the firm borrows in equilibrium without financial risk management, engaging in financial risk management decreases the expected borrowing level, and the creditor increases the unit financing cost to compensate for this reduction. This is in contrast with the documented positive strategic value of financial risk management. Smith and Stulz (1985) demonstrate that when the firm uses financial risk management to hedge the operating cash flows *after the loan is taken*, then engaging in financial risk management decreases the equilibrium level of financing cost by reducing the expected default risk. In our case, financial risk management is effective *before the loan is taken*, and increases the equilibrium level of financing cost by decreasing the expected earnings of the creditor. The total value Δ is determined by the comparison between the positive static value and negative strategic value.

8 Effect of Capital Market Imperfections on Firm's Decisions and Performance – The Two-Product Case

In Section 7, we analyzed the effect of capital market imperfections in the single product setting in which the technology choice of the firm is irrelevant. In this section, we focus on the effect of capital market imperfections in the two-product setting where technology choice is a non-trivial question and demand correlation matters. In §8.1, we investigate

the effect of product market uncertainty (demand correlation ρ and demand variability σ) on the optimal capacity investment level and the operational performance of the firm with each technology. In §8.2, we analyze the impact of capital market imperfections on the firm's technology choice. We use the perfect market benchmark that we developed in §6 to delineate the effect of capital market imperfections and highlight the new trade-offs that arise. Our main results are summarized in Table 2.

In this section, to carry out comparative statics analysis with respect to ρ and σ , we assume that ξ has a bivariate normal distribution. We use the same parameter set ($c_F = c_D = 2, P = 220, b = -2, \bar{\xi}_1 = \bar{\xi}_2 = 20, \bar{B} = 5, U = 1$) for the numerical examples throughout this section. As in Section 7, we focus on the firms that engage in financial risk management throughout this section. All the analytical results continue to hold for the firms that do not use financial risk management.

		Perfect Market	Imperfect Market
Dedicated Technology	Increase in variability	Does not impact capacity level and equity value	Decreases capacity level and equity value
	Increase in correlation	Does not impact capacity level and equity value	Decreases capacity level and equity value
Flexible Technology	Increase in variability	Increases capacity level and equity value	May decrease capacity level and equity value
	Increase in correlation	Decreases capacity level and equity value	May increase capacity level and equity value
Risk Pooling Value ($c_F = c_D$)	Increase in variability	Increases risk pooling value	May decrease risk pooling value
	Increase in correlation	Decreases risk pooling value	May decrease or increase risk pooling value
Technology choice	$c_F = c_D$	Flexible is always preferred	Dedicated may be preferred to flexible

Table 2: Differences between perfect and imperfect markets in two-product investments with bivariate normal product market uncertainty

8.1 The Effect of Capital Market Imperfections for a Given Technology

Dedicated Technology. Similar to the single product case, in perfect capital markets, the firm's capacity decision and the expected (stage 0) equity value with the dedicated technology depend only on the mean demand vector $\bar{\xi}$ and not on the covariance matrix Σ . In imperfect capital markets, if the firm borrows in equilibrium elements of the covariance matrix (ρ and σ) also matters.

Proposition 16 *If the product market uncertainty ξ has symmetric bivariate normal distribution, then the optimal expected capacity investment level and (stage 0) equity value of the firm using dedicated technology decrease in the demand correlation ρ and the demand variability σ through an increase in the equilibrium level of unit financing cost.*

As in the single product case, higher variability increases the default risk of the firm, which reduces the expected return of the creditor for an arbitrary financing cost a_D . The creditor increases the equilibrium level of financing cost to compensate for this reduction.

The result with respect to demand correlation follows from a financial risk-pooling argument. The firm's default probability for a given capacity choice depends on the variability in operating revenues. Operating in two markets creates a *diversification benefit* for the firm: When the product markets are negatively correlated, the firm generates high returns from one market and low returns from the other, reducing revenue variability and hence default risk. With high positive correlation, the firm generates similar revenues from both markets, increasing its default risk. Therefore, as correlation increases, the creditor increases the equilibrium level of financing cost to compensate for the increase in expected bankruptcy costs. Increase in the equilibrium level of financing cost leads to a reduction in the total capacity investment level and the expected (stage 0) equity value of the firm as Figure 1 demonstrates.

The financial risk-pooling effect discussed in this section is different from the risk-pooling effect of flexible technology that comes from the ability to switch capacity between products. The former effect only exists in imperfect capital markets, whereas the latter is a product-market effect and also exists in perfect capital markets.

Flexible Technology. In perfect capital markets, the firm's capacity investment decision and its expected (stage 0) equity value with flexible technology do depend on the covariance matrix Σ of ξ through the term $M_F \doteq E \left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$ (Proposition 10). This term

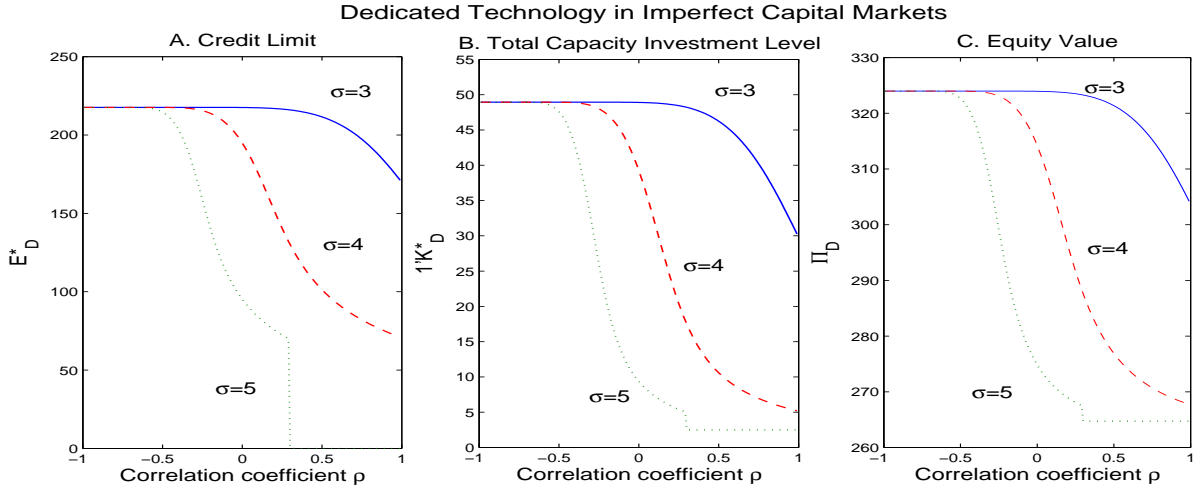


Figure 1: Effect of demand correlation ρ and demand variability σ on the dedicated technology investment in imperfect markets: Higher ρ and σ leads to lower credit limit E_D^* in equilibrium (Panel A) and this decreases the total capacity investment level (Panel B) and the expected (stage 0) equity value of the firm (Panel C).

captures the risk-pooling value of flexible technology that comes from the ability to switch production between two products after demand uncertainty is resolved. Under general correlation and variability measures, M_F decreases with increasing ρ and decreasing σ (Boyabatlı and Toktay 2006). Therefore, the optimal expected capacity investment level and (stage 0) equity value of the firm decreases with increasing ρ and decreasing σ in perfect capital markets. We call this the *direct (static) effect* of ρ and σ .

In imperfect capital markets, demand correlation ρ and the demand variability σ also have an *indirect (strategic) effect* as they alter the equilibrium level of financing cost. Two different drivers give rise to the indirect effect of ρ and σ : the expected value of production switching for a given capacity investment level and the optimal capacity investment level that incorporates this switching value. These two drivers have different effects on the equilibrium level of financing cost.

As the value of the switching option at a given capacity level decreases, the expected default probability of the firm increases. An increase in ρ or a decrease in σ typically decreases the value of production switching and increases the firm's default probability for

a given capacity investment level³. This acts to increase the equilibrium financing cost.

Anticipating the expected production switching value, the firm optimally adjusts its capacity investment level to exploit the benefits of production switching. With an increase in ρ and a decrease in σ , the firm optimally invests less in capacity. A lower capacity investment level means lower expected earnings for the creditor but also a lower expected default cost. Lower expected earnings (without default) act to increase the equilibrium financing cost, while a lower expected default cost acts to decrease it. The net effect is indeterminate.

To summarize, the overall effect of demand correlation ρ and demand variability σ on the optimal capacity investment level and the expected (stage 0) equity value of the firm with flexible technology is indeterminate in imperfect capital markets. This effect is a combination of the indirect effect, where the two major drivers may work in opposite directions, and the direct effect, which is the same as in the perfect market case. Figure 2 demonstrates that any effect can dominate: Different from perfect market setting, the (stage 0) equity value of flexible technology may increase with an *increase* in demand correlation and a *decrease* in demand variability.

8.2 The Effect of Capital Market Imperfections on Technology Choice

For each technology cost pair (c_F, c_D) , there exists a unique unit cost threshold $\bar{c}_F(c_D, a_F^*(c_F), a_D^*(c_D))$ ⁴ that determines the optimal technology choice of the firm (Propositions 4 and 7). The optimal technology choice captured by this threshold is based on comparing the higher investment cost of flexible technology against its (potential) production switching value relative to dedicated technology. As we shall see, putting these two effects together, there are cases where flexible technology is preferred in perfect markets, but the firm chooses dedicated technology in imperfect capital markets. In this section, we explain the main driver for this result: the strategic effects of investment cost and production switching capability on financing costs in equilibrium.

We start with analyzing the production switching value of flexible technology under identical unit capacity investment costs ($c_F = c_D$), defined as the stage 0 equity value

³It can be shown that the default probability increases for ξ that has a bivariate normal distribution when we go from $\rho = -1$ to $\rho = 1$ and $\sigma > 0$ to $\sigma = 0$.

⁴We drop the argument \mathbf{H}^* in the cost threshold, because the firm optimally fully hedges for any c_T, a_T for technology $T \in \{D, F\}$.

Flexible Technology in Imperfect Capital Markets

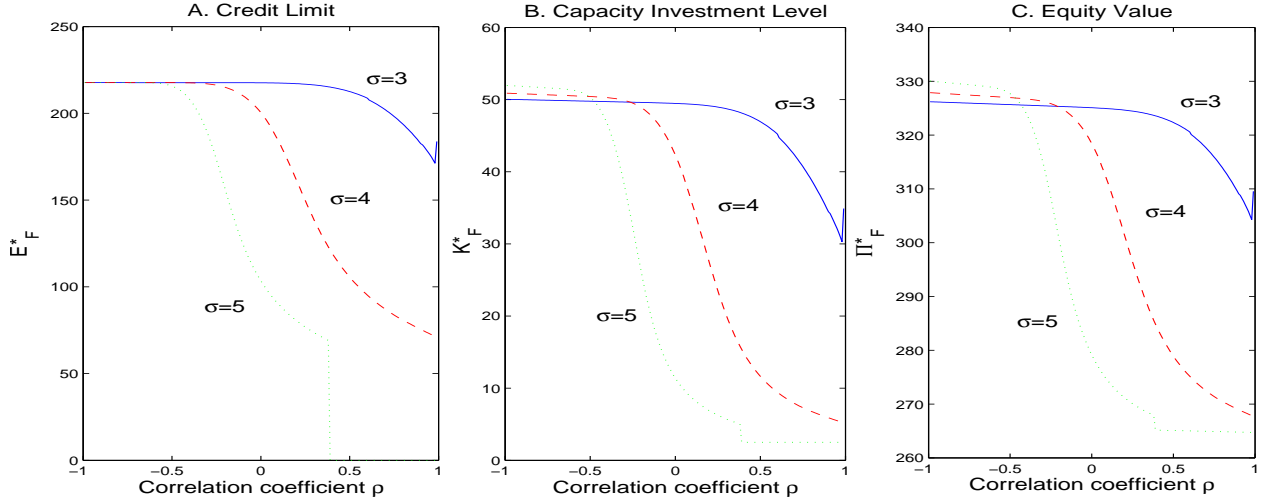


Figure 2: Effect of demand correlation ρ and demand variability σ on the flexible technology investment in imperfect markets: In equilibrium, higher σ leads to lower credit limit E_F^* (Panel A), higher ρ leads to lower credit limit E_F^* for low correlations (Panel A) and may lead to higher credit limit E_F^* for close to perfect positive correlations (Panel A, $\sigma = 3$). Sufficiently large decrease in E_F^* may lead to lower capacity investment (Panel B) and expected (stage 0) equity value (Panel C) for increasing σ . For close to perfect positive correlations, higher ρ may lead to higher capacity investment (Panel B, $\sigma = 3$) and expected (stage 0) equity value (Panel C, $\sigma = 3$).

difference between the two technology choices:

$$\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_D(\mathbf{K}_D^*; a_D^*(\mathbf{K}_D^*)) = \tag{8}$$

$$\underbrace{[\pi_F(K_F^*; a_D^*(\mathbf{K}_D^*)) - \pi_D(\mathbf{K}_D^*; a_D^*(\mathbf{K}_D^*))]}_{\text{Static Value}} + \underbrace{[\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_D^*(\mathbf{K}_D^*))]}_{\text{Strategic Value}}$$

The value of production switching is a combination of two terms, the *static value* and the *strategic value*. The static value is the value of production switching at a given unit financing cost (the equilibrium unit financing cost $a_D^*(\mathbf{K}_D^*)$ under dedicated technology in this case). The strategic value captures the effect of production switching on the creditor's expected returns and, hence, the change in the equilibrium level of financing cost with flexible technology.

In perfect markets, the static value of production switching is always positive (as follows from Proposition 10) and the strategic value does not exist (as follows from Proposition 9,

we have $a_D^*(\mathbf{K}_D^*) = a_F^*(K_F^*) = 0$ in perfect markets). Production switching enables the firm generate to higher revenues for a given capacity level at stage 2; and the optimal capacity investment decision of the firm increases the expected (stage 0) equity value of the firm even more by optimally exploiting the production switching capability.

In imperfect capital markets, similar to perfect market case, the static value of production switching is always positive. To delineate the strategic value of production switching, we focus on the two fundamental drivers of production switching that we discussed in the previous section: the expected value of production switching for a given capacity investment level and the optimal capacity investment level that incorporates this switching value.

$$\begin{aligned} \pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_D^*(\mathbf{K}_D^*)) = & \quad (9) \\ [\pi_F(K_F^*; a_F^*(\mathbf{K}_D^*)) - \pi_F(K_F^*; a_D^*(\mathbf{K}_D^*))] + [\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_F^*(\mathbf{K}_D^*))] \end{aligned}$$

In (9), the first term in brackets captures the effect of production switching on the equilibrium level of financing cost at a given capacity investment level (\mathbf{K}_D^*) with flexible technology. The second term in brackets demonstrates the effect of the optimal capacity investment decision that incorporates the production switching value, K_F^* , on the equilibrium financing cost.

The first term in brackets in (9) is always positive. At a given capacity investment level, the production switching capability of flexible technology decreases the default risk of the firm relative to dedicated technology. This can be observed by noting that for any ξ realization, we have $H_F(\xi) \geq H_D(\xi)$ in Proposition 6. Therefore, the expected default cost with flexible technology is lower and all else being equal, the creditor would charge a lower financing cost for the flexible technology: $a_F^*(\mathbf{K}_D^*) \leq a_D^*(\mathbf{K}_D^*)$.

The second term in brackets in (9) can be positive or negative. Due to the expected production switching value, the firm optimally increases its capacity investment level with flexible technology compared to dedicated technology ($K_F^* \geq \mathbf{1}'\mathbf{K}_D^*$) and optimally borrows more. This induces the creditor to make higher expected returns at $a_F^*(\mathbf{K}_D^*)$, and acts to decrease the financing cost. On the other hand, the higher borrowing level increases the default risk of the flexible technology for a fixed $H_T(\xi)$. This increases the expected default cost of the creditor and acts to increase the financing cost.

The overall strategic effect of these two value drivers of production switching and consequently, the total production switching value that also incorporates the static value are

indeterminate. Panel C of Figure 3 demonstrates the strategic value can be negative and dedicated technology can be preferred over flexible technology with identical unit capacity investment cost. Panel D of Figure 3 shows that the strategic value of risk pooling may always be positive for any correlation level.

In the limiting case, we can show that the strategic value of production switching does not exist when there is no static value:

Proposition 17 *If the product markets are perfectly positively correlated ($\rho = 1$), for technologies with identical unit capacity investment cost ($c_F = c_D$), the creditor offers the same unit financing cost for both technologies ($a_D^*(\mathbf{K}_D^*) = a_F^*(K_F^*)$), the firm's optimal total capacity investment decision is the same with both technologies ($K_F^* = \mathbf{1}'\mathbf{K}_D^*$), and there is no production switching value of the flexible technology ($\pi_F(K_F^*; a_F^*(K_F^*)) = \pi_D(\mathbf{K}_D^*; a_D^*(\mathbf{K}_D^*))$).*

With perfect positive correlation, there is no static value for production switching as the traditional models in capacity investment suggest. It follows from Proposition 17 that both terms in (9) are zero and there is no strategic value either. Although the effects of

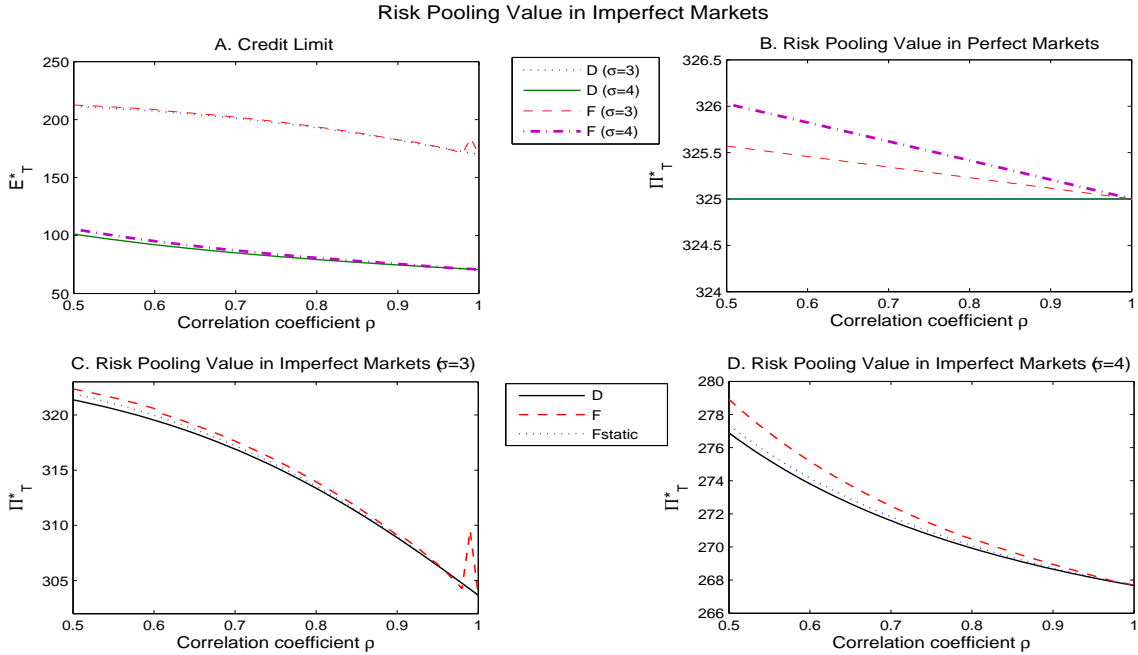


Figure 3: Value of Risk Pooling in Imperfect Markets

production switching are best understood when two technology costs are identical ($c_F = c_D$), in general, the firm is concerned with comparing technologies at any given technology cost

pair (c_F, c_D) . For the technology cost structure $c_F > c_D$, not only the static effect but also the strategic effect of production switching in (9) is altered by the higher investment cost of flexible technology. In imperfect capital markets, the sign of the static value of production switching for an arbitrary cost pair (c_F, c_D) coincides with the sign of the static value in perfect capital markets⁵. This property underlines the importance of the strategic value in determining technology choice in imperfect capital markets. As the following proposition demonstrates, the negative strategic value of production switching that exists only in imperfect capital markets can direct the firm towards dedicated technology in these markets.

Proposition 18 *For product market correlation $\rho \approx 1$, and unit capacity investment cost pair $(c_F^S(c_D; \rho \approx 1), c_D)$, the creditor offers a lower unit financing cost for the dedicated technology ($a_D^* < a_F^*$), and the firm chooses dedicated technology in equilibrium ($T^* = D$).*

The unit cost of flexible technology in this technology cost pair is the threshold derived in Proposition 4: The firm is indifferent between the two technologies in perfect markets. What Proposition 18 shows is that the firm chooses dedicated technology with these same costs if there are capital market imperfections. We explain this as being the result of the negative strategic value of production switching, sufficient to make the total value of the flexible technology negative. To see this, suppose that the creditor offers identical financing costs for each technology. Then the firm borrows the same amount from the creditor, and hence the expected earnings of the creditor (without default) are identical with each technology. However, the default risk with dedicated technology is lower: Production switching is not of high value (because of high correlation) and the firm optimally invests higher capacity with the dedicated technology ($\mathbf{1}'\mathbf{K}_D^* > K_F^*$). Higher total capacity investment enables the firm to generate sufficient revenues to avoid default with the dedicated technology for some demand realizations in which the firm defaults with the flexible technology. This means that in the Pareto equilibrium that achieves expected return U for the creditor, the financing cost with dedicated technology must be lower. Consequently, the strategic value of production switching is negative, and the firm chooses the dedicated technology in equilibrium.

⁵The sign of the static value is determined by the unit cost threshold $\bar{c}_F^S(c_D)$ as we demonstrated in Proposition 4.

9 Conclusion

This paper contributes to the capacity investment literature by taking capital market imperfections into account and analyzing the interaction of a number of operational and financial decisions in a capacity investment setting. In our model, the firm makes three sequential decisions: technology choice (flexible or dedicated technology), capacity investment and production quantities. The firm's limited internal endowment depends partly on a tradable asset. The firm can borrow from a creditor to finance its operational investments in the capacity investment stage. It can also undertake financial risk management to engineer its internal endowment and reduce its borrowing needs.

The creditor offers two separate secured loan commitment contracts, one for each technology, the terms of which are determined in a Stackelberg equilibrium. If the firm defaults on its loan, the creditor is exposed to bankruptcy costs. The creditor asks for a positive expected return from lending, which we interpret as the underwriter fee. The bankruptcy costs and the underwriter fee are the capital market imperfections captured in our model and create deadweight costs of external financing for the firm.

In a parsimonious model, we solve for the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm and the creditor's optimal contracting decision in equilibrium. We characterize a perfect capital benchmark that arises naturally from our framework. In perfect capital markets, the operational and financial decisions decouple and do not interact. Using this perfect market benchmark, we delineate the effect of capital market imperfection costs and analyze the interactions between operational and financial decisions. Our main results are the following:

1. An increase in capital market imperfection costs decreases the the optimal capacity investment level and operational performance of the firm.

Driver: Higher imperfection costs lead to higher financing costs in equilibrium.

2. In a single product setting, an increase in demand variability decreases the optimal capacity investment level and the operational performance of the firm; this effect does not exist in perfect capital markets.

Driver: Higher variability leads to higher default risk and hence higher financing costs in equilibrium.

3. In a two-product setting with dedicated technology, an increase in demand variability or correlation decreases the optimal capacity investment level and the operational performance of the firm; optimal capacity investment level and the operational performance of the firm are independent of these parameters in perfect capital markets.
Driver: An increase in correlation reduces diversification benefits, which leads to a higher default risk and hence higher financing costs in equilibrium.
4. Flexible technology may have negative strategic production switching value; this value does not exist in perfect capital markets.
Driver: The adjustment in the optimal capacity investment level to exploit production switching leads to a higher default risk and hence higher financing costs in equilibrium.
5. Financial risk management has positive static value and negative strategic value; financial risk management has no value in perfect capital markets.
Driver: Financial risk management decreases the expected borrowing level of the firm for a given financing cost (static value). It also leads to a loss of revenue for the lender due to the lower expected borrowing level and hence an increase in financing costs in equilibrium (strategic value).

With these results, we contribute to the growing operations management literature that incorporates financial considerations in operational decision making. Our analysis demonstrate undocumented tradeoffs in the capacity and technology investment decisions and provide guidelines for managers concerning capacity management.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem and highlights trade-offs undocumented in this literature. In turn, by modelling operations in more detail than the finance literature, we provide novel insights on issues discussed in this literature. For example, in contrast to arguments summarized in McKay (2003), we show that operational flexibility may increase the financing costs in equilibrium. We add to the argument that financial risk management has positive strategic value (Smith and Stulz 1985) by showing that it can have negative strategic value.

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A Appendix A

Proof of Proposition 1: The proof follows from Proposition 1 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$. ■ **Proof of Proposition 2:** The proof follows from Proposition 2 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$, $E = E_T$. The open-form expressions for $\mathbf{K}_T^*(\tilde{B})$ is given by

$$\begin{aligned}
\mathbf{K}_D^0 &= \left(\left(\frac{\bar{\xi}(1 + \frac{1}{b})}{c_D} \right)^{-b}, \left(\frac{\bar{\xi}(1 + \frac{1}{b})}{c_D} \right)^{-b} \right) \\
\bar{\mathbf{K}}_D &= \left(\frac{B}{2c_D}, \frac{B}{2c_D} \right) \\
\mathbf{K}_D^1 &= \left(\left(\frac{\bar{\xi}(1 + \frac{1}{b})}{c_D(1 + a_D)} \right)^{-b}, \left(\frac{\bar{\xi}(1 + \frac{1}{b})}{c_D(1 + a_D)} \right)^{-b} \right) \\
\bar{\bar{\mathbf{K}}}_D &= \left(\frac{E_D + B}{2c_D}, \frac{E_D + B}{2c_D} \right) \\
K_F^0 &= \left(\frac{M_F(1 + \frac{1}{b})}{c_F} \right)^{-b} \\
\bar{K}_F &= \frac{B}{c_F} \\
K_F^1 &= \left(\frac{M_F(1 + \frac{1}{b})}{c_F(1 + a_F)} \right)^{-b} \\
\bar{\bar{K}}_F &= \frac{E_F + B}{c_F}.
\end{aligned}$$

where $M_F = \mathbb{E} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$. ■

Proof of Corollary 1: The proof follows from Corollary 1 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$, $E = E_T$. The expected (stage 1) equity value of the firm with a given budget level \tilde{B} follows directly from Proposition 2:

$$\pi_T(\tilde{B}) = \begin{cases} \tilde{B} + \frac{c_T \mathbf{1}' \mathbf{K}_T^0}{-(b+1)} + P & \text{if } \tilde{B} \in \Omega_T^0 \\ M_T \left(\frac{\tilde{B}}{c_T} \right)^{1 + \frac{1}{b}} + P & \text{if } \tilde{B} \in \Omega_T^1 \\ \tilde{B}(1 + a_T) + \frac{c_T \mathbf{1}' \mathbf{K}_T^1(1 + a_T)}{-(b+1)} + P & \text{if } \tilde{B} \in \Omega_T^2 \\ -E_T(1 + a_T) + M_T \left(\frac{E_T + \tilde{B}}{c_T} \right)^{1 + \frac{1}{b}} + P & \text{if } \tilde{B} \in \Omega_T^3 \end{cases} \quad (10)$$

where $M_F = \mathbb{E} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$ and $M_D = \left(\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b} \right)^{-\frac{1}{b}}$. ■

Proof of Proposition 3: The proof follows from Case (i) of Proposition 3 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$, $F_{FRM} = 0$ and $a = a_T$, $E = E_T$. ■

Proof of Proposition 4: The existence and the uniqueness of the variable cost threshold $\bar{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$, and the closed-form characterization of the symmetric case ($\bar{c}_F^S(c_D)$) follow from Proposition 4 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$, $F_{FRM} = 0$ and $a = a_T$, $E = E_T$. The dominance of investing in T^* over not making any technology investment follows from Case (i) of Proposition 11 of Boyabatlı and Toktay (2006a) by using $F_{FRM} = 0$. ■

Proof of Proposition 5: We only demonstrate the proof for the comparative static result with respect to a_T . We have

$$\frac{\partial EE_T}{\partial a_T} = -\bar{B} + c_T \mathbf{1}' \mathbf{K}_T^0 \frac{1 + \frac{a_T b}{1+a_T}}{(1+a_T)^{-b}} \quad (11)$$

for $\bar{B} \in \Omega_T^2$. We define $x \doteq 1 + a_T$ and after some algebra to (11), we obtain $P(x) = -\frac{\bar{B}}{c_T \mathbf{1}' \mathbf{K}_T^0} x^{-b+1} + (b+1)x - b$. We have

$$\frac{\partial}{\partial x} P(x) = -(b+1) \left[-\frac{\bar{B}}{c_T \mathbf{1}' \mathbf{K}_T^0} x^{-b} - 1 \right] < 0.$$

Since $P(1) > 0$ and $\lim_{x \rightarrow \infty} P(x) \rightarrow -\infty$, it follows that $\exists \underline{a}_T > 0$ (that solves $\frac{\partial}{\partial a_T} EE_T = 0$) such that $\frac{\partial}{\partial a_T} EE_T > 0$ for $a_T < \underline{a}_T$ and $\frac{\partial}{\partial a_T} EE_T < 0$ for $a_T > \underline{a}_T$. ■

Proof of Proposition 6: P_T is obtained from (6) after some algebra. We define $C_T \doteq O_T(1 + \frac{1}{b}) \left[1 - \frac{\bar{B}}{c_T \mathbf{1}' \mathbf{K}_T^0} \right]$ for technology $T \in \{D, F\}$. The proof follows by establishing $\frac{\partial}{\partial \bar{B}} C_T < 0$, $\frac{\partial}{\partial c_T} C_T < 0$ and $\frac{\partial}{\partial a_T} C_T < 0$. ■

Proof of Proposition 7: We will only consider the case $\bar{B} < c_T \mathbf{1}' \mathbf{K}_T^0$, otherwise the firm does not borrow from the creditor for any $a_T \geq 0$. Since the firm only borrows from the creditor if $\bar{B} < c_T \mathbf{1}' \mathbf{K}_T^0 (1 + a_T)^b$ (as follows from (6)), for Pareto-optimality, it is sufficient to show that the expected (stage 0) equity value of the firm with technology T strictly decreases in the unit financing cost a_T . This follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T$, $E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$. Therefore the firm prefers the smallest a_T that satisfies $\mathbb{E}[\Lambda_T(a_T)] = U$.

If there exists a feasible a'_T ($\bar{B} < c_T \mathbf{1}' \mathbf{K}_T^0 (1 + a'_T)^b$) such that $\mathbb{E}[\Lambda_T(a'_T)] \geq U$, since $\mathbb{E}[\Lambda_T(0)] < 0$ and $\mathbb{E}[\Lambda_T(a_T)]$ is a continuous function of a_T (which can be easily verified), it follows from the Mean-value theorem that such $a_T^* \leq a'_T$ always exists and is unique. If there does not exist a feasible a_T that satisfies $\mathbb{E}[\Lambda_T(a_T)] = U$ then in equilibrium the creditor does not offer any contract. ■

Proof of Proposition 8: To prove the first part of the proposition, since $E_T^* = \frac{P}{1+a_T^*}$, it is sufficient to focus on a_T^* . We will only provide the proof for the results related to the bankruptcy cost BC . The results related to the underwriter fee U follow in a similar fashion. Let $a_T^0 < \infty$ and a_T^1 denote the equilibrium financing cost with bankruptcy cost BC^0 and BC^1 with $BC^1 > BC^0$, respectively. It follows from Proposition 7 that $a_T^0 = \operatorname{argmin}_{a_T \geq 0} \mathbb{E} [\Lambda_T(a_T, BC^0)] = U$. Let us assume that the unit financing cost decreases in BC , i.e. $a_T^1 < a_T^0$. We will show by contradiction that this is not possible. Since $a_T^0 < \infty$, we also have $a_T^1 = \operatorname{argmin}_{a_T \geq 0} \mathbb{E} [\Lambda_T(a_T, BC^1)] = U$. From $\frac{\partial}{\partial BC} \mathbb{E} [\Lambda_T(a_T, BC^0)] = -P_T < 0$, we obtain $U = \mathbb{E} [\Lambda_T(a_T^1, BC^1)] < \mathbb{E} [\Lambda_T(a_T^1, BC^0)]$. It follows from *i)* $\mathbb{E} [\Lambda_T(0, BC^0)] < 0$, *ii)* the continuity of $\mathbb{E} [\Lambda_T(a_T, BC^0)]$ in a_T , and *iii)* the Mean-value theorem that there exist $a_T^2 < a_T^1$ such that $\mathbb{E} [\Lambda_T(a_T^2, BC^0)] = U$. This is a contradiction with a_T^0 being the Pareto-optimal equilibrium for BC^0 . Therefore, if there exists a_T^1 that satisfies $\mathbb{E} [\Lambda_T(a_T^1, BC^1)] = U$, we have $a_T^1 > a_T^0$. If such a_T^1 does not exist, then we have $a_T^1 = \infty > a_T^0$. In the same line of reasoning, it can be shown that if a feasible contract does not exist with BC^0 , i.e. $a_T^0 = \infty$, we also have $a_T^1 = \infty$. This concludes the proof of the first part.

For the second part, the effect of increasing BC on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T$, $E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted. ■

Proof of Proposition 9: When $U = BC = 0$, it follows from (5) and Proposition 7 that $a_T^* = \operatorname{argmin}_{a_T \geq 0} a_T \mathbb{E} [e_T] = 0$. This implies that $a_T^* = 0$ (minimum feasible unit financing cost) and from Proposition 7, we obtain $E_T^* = P$. ■

Proof of Proposition 10: If the capital markets are perfect, Proposition 9 states that $a_T^* = 0$ and $E_T^* = P$. Since $P > c_T \mathbf{1}' \mathbf{K}_T^0$ by assumption, with this financing cost scheme we obtain from Proposition 2 that $\Omega_T^1 = \Omega_T^3 = \emptyset$, $\mathbf{K}_T^1 = \mathbf{K}_T^0$, and for $\tilde{B} \in \Omega_T^2$, the expected (stage 1) equity value is $\tilde{B} + \frac{c_T \mathbf{1}' \mathbf{K}_T^0}{-(b+1)} + P$. Therefore, we have from Proposition 2 that the firm invests in the budget-unconstrained capacity investment level for any budget realization $\mathbf{K}_T^*(\tilde{B}) = \mathbf{K}_T^0$, and borrows to finance this capacity level $e_T^*(\tilde{B}) = [c_T \mathbf{1}' \mathbf{K}_T^0 + -B]^+$. It follows that expected (stage 1) equity value at each budget state $\tilde{B} > 0$ is $\tilde{B} + \frac{c_T \mathbf{1}' \mathbf{K}_T^0}{-(b+1)} + P$, and the expected (stage 0) equity value is $\bar{B} + \frac{c_T \mathbf{1}' \mathbf{K}_T^0}{-(b+1)} + P$. From Corollary 2 of Boyabatlı and Toktay (2006a), financial risk management does not have any value. With $a_D^* = a_F^* = 0$,

and $E_D^* = E_F^* = P$, Proposition 4 implies that the technology choice T^* is determined by the variable cost threshold $\bar{c}_F^P(c_D) = \bar{c}_F^S(c_D)$ and T^* is more profitable than not investing in technology option. ■

Proof of Proposition 11: With uniform $[0, 2\bar{\xi}]$ distribution of ξ , the expected returns of the creditor can be written as

$$\mathbb{E}[\Lambda(a)] = \left(cK^0(1+a)^b - \bar{B} \right) \left(a - \frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b} \right) \quad (12)$$

for a given a satisfying $\bar{B} \in \Omega_2(a)$. We first focus on the second term, the expected unit marginal profit of lending. We define $x \doteq 1+a$ and after some algebra to the second term in (12), we obtain $G(x) = -\frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}x^{-b} + x - 1$. We have $\frac{\partial}{\partial x}G(x) > 0$ for $x < \bar{x} = \left(\frac{2\bar{\xi}cK^0}{-bBC \bar{\xi}(1+1/b)} \right)^{\frac{-1}{b+1}}$ and $\frac{\partial}{\partial x}G(x) < 0$ for $x > \bar{x}$. Note that if $G(\bar{x}) < 0$ then the marginal profit is always negative and the creditor does not lend in equilibrium. This is the case if $BC > \widehat{BC} = \left(\frac{\bar{\xi}(1+1/b)^2}{c} \right)^{-b-1} \frac{2\bar{\xi}}{-b}$. We now show that for $BC \leq \widehat{BC}$ and for sufficiently small U and \bar{B} , the creditor always offers a contract and the firm borrows in equilibrium. We obtain $\lim_{x \rightarrow \infty} P(x) = -\infty$ and $\frac{\partial}{\partial x}G(x)|_{x=1} > 0$ for $BC \leq \widehat{BC}$; therefore there exist two positive roots x_1, x_2 such that $1 < x_1 < \bar{x} < x_2$. Since we focus on Pareto-optimal equilibrium, we are interested in the smallest root x_1 . Since the feasible set of the unit financing cost a is $[0, \left(\frac{cK^0}{\bar{B}} \right)^{-1/b} - 1]$, we are interested in the roots of $G(x) = 0$ in the range of $1 \leq x \leq \bar{x}$ where $\bar{x} = \left(\frac{cK^0}{\bar{B}} \right)^{-1/b}$. We now check if $a = x_1 - 1$ is feasible. For x_1 to be infeasible, i.e. $\bar{x} < x_1$, the conditions

$$\begin{aligned} G(\bar{x}) &= -\frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0} \left(\frac{cK^0}{\bar{B}} \right) + \left(\frac{cK^0}{\bar{B}} \right)^{-1/b} - 1 < 0, \\ \frac{\partial G(x)}{\partial x} \Big|_{x=\bar{x}} &= \frac{b BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0} \left(\frac{cK^0}{\bar{B}} \right)^{\frac{b+1}{b}} + 1 > 0 \end{aligned}$$

should be satisfied. The second condition is equivalent to $\bar{B} > \widehat{B} = c \left(\frac{-b BC}{2\bar{\xi}} \right)^{\frac{b}{b+1}}$. Two cases can arise:

Case i, $\bar{B} \leq \widehat{B}$: We have $\frac{\partial}{\partial x}G(x)|_{x=\bar{x}} < 0$, therefore x_1 is feasible. Since $x_1 < \bar{x}$, for $U = 0$, $a^* = x_1 - 1 < \left(\frac{cK^0}{\bar{B}} \right)^{-1/b} - 1$ is the equilibrium financing cost. For $U > 0$, if U is sufficiently small then there exists an equilibrium financing cost that satisfies $a^* > x_1 - 1$ and $a^* < \left(\frac{cK^0}{\bar{B}} \right)^{-1/b} - 1$. If U is large enough, it is not feasible to generate U because both terms in (12) are bounded.

Case ii, $\bar{B} > \hat{B}$: We have $\frac{\partial}{\partial x}G(x)|_{x=\bar{x}} > 0$. Using $G(\bar{x})$, we define

$$H(\bar{B}) \doteq -\frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0} \left(\frac{cK^0}{\bar{B}}\right) + \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1.$$

It is easy to establish that $\frac{\partial}{\partial B}H(\bar{B}) > 0$ for $\bar{B} < \hat{B}$ and $\frac{\partial}{\partial B}H(\bar{B}) < 0$ for $\bar{B} > \hat{B}$.

For $BC = \widehat{BC}$, we obtain $H(\hat{B}) = 0$ and it follows that for $BC = \widehat{BC}$, $H(\bar{B}) < 0$ for $\bar{B} > \hat{B}$ and hence $G(\bar{x}) < 0$ is satisfied for such \bar{B} . Therefore, x_1 is infeasible. In this case, the creditor offers $a^* = \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm does not borrow for $U = 0$, and the creditor does not offer a contract for $U > 0$.

For $BC < \widehat{BC}$, we have $H(\hat{B}) = 0$, therefore for some $\bar{B} > \hat{B}$, we have $G(\bar{x}) > 0$, and x_1 is feasible. For such \bar{B} , the creditor offers $a^* = x_1 - 1 < \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ for $U = 0$, and $a^* > x_1 - 1$ and $a^* < \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ for sufficiently small $U > 0$. For significantly large U , the creditor does not offer any contract. Since $\frac{\partial}{\partial B}H(\bar{B}) < 0$ for $\bar{B} > \hat{B}$, after sufficient increase in \bar{B} , we may have $G(\bar{x}) < 0$, and x_1 becomes infeasible. In this case, the creditor offers $a^* = \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm does not borrow for $U = 0$, and the creditor does not offer a contract for $U > 0$.

In summary,

1. If $BC > \widehat{BC}$ then for $U = 0$, the creditor offers a $a^* = \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm does not borrow in equilibrium; and the creditor does not offer any contract for $U \geq 0$,
2. If $BC \leq \widehat{BC}$ then
 - i. for $U = 0$, if \bar{B} is sufficiently small then the creditor offers a $a^* < \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm borrows in equilibrium,
 - ii. for $U > 0$, if \bar{B} and U is sufficiently small then the creditor offers a $a^* < \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm borrows in equilibrium,
 - iii. for $U > 0$, if \bar{B} is sufficiently small and U is sufficiently large then the creditor does not offer any contract,
 - iv. for $U = 0$, if \bar{B} is sufficiently large then the creditor offers $a^* = \left(\frac{cK^0}{\bar{B}}\right)^{-1/b} - 1$ and the firm does not borrow in equilibrium,
 - v. for $U > 0$, if \bar{B} is sufficiently large then the creditor does not offer any contract.

■

Proof of Proposition 12: We prove this result for general uniform distributions with mean $\bar{\xi}$ and support $[\bar{\xi} - a, \bar{\xi} + a]$ where $a \leq \bar{\xi}$. We use the mean-preserving spread of the uniform distribution to characterize an increase in the product market variability. For uniform distributions, this can be achieved by symmetrically increasing the support by keeping the mean constant, i.e. $[\bar{\xi} - a - \epsilon, \bar{\xi} + a + \epsilon]$ for $\epsilon > 0$. Higher ϵ leads to a higher variance of ξ_1 ⁶. Similar to (12), we have

$$\mathbb{E}[\Lambda(a)] = \left(cK^0(1+a)^b - \bar{B} \right) a - BC \left[\frac{\bar{\xi}(1+1/b) \left(1 - \frac{\bar{B}}{c_D \mathbf{1}' \mathbf{K}_D^1} \right) - (\bar{\xi} - a - \epsilon)}{2(\bar{\xi} + a)} \right]$$

for a given a satisfying $\bar{B} \in \Omega_2(a)$. We obtain

$$\frac{\partial \mathbb{E}[\Lambda(a)]}{\partial \epsilon} = \frac{BC}{2(\bar{\xi} + a)^2} \left[\bar{\xi}(1+1/b) \left(1 - \frac{\bar{B}}{c_D \mathbf{1}' \mathbf{K}_D^1} \right) - \bar{\xi} \right] < 0$$

for any a . Since the expected return of the creditor decreases in ϵ , within similar arguments of Proposition 8, it follows that $\frac{\partial}{\partial \epsilon} a^* \geq 0$. The effect of increasing a^* on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and using the identity $E = \frac{P}{1+a}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted. ■

Proof of Proposition 13: From (12), we obtain

$$\frac{\partial \mathbb{E}[\Lambda(a)]}{\partial \bar{B}} = - \left(a - \frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi} c K^0(1+a)^b} \right) \leq 0$$

for any a satisfying $\bar{B} \in \Omega_2(a)$. Since the expected return of the creditor decreases in \bar{B} , within similar arguments of Proposition 8, it follows that $\frac{\partial}{\partial \bar{B}} a^* \geq 0$. From Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and using the identity $E = \frac{P}{1+a}$, it follows that the expected (stage 0) equity value of the firm decreases in a . If the firm borrows in equilibrium, i.e. $\bar{B} \in \Omega_2(a^*)$, then the expected optimal capacity investment level $K^* = \left(\frac{\bar{\xi}(1+\frac{1}{b})}{c(1+a^*)} \right)^{-b}$ decreases in \bar{B} because of increasing a^* . If the firm does not borrow in equilibrium, i.e. $\bar{B} \in \Omega_0$, then the expected optimal capacity investment level

$$K^* = \begin{cases} \left(\frac{\bar{\xi}(1+\frac{1}{b})}{c(1+a^*)} \right)^{-b} & \text{if } \bar{B} \in \Omega_F^0 \\ \frac{\bar{B}}{c} & \text{if } \bar{B} \in \Omega_F^1 \end{cases}$$

⁶Variance is the correct indicator of risk for uniform distributions in the Rothschild-Stiglitz sense (Eckhoudt and Gollier 1995, p.82)

increases in \bar{B} . It is easy to establish that the expected (stage 0) equity value of the firm increases in \bar{B} without considering the effect on the equilibrium level of financing cost. For the effect on the equity value, we have two drivers, \bar{B} and a^* that work in opposite directions. Figure 4 demonstrates that either effect may dominate; therefore expected (stage 0) equity value of the firm may increase or decrease in the expected budget level \bar{B} . The numerical example is generated by using the parameter levels $c = 1, b = -2, P = 650, \bar{\xi} = 25, BC = 10, U = 10$. ■

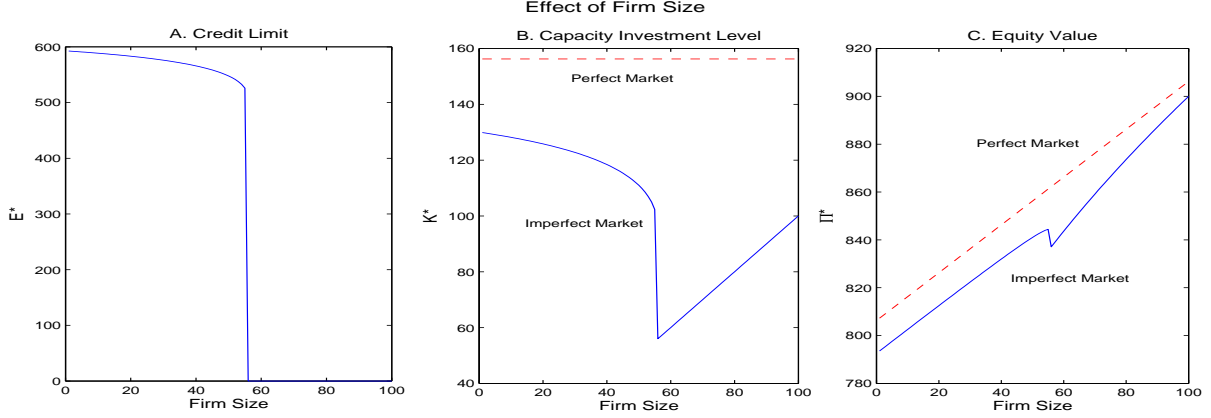


Figure 4: Increasing firm size (expected budget level) increases a^* and decreases E^* (Panel A). The creditor does not offer a loan contract after sufficient increase in \bar{B} . The capacity level decreases in \bar{B} when the firm borrows and increases in \bar{B} when the firm does not borrow (Panel B). For small levels of \bar{B} , the positive effect of an increase in \bar{B} dominates the negative effect of an increase in a^* and the equity value increases in \bar{B} . With a sharp increase in a^* , the equity value decreases (Panel C).

Proof of Proposition 14: From (12), we obtain

$$\frac{\partial \mathbb{E}[\Lambda(a)]}{\partial c} = (b+1)K^0(1+a)^b \left(a - \frac{BC\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b} \right) + \left(cK^0(1+a)^b - \bar{B} \right) \frac{(b+1)BC\bar{\xi}(1+1/b)}{2\bar{\xi}c^2K^0(1+a)^b} < 0$$

for any a satisfying $\bar{B} \in \Omega_2(a)$. Since the expected return of the creditor decreases in ϵ , within similar arguments of Proposition 8, it follows that $\frac{\partial}{\partial c} a^* \geq 0$. The proof for the effect of increasing a^* on the expected optimal capacity investment and (stage 0) equity value of the firm is similar to Proposition 12 and is omitted. ■

Proof of Proposition 15: Without financial risk management, the expected return of

the creditor is given by

$$\mathbb{E}[\Lambda(a)] = \left(a - \frac{BC \bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b} \right) \int_{\omega_0}^{cK^0(1+a)^b} \left(cK^0(1+a)^b - \tilde{B} \right) dR_{B(0)}(\tilde{B}) \quad (13)$$

where $R_{B(0)}(\tilde{B}) = R_{\alpha_1} \left(\frac{\tilde{B} - \omega_0}{\omega_1} \right)$ for $\tilde{B} \geq \omega_0$ as follows from Proposition ???. We analyze the equilibrium financing cost a_{-FRM}^* by focusing on each case of a_{FRM}^* equilibrium that we demonstrated in Proposition 11 separately.

1. For $BC > \widehat{BC}$ and $U = 0$, we have $a_{FRM}^* = \left(\frac{cK^0}{B} \right)^{-1/b} - 1$. Since the first term in (13), the expected marginal profit, is identical with financial risk management case; it follows that $a_{-FRM}^* = \left(\frac{cK^0}{\omega_0} \right)^{-1/b} - 1$ and the firm does not borrow in equilibrium. Therefore there is no strategic value of financial risk management. For $U > 0$, similar to Proposition 11, the creditor does not offer any contract and there is no strategic value of financial risk management.

2. If $BC \leq \widehat{BC}$ then

i. for $U = 0$ and sufficiently small \bar{B} , a_{FRM}^* is the financing cost that makes the marginal profit term 0 ($a_{FRM}^* + 1$ is a solution to $G(x)$ as defined in Proposition 11). Since $\left(\frac{cK^0}{\omega_0} \right)^{-1/b} - 1 > \left(\frac{cK^0}{B} \right)^{-1/b} - 1$ and the expected marginal profit term is independent of budget level, we have $a_{FRM}^* = a_{-FRM}^*$, the firm borrows in equilibrium and there is no strategic value of financial risk management.

ii. for $U > 0$, if U and \bar{B} are sufficiently small such that a finite a_{FRM}^* exists then we have $a_{-FRM}^* < a_{FRM}^*$ and there is negative strategic value of financial risk management. To prove this result, we define

$$f(\tilde{B}) \doteq \begin{cases} c_T \mathbf{1}' \mathbf{K}_T^1 - \tilde{B} & \text{if } \tilde{B} \leq c_T \mathbf{1}' \mathbf{K}_T^1 \\ 0 & \text{if } \tilde{B} > c_T \mathbf{1}' \mathbf{K}_T^1 \end{cases}$$

Since $f(\tilde{B})$ is a convex function, we have $\mathbb{E}[f(\tilde{B})] \geq f(\bar{B})$ therefore the expected amount of lending with financial risk management is lower than without financial risk management. This induces the creditor to charge $a_{-FRM}^* < a_{FRM}^*$ in equilibrium. Since the expected (stage 0) equity value is decreasing in a , it follows that $\Pi^{FRM}(a_{FRM}^*) < \Pi^{FRM}(a_{-FRM}^*)$ and there is negative strategic value.

iii. for $U > 0$, if \bar{B} is sufficiently small and U is sufficiently large such that a finite a_{FRM}^* does not exist then three cases may happen. If U is so large, even if expected lending

is higher without financial risk management and the creditor needs to charge lower a to attain U , such an a may not be feasible and the creditor does not lend in equilibrium without financial risk management either, i.e. $a_{FRM}^* = a_{-FRM}^* = \infty$. In this case, there is no strategic value of financial risk management. Since expected lending amount is higher without financial risk management, a finite a_{-FRM}^* may exist. Since $a_{FRM}^* \rightarrow \infty$ and the firm does not borrow, such a_{FRM}^* has the same effect with $a_{FRM}^* = \left(\frac{cK^0}{B}\right)^{-1/b} - 1$ because the firm does not borrow at this financing cost either. If $a_{-FRM}^* > \left(\frac{cK^0}{B}\right)^{-1/b} - 1$, then we have $\Pi^{FRM}(a_{FRM}^*) - \Pi^{FRM}(a_{-FRM}^*) = 0$ because the firm does not borrow at each financing cost with financial risk management. Therefore, there is no strategic value. If the increase in expected lending is sufficiently high, then we may have $a_{-FRM}^* < \left(\frac{cK^0}{B}\right)^{-1/b} - 1$. In this case, the firm borrows at a_{-FRM}^* with financial risk management, and since $a_{-FRM}^* < a_{FRM}^*$ we have negative strategic value of financial risk management.

iv. for $U = 0$, if \bar{B} is sufficiently large such that there is no feasible a that makes the marginal profit term with financial risk management equal to 0, two cases may happen. Recall from Proposition 11) that $\bar{B} > \hat{B} = c \left(\frac{-bBC}{2\xi}\right)^{\frac{b}{b+1}}$ should hold for this case to be satisfied. We have $\bar{B} = \omega_0 + \bar{\alpha}_1\omega_1$. If ω_0 is sufficiently large, an equivalent condition $\omega_0 > c \left(\frac{-bBC}{2\xi}\right)^{\frac{b}{b+1}}$ can be satisfied without financial risk management. If ω_0 also satisfies the second necessary condition for infeasibility, $-\frac{BC\xi(1+1/b)}{2\xi cK^0} \left(\frac{cK^0}{\omega_0}\right) + \left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1 < 0$ then there is no feasible a that makes the marginal profit term without financial risk management equal to 0. In this case we obtain $a_{-FRM}^* = \left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1$, the firm does not borrow in equilibrium and there is no strategic value of financial risk management. If ω_0 is not very large, then the infeasibility conditions cannot be satisfied. In this case there exists a $a_{-FRM}^* < \left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1$. We have $a_{FRM}^* < a_{-FRM}^*$ otherwise there should be a $a_{FRM}^* < \left(\frac{cK^0}{B}\right)^{-1/b} - 1$ that makes the marginal profit term with financial risk management equal to 0. Similar to Case *iii*, there is no strategic value of financial risk management.

iv. for $U > 0$, if \bar{B} is sufficiently large such that there is no feasible a that makes the marginal profit term with financial risk management equal to 0, without financial risk management either there is a finite $a_{-FRM}^* < \left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1$ or the creditor does not offer a contract. The proof is similar to Case *iv* and is omitted. In either case, there

is no strategic value of financial risk management.

In the proposition, 1 corresponds to Cases *i* and *iv*, 2 corresponds to Case *ii*, 3 to Case *iii* and 4 to Case *v*. This concludes the proof. ■

Proof of Proposition 16: It follows from (10) that the expected (stage 0) equity value of the firm is independent of the covariance matrix Σ for a given financing cost scheme (a_T, E_T) . From Propositions 5 and 6, we obtain

$$\mathbb{E}[\Lambda_D(a_D)] = (c_D \mathbf{1}' \mathbf{K}_D^1 - \bar{B}) a_D - BC \Pr \left(\xi_1 + \xi_2 < 2\bar{\xi} \left(1 + \frac{1}{b} \right) \left[1 - \frac{\bar{B}}{c_D \mathbf{1}' \mathbf{K}_D^1} \right] \right)$$

where $\mathbf{K}_D^1 = \left(\left(\frac{\bar{\xi}(1+\frac{1}{b})}{c_D(1+a_D)} \right)^{-b}, \left(\frac{\bar{\xi}(1+\frac{1}{b})}{c_D(1+a_D)} \right)^{-b} \right)$. The first term (EE_D) is also independent of Σ and the effect of the demand correlation ρ and the demand variability σ is inherent in the expected default cost ED_D . If (ξ_1, ξ_2) is bivariate normal with $N(\boldsymbol{\xi}, \Sigma)$, $\xi_1 + \xi_2$ is also Normally distributed with mean $2\bar{\xi}$ and standard deviation $\bar{\sigma} = \sigma\sqrt{2(1+\rho)}$. Recall from the proof of Proposition 6, C_D denotes the righthand side of the default probability P_D . Since $b < -1$ and $\bar{B} < c_D \mathbf{1}' \mathbf{K}_D^1$, we obtain $C_D < 2\bar{\xi}$. We have $P_D = \Pr(\xi_1 + \xi_2 < C_D) = \Phi\left(\frac{C_D - 2\bar{\xi}}{\bar{\sigma}}\right)$ where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable. For an arbitrary a_D that satisfies $\bar{B} \in \Omega_D^2(a_D)$, since $\frac{\partial}{\partial \rho} \bar{\sigma} = \frac{\sigma}{\bar{\sigma}} > 0$, $\frac{\partial}{\partial \sigma} \bar{\sigma} = \sqrt{2(1+\rho)} > 0$, and $C_D < 2\bar{\xi}$ it follows that

$$\begin{aligned} \frac{\partial \mathbb{E}[\Lambda_D(a_D)]}{\rho} &= -BC \phi\left(\frac{C_D - 2\bar{\xi}}{\bar{\sigma}}\right) \left(\frac{2\bar{\xi} - C_D}{\bar{\sigma}^2}\right) \frac{\partial \bar{\sigma}}{\partial \rho} < 0, \\ \frac{\partial \mathbb{E}[\Lambda_D(a_D)]}{\sigma} &= -BC \phi\left(\frac{C_D - 2\bar{\xi}}{\bar{\sigma}}\right) \left(\frac{2\bar{\xi} - C_D}{\bar{\sigma}^2}\right) \frac{\partial \bar{\sigma}}{\partial \sigma} < 0 \end{aligned}$$

where $\phi(\cdot)$ is the density function of the standard normal random variable. Since the expected return of the creditor decreases in ρ and σ , within similar arguments of Proposition 8, it follows that $\frac{\partial}{\partial \rho} a_D^* \geq 0$ and $\frac{\partial}{\partial \sigma} a_D^* \geq 0$. The effect of increasing a_D^* on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabathı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and $a = a_D$, $E = E_D$ and using the identity $E_D = \frac{P}{1+a_D}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted.

■

Proof of Proposition 17: Since $\boldsymbol{\xi}$ has a symmetric bivariate distribution, with perfect position correlation ($\rho = 1$), we have $\Pr(\xi_1 = \xi_2) = 1$. In this case, we obtain from Proposition 6 that $H_F(\boldsymbol{\xi}) = H_D(\boldsymbol{\xi})$ for any $\boldsymbol{\xi}$ realization and $M_F = M_D$. With the identical

financing cost scheme $a_F = a_D$ and $E_F = E_D$, we have $\bar{c}_F^S(c_D) = c_D$ from Proposition 4 and $c_D \mathbf{1}' \mathbf{K}_D^1 = c_F K_F^1$ for $c_F = \bar{c}_F^S(c_D)$ from the proof of Lemma 7 of Boyabatlı and Toktay (2006a). Therefore, we have $P_F = P_D$ and $EE_D = EE_F$ for an arbitrary $a_D = a_F$. Since the expected returns of the creditor is identical with both technologies for a given unit financing cost, the creditor offers the same unit financing cost for both technologies ($a_D^*(\mathbf{K}_D^*) = a_F^*(K_F^*)$) in equilibrium. From Lemma 7 of Boyabatlı and Toktay (2006a), we conclude that the firm's optimal total capacity investment decision $\mathbf{1}' \mathbf{K}_T^*$, and the expected (stage 0) equity value of the firm $\pi_T(\mathbf{K}_T^*; a_T^*(\mathbf{K}_T^*))$ is identical for each technology. ■

Proof of Proposition 18: With the identical financing cost scheme $a_F = a_D$ and $E_F = E_D$, for $c_F = \bar{c}_F^S(c_D)$, the firm is indifferent between two technologies (which follows from Proposition 4) and we have $c_D \mathbf{1}' \mathbf{K}_D^1 = c_F K_F^1$ from the proof of Lemma 7 of Boyabatlı and Toktay (2006a). Therefore, with identical financing cost scheme, the expectation earning with each technology is the same ($EE_D = EE_F$) for the creditor. The default risk P_T comparison at the identical financing cost scheme (off-the-equilibrium path) between two technologies determines the ordering between the equilibrium level of unit financing costs, which in turn determines the technology choice of the firm in equilibrium. Figure 5

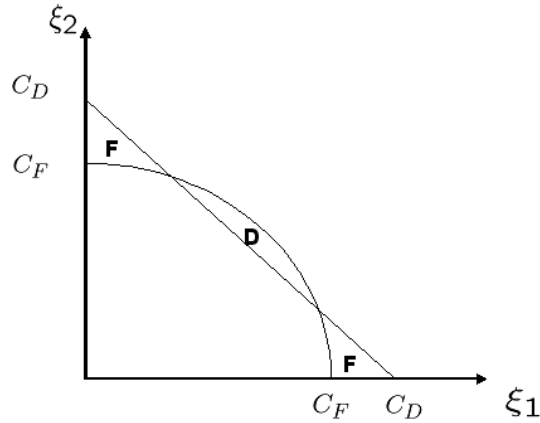


Figure 5: Default regions in (ξ_1, ξ_2) space with each technology: C_D and C_F are the right-hand side terms in the default probability as defined in the proof of Proposition 6. The area below the straight line (curve) is the default region with the dedicated (flexible) technology. F (D) represents the ξ realizations that the firm does not default with the flexible (dedicated) technology and defaults with the other technology.

demonstrates the default region with each technology with identical financing cost scheme for $c_F = \bar{c}_F^S(c_D)$. The overall default probability P_T is determined by superimposing the

ξ distribution and taking the expectation over the regions. For the technology cost pair $(c_F^S(c_D; \rho), c_D)$, since $(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} < ((\xi_1 + \xi_2)^{-b})^{-\frac{1}{b}}$ for any ξ realization, it follows that $M_F < 2\bar{\xi}$ and we have $C_D = C_F \frac{2\bar{\xi}}{M_F} > 1$. Therefore the region denoted with F always exists around the points $(0, C_D)$ and $(C_D, 0)$. The point $(\frac{C_D}{2}, \frac{C_D}{2})$ that is on the default line of the dedicated technology is in the default region of the flexible technology if $(\frac{C_D}{2}^{-b} + \frac{C_D}{2}^{-b})^{-\frac{1}{b}} \leq C_F$. For the technology cost pair $(c_F^S(c_D; \rho), c_D)$ this condition is equivalent to $2^{-\frac{1}{b}}\bar{\xi} \leq M_F$. It follows from Proposition 4 of Boyabatlı and Toktay (2006a), this condition is satisfied with equality only if $\rho = 1$, otherwise the inequality is always satisfied for $\rho \neq 1$. Therefore, D region only exists if $\rho \neq 1$. With close to perfect positive correlation, all the ξ realizations are located around $\xi_1 = \xi_2$ line (which also passes through the point $(\frac{C_D}{2}, \frac{C_D}{2})$). It follows that after taking the expectation over the default regions, we obtain $P_D < P_F$ because of the existence of region D . Since we have $P_D < P_F$ for identical financing cost scheme, the creditor charges lower financing cost for the dedicated technology in equilibrium $a_D^* < a_F^*$. It follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T, E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$ that the expected (stage 0) equity value of the firm decreases in a_T ; therefore the firm chooses the dedicated technology in equilibrium. ■

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