Predestination and the Protestant Ethic

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Abstract

This paper relates the secular premise that wealth accumulation is a moral obligation and the religious dogma that salvation is immutable and preordained by God. This result formalizes Weber’s renowned thesis on the connection between the worldly asceticism of Protestants and the religious doctrines of Calvinism.

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1 Introduction

This paper relates Weber’s thesis on the religious origins of the Protestant ethic and the Kreps-Porteus (1978) model of preferences for early resolution of uncertainty. In The Protestant Ethic and the Spirit of Capitalism (2002, first published as a two part article 1904-05), Max Weber argues that Protestantism fosters an environment conducive to economic development because it instills the idea that earning and accumulating wealth is a moral obligation. He further argues that the Protestant ethic originated in religious beliefs and, in particular, in the Calvinist doctrine of predestination: the idea that some humans are in a state of grace and destined to salvation while others are damned, the choice being immutable and preordained by God.

Weber’s thesis is one of the most celebrated, and polemic, works of social science because it puts forward an appreciation of metaphysical ideas as effective forces in the formation and development of economies (see, for instance, Acemoglu, Johnson and Robinson (2005a)). Weber’s work has formed the basis of a large empirical and historical literature (see Guiso, Sapienza and Zingales (2006) and McCleary and Barro (2006)). In addition, models of the Spirit of Capitalism – i.e., models which include non-instrumental utility of wealth – have also contributed to the understanding of asset prices (Bakshi and Chen (1996)), business cycles (Karnizova (2010)), capital accumulation and savings patterns ((Zou (1995), Carroll (2000), Francis (2009)), and economic growth (Smith (1999)).

Weber notes that the Protestants’ pursuit of wealth for its own sake is perplexing to the point of appearing irrational. But while these preferences may seem odd, it is straightforward to capture them formally. It suffices to assume a direct expected utility for wealth, independent of consumption or any other benefit that wealth may bring about (such as power, security or prestige). We model the intrinsic value of wealth with utility functions that takes the form

\[ Ev(w^i), \]

where \( v \) is an increasing function and \( w^i \) is agent \( i \)'s earned wealth.\(^1\) This intrinsic utility for earned wealth is assumed to be above and beyond the instrumental value that wealth also delivers in procuring consumption and other benefits. It captures a moral obligation to accumulate wealth or, more broadly, any motivation for the pursuit of wealth for its own sake. Hence, a model of pursuit of wealth for its own sake can be easily achieved within the standard von Neumann-Morgenstern expected utility framework.

\(^1\)Wealth must be earned, according to this form of ethics- receiving wealth through a transfer, for instance, would not enter this function.
It is common to refer to Weber to motivate utility functions of the form (1). However, the centerpiece of Weber’s work is the connection between these utility functions and religious ideas. Hence, to provide foundations for utility functions that take the form (1), it is essential to show that they are related to Weber’s description of Calvinism. A proper model of Calvinism is less straightforward than a proper model of the Protestant work ethics. One difficulty is that Calvinists often lived a frugal life of hard work and, according to their faith, an ascetic lifestyle does not deliver any reward of salvation. This follows because Weber’s focus is on the doctrine of predestination for which salvation is predetermined by God and not influenced by any action that the Church or the individual might take. Thus, what motivated Calvinists to austerity is not obvious. We follow Weber’s perspective and assume that Calvinists believe that grace is conducive to success in business; that wealth is a sign, never a means, of grace; and that the signs of grace are more informative if more wealth is earned, saved and reinvested as capital in risky enterprises. Finally, Calvinists are motivated by the need to obtain informative signs of grace in their lifetime. The Kreps and Porteus (1978) model of recursive expected utility is crucial to formally model these ideas because it can accommodate a strict preference for early resolution of uncertainty (e.g., a preference for informative signs of grace during one’s lifetime rather than in the afterlife). This preference for early resolution of uncertainty is above and beyond any instrumental value of early acquisition of information, as it must be in the case of an immutable state such as grace. Accordingly, we model Calvinists with Kreps-Porteus utility functions of the form

\[ Eu_C(p(g^i = 1|\gamma^i, s^i)) , \]

where \( u_C \) is an increasing and convex function, and \( p(g^i = 1|\gamma^i, s^i) \) is agent \( i \)'s Bayesian posterior belief of grace, contingent on his savings and the idiosyncratic shock on his investments. This specification captures the idea that Calvinists are anxious about their uncertainty over being in grace or not, that they prefer resolving this uncertainty sooner rather than later, and that they learn about grace through their successes.

The formal models given by (1) and (2) permit a formal comparison between the preferences of secular Protestants and religious Calvinists. This connection is the central point in Weber’s work and it is also the focus of this paper. However, Weber claims that the preferences of secular Protestants originated from the religious premises of Calvinism.

\footnote{Unlike the existing literature, in our model, Calvinists do \textit{not} learn about their state of grace directly by their actions. For example, a high saving rate, by itself, is not a sign of grace. Instead, actions may change the informativeness of the signs of grace (that arise from the results of business projects).}

\footnote{These assumptions model Weber’s perspective on practiced Calvinism. They do not model the theology of Calvin which did not allow for a sign of grace, with the notable exception of faith itself.}
We do not examine any causality link between Calvinism and Protestantism. Instead, we compare the logical structure of their preferences and behavior. Thus, our results are broader than the interpretation we provide. For example, the preferences of any agent who pursues wealth for its own sake (whether or not this agent finds it moral) can be captured by (1). We focus on a narrow interpretation of (1) and (2) to emphasize the connection with Weber’s thesis.

Model (1) of Protestant Ethics captures a desire of wealth for its own sake and stays within the bounds of standard expected utility theory. In contrast, model (2) of Calvinism must step outside standard expected utility theory to capture a desire for early signs of grace. This critical aspect of Calvinism makes their preferences seem behaviorally distinguishable from Protestant preferences.

The fundamental difference between Protestants and Calvinists is expressed by the distinct axiomatic structures of models (1) and (2). The Kreps-Porteus foundation is not the same as that of the more restrictive von Neumann-Morgenstern expected utility. In addition, a direct glance at (1) and (2) may suggest further differences (e.g., $u_c$ must be convex, but $v$ need not be). The key difficulty is that Weber’s work is centered at the connection between the behavior of Protestants and Calvinists. However, the logical structures behind the two models seem at odds with each other.

Our analysis resolves this difficulty. Our main result shows that despite the differences in interpretation, axiomatic foundations and functional forms between models (1) and (2), there is an exact equivalence between the two classes of utility functions. This result formalizes Weber’s thesis of a deep connection between secular agents who abide by the Protestant ethic and religious agents who believe in the Calvinist doctrine of predestination. Under suitable assumptions, their behavior cannot be distinguished. This holds in spite of the robustness and the lack of ad-hoc assumptions in the broad class of utility functions included here. Thus, the seemingly naive, but highly tractable, model given by (1) has foundations based on Weber’s thesis. The intuition behind this result is rooted in the nature of religious premises. In particular, the fact that grace is not directly observable is critical for the result.

Both economic history studies of Calvinism and a contemporary analysis of societies influenced by the Protestant ethic can make use of the simple model given by (1). The behavior of Calvinists need not be understood with the unusual utility functions such as those given by (2). Instead, their behavior can be understood with traditional expected utility functions. Our equivalence theorem provides the logical structure of this connection, and therefore provides a foundation for the vast literature that uses direct preferences for wealth. The connection, which holds to a surprising extent, between the
utility functions of Calvinists and Protestants naturally raises the question of whether our model can be modified so that Calvinist preferences can be distinguished from Protestant work ethic preferences. We discuss modeling choices which could be pursued to make this distinction in Section 3.1.

Models of the Spirit of Capitalism often use utility over relative wealth; see, for instance, Bakshi and Chen (1996) and Zou (1995). ‘Keeping up with the Joneses’ preferences are useful to accommodate several well-known patterns in macroeconomics and finance (e.g., Campbell and Cochrane (1995)). A simple variation of our analysis provides a foundation for these preferences as well.

Finally, our model allows for a broader interpretation than the one we focus on. In particular, abstracting away from Calvinism and the Protestant Ethics, our result shows that under the assumptions of our model, a preference for earned wealth is behaviorally indistinguishable from a preference for reducing uncertainty over a binary variable which the agent cannot directly observe (e.g. untested ability).

This paper is structured as follows: Section 1.1 presents a brief literature review and Section 2 introduces the model and the different types of agents. Section 3 presents the main equivalence result. Section 4 discusses the assumptions on Calvinists and Section 5 extends the model to include relative wealth aspects. Section 6 concludes.

1.1 Related literature

A large literature, which we do not survey here, studies Weber’s thesis from an empirical and historical viewpoint (see, among many contributions, Guiso, Sapienza and Zingales (2003, 2006) for a discussion on culture and economic outcomes and Blum and Dudley (2001), McCleary and Barro (2006) for a survey on religion and economics, Ekelund, Hebert and Tollison (2002), Cavalcanti, Parente and Zhao (2007), Becker and Woessman (2009), Cantonii (2015), Arruñada (2010) for discussions, literature reviews and empirical analyses of Weber’s thesis. See also Glaeser and Glendon (1998) for a model and empirical study of predestination compared to free will). These papers frequently use non-instrumental utility of wealth. Our model provides a justification, based on Weber’s thesis, for the use of these preferences. Recent theoretical work has also yielded valuable insight into understanding Calvinism. In particular, Levy and Razin (2014a) examine self versus social signaling for Calvinists’ players in a repeated game, and Dal Bó and Terrvio (2013) consider a model of internal reputation in which moral choices directly affect self-esteem.

One view of Calvinists is that their anxieties over their salvation led them to confuse
the causes and consequences of their actions, thereby succumbing into a form of “magical thinking” (see Elster (2007) and Quattrone and Tversky (1984)). In contrast, Weber upholds that the intense psychological impact of Calvinism was in part due to its “iron consistency” and lack of vacillation between competing dogmas. Weber’s perspective on the rationality of Calvinism found support in the recent literature. For example, Bodner and Prelec (2003) note that their model of self-diagnosis may apply to Calvinists if they had a preference for self-signaling. Benabou and Tirole (2004, 2006, 2011) discuss an application of their framework to Calvinism, under the interpretation that increases in effort is directly indicative of grace. These models show that Calvinists need not be seen as irrational. Instead, Calvinists are interpreted as dual selves agents in which one self of the agent learns about himself from the behavior of his other self (see also Brunnermeier and Parker (2005) and Koszegi (2006) for planner-doer self models). While our approach does not rely on multiple selves, we also follow Weber’s perspective and do not allow for inconsistencies in the preferences, beliefs and behavior of the Calvinists.


Our main objective differs from the existing literature. We formalize Weber’s thesis on the connection between the religious doctrine of predestination and the secular Protestant ethic. This motivation is not shared by any theoretical paper that we know of.

This paper also relates to models in which agents have an intrinsic preference for
information. While we do not provide an exhaustive review of the literature, the seminal paper in this field is the Kreps-Porteus (1978) framework that we use. Grant, Kajii and Polak (1998) generalize the Kreps-Porteus (1978) model, and characterize the relation between a preference for early resolution of uncertainty and risk aversion. Wu (1999) models a notion of utility anxiety with a representation that takes a rank-dependent utility representation with iteratively connected weighting functions. Caplin and Leahy (2001) introduce a psychological expected utility (PEU) model in which agents have anticipatory utility, such as anxiety. Koszegi (2003) uses the PEU expected utility model to explain why patients would avoid even free information about their health. Barigozzi and Levaggi (2010) use the PEU model with a quadratic loss function to analyze the decision maker’s choice of information accuracy. They find that anticipatory utility need not be monotone in information accuracy, and full information, full ignorance or partial information can be optimal, depending on the parameters. Eliaz and Spiegler (2006) show that there are behavioral anomalies that cannot be accommodated by models in which beliefs are incorporated directly into utility functions, preserving Bayesian updating. Epstein (2008) provides a model in which anxiety is allowed and accommodates flexible attitudes towards acquisition of information.

Some of the papers mentioned above link different non-standard models and intrinsic preference for information acquisition. Our equivalence result links a non-standard Kreps-Porteus (1978) model of recursive expected utility with a standard expected utility model. This is possible because of limitations in the dataset, due to the unobservability of the state of grace.

2 Model

There are two periods and a continuum of agents normalized to 1. Each individual lives in both periods. Agent $i$ is endowed with $l$ units of time that is used, in the first period, for labor $(l^i)$ and leisure $(\bar{l} - l^i)$. Agents live in autarchy and they each have the same strictly increasing and concave production function $f$ with labor as the input. At the beginning of period 1, agent $i$ produces $f(l^i)$, where $f(0) = 0$. At the end of period 1, he saves all that he produces, $s^i = f(l^i).$ In period 2 he receives wealth $w^i = s^i + \gamma^i$, where $\gamma^i \in \gamma$, $\gamma > 0$ is agent $i$’s individual shock, which belongs to a finite set. The distribution of the individual shock $\gamma^i$ is identical for all agents $i$ and independent across agents. This distribution is a primitive of the model, and is known by the agents. The

\footnote{For the more general model with consumption, see the Appendix and the working paper (Alaoui and Sandroni (2015)).}
mean of $\gamma^i$ is 1. The probability of $\gamma^i$ is denoted by $q(\gamma^i)$.

Individuals can perfectly infer their idiosyncratic shock $\gamma^i$ from the difference $w^i - s^i$. Aggregate variables are not required for this stage of the analysis, and are introduced in Section 5 only, which extends the model to include relative aspects. Our analysis for the baseline model, therefore, can be viewed as a purely single-agent problem. We also assume that the lowest individual shock, $\underline{\gamma}$, is sufficiently low.

**Assumption R1** The lowest individual shock $\underline{\gamma}$ satisfies $\underline{\gamma} < \gamma - 2f(l)$ for all $\gamma > \gamma$, where $\gamma \in \{\gamma, \ldots, \gamma\}$.

The probability of the lowest shock $\underline{\gamma}$ can be arbitrarily small. Hence, this is a common assumption of rare disasters, adapted to our purposes (see Barro (2009)). This technical assumption simplifies the analysis and several parts of our main result do not require it. Assumption R1 can also be replaced with the assumption that the highest shock is sufficiently high. Specifically, the alternative assumption is that $\overline{\gamma} > \gamma + 2f(l)$ for all $\gamma < \overline{\gamma}$, where $\gamma \in \{\gamma, \ldots, \gamma\}$. We discuss the role of this technical assumption after our main theorem and in greater detail in the Appendix. We make this assumption mainly for simplicity. A similar, but more involved, result without this assumption can be provided from the authors upon request.

### 2.1 General preferences

All decision-makers have a (strictly increasing and convex) disutility function $d$ in labor and an additional term $V$.\(^5\) So, a generic agent $i$ maximizes:

$$U^i = V^i(s^i, w^i) - d(l^i).$$

All functions in our analysis are smooth and allow for an interior solution.

The production function $f(l^i)$, the disutility of labor $d(l^i)$ and the distribution of the idiosyncratic shocks $q(\gamma^i)$ are common to all decision-makers. We refer to standard agents as those for whom $V^i(s^i, w^i) = 0$. These benchmark standard agents do not work or save, i.e., $l^i = s^i = 0$, since it provides them with no utility. Assumptions over $V^i$ are the key differences between the agents.

\(^5\)In the more general model with consumption, the term $U^i$ also includes common utility over wealth, $u(c_1^i) + \beta u(c_2^i)$. Agents then have the choice at the end of period 1 over how much to save and how much to consume, i.e., $s^i = f(l^i) - c_1^i$, and in period 2 they consume all their wealth, $c_2^i = w^i$ (Alaoui and Sandroni (2015)). All the results in this paper hold with consumption as well.
2.2 Protestant Ethics

Protestant ethics agents are assumed to be secular and indifferent towards religious concepts. Their main distinguishing feature is their determination to accumulate wealth for its own sake, rather than for the rewards (e.g., consumption, power, prestige) that wealth may bring about. Protestants believe that it is a moral obligation to accumulate wealth. Let the Protestant ethics utility class $\mathcal{U}_{PE}$ be all utility functions $U^i$ such that

$$V^i = Ev(w^i),$$

where $v' > 0$. The expectation operator $E$ is taken over the individual shock $\gamma^i$. In this case, $Ev(w^i) = \sum_{\gamma^i} q(\gamma^i)v(w^i|\gamma^i)$. So, a Protestant ethics agent utility function is in $\mathcal{U}_{PE}$.

As we have not assumed utility over consumption, Protestants only have non-instrumental value for wealth. That is, Protestants still accumulate some wealth even when they do not have any intention of using this wealth in any state of nature. This is the puzzling aspect of the Protestants’ behavior that has interested Weber and is captured here. The renowned work ethic of Protestants follows from their preferences. For the remark below, recall that the only difference between a Protestant ethics agent and a standard agent is that $V^i = Ev(w^i)$ with $v' > 0$ for the Protestant ethics agent and $V^i = 0$ for the standard agent. All other functions, distribution of idiosyncratic shocks and constraints are common to both.

Here, standard agents do not work or save, since there is no consumption. We do not introduce consumption in this model because consumption has no effect on the main result, i.e., the equivalence between Protestant agent utility functions and Calvinist agents’ utility functions. This follows if the utility functions for consumption are common for all agents. So, introducing consumption in this model presents no difficulty, and can be found both in the Appendix of this paper and in our working paper (Alaoui and Sandroni (2015)). In particular, all remarks and results hold if agents have a common utility over consumption.

Remark 1. Any Protestant ethics agent works more and has higher expected wealth than a standard agent.

So, an economy consisting of Protestant agents produces more than an economy comprised of standard agents.

2.3 Calvinists

While the Protestants’ outlook is directed towards worldly affairs, Calvinists are concerned with the afterlife. The central tenet of Calvinism is the doctrine of predestination: an
individual $i$ is either in a state of grace ($g^i = 1$), and, hence, saved from damnation, or he is not ($g^i = 0$). The choice is not influenced by any action taken. A Calvinist does not know whether he is in grace. However, earned wealth can be seen as a signal, not a means, of grace.

2.3.1 Earned wealth as a signal of grace

Rational inferences about grace follow from two Calvinist premises: Grace is a tool of divine will and, hence, is conducive to success. Acquiring, saving and reinvesting wealth leads to better signals of grace. We formalize these two claims as assumptions WI and WII, respectively, but a simple example may deliver the gist of the idea. Consider the extreme case in which an individual shock can be either high or low and that a Calvinist believes: that his prior probability of grace is 0.5; that if his savings are low then both shocks have equal odds regardless of his state of grace; and that if his savings are high then he gets a high shock when he is in grace and a low shock when he is not (the model is far more general than this example). It follows from these premises that if his savings are low then he learns nothing about his state of grace (i.e., his posterior belief over his state of grace is still 0.5) and if his savings are high then he learns his state of grace perfectly (i.e., if his shock is high he is in grace and if his shock is low he is not). Thus, Calvinists do not receive a signal of grace from their choice of savings. Savings only increases the informativeness of the signal. This is illustrated in Figure 1.6

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Figure 1: Left: low savings, no information on grace; right: high savings; perfect information on grace. The ex-ante probability of grace is 0.5 in both cases.

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6Calvinists believe that hard work leads to better signals of grace. We focus on savings for simplicity.
Calvinists make rational inferences given their theological premises and these premises cannot be empirically tested. A claim of correlation between grace and success cannot be tested because grace is not observable even by proxy, unless it is assumed that some other observable variable is also correlated with grace. Finally, all agents (Calvinist or not) place the same odds over all observable variables. The beliefs over the joint distribution of grace and the (observable) idiosyncratic shock \( \gamma^i \) is subjective and can vary across agents, with the requirement that the unconditional distribution over \( \gamma^i \) is equal to the correct distribution. The formula used to update beliefs is given by Bayes’ rule, and so the agents are rational in the standard economics sense, and cannot manipulate beliefs in any way. The updating rule is therefore not part of the representation in our equivalence theorem. It is fully determined by Bayes’ rule.

The general signal structure is formalized as follows: Let \( p_g \in (0, 1) \) be the prior probability of grace. We take \( p_g \) to be the same for all Calvinists. Therefore, \( p_g \) is exogenously given and it is not a free parameter. It follows that our results also hold in the alternative case that \( p_g \) varies and is part of the representation.

Let \( q(\gamma^i|g^i, s^i) \) be the probability of receiving shock \( \gamma^i \), given a state of grace \( g^i \) and savings \( s^i \). (Recall that the agent can perfectly infer \( \gamma^i \) when forming posterior beliefs regarding \( g^i \).) The prior beliefs over the joint distribution of individual shocks \( \gamma^i \), grace \( g^i \) and savings \( s^i \), denoted \( q(\gamma^i, g^i, s^i) \), is not necessarily the same for all Calvinist agents, but the marginal distribution over \( \gamma^i \) (the observable part of this distribution) is constant across all Calvinists and coincides with the exogenously given correct distribution.\(^7\) The updating rule is always Bayes’ rule. Similarly, \( p(g^i = 1|\gamma^i, s^i) \) denotes the updated probability of grace given realized shock \( \gamma^i \) and savings \( s^i \).

The motivation for the two central assumptions over the information received by Calvinists is discussed in Section 4. The assumption that Calvinists associate grace with a higher likelihood of success is captured by the following version of the monotone likelihood ratio property:

**Assumption WI (MLRP):** Given any individual savings \( s^i \in [0, f(\bar{I})] \), and individual shocks \( \gamma^i, \gamma'^i \in \{\gamma, \ldots, \overline{\gamma}\} \), \( \gamma^i > \gamma'^i \iff \frac{q(\gamma^i|g^i=1, s^i)}{q(\gamma^i|g^i=0, s^i)} > \frac{q(\gamma'^i|g^i=1, s^i)}{q(\gamma'^i|g^i=0, s^i)} \).

So, Calvinists believe that good individual shocks are more likely when in grace.

**Assumption WII (Informativeness):** For any individual savings \( s^i > s'^i \), there is a nonnegative function \( h \) on \( \{\gamma, \ldots, \overline{\gamma}\}^2 \) such that: (i) \( \sum_{\gamma'} h(\gamma', \gamma) = 1 \) for all \( \gamma, \gamma' \in \{\gamma, \ldots, \overline{\gamma}\} \).

\(^7\)Note that savings do not affect the probability of \( \gamma^i \), i.e., \( q(\gamma^i) = q(\gamma^i|s^i) \) for all \( s^i \) and \( \gamma^i \).
and (ii) \( q(\gamma_i | g^i = x, s^i) = \sum_{\gamma} h(\gamma_i, \gamma) q(\gamma | g^i = x, s^i) \) for \( x \in \{0, 1\} \).

That is, Calvinists believe that the informativeness of the signals of grace (in the usual Blackwell sense) is higher for larger savings.\(^8\) This does not imply that savings alter the distribution of individual shocks, which is fixed and unaffected by savings.

Calvinists, as we discuss below, aim to reduce uncertainty over their state of grace. By assumption WII, reducing this uncertainty means saving (and working) more. If instead, higher savings led to a less accurate signal, their incentives would be to work less to learn about grace, rather than more.

### 2.3.2 Predestination and preferences for early resolution of uncertainty

We use the Kreps and Porteus (1978) model to capture Calvinists’ preferences because it allows for preferences for early resolution of uncertainty. Specifically, we assume that a Calvinist agent \( i \) has a utility function \( V_i \) of the form

\[
V_i = E u_C(p(g^i = 1 | \gamma^i, s^i)),
\]

(4)

where \( u_C \) is strictly increasing and convex and the expectation operator \( E \) is taken over the idiosyncratic shock, as in the case of the Protestants ethics agents.\(^9\)

The assumption that \( u_C \) is increasing follows from the notion that Calvinists prefer being in grace than not. The convexity of the function \( u_C \) implies that a Calvinist prefers to learn about his state of grace sooner rather than later. To see this point, consider the case in which the agent has the choice between discovering his state of grace in period 2 or remaining with his prior over grace (\( p_g \)) in his lifetime. If he knows he will learn his state of grace in period 2, his expected utility is \( p_g u_C(1) + (1 - p_g) u_C(0) \). This follows because with probability \( p_g \) he learns in period 2 that he is in grace and with probability \( 1 - p_g \) he learns that he is not. If, instead, he remains with his prior then his expected utility is \( u_C(p_g) \). Thus, if \( u_C \) is convex then the agent prefers to learn about his state of grace earlier. Extending this reasoning, it follows that \( u_C \) is convex if and only if the agent prefers early resolution of uncertainty, as is shown by Kreps-Porteus (1978).

Let \( U_C \), the utility class of Calvinists, be such that (4) holds, \( u'_C > 0 \) and \( u''_C > 0 \) and

\(^8\)For a more thorough discussion of Blackwell’s theorem, see Marschak and Radner (1972), Kim (1995) and Grant, Kajii and Polak (1998).

\(^9\)The Recursive Expected Utility model of Kreps and Porteus (1978) has formed the basis of several established frameworks. See, for instance, Epstein-Zin (1989). The standard REU representation is more complex than the one we use. If only two outcomes have positive probability then preferences for early resolution can be reduced to the simpler function that we use (this is shown in the Appendix). Furthermore, our representation is compatible with extensions of the Kreps-Porteus framework.
assumptions WI and WII hold. The functional form in (4) is different from expected utility theory. However, the Kreps-Porteus model is now widely accepted as a coherent model of decision making. So, our perspective of Calvinism as a rational system of belief and action is congruent with Weber’s view that Calvinists’ doctrines are uniquely rational in the sense that, once their premises are accepted, they contain no inner contradictions.

Although the Kreps-Porteus model is typically not associated with religion, it provides the added richness of preferences to accommodate Calvinism. This is due to a key axiom that the standard von Neumann-Morgenstern expected utility model assumes and Kreps-Porteus model does not. Von Neumann-Morgenstern expected utility assumes reduction: the decision maker is indifferent between two alternatives that have the same overall probability of reaching each final outcome, regardless of the timing of the resolution of uncertainty. The Kreps-Porteus model does not make this assumption.

The relaxation of the reduction axiom (together with a restriction on preferences that delivers the convexity assumption on $u_C$) allows for Calvinist preferences because it allows for a preference for early signs of grace. The added richness of the Kreps-Porteus model has behavioral content because it can accommodate decisions that are inconsistent with the von Neumann-Morgenstern expected utility model, such as a strict preference for the lottery on the right in Figure 2. It is precisely this additional flexibility that is required to capture a preference for early signs of grace that makes Calvinist preferences appear to be logically distinct from the preferences of Protestants. However, our equivalence result below shows that despite the differences in behavior that are allowed by the Kreps-Porteus model (and not allowed by expected utility theory), Calvinists still behave in identical ways to secular agents who abide by the Protestant ethic.

3 The Equivalence Result

Before presenting our main result, we review explicitly which variables of the model are taken to be given and which are part of the representation. The disutility function of labor $d$, the production function $f$ and the distribution $q$ of the idiosyncratic shocks are common to all agents and exogenously given. Hence, these are not free parameters in

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10 We rely on preferences for early resolution, which can be found in a large class of preferences that includes not only the Kreps-Porteus model, but also other models of anticipatory utility. We do not aim to discriminate among these theories.

11 Both standard (von Neumann-Morgenstern) and Recursive (Kreps-Porteus) Expected Utility theory can be used to study religious choices. Standard Expected Utility theory can accommodate incentives common to many religions, in which pious behavior increases the chances of going to heaven (see McCleary and Barro (2006)). But the Kreps-Porteus enrichment of expected utility is critical to accommodate Calvinism, whose incentives are fundamentally different and based on the need for early signs of grace.
Figure 2: Left: lottery with no resolution of uncertainty in period 2; right: full resolution of uncertainty in period 2. The ex-ante probability of grace is 0.5 in both cases, as in Figure 1.

the equivalence result. For the Calvinists, the beliefs are based on unobservable variables (grace) and are subjective. Hence, these are free parameters obtained by the representation. However, the updating rule is required to be Bayesian. Assumptions WI and WII must be satisfied, and all observable probabilities must be correct. The probability $p_g$ is unobservable and impossible to infer. Hence, in principle $p_g$ could also be a free parameter, but this degree of freedom is not required for the proof. So, we take $p_g$ to be exogenously given. For Calvinists, utility function $u_C$ over grace is a free parameter as well. For (non-Calvinist) Protestants, the free parameter is the function $v$ over wealth.

In summary, our free parameters are Calvinist utility function $u_C$ and the joint distribution $q(\gamma^i, g^i, s^i)$ over idiosyncratic shock, grace, and savings (subject to all constraints), and utility function $v$ for Protestant agents (since $p_g$ can be taken to be fixed, the conditional distribution $q(\gamma^i | g^i, s^i)$ rather than the joint distribution can be viewed as the free parameter). Our main theorem is as follows.

**Main Result.** Assume $R1$. The Calvinist and Protestant ethics utility subclasses are identical. That is, $\mathcal{U}_{PE} = \mathcal{U}_C$.

So, for every utility function $v$ of Protestant agents, there is a Calvinist utility function $u_C$ and joint distribution $q$ that matches it. Conversely, for every Calvinist utility function $u_C$ and joint distribution $q$, there is a matching Protestant utility function $v$. Hence, there is no qualitative distinction between Calvinists and Protestant ethics utility functions from a revealed preference perspective.

The equivalence between Calvinism and Protestant ethics preferences is central in our analysis. It formalizes Weber’s thesis of a deep connection between the secular Protestant ethic and the religious dogmas of Calvinism. In particular, the highly tractable utility functions of Protestant ethics given by (3) have foundations based on Weber’s ideas.

Our equivalence result demonstrates that despite the differences in the axiomatic foun-
dations of the Kreps-Porteus model and the von Neumann-Morgenstern expected utility model, the Calvinism model and the Protestant ethics model are behaviorally equivalent everywhere. Hence, this result provides decision theoretic foundations for Weber’s thesis.

This equivalence is tied to the unobservability of grace. In a typical setting, we can distinguish whether an individual has preferences for early resolution of uncertainty by submitting temporal choices to him. In Figure 2, for instance, the Calvinist prefers the lottery on the right. But here, we cannot detect these preferences because we cannot offer these choices since they involve promises of grace. The data, therefore, is restricted by the religious motives.

**Intuition of the Proof.** The proof of the main result is elaborate and left to the Appendix, but we briefly describe the intuition behind it. We focus first on showing that any Protestant ethics utility falls within the Calvinist utility subclass $U_C$. Take any increasing utility function $v$ in (3). To show that it falls within $U_C$, we begin by defining a function $\tilde{v}(s^i) \equiv E v(w^i)$ for all $s^i$. Our objective is then to find $u_C$ and a prior distribution of beliefs over grace and idiosyncratic shock to fit $\tilde{v}(s^i)$. We take a function $u_C$ that is strictly increasing and convex to satisfy the required properties. While the exact specification of $u_C$ is not crucial for our proof, we take $u_C$ to be quadratic, specifically $a((s^i)^2) + b$, where $a > 0$. Constant $a$ must be sufficiently large. Notice that while $u_C$ is convex, function $v$ need not be. But we will use our remaining degree of freedom in specifying the joint distribution of grace and idiosyncratic shock (that satisfy all constraints) to match $v$ precisely. By way of illustration, suppose that there are only two shocks, $\gamma$ and $\bar{\gamma}$. In that case, the joint distribution is specified in such a way that the probability of being in a state of grace $p(g^i = 1|\gamma^i, s^i)$ goes up when $\gamma^i = \bar{\gamma}$ occurs, and down when $\gamma^i = \gamma$ occurs, for any savings $s^i$. This ensures that assumption $WI$ holds, as it can easily be shown to imply that the high idiosyncratic shock is more likely when in a state of grace, and the low idiosyncratic shock less likely. Moreover, we specify the joint distribution in such a way that $p(g^i = 1|\gamma, s^i)$ is increasing in $s^i$, and correspondingly $p(g^i = 1|\gamma, s^i)$ is diminishing (the exact decrease of $p(g^i = 1|\gamma, s^i)$ is fully determined by the increase in $p(g^i = 1|\gamma, s^i)$). We show that this entails that assumption $WII$ holds. Intuitively, this will entail a wider spread over posterior beliefs of $q(\gamma^i|g^i = 1, s^i)$, which means a more informative signal. Lastly, we choose the joint distribution of grace and idiosyncratic way so that the rate of the increase in $p(g^i = 1|\gamma, s^i)$ (and corresponding decrease in $p(g^i = 1|\gamma, s^i)$) leads $a u_C((s^i)^2) + b$ to match $\tilde{v}(s^i)$ precisely for every $s^i$.

Next, we show that any Calvinist agent falls within the Protestant ethics subclass $U_{PE}$. For any Calvinist agent, we begin by defining function $\tilde{u}_C(s^i) \equiv E u_C(p(g^i = 1|\gamma^i, s^i))$, for all $s^i$. By assumption $WII$, $\tilde{u}_C > 0$. This can easily be shown directly, but it also follows
from a known result in Grant, Kajii and Polak (1998). While it would be immediate to find a utility function \( v(Ew^i) \) over expected wealth that would coincide with the Calvinists’ preferences, the main challenge is to find a utility function over ex-post wealth that would match Calvinists’ utility, i.e., a function \( v(w^i) \) such that \( Ev(w^i) = \tilde{u}_C(s^i) \) for all \( s^i \), and for which \( v' > 0 \). We do so by directly constructing the appropriate utility function \( v \). We first find \( v \) with the required properties on a restricted interval in the neighborhood of the highest possible savings. Such a \( v \) is not uniquely defined, and at this stage we have available degrees of freedom. As we then move outside of this interval, we pin down the definition of \( v \) from the original interval and parameters. In the last stage, we choose all parameters to ensure that \( v' > 0 \) everywhere.

Assumption \textbf{R1} is used only in this direction of the proof. It simplifies the construction of the utility function \( v \) by making the range of \( \gamma \) larger. If we dispense with \textbf{R1}, then we would have a uniform approximation result. That is, without R1, any Calvinist utility function and beliefs can be uniformly approximated by a utility function \( v(W) \).

The simple utility functions in (3) can be utilized to provide qualitative insights on Calvinism. While standard agents in our baseline model do not work or save because there is no utility over consumption, the following result holds in the case such that consumption has utility.

**Corollary 1.** A Calvinist works more, saves more and has higher expected wealth than a standard agent.

Calvinists work harder than standard agents because hard work and a frugal life is a way to obtain early signs of grace.\(^{12}\) Hence, Calvinists produce higher expected wealth than standard agents.

In closing this section, we note that our central result allows for a broader interpretation than the one we take. Preferences for wealth-accumulation purely for its own sake - moral or not - falls within the class of preferences we analyze. We focus on the specific relation between Calvinism and the Protestant work ethic because of its importance in the study of modern economic growth. Weber’s thesis is widely argued to be the “most famous link between culture and economic development” (Acemoglu, Johnson, and Robinson (2005a)), and it is central to a rich empirical literature. A formal theoretical understanding of the connection between Calvinism and the Protestant work ethic is therefore important.\(^{13}\)

\(^{12}\)Corollary 1 does not make use of assumption \textbf{R1} and does not require the full force of the main result.

\(^{13}\)Models that aim to specifically distinguish Calvinistic-based beliefs from other preferences in which individuals have utility for non-instrumental wealth accumulation can explore other domains: see Levy and Razin (2014b) for a theoretical analysis of a social ethic, and Arruñada (2010) for an empirical study.
But we can abstract from the Calvinist interpretation of agents in the $\mathcal{U}_C$ class, and view these agents as having preferences for reducing uncertainty over a binary unobservable variable. This could be, for instance, a notion of ability rather than grace. Under this interpretation, given our assumptions, the agent with these preferences is behaviorally indistinguishable from an agent with utility over acquired wealth.\textsuperscript{14}

### 3.1 Distinguishing Calvinist and Protestant ethics preferences

The connection between Calvinists and those who abide by a Protestant ethics holds to a surprising extent. This supports Weber’s thesis and the large literature that uses preferences of the form $Ev(w^i)$, where $w^i$ is earned wealth, but it also raises the question of whether –and if so, how– Calvinism and Protestantism can be differentiated from a revealed preference approach. This is an open (and, we believe, difficult) question.

It may seem plausible that making simple extensions to our model such as the inclusion of both consumption and wealth transfers would provide this distinction. This is not the case. Wealth transfers would only enter the Protestant ethics value function through consumption utility, since the utility of wealth term is over earned wealth, and does not include transfers. In the same way, non-earned wealth is not a sign of grace. So, windfall wealth transfers would therefore not lead to an observable difference in choices, or even (unobserved) utility. Instead, a promising avenue is to consider a more radical departure from our model and include social ethics in Calvinist preferences (see, for instance, Levy and Razin (2014b)). Whether this added richness would provide a meaningful distinction between the behavior of Calvinists and Protestant ethics agents is left to future research.

### 4 Discussion of the Assumptions for Calvinists

We expand here on the rationale for the assumptions concerning Calvinists in greater detail. The main tenet of Calvinism is the doctrine of predestination. If it is impossible to influence God’s choices then it is natural to ask about the incentives to follow religious teachings or to acquire costly information about grace. Weber points out that salvation was a critical concern of believers and so it became psychologically necessary to obtain means of recognizing grace. Even if grace is immutable, Calvinists still prefer to obtain signs of grace during their lifetime rather than to have it revealed only in the afterlife. In Weber’s words, “The question, Am I one of the elect? must sooner or later have arisen for

\textsuperscript{14}This can be viewed as a notion of Bayesian persuasion in that the agent has some control over the signal structure, see Kamenica and Gentzkow (2011).
every believer and have forced all other interests in the background.” (Weber (2002)).

The Kreps-Porteus model is critical to capture this aspect of Calvinism because it allows for preferences for early resolution of uncertainty. In our model, this captures Calvinists’ anxiety over being in a state of grace or not and it allows for Calvinists’ preferences to resolve this uncertainty sooner rather than later, i.e., in their lifetimes and not in the afterlife.

In addition, we have assumed WI and WII on Calvinists. These take the content and the informativeness of the signals of grace to depend on savings, as a proxy for hard work and austerity. We use savings for simplicity; the crucial modeling assumption is that there exists some measure of hard work and austerity that Calvinists believe to be correlated with signals of grace. Assumptions WI and WII (and also those concerning preferences for early resolution of uncertainty) do not necessarily stay close to the theology of Calvin himself, who does not stress wealth as a signal of grace. Rather, we base our approach on Weber’s thesis of Calvinist societies, and the way in which Calvin’s teachings were adapted and popularized.

Weber, as Gibbens explains, “is interested not just in Calvin’s doctrines as such but in their later evolution within the Calvinist movement [...] success in a calling eventually came to be regarded as a ‘sign’- never a means- of being one of the elect” (preface to Weber (2002)). The idea that success is a sign of grace is captured by assumption WI.

The notion that the informativeness of the signal of grace depends on effort and thrift, and hence expected wealth, is also close to Weber’s views, who states, for instance, that “God Himself blessed His chosen ones through the success of their labours” (Weber (2002)). Hence, it is the success of labor, not any type of success, that delivers an indication of grace. This is captured by assumption WII. Moreover, if there were no informational value to accumulating wealth, then the Calvinist agent would not have any added motivation to work harder. That is, if Calvinists believed only success were indicative of grace independently of their attempts to be successful (i.e., if the actions they take had no impact on the signal), then their value function would be the same as that of the standard agents. Assumption WII therefore serves here to capture in a simple way the idea that Calvinists will work harder and save more to obtain information through their success over their state of grace. This picks up the “ideas essential to [Weber’s] thesis” that “the methodological development of one’s own state of grace to a higher and higher degree of certainty [...] was a sign of grace; [...] that He gives them His signs if they wait

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15Weber also discusses the importance of “conviction”, the “doctrine of proof”, in the search for a “sign of election”, and the powerful “motive to rationalize worldly activity [...] of the Calvinistic elect for proof with their exclusive preoccupation with the beyond” (Weber (2002)).
patiently and deliberate methodically” (Weber (2002)).

The main results would hold if Assumption WII were modified so long as we maintain the Weberian perspective of “a set of beliefs which emphasized hard work, thrift, saving, and where economic success was interpreted as consistent with (if not actually signaling) being chosen by God” (Acemoglu, Johnson and Robinson (2005a)).

5 The Spirit of Capitalism and Keeping up with the Joneses

It is common for utility to be defined over relative wealth (see, for instance, Bakshi and Chen (1996)). Keeping up with the Joneses (KUJ) preferences, in particular, play an important role in the analysis of modern economic behavior and are commonly used in macroeconomics and finance to fit empirical regularities. Campbell and Cochrane (1995) use preferences with KUJ properties to accommodate patterns of modern economies such as procyclical variation of stock prices, long-run predictability of excess stock returns, countercyclical variations of stock volatility, short and long-run equity puzzles despite low and constant interest rates. Abel (1990) argues that Catching up with the Joneses preferences can explain empirical puzzles in asset price trends; Gali (1994) analyzes the connection between consumption externalities and asset prices; Dupor and Liu (2003) consider the effects of jealousy on overconsumption; and Ljungqvist and Uhlig (2000) analyze tax policies under “Keeping up” and “Catching up” with the Joneses.\(^{16}\)

We now show how our analysis can be modified to provide foundations for these preferences. Let \( S \) denotes the aggregate savings and \( W \) the aggregate wealth, and assume that these aggregate variables are observed by all agents (in each period).\(^{17}\) Let the Protestant ethics utility class with relative wealth \( \mathfrak{U}_{PER} \) consist of all utility function \( U^i \) such that

\[
V^i = E v(w^i - W), \tag{5}
\]

where \( v' > 0 \). So, in the Protestant utility functions in \( \mathfrak{U}_{PER} \), average wealth is a benchmark used to measure performance.

Now assume that Calvinists’ signals over state of grace are taken over relative, rather than absolute, savings. That is, savings are also measured against a benchmark (as in

\(^{16}\)This is a large literature that we do not survey, but we refer the reader to Rege (2008) and Hopkins and Kornienko (2004) for models of relative wealth and social status.

\(^{17}\)Formally, \( S = \int_{i \in [0,1]} s^i di \) and \( W = \int_{i \in [0,1]} w^i di \). Since the continuum of agents has been normalized to 1, \( S \) and \( W \) also represent average wealth and savings, respectively.
social comparison theory, see Festinger (1954)) given by average savings $S$. Formally, Assumptions WI and WII are modified as follows.

**Assumption WIR:** Given any relative savings $s^i - S \in [-f(\bar{L}), f(\bar{L})]$, and individual shocks $\gamma^i, \gamma'^i \in \gamma^i \in \{\gamma, \ldots, \tau\}$, $\gamma^i > \gamma'^i \iff \frac{q(\gamma^i | \gamma^i = 1, s^i - S)}{q(\gamma'^i | \gamma^i = 0, s^i - S)} > \frac{q(\gamma'^i | \gamma^i = 1, s^i - S)}{q(\gamma'^i | \gamma^i = 0, s^i - S)}$.

**Assumption WIIR:** For any relative savings $s^i - S > s'^i - S'$, there is a nonnegative function $h$ on $\{\gamma, \ldots, \tau\}^2$ such that: (i) $\sum_{\gamma} h(\gamma', \gamma) = 1$ for all $\gamma, \gamma' \in \{\gamma, \ldots, \tau\}$ and (ii) $q(\gamma'^i | g^i = x, s'^i - S') = \sum_{\gamma} h(\gamma', \gamma) q(\gamma | g^i = x, s^i - S)$ for $x \in \{0, 1\}$.

Let the utility class of Calvinists with relative signals $\mathcal{U}_{CR}$ be given by $\mathcal{U}_C$, but replacing WI and WII with WIR and WIIR, and hence $V^i = \mathbb{E}u_C(p(g^i = 1 | \gamma^i, s^i))$ with $V^i = \mathbb{E}u_C(p(g^i = 1 | \gamma^i, s^i - S))$.

**Equivalence Theorem 2.** Assume R1. The Calvinist with relative signals utility subclass and Protestant ethics with relative wealth utility subclass are identical. That is, $\mathcal{U}_{PER} = \mathcal{U}_{CR}$.

So, models of Protestant Ethics based on relative wealth are also equivalent to models of Calvinism based on relative signals. In particular, the utility functions of Calvinists with relative signals have the main features of the ‘Spirit of Capitalism’ preferences: utility increases in own wealth and diminishes in aggregate wealth (see Bakshi and Chen (1996) and Robson (1992)). These features are also central in $KUJ$ preferences.\(^18\)

$KUJ$ preferences are generally associated with other-regarding sentiments such as envy, jealousy and social status.\(^19\) These sentiments are quite distinct from those typically associated with Calvinism and the Protestant work ethic, in which conspicuous display of wealth are discouraged. Interestingly, however, while Weber does not explicitly discuss $KUJ$ preferences, he recognizes the signaling aspect of uneven wealth distribution, and notes “the comforting assurance that the unequal distribution of the goods of this world was a special dispensation of Divine Providence” (Weber (2002)). Similarly, Calvinists may not derive utility from displaying wealth to others, but may receive signs of grace by comparing themselves to others. This comparative mechanism may lead Calvinism to utility of relative wealth.

\(^18\)Typically, $KUJ$ preferences are defined in terms of consumption and not wealth. As long as aggregate wealth and consumption are positively related, the utility of Protestant ethics agents decreases when aggregate consumption increases. We also note that a distinction is sometimes made between utility that is diminishing in aggregate consumption and an increase in marginal utility of consumption as aggregate consumption increases (see Dupor and Lui (2003)). We focus on the former, but conditions for the latter are provided in the Appendix.

\(^19\)And, to some degree, conspicuous consumption (see Bagwell and Bernheim (1996)). Conspicuous consumption is chastised by Calvinists, and by ascetic Protestants.
Finally, we note that if we introduced aggregate shocks, a second mechanism may lead Calvinists to have utility over relative wealth. Suppose that the agent observes his own wealth and aggregate wealth, but he does not observe the shocks independently. Then, higher aggregate wealth signals lower individual shock. For a Calvinist, this lowers his assessment of salvation. This ex-post mechanism is independent of the assumption of relative signals.\textsuperscript{20} The analysis is deferred to the Appendix.

6 Conclusion

Modeling Weber’s thesis on Calvinism societies requires an important departure from expected utility theory, while modeling his description of the Protestant ethics does not. These are deeply-rooted differences that cast doubt on Weber’s argument of a connection between Protestantism and Calvinism. However, we show that Weber’s thesis of a connection between Calvinism and secular Protestant ethics holds. This result also provides for foundation for the continued use, in both theoretical and empirical work, of simple functional form for the Protestant ethics. We hope that these results will further motivate the use of decision theory for understanding religious and metaphysical concepts. Lastly, our model allows for a broader interpretation than the one we have followed in this paper. Our result can be viewed as linking utility over earned wealth with a preference for reducing uncertainty over an unobservable variable.

\textsuperscript{20}In addition, even if agents were to use absolute standards, higher aggregate savings would negatively impact the informativeness of individual signals whenever aggregate variables were not perfectly observable. The use of relative standards simplifies the analysis because it does not require imperfect observations of aggregate variables.
Appendix

Below, we use the notation $s^i - S$ and $\Delta s^i \equiv s^i - S$ interchangeably, and we use $w^i - W$ and $\Delta w^i \equiv w^i - W$. We also use the notation $[-f(l), f(l)]$ and $[\Delta S, \Delta \bar{S}]$ interchangeably, where $\Delta S = -f(l)$ and $\Delta \bar{S} = f(l)$.

Recall, in the proofs that follow, that the probability of shock $\gamma_i$ are not a function of individual’s choices; hence we write $q(\gamma_i)$ and not $q(\gamma_i|s^i, S, ...)$, without loss.

We also note, as discussed in the text, that the Kreps-Porteus utility representation can be written as function $Eu_C(p(g^i = 1|\gamma_i, s^i - S))$ when there are only two attainable states of the world. The standard Kreps-Porteus representation for two periods is

$$Eu_{C,e} \left( u_{C,1}^{-1}(p(g^i = 1|\gamma_i, s^i - S))u_{C,1}(g^i = 1) + (1 - p(g^i = 1|\gamma_i, s^i - S))u_{C,1}(g^i = 0) \right) ,$$

where $u_{C,e}$ is the utility associated with the first stage, and $u_{C,1}$ is the utility associated with the second stage. Normalizing $u_{C,1}(g^i = 1) = 1$ and $u_{C,1}(g^i = 0) = 0$, the representation $Eu_C(p(g^i = 1|\gamma_i, s^i - S))$ follows immediately, defining $u_C \equiv u_{C,e} \circ u_{C,1}^{-1}$. Moreover, the assumption of a preference for early resolution of uncertainty in the Kreps-Porteus representation requires that $u_C = u_{C,e} \circ u_{C,1}^{-1}$ be convex, as we have assumed.

Proofs

We only prove results for the relative cases (i.e., for utility class of Calvinists with relative signals, and Protestant ethics with relative wealth), as in Section 5. It is immediate that these proofs can be modified for the simpler case discussed in the paper in which agents do not have relative aspects. We omit them for brevity, but they are available upon request. Moreover, all our proofs are for the extension in which there is utility of consumption; it is simple to see that the proofs all hold for the case for which this utility is zero. All proofs (aside from those for the last remarks, Remarks 2 and 3, which are not in the main text) are for the case without aggregate shocks; see the working paper for proofs in which aggregate shocks are included. Notice that even without aggregate shocks, aggregate variables are still taken into account when signals are relative. Formally, we extend the model to include strictly increasing and concave utility function $u$ of consumption, and
the maximization problem is as follows

\[ U^i = \left[ u(c^i_1) + \beta u(c^i_2) - d(l^i) \right] + V^i, \]

s.t. budget constraint [BC] :

\[ c^i_1 = f(l^i) - s^i; \quad w^i = s^i + \gamma^i \theta; \]
\[ c^i_2 = w^i, \quad 0 \leq l^i \leq \bar{l}; \quad 0 \leq s^i \leq f(l^i), \]

where \( \beta \in (0, 1] \), where all functions are such that an interior solution holds, and where \( V^i \) for different agents (standard, Calvinists, and Protestant ethics agents) are as defined in the text. All terms aside from \( V^i \) are common to all agents (in particular, functions \( u, d, f, \beta \) and the budget constraint).

**Remark 1.** Given any values of the aggregate variables, any Protestant ethics (with relative wealth) agent works more, saves more and has higher expected wealth than a standard agent.

**Proof.** Letting \( \mu \in [0, 1] \), we consider the following maximization problem.

\[ U^i = \left[ u(c^i_1) + \beta u(c^i_2) - d(l^i) \right] + \mu E_v(w^i - W) \]

s.t. [BC] :

\[ c^i_1 = f(l^i) - s^i; \quad w^i = s^i + \gamma^i; \]
\[ c^i_2 = w^i, \quad 0 \leq l^i \leq \bar{l}; \quad 0 \leq s^i \leq f(l^i). \]

\( \mu = 0 \) corresponds to the standard agent’s problem, and \( \mu = 1 \) corresponds to the Protestant ethics (with relative wealth) agent problem. We proceed by using the implicit function theorem, and show that \( \frac{ds^*(S, L)}{d\mu} > 0 \) and \( \frac{dl^*(S, L)}{d\mu} > 0 \), where \( s^*(S, L), l^*(S, L) \) are the optimal savings and labor chosen by agent \( i \), given aggregate variables \( S \) and \( L \) (note that \( S \) and \( L \) completely characterize the ex-ante aggregate decisions). The first order conditions with respect to \( s^i \) and \( l^i \) are:

**First order conditions**

\[ F_s \equiv -u'(c^i_1) + \beta u'(c^i_2) + \mu E_v'(w^i - W) = 0 \quad (6) \]
\[ F_l \equiv u'(c^i_1)f'(l^i) - d'(l^i) = 0. \quad (7) \]
The Hessian is then \[
\begin{bmatrix}
F_{ss} & F_{ls} \\
F_{ls} & F_{ll}
\end{bmatrix},
\]
where
\[
F_{ss} \equiv u''(c_i) + \beta E u''(c_2) + \mu E v''(w^i - W)
\]
\[
F_{ls} \equiv -u''(c_i)f'(l^i)
\]
\[
F_{ll} \equiv u''(c_i)(f'(l^i))^2 + f''(l^i)u'(c_i) - d''(l^i).
\]

Applying the implicit function theorem, we have:
\[
\begin{bmatrix}
\frac{ds^*(S,L)}{d\mu} \\
\frac{d''(S,L)}{d\mu}
\end{bmatrix} = -\frac{1}{\Delta} \begin{bmatrix}
F_{ll} & -F_{ls} \\
-F_{ls} & F_{ss}
\end{bmatrix} \begin{bmatrix}
\frac{dF_{ss}}{d\mu} \\
0
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
-F_{ll} E v'(w^i - W) \\
F_{ls} E v'(w^i - W)
\end{bmatrix},
\]
where \(\Delta\) is the determinant of the Hessian.

Given our assumptions of an interior solution, the Hessian is negative definite, and therefore (as it is 2 \(\times\) 2) \(\Delta > 0\). It is clear that \(F_{ll} < 0\), \(F_{ls} > 0\) and \(E v'(w^i - W) > 0\), since \(u''(c_i) < 0\), \(f''(l^i) < 0\), \(u'(c_i) > 0\), \(d''(l^i) > 0\) and \(v'(w^i - W) > 0\). It follows that \(\frac{ds^*(S,L)}{d\mu} > 0\) and that \(\frac{d''(S,L)}{d\mu} > 0\). It is then immediate that \(\frac{dE w^*(S,L)}{d\mu} = \frac{ds^*(S,L)}{d\mu} > 0\).

Finally, since the result holds for any aggregate variables \(S\) and \(L\), this concludes the proof.

The following lemma will be used in the results that follow.

**Lemma 1.** For a Calvinist (with relative signals) agent with function \(V^i = Eu_C(p(g^i = 1|\gamma^i, s^i - S))\), define function \(\tilde{u}_C : [\Delta \bar{s}, \Delta \bar{s}] \to \mathbb{R}\) to be \(\tilde{u}_C(s^i - S) = Eu_C(p(g^i = 1|\gamma^i, s^i - S))\), for all \(s^i - S \in [\Delta \bar{s}, \Delta \bar{s}]\). Then \(\tilde{u}_C' > 0\).

**Proof.** By assumption WIIR, the Calvinist’s signal over grace becomes more informative as \(s^i - S\) increases. While this can be proven directly, we instead apply Proposition 1 of Grant, Kajii and Polak (1998). Specifically, by statement (V) of the proposition, SAIL holds, and the result then follows from the equivalence of statements A and B of the same proposition.

Recall that we only provide proofs for the relative cases; hence, we prove below Equivalence Theorem 2, but it is trivial to modify the argument for the Main Result.
**Equivalence Theorem 2.** Assume \( R1 \). The Calvinist with relative signals utility subclass and Protestant ethics with relative wealth utility subclass are identical. That is, \( \mathcal{U}_{\text{PER}} = \mathcal{U}_{\text{CR}} \).

**Proof.** We first show that any Protestant ethic agent falls within the Calvinist utility subclass \( \mathcal{U}_{\text{C}} \), and then show that any Calvinist agent falls within the Protestant ethic subclass.

**Protestant ethics agent falls within Calvinist subclass**

Take any Protestant agent function \( U^i = Ev(w^i - W) \), with \( v' > 0 \).

We first define function \( \tilde{v} : [\Delta s^i, \bar{\Delta s}] \Rightarrow \mathbb{R} \) as follows:

\[
\tilde{v}(\Delta s^i) \equiv Ev_W(w^i - W) = \sum_{\gamma^i} q(\gamma^i) v(w^i - W) = \sum_{\gamma^i} q(\gamma^i) v(\Delta s^i + \gamma^i - 1).
\]

Note that \( \tilde{v} \) is well defined and strictly increasing in \( \Delta s^i \), since \( v \) is strictly increasing everywhere. We now proceed by construction. We specifically define a \( u_C \) function that is strictly increasing and convex, as required. We also define the (Calvinist) agent’s joint distribution \( q(\gamma^i, g^i, \Delta s^i) \) over the idiosyncratic shock \( \gamma^i \), grace \( g^i \) and relative savings \( \Delta s^i \). Maintaining Bayes’ rule for updating beliefs, we then show that the marginal beliefs match the given distributions, i.e. that the marginal beliefs over the shocks \( \gamma^i \) are equal to the given \( q(\gamma^i) \) and that the marginal belief over prior state of grace being \( g^i = 1 \) is equal to \( p_g \). We proceed to show that all the properties required by the Calvinist problem are satisfied. Lastly, we show that \( Eu_C(p(G^i = 1|\Delta s^i, \gamma^i)) \) is exactly equivalent to function \( \tilde{v}(\Delta s^i) \) for all \( \Delta s^i \in [\Delta s, \bar{\Delta s}] \).

**Preferences \( u_C \)**

We define \( u_C \) to be \( u_C(\cdot) = a((\cdot)^2 - p_g^2) + b \), where \( a > 0 \), and \( b < \tilde{v}(\Delta s) \). (The agent’s prior over grace, \( p_g \), is exogenous and common to all agents.) This specific form is only used for convenience, and can defined in another way. It is useful as a simple function for which the requirements that \( u_C \) be strictly increasing and convex are met. Moreover, notice that \( b \) and the term \(-ap_g^2 \) are both constant and would not affect the preferences if removed, so that they can be dropped from the definition of \( u_C \). We maintain them only to have the equivalence of functions be exact rather than up to a constant. We will further restrict \( a \) below.
Beliefs

First, partition the set of $\gamma$ into $\{\gamma_1,h,...,\gamma_{n,h},h\}$ and $\{\gamma_{1,l},...,\gamma_{n,l},l\}$, where $1 < \gamma_{1,h} < ... < \gamma_{n,h}$, and $1 \geq \gamma_{1,l} > ...\gamma_{n,l}$. Both sets are well-defined and non-empty.

The agent’s joint distribution $q(\gamma^i, g^i, \Delta s^i)$ is defined as follows. For any $\gamma^i \in \{\gamma,...,\gamma\}$ and $\Delta s^i \in [\Delta s, \Delta s]$, 

$$q(\gamma^i, g^i = 1, \Delta s^i) = (p_g + t(\Delta s^i, \gamma^i))q(\gamma^i)$$

and

$$q(\gamma^i, g^i = 0, \Delta s^i) = (1 - p_g - t(\Delta s^i, \gamma^i))q(\gamma^i),$$

where $t$ is defined in the following way. We first define:

$$t(\Delta s^i, \gamma_{n,h},h) = \left( \frac{\hat{v}(\Delta s^i) - b}{a \left( q(\gamma_{n,h},h) + \epsilon^2 \sum_{j=1}^{n_h-1} q(\gamma_{j,h})j^2 + \nu^2(q(\gamma_{n,l},l) + \epsilon^2 \sum_{j=1}^{n_l-1} q(\gamma_{j,l})j^2) \right) \right)^{1/2}$$

where $\epsilon$ is an arbitrarily small positive constant (specifically, $\epsilon \in (0, \min\{(n_l - 1)^{-1}, (n_h - 1)^{-1}\})$), and:

$$\nu = \frac{q(\gamma_{n,h},h) + \epsilon \sum_{j=1}^{n_h-1} q(\gamma_{j,h})j}{q(\gamma_{n,l},l) + \epsilon \sum_{j=1}^{n_l-1} q(\gamma_{j,l})j}.$$

For other values of $\gamma^i$, $t$ is defined as follows:

$$t(\Delta s^i, \gamma^i) = \begin{cases} \epsilon t(\Delta s^i, \gamma_{n,h},h) & \text{if } \gamma^i = \gamma_{j,h}, j \in \{1, ..., n_h - 1\} \\ -\nu t(\Delta s^i, \gamma_{n,h},h) & \text{if } \gamma^i = \gamma_{n,l} \\ \epsilon t(\Delta s^i, \gamma_{j,l},l) & \text{if } \gamma^i = \gamma_{j,l}, j \in \{1, ..., n_l - 1\}. \end{cases}$$

Moreover, assume that $a$ is high enough that $p_g + t(\Delta s^i, \gamma^i), 1 - p_g - t(\Delta s^i, \gamma^i) \in (0,1)$ for all values of $\Delta s^i \in [\Delta s, \Delta s]$ and attainable values of $\gamma^i$. Notice that such a value exists, since $t(\Delta s^i, \gamma_{n,h},h)$ and $t(\Delta s^i, \gamma_{n,l},l)$, the maximum and minimum values, respectively, of $t$, go to zero as $a$ goes to infinity. Note that the signal received by the agent is only a
function of \( \Delta s^i \) and \( \gamma^i \), and note also that \( b < \hat{v}(\Delta s) \) ensures that the term \( \hat{v}(\Delta s^i) - b \) is non-negative.

To show that the marginal beliefs match the exogenously given distributions, we have that for any \( \gamma^i \) and \( \Delta s^i \), \( q(\gamma^i, \Delta s^i) = (p_g + t(\Delta \bar{s}^i, \gamma^i))q(\gamma^i) + (1 - p_g - t(\Delta \bar{s}^i, \gamma^i))q(\gamma^i) = q(\gamma^i) \), which is indeed correct. Concerning the marginal beliefs over \( \Delta s^i \), we first have that for any \( \gamma^i \) and \( \Delta s^i \),

\[
q(\gamma^i | g^i = 1, \Delta s^i) = \frac{q(\gamma^i, g^i = 1, \Delta s^i)}{p_g} = \frac{(p_g + t(\Delta s^i, \gamma^i))q(\gamma^i)}{p_g}
\]

and

\[
p(g^i = 1 | \gamma^i, \Delta s^i) = \frac{q(\gamma^i | g^i = 1, \Delta s^i)p_g}{q(\gamma^i)} = \frac{q(\gamma^i, g^i = 1, \Delta s^i)}{q(\gamma^i)} = p_g + t(\Delta s^i, \gamma^i),
\]

in accordance with Bayesian updating. Next, we have that for any \( \Delta s^i \), the marginal belief over being in a state of grace (\( g^i = 1 \)) is:

\[
\sum_{\gamma^i} q(\gamma_i)(p_g + t(\Delta s^i, \gamma^i)) = p_g + \sum_{\gamma^i} q(\gamma_i)t(\Delta s^i, \gamma^i)
\]

\[
= p_g + \sum_{\gamma^i} q(\gamma_n,h) t(\Delta s^i, \gamma_n,h) + \sum_{\gamma^i} q(\gamma_m,l) t(\Delta s^i, \gamma_m,l)
\]

\[
+ \epsilon \left( \sum_{j=1}^{n_h-1} q(\gamma_j,h) t(\Delta s^i, \gamma_n,h) + \sum_{j=1}^{n_l-1} q(\gamma_j,l) t(\Delta s^i, \gamma_m,l) \right)
\]

\[
= p_g + t(\Delta s^i, \gamma_n,h) \left( q(\gamma_n,h) + \epsilon \sum_{j=1}^{n_h-1} q(\gamma_j,h) j - \nu \left( q(\gamma_m,l) + \epsilon \sum_{j=1}^{n_l-1} q(\gamma_j,l) j \right) \right)
\]

\[
= p_g + t(\Delta s^i, \gamma_n,h) \left( q(\gamma_n,h) + \epsilon \sum_{j=1}^{n_h-1} q(\gamma_j,h) j - q(\gamma_n,h) + \epsilon \sum_{j=1}^{n_h-1} q(\gamma_j,h) j \right) = p_g,
\]

(where we have used the definition of \( \nu \) and \( t \)) and is therefore correct.

**Properties of the signal**

We now show that Assumptions **WIR** and **WIIR** hold.

1. Assumption **WIR** (MLRP), for which it suffices to show that a higher \( \gamma^i \) leads to a more positive signal of state of grace \( g^i \). Specifically, the condition \( \frac{q(\gamma^i | g^i = 1, \Delta s^i) > q(\gamma^i | g^i = 0, \Delta s^i)}{q(\gamma^i | g^i = 0, \Delta s^i)} \) if \( \gamma^i > \gamma^i \) (for all \( \Delta s^i \in [\Delta s^i, \Delta \bar{s}^i] \)) is trivially equivalent, in this setting, to \( p(g^i = 1 | \gamma^i, \Delta s^i) > p(g^i = 1 | \gamma^i, \Delta s^i) \) for \( \gamma^i > \gamma^i \) (for all \( \Delta s^i \in [\Delta s^i, \Delta \bar{s}^i] \)). Then, to prove that a higher \( \gamma^i \) leads to a more positive signal of grace, note first that
t(\Delta s^i, \gamma^i) \) increases in \( \gamma^i \) for all \( \Delta s^i \in [\Delta s, \Delta \bar{s}] \). Hence, \( p(g^i = 1|\gamma^i, \Delta s^i) = p_g + t(\Delta s^i, \gamma^i) > p_g + t(\Delta s^i, \gamma^i) = p(g^i = 1|\Delta s^i, \gamma^i) \) for \( \gamma^i > \gamma^i \), for all \( \Delta s^i \in [\Delta s, \Delta \bar{s}] \).

2. Assumption \textbf{WIIR}, that for higher \( \Delta s^i \), the agent receives a more informative signal, in the Blackwell sense. To show that this property holds, note first that the lottery characterized by \( \{q(\gamma), p_g + t(\Delta s^i, \gamma); \ldots; q(\bar{\gamma}), p_g + t(\Delta s^i, \bar{\gamma})\} \) is a mean-preserving spread of the lottery characterized by \( \{q(\gamma), p_g + t(\Delta s^i, \gamma_1); \ldots; q(\bar{\gamma}), p_g + t(\Delta s^i, \gamma_i)\} \), where \( \Delta s^i > \Delta s^i \). As there are only two states of the world, it follows immediately that Assumption \textbf{WIIR} holds (see, for instance, Gámiz and Penalva (2010), Proposition 3: the signals are ranked by integral precision, which is equivalent to Blackwell informativeness in a context of dichotomies).

Lastly, we show that \( Eu_{C}(p(g^i = 1|\Delta s^i, \gamma^i) \) is exactly equivalent to function \( \bar{v}(\Delta s^i) \) for all \( \Delta s^i \in [\Delta s, \Delta \bar{s}] \):

\[
Eu_{C}(p(G^i = 1|\Delta s^i, \gamma^i)) = \sum_{\gamma^i} q(\gamma^i)a((p_g + t(\Delta s^i, \gamma^i))^2 - p_g^2) + b \\
= \sum_{\gamma^i} aq(\gamma^i)t(\Delta s^i, \gamma^i)^2 - 2ap_g\sum_{\gamma^i} q(\gamma^i)t(\Delta s^i, \gamma^i)) + b = \sum_{\gamma^i} aq(\gamma^i)t(\Delta s^i, \gamma^i)^2 + b \\
= a(q(\gamma_{n_h,h})t(\Delta s^i, \gamma_{n_h,h})^2 + \sum_{j=1}^{n_h-1} q(\gamma_{j,h})(\epsilon^j t(\Delta s^i, \gamma_{n_h,h}))^2 + q(\gamma_{n_l,l})t(\Delta s^i, \gamma_{n_l,l})^2 \\
+ \sum_{j=1}^{n_l-1} q(\gamma_{j,l})(\epsilon^j t(\Delta s^i, \gamma_{n_l,l}))^2) + b \\
= at(\Delta s^i, \gamma_{n_h,h})^2 \left(q(\gamma_{n_h,h}) + \epsilon^2 \sum_{j=1}^{n_h-1} q(\gamma_{j,h})j^2 + \nu^2 \left(q(\gamma_{n_l,l}) + \epsilon^2 \sum_{j=1}^{n_l-1} q(\gamma_{j,l})j^2 \right)\right) + b \\
= \bar{v}(\Delta s^i) - b + b = \bar{v}(\Delta s^i).
\]

Note that we have used that \( \sum_{\gamma^i} q(\gamma^i)t(\Delta s^i, \gamma^i) = 0 \), as had been shown in the proof that the marginal belief that \( g^i = 1 \) is indeed \( p_g \). We have also used the definition of \( \nu \) and \( t \). All the properties are satisfied, which completes this direction of the proof. We now turn to the second part of the proof.

**Calvinist agent falls within Protestant subclass**

We proceed by construction. First, we apply Lemma 1, and write Calvinist function \( V^i = Eu_{C}(p(g^i = 1|\gamma^i, \Delta s^i)) \) as strictly increasing function \( \bar{u}_{C}(\Delta s^i) = Eu_{C}(p(g^i = 1|\gamma^i, \Delta s^i)) \), for all \( \Delta s^i \in [\Delta s, \Delta \bar{s}] \). We construct a function \( \bar{v} : [\Delta s - (1 - \gamma), \Delta \bar{s} + (\bar{\gamma} - 1)] \to \mathbb{R} \)
such that $Ev(w^i - W|\Delta s^i) = \tilde{u}_C(\Delta s^i)$ for all $\Delta s^i \in [\Delta \bar{s}, \Delta \bar{s}]$. We then show that $v' > 0$ everywhere.

We first define $t$ to be an arbitrary smooth and strictly increasing function on the compact interval $[\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} + (\overline{\gamma} - 1)]$, where the maximum derivative of $t$ on this interval is $t'_{\text{max}} < \infty$. We write the lower bound of the interval in this manner (instead of $\Delta \bar{s} + (\overline{\gamma} - 1)$) as a reminder that $\gamma < 1$. We define, on this interval, function

$$v(\Delta w^i) = at(\Delta w^i) + b,$$  \hspace{1cm} (9)

where $a > 0$, and constant $b$ is chosen such that $Ev(\Delta w^i|\Delta \bar{s}) = \tilde{u}_C(\Delta \bar{s})$. Specifically, let $b = \tilde{u}_C(\Delta \bar{s}) - a \sum_{\gamma^i} q(\gamma^i)t(\Delta \bar{s} + (\gamma^i - 1))$.

Consider any $\Delta \bar{s} \leq \Delta s^i < \Delta \bar{s}$. We require that $Ev(\Delta w|\Delta s^i) = \tilde{u}_C(\Delta s^i)$. That is, we require that

$$Ev(\Delta w|\Delta s^i) = \sum_{\gamma^i} q(\gamma^i)v(\Delta s^i + (\gamma^i - 1))$$ \hspace{1cm} (10)

$$= q(\gamma)v(\Delta s^i - (1 - \gamma)) + \sum_{\gamma^i \neq \gamma} q(\gamma^i)v(\Delta s^i + (\gamma^i - 1))$$ \hspace{1cm} (11)

$$= q(\gamma)v(\Delta s^i - (1 - \gamma)) + \sum_{\gamma^i \neq \gamma} q(\gamma^i)(at(\Delta s^i + (\gamma^i - 1)) + b) = \tilde{u}_C(\Delta s^i)$$ \hspace{1cm} (12)

where we have used, for the second term of line 12, that $v$ has already been defined in (9) on that range. We note, for $\gamma^i \in \{\gamma, ..., \overline{\gamma}\} \setminus \{\gamma\}$, that $\Delta s^i + (\gamma^i - 1) \in (\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} + \overline{\gamma} - 1)$] follows from assumption R1. To see this, starting with the upper bound, it is immediate that $\Delta s^i + (\gamma^i - 1) \leq \Delta \bar{s} + \overline{\gamma} - 1$. Concerning the lower bound, $\Delta \bar{s} - (1 - \gamma) < \Delta s^i + (\gamma^i - 1)$ holds if $\Delta \bar{s} - \Delta s^i \leq \gamma^i - \gamma$. But we know from assumption R1 that $2f(\overline{l}) = \Delta \bar{s} - \Delta \bar{s} < \gamma^i - \overline{\gamma}$. Since $\Delta \bar{s} - \Delta s^i \leq \Delta \bar{s} - \Delta \bar{s}$, it follows that $\Delta \bar{s} - \Delta s^i < \gamma^i - \overline{\gamma}$.

While we have not yet explicitly defined $v$ on $\Delta w^i = \Delta s^i - (1 - \gamma)$ for $s^i \in [\Delta \bar{s}, \Delta \bar{s}]$, i.e. on the remaining interval $[\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} - (1 - \gamma)]$, it is clear from (12) that we are fully constrained in the specification of $v$ on this interval. Specifically,

$$v(\Delta s^i - (1 - \gamma)) = (\tilde{u}_C(\Delta s^i) - \sum_{\gamma^i \neq \gamma} q(\gamma^i)(at(\Delta s^i + (\gamma^i - 1)) + b))/q(\gamma).$$ \hspace{1cm} (13)

Using (13), we define $v$ as a function of $\Delta w^i$, i.e. letting $\Delta w^i = \Delta s^i - (1 - \gamma)$, to be:

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\[ v(\Delta w^i) = (\tilde{u}_C (\Delta w^i + (1 - \gamma)) - \sum_{\gamma^i \neq \gamma} q(\gamma^i) (at (\Delta w^i + (\gamma^i - \gamma)) + b))/q(\gamma). \] (14)

We define \( v \) according to (14) for all \( \Delta w^i \in [\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} - (1 - \gamma)] \).

By construction, we therefore have that \( Ev(\Delta w^i | \Delta s^i) = \tilde{u}_C(\Delta s^i) \) everywhere. We now guarantee that \( v' > 0 \). First, \( v \) is increasing on \([\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} + (\gamma - 1)]\) by construction. Second, for \( \Delta w^i \in [\Delta \bar{s} - (1 - \gamma), \Delta \bar{s} - (1 - \gamma)] \), the condition \( v'(\Delta w^i) > 0 \) is satisfied if:

\[
(\tilde{u}'_C (\Delta w^i + (1 - \gamma)) - \sum_{\gamma^i \neq \gamma} aq(\gamma^i)t' (\Delta w^i + (\gamma^i - \gamma))/q(\gamma) > 0,
\]

for which it suffices that

\[
\tilde{u}'_C (\Delta w^i + (1 - \gamma)) - a(1 - q(\gamma))t'_{\text{max}} > 0
\]

\[
\Rightarrow a < \frac{\tilde{u}'_C (\Delta w^i + (1 - \gamma))}{(1 - q(\gamma))t'_{\text{max}}}. \] (15)

Define \( \tilde{u}'_{C, \text{min}} \equiv \min \{\tilde{u}'_C (\Delta w^i + (1 - \gamma))\} \) on \([\Delta \bar{s}, \Delta \bar{s}]\). Letting \( a < \frac{\tilde{u}'_{C, \text{min}}}{(1 - q(\gamma))t'_{\text{max}}} \) guarantees that condition (15) is always satisfied, and hence that \( v' > 0 \) everywhere.

Finally, we note that \( v \) is guaranteed to be differentiable at every point except at \( \Delta w^i = \Delta \bar{s} - (1 - \gamma) \), and that it is trivial to show that \( t \) can be chosen to guarantee differentiability at this point as well.

We have shown that every Protestant ethics (with relative wealth) agent falls within the Calvinist (with relative signals) subclass, and that every Calvinist (with relative signals) agent falls within the Protestant ethics (with relative wealth) subclass, hence \( \Upsilon_{PER} = \Upsilon_{CR} \). This concludes the proof.

**Corollary 1.** Given any values of the aggregate variables, any Calvinist (with relative signals) works more, saves more and has higher expected wealth than a standard agent.

**Proof.** Applying Lemma 1, we can define, for any Calvinist, function \( \tilde{u}_C(s^i - S) = Eu_C(p(g^i = 1 | \gamma^i, s^i - S)) \), where \( \tilde{u}'_C > 0 \). The rest of the proof then follows closely the proof for Remark 1, and makes use of the implicit function theorem. In particular,
we do not make use of the rare disasters assumption R1. Letting \( \mu \in [0, 1] \), we consider maximization problem 
\[
U^i = [u(c^i_1) + \beta Eu(c^i_2) - d(l^i)] + \mu \tilde{u}_C(s^i - S),
\]
subject to the budget constraints \([BC]\). We note that \( \mu = 0 \) corresponds to the standard agent’s problem, and \( \mu = 1 \) corresponds to the Calvinist problem.

**First order conditions**

\[
F_s \equiv -u'(c^i_1) + \beta Eu'(c^i_2) + \mu \tilde{u}_C'(s^i - S) = 0 \quad (16)
\]
\[
F_l \equiv u'(c^i_1) f'(l^i) - d'(l^i) = 0. \quad (17)
\]

The Hessian is then
\[
\begin{bmatrix}
F_{ss} & F_{ls} \\
F_{ls} & F_{ll}
\end{bmatrix},
\]
where

\[
F_{ss} \equiv u''(c^i_1) + \beta Eu''(c^i_2) + \mu \tilde{u}_C''(s^i - S)
\]
\[
F_{ls} \equiv -u''(c^i_1) f'(l^i)
\]
\[
F_{ll} \equiv u''(c^i_1)(f'(l^i))^2 + f''(l^i)u'(c^i_1) - d''(l^i).
\]

Applying the implicit function theorem, we have:
\[
\begin{bmatrix}
\frac{ds^*}{d\mu}(S,L) \\
\frac{dt^*}{d\mu}(S,L)
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
-F_{ll} \frac{dE}{d\mu} \\
F_{ls} \frac{dE}{d\mu}
\end{bmatrix} \begin{bmatrix}
-F_{ll} \tilde{u}_C'(s^i - S) \\
F_{ls} \tilde{u}_C'(s^i - S)
\end{bmatrix},
\]
(18)

where \( \Delta \) is the determinant of the Hessian.

The 2 \( \times \) 2 Hessian is negative definite and hence \( \Delta > 0 \). It is clear that \( F_{ll} < 0 \) and that \( F_{ls} > 0 \), and since \( \tilde{u}_C'(s^i - S) > 0 \), it follows that \( \frac{ds^*(S,L)}{d\mu} > 0 \) and that \( \frac{dt^*(S,L)}{d\mu} > 0 \). It is then immediate that \( \frac{dEw^*(S,L)}{d\mu} = \frac{ds^*(S,L)}{d\mu} > 0 \). Finally, since the result holds for any aggregate variables, this concludes the proof.

In closing Section 4, we have discussed a second mechanism that would induce Calvinists to have negative utility of relative wealth. In particular, we have mentioned that a Calvinist’s utility diminishes because a higher (ex-post) aggregate wealth indicates that he has obtained a lower individual shock. The precise statement is provided in Remark 2 below. We first introduce aggregate shocks, so that the maximization function is now:
\[ U^i = [u(c_1^i) + \beta Eu(c_2^i) - d(l^i))] + Eu_C(p(g^i = 1|\gamma^i, s^i - S)) \]

s.t. budget constraint [BC] :
\[
\begin{align*}
c_1^i & = f(l^i) - s^i; \quad w^i = s^i + \gamma^i \theta; \\
c_2^i & = w^i, \quad 0 \leq l^i \leq \bar{l}; \quad 0 \leq s^i \leq f(l^i),
\end{align*}
\]

where \( \theta \in \{\theta, ..., \bar{\theta}\} \). Here too, while the individual does not observe the individual shock or aggregate shock directly, he can infer them. In particular, he can infer them from \( s^i, w^i, S \) and \( W \), which he does observe. Remark 2 then follows immediately.

**Remark 2.** The utility of a Calvinist (with relative signals) diminishes ex-post with realized aggregate wealth \( W \).

**Proof.** Since ex-post wealth \( w^i = s^i + \gamma^i \theta \) and \( W = S + \theta \), it follows that:
\[
\gamma^i = \frac{w^i - s^i}{\theta} = \frac{w^i - s^i}{W - S}.
\]

Hence, a higher \( W \) implies a lower \( \gamma^i \). By assumption \( \text{WIR (MLRP)} \), it is immediate that a lower \( \gamma^i \) leads to a more negative signal of grace \( g^i \), as previously mentioned. Hence, \( p(g^i = 1|\gamma^i, s^i - S) \) is lower for a higher \( W \), which in turn implies that \( u_C((p(g^i = 1|\gamma^i, s^i - S))) \) is lower. Since no other term in the utility function is affected, this concludes the proof.

Some models in the existing literature associate \( KUJ \) preferences with an agent’s propensity to increase his own consumption as aggregate consumption increases; the next result provides the conditions under which this occurs for a Calvinist. This property is a function of the rate at which informativeness increases with \( s^i - S \) relative to the preferences for early resolution of uncertainty, although we abstract from this point.

**Remark 3.** Given any values of the aggregate variables, the following statement holds. If, for Calvinist agent \( i \), \( \tilde{u}_C \) (as defined in Lemma 1) is concave, then Calvinist \( i \) works more, saves more and has higher expected consumption \( Ec_2^i \) as expected consumption \( EC_2 \) increases.

**Proof.** We again apply the implicit function theorem. Using the first order conditions and the Hessian from Corollary 1, we have:
\[
\begin{bmatrix}
\frac{ds^*(S,L)}{dEC_2} \\
\frac{dl^*(S,L)}{dEC_2}
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
-F_{ll}\frac{dF_s}{dEC_2} \\
F_{ls}\frac{dF_s}{dEC_2}
\end{bmatrix}
= \frac{1}{\Delta} \begin{bmatrix}
F_{ll}\tilde{u}''_C(s^i - S) \\
-F_{ls}\tilde{u}''_C(s^i - S)
\end{bmatrix}
\] (19)

We have that \(\Delta > 0, F_{ll} < 0\) and \(F_{ls} > 0\). Moreover, by the concavity assumption, \(\tilde{u}''_C(s^i - S) < 0\), and it therefore follows that \(\frac{ds^*(S,L)}{dEC_2} > 0\) and that \(\frac{dl^*(S,L)}{dEC_2} > 0\). It is then immediate that \(\frac{dEc^*_2(S,L)}{dEC_2} > 0\). The result holds for any aggregate variables, which concludes the proof.

## References


