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A Stochastic Dynamic Programming Approach to Revenue Management in a Make-to-Stock Production System

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7th Annual INFORMS Revenue Management and Pricing Section Conference
June 29 2007, Barcelona

Agenda

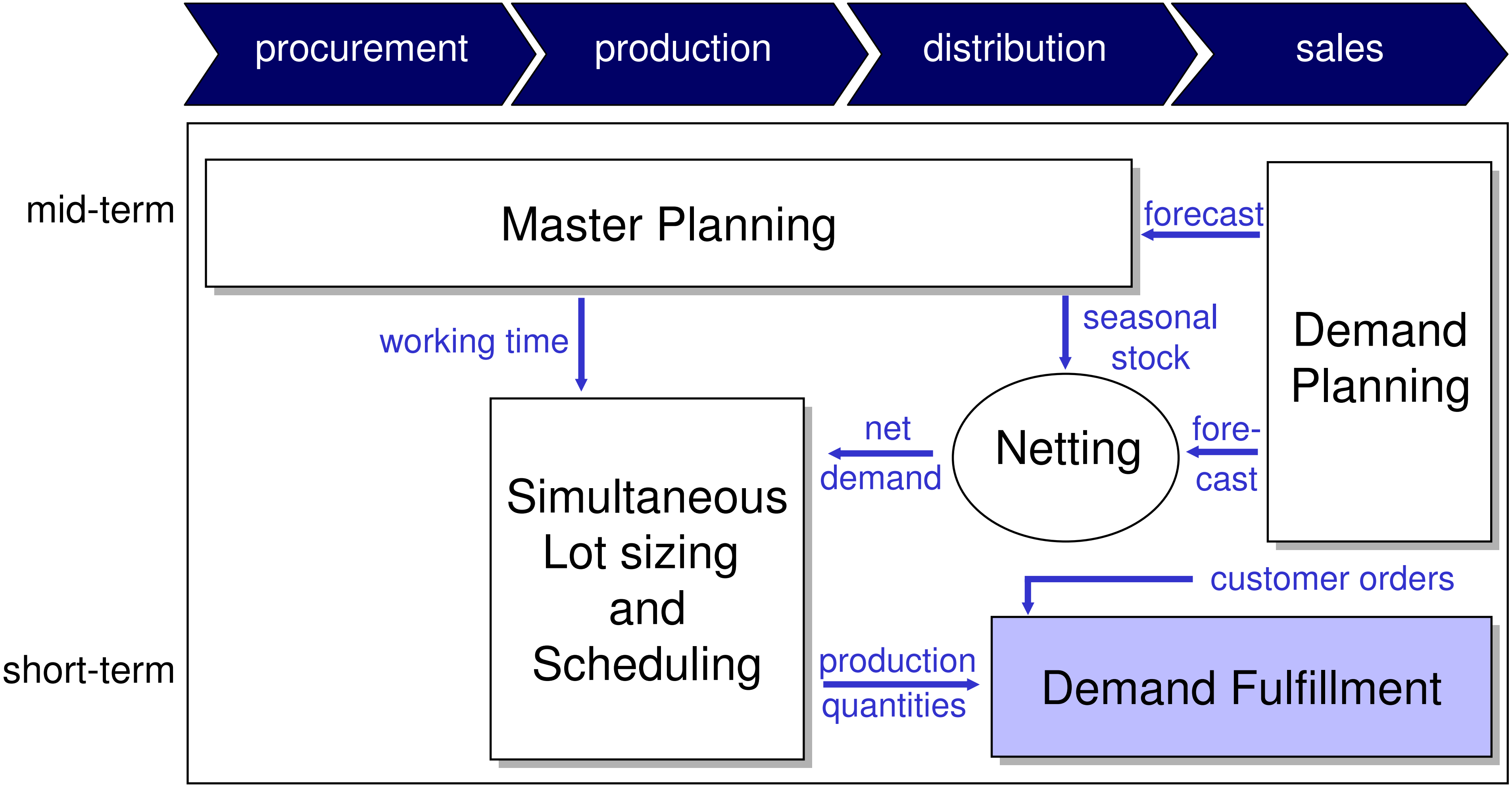
- Introduction
  - Problem description
  - Motivation
- Mathematical formulation
- Structural properties
- Outlook

Description of the problem

- **Scarce** capacity
- **Make-to-stock** manufacturing environment
- Consideration of **future incoming supply**
- Customers with **different priorities**
- Immediate order confirmation required
- Refusal or acceptance of incoming customer orders
- Service level of 98 % ⇨ 2% backorders or lost sales

General question to be answered:
Is it better to refuse a low profit customer order in expectance of future more profitable orders?

Example: Use of APS in Make-to-Stock Supply Chains



Fleischmann / Meyr / Wagner (2004)

Research motivation:

- Demand Fulfillment in **A**dvanced **P**lanning **S**ystems (APS)
  - Partly simple solution methods (as FCFS)
  - Mostly deterministic
- Why not using methods of **Revenue Management**?

Related literature:

- ATP research: (e.g. Fleischmann / Meyr 2004)
  - **Deterministic** OR methods or simple rules
  - Mostly concentrates on **batch ordering**
- Revenue Management: (e.g. Talluri / van Ryzin 2004)
  - Mostly in service industries and retail
  - Uses **stochastic** demand distributions
  - Generally **fixed capacity** and no storage of inventory

Dynamic Programming Formulation

- **State** $\bar{x} = (x_1,\ldots,x_T)$:

  Remaining quantities of arriving supplies

  (Note that $\bar{x}$ has as many elements as there are number of periods)

- **Action** $\bar{u} = (u_1,\ldots,u_T)$:

  Quantities to accept from each arriving supply

- **Value function** $V_t(\bar{x})$:

  Maximum expected profit-to-go

Dynamic Programming Formulation

$$V_t(\bar{x}) =$$

$$E[\max_{\substack{\bar{u} \\ 0 \le u_i \le x_i, \sum_i^T u_i \le d}} \{\sum_i^T (u_i P(t,i,c) - h(x_i - u_i)\delta_{it}) + V_{t+1}(\bar{x} - \bar{u})\}]$$

$$P(t,i,c) = p_c - b(i-t)^+ \qquad \delta_{it} = \begin{cases} 1 & \text{if } i \le t, \\ 0 & \text{otherwise.} \end{cases} \qquad V_{T+1}(\bar{x}) = 0 \quad \forall \bar{x}$$

d = 1,…,D	Demand **(Random variable)**
c = 1,…,C	Customer classes **(Random variable)**
P(t,i,c)	Profit dependent on period t, supply arriving in period i and customer class c
b	**Backlogging costs** for one unit of demand in one period
h	**Holding costs** for one unit of supply
p_c	**Price** paid by customer class c
$\bar{e}_i$	i-th unit vector

Model Assumptions

- Combination of **static** and **dynamic** RM-models*
  - Static models
    - Sequential arrival of customer classes
    - More than one arrival per period
  - Dynamic models
    - One arrival per period
    - Arbitrary arrival of customers
- **Arbitrary arrival** of customers classes
- **Partial fulfillment** allowed
- Incoming date = Desired delivery date
- Only one arrival per period, **order quantity can be larger than one**

*(Lautenbacher / Stidham 1999)

Redraft of Value Function in Terms of Marginal Profits

$$V_t(\bar{x}) = E[\max_{\substack{\bar{u} \\ 0 \le u_i \le x_i, \sum_i^T u_i \le d}} \{\sum_i^T (u_i P(t,i,c) - h(x_i - u_i)\delta_{it}) + V_{t+1}(\bar{x} - \bar{u})\}] =$$

$$= V_{t+1}(\bar{x}) - \sum_i^T (h x_i \delta_{it}) + E[\max_{\substack{\bar{u} \\ 0 \le u_i \le x_i, \sum_i^T u_i \le d}}$$

$$\{\sum_i^T (\sum_{z=1}^{u_i} (P(t,i,c) + h\delta_{it} - \Delta_i V_{t+1}(\bar{x} - \bar{e_i}(z-1) - \sum_{j=0}^{i-1} (\bar{e_j} u_j)))\}]$$

Definition: $\Delta_i V_t(\bar{x}) \equiv V_t(\bar{x}) - V_t(\bar{x} - \bar{e_i})$

Property I:

$$\sum_{i}^{I} u_i^*(\bar{x} + \Delta\bar{x}) \geq \sum_{i}^{I} u_i^*(\bar{x})$$

"Never sell less when you have more"

Property II:

$$x_i \leq x_i^{'} \; \forall i$$

$$V_t(\bar{x} + \Delta\bar{x}) - V_t(\bar{x}) \geq V_t(\bar{x}^{'} + \Delta\bar{x}) - V_t(\bar{x}^{'})$$

Decreasing differences: "The more you have, the less the marginal value of each unit"

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Property III: $\forall m < n$

$$\Delta_m V_t(\bar{x}) - \Delta_n V_t(\bar{x}) \leq b(n-m+(t-n)^+ + (m-t)^-)$$

The difference in marginal values of different supplies is bounded. (Required to prove property IV)

Property IV: $\forall m < n$

$$P(t,m,c) - \Delta_m V_{t+1}(\bar{x}) + h\delta_{mt} \geq P(t,n,c) - \Delta_n V_{t+1}(\bar{x}) + h\delta_{nt}$$

"It is always better to take a unit of the soonest arriving supply or the one on hand (supply m), instead of taking one arriving later on (supply n)"

$$\Delta_m V_t(\bar{x}) - \Delta_n V_t(\bar{x}) \leq b(n - m + (t-n)^+ + (m-t)^-) \Rightarrow$$

$$V_t(\bar{x} - \bar{e_n}) - V_t(\bar{x} - \bar{e_m}) \leq b(n - m + (t-n)^+ + (m-t)^-)$$

Let $\bar{u}^*$ be an **optimal decision** for a capacity vector $\bar{x}$ - $\bar{e}_n$ in period *t:*

$$V_t^{\bar{u}^*}(\bar{x} - \bar{e_n})$$

Case (I): $u_m^* > 0$

Let $\bar{u}^*$ - $\bar{e}_m$ + $\bar{e}_n$ be a **feasible decision** for a capacity vector $\bar{x}$ - $\bar{e}_m$ in period t:

$$V_t^{\bar{u}^*}(\bar{x} - \bar{e_n}) - V_t^{\bar{u}^* - \bar{e_m} - \bar{e_n}}(\bar{x} - \bar{e_m}) =$$

$$E[P(t,m,c) - P(t,n,c)] = b(n - m + (t-n)^+ + (m-t)^-)$$

Any feasible solution is less or equal to the optimal solution:

$$\Rightarrow V_t(\bar{x} - \bar{e_n}) - V_t(\bar{x} - \bar{e_m}) \leq b(n - m + (t-n)^+ + (m-t)^-)$$

Sketch of the Proof (contd.)

Case (II): $u_m^* = 0$

Let $\bar{u}^*$ be an **optimal decision** for a capacity vector $\bar{x} - \bar{e}_n$ in period t, and

let $\bar{u}^*$ be a **feasible decision** for a capacity vector $\bar{x} - \bar{e}_m$ in period t.

$$V_t^{\bar{u}^*}(\bar{x} - \bar{e}_n) =$$

$$\mathop{E}_{\bar{u}^*}[\cdots + u_m P(t,m,c) - h(x_m - u_m)\delta^{\leq}_{mt} + \ldots$$

$$+u_n P(t,n,c) - h(x_n - 1 - u_n)\delta^{\leq}_{nt} + \cdots + V_{t+1}(\bar{x} - \bar{e}_n - \bar{u}^*)]$$

$$V_t^{\bar{u}^*}(\bar{x} - \bar{e}_m) =$$

$$\mathop{E}_{\bar{u}^*}[\cdots + u_m P(t,m,c) - h(x_m - 1 - u_m)\delta^{\leq}_{mt} + \ldots$$

$$+u_n P(t,n,c) - h(x_n - u_n)\delta^{\leq}_{nt} + \cdots + V_{t+1}(\bar{x} - \bar{e}_m - \bar{u}^*)]$$

≤ 0

Induction Assumption

Structure of Optimal Control

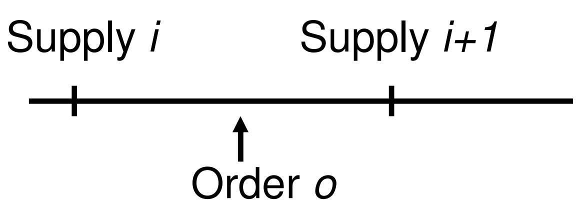
Fulfill if the following holds:

$$P(t,i,c) - \Delta_i V_{t+1}(\bar{x}) + h\delta_{it} \geq 0$$

Order Fulfillment Algorithm:

o = Arriving order with immediate delivery
i = supply on hand or soonest arriving supply

REPEAT WHILE $P(t,i,c) - \Delta_i V_{t+1}(\bar{x}) + h\delta_{it} \geq 0$ AND *o* is not fulfilled
    IF there are still units available from supply *i*
        Fulfill one unit of order *o*
    ELSE
        i = Next available supply
    END IF
END REPEAT



Outlook

- Complete the proofs
- Inventory and backlogging costs with dependence on customer classes
- Comparing results with existing deterministic approaches

Literature

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Thank you!

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Research funded by the
Vienna Science and Technology Fund

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Properties and Optimal Policy

u_2 \ u_1	0	1	2	3
0	20	25	30	28
1	24	29	26	25
2	28	25	24	22
3	23	22	21	20

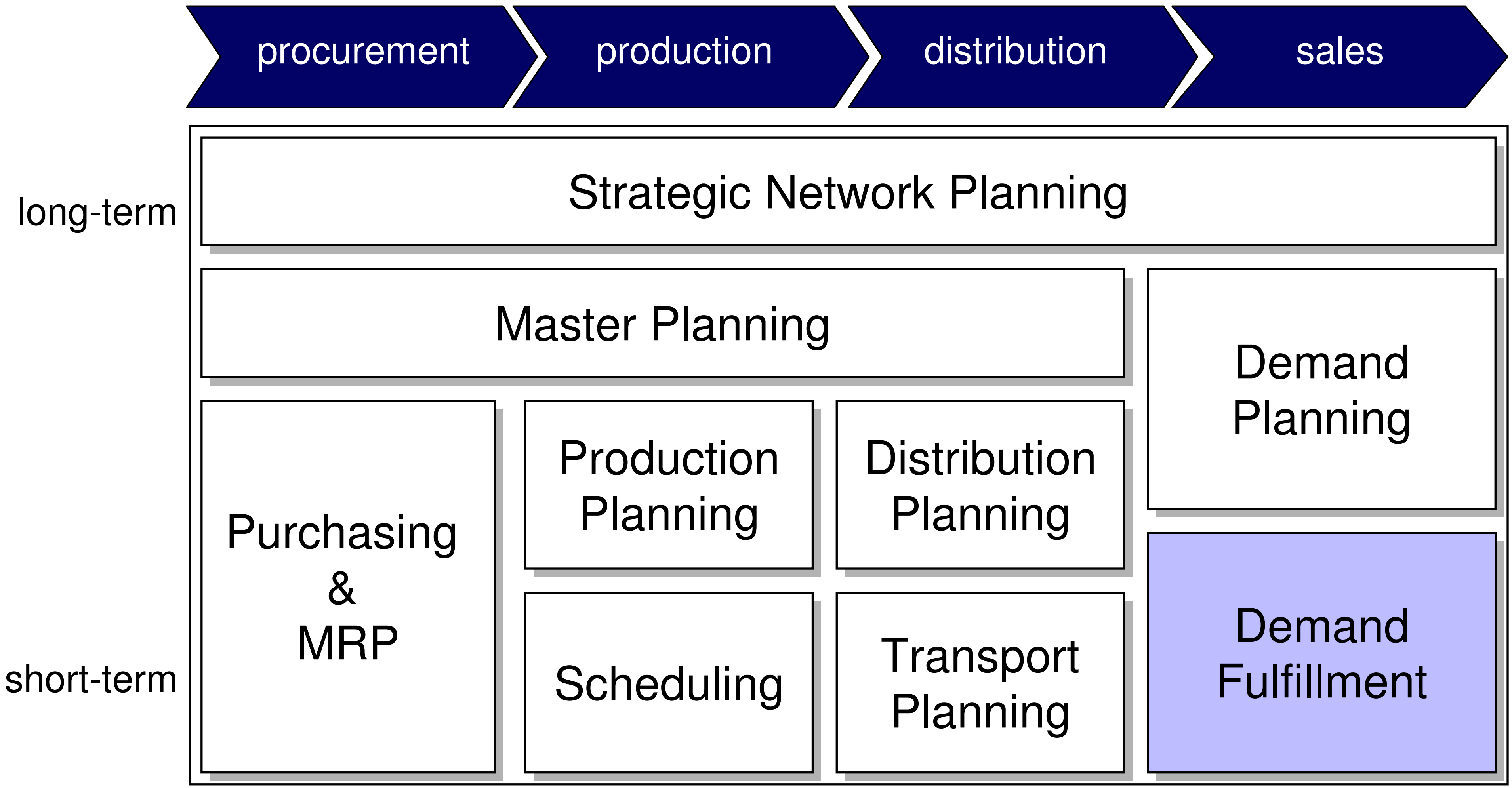
Search path to find optimum

Property II:
"In each row and each column there is a unique maximum"

Property 4:
"It is always better or at least equally good to take an earlier supply than a later one"

Note that it is not possible that the maximum lies inside the table!

Structure of Advanced Planning Systems: *Supply Chain Planning Matrix*



Rohde / Meyr / Wagner (2000)