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Imitation—theory and experimental evidence

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Abstract

We introduce a generalized theoretical approach to study imitation and subject it to rigorous experimental testing. In our theoretical analysis we find that the different predictions of previous imitation models are mainly explained by different informational assumptions, and to a lesser extent by different behavioral rules. In a laboratory experiment we test the different theories by systematically varying information conditions. We find significant effects of seemingly innocent changes in information. Moreover, the generalized imitation model predicts the differences between treatments well. The data provide support for imitation on the individual level, both in terms of choice and in terms of perception. Furthermore, individuals' propensity to imitate more successful actions is increasing in payoff differences.

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1. Introduction

Everyone who watches children grow up will attest that imitation is one of the main sources of learning. And introspection shows that imitation plays a significant role also for adult learning. In fact, imitation is prevalent in much of everyday decision making, in particular when the environment is complex or largely unknown. Openings in chess games are a good example or finding routes through traffic, or buying complex consumer items like cars, laptop computers, or digital cameras. But, while social scientists and psychologists have long recognized the importance of

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imitation (see Asch [2] for an early example), imitation has only recently moved into the focus of economists.

Important theoretical advances towards understanding imitation have been made by Vega-Redondo [23] and Schlag [18,19]. Both approaches are based on the idea that individuals who face repeated choice problems will imitate others who obtained high payoffs. But despite this basic similarity, the two theories imply markedly different predictions when applied to specific games. For example, for games with a Cournot structure, Schlag's model predicts Cournot–Nash equilibrium play, while Vega-Redondo's model predicts the Walrasian outcome. The latter prediction is also obtained by Selten and Ostmann's [21] notion of an "imitation equilibrium", while Cournot–Nash is also predicted by imitation models with large populations as studied by Björnerstedt and Weibull [3].

The current paper makes two main contributions. First, it introduces a generalized theoretical approach to imitation, which enables us to analyze why the models of Vega-Redondo [23] and Schlag [18,19] come to such different predictions. Basically, the models differ along two different dimensions, the informational structure ("whom agents imitate") and the behavioral rule ("how agents imitate"). While agents in Vega-Redondo's model observe their immediate competitors, in Schlag's model they observe others who are just like them but play in different groups against different opponents. Additionally, agents in Vega-Redondo's model copy the most successful action of the previous period whenever they can. In contrast, Schlag's agents only imitate in a probabilistic fashion and the probability with which they imitate is proportional to the observed difference in payoffs between own and most successful action. We show that the difference between the two models is mainly due to the different informational assumptions rather than the different adjustment rules. In that sense, it is more important *whom* one imitates than *how* one imitates (although, as we shall see below, some aspects of "how" one imitates do matter theoretically). In particular, if one imitates one's own opponents, outcomes become very competitive. If, on the other hand, one imitates other players who face the same problem as oneself but play against different opponents, Nash equilibrium play is obtained.

The second objective of our paper is to present rigorous experimental tests of the different imitation models. We chose to study imitation in a normal form game with the payoff structure of a simple discrete Cournot game. This has the advantage that the theoretical predictions of the various imitation models are very distinct. Both traditional benchmark outcomes of oligopoly models (Cournot–Nash equilibrium and Bertrand equilibrium) are supported by at least one imitation model. Also, the games are easy to implement in an experiment, and we have a good understanding of how Cournot markets operate in laboratory environments under different circumstances.[1] The key design feature of our experiment is that we vary the feedback information subjects receive between rounds of play. In one treatment they observe their competitors' actions and profits, in another they observe the actions and profits of others who are like them but play against different opponents. Finally, there is a treatment where agents have access to both types of information.

On some level, these variations appear to be very innocent and many (learning) models would not predict any difference between them. In that sense, the experimental part of our study examines whether (and if so how) slight variations in the informational structure of a repeated-game setting have an impact on behavior. We find that the variations indeed have significant effects. Moreover, the directions of these effects are well organized by the generalized imitation model. Specifically, average profits are ranked according to the theoretical predictions and significantly so: the treatment in which opponents can be observed is the most competitive. The treatment in

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[1] See e.g. Plott [17], Holt [10], and Huck et al. [13] for surveys.

which only subjects in other groups can be observed is roughly in line with the Cournot–Nash equilibrium prediction and is the least competitive. Intermediate outcomes result if subjects have access to both types of information.

Analyzing individual adjustments, we find strong support for imitative behavior. Simple imitation can explain a surprisingly large fraction of subjects' decisions. But subjects differ in their propensity to imitate. While some do not imitate more often than a randomization device would, others are almost pure imitators. In general though, we find that, much in line with Schlag's model, the likelihood of imitation increases in the difference between the highest payoff observed and the own payoff. In addition, we find that imitation is more pronounced when subjects observe their direct competitors—rather than others who have the same role but play in different groups.

All these results are obtained from studying choice data. Subjects do imitate and they do it in specific ways. Whether or not subjects are aware of this, is a different issue on which we shed some light by analyzing replies to a post-experimental questionnaire. Interestingly, many replies quite clearly reveal that subjects know what they are doing. Quite a number of subjects perceive themselves as imitating.

Despite being inherently "behavioral", there have been few prior experiments on imitation. In particular, Schlag's imitation model has not been experimentally tested at all, while the models of Vega-Redondo and Selten and Ostmann have been subject to isolated experiments. Huck et al. [11,12] and Offerman et al. [16] find experimental support for Vega-Redondo's model, while Bosch-Domenech and Vriend [4] conclude that imitation was not significantly present in their data. Also, Abbink and Brandts [1] provide data that are well-organized by a model closely related to Vega-Redondo's. Finally, Selten and Apesteguia [20] find some experimental support for Selten and Ostmann's [21] static model of imitation.

The remainder of the paper is organized as follows. Section 2 introduces the games and the experimental details. In Section 3 we review the imitation models, introduce a general framework, and derive theoretical results. In Section 4 the experimental results are reported. Section 5 reports results from a control experiment in which subjects received more information about the payoff function. Finally, Section 6 concludes. In the online supplement[2] to this paper we report all proofs (Appendix A), a treatment of Selten and Ostmann's imitation equilibrium (Appendix B), the instructions for the experiment (Appendix C), and additional regression results (Appendix D).

2. The experimental environment

In our experiments subjects repeatedly play simple 3-player normal form games, with a payoff structure that is derived from a symmetric Cournot game. All players have five pure strategies with identical labels, a, b, c, d, and e. Subjects are, however, not told anything about the game's payoff function apart from the fact that their payoff deterministically depends on their own choice and the choices of two others, and that the payoff function is the same throughout all of the experiment.[3]

Interaction in the experiment takes place in populations of nine subjects. Each subject has a *role* and belongs to a *group*. There are three roles, labelled X, Y, and Z, filled by three subjects each. Roles are allocated randomly at the beginning of the session and then kept fixed for the entire session. Sessions last for 60 periods. In each period, subjects are randomly matched into

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[2] Available at `http://www.nyu.edu/jet/supplementary.html`.

[3] See the translated instructions in the online Appendix C. Section 5 presents a variant where subjects do have access to the full information about payoff functions.

Table 1
Payoff table

	Action combination of other players in group														
	aa	*ab*	*ac*	*ad*	*ae*	*bb*	*bc*	*bd*	*be*	*cc*	*cd*	*ce*	*dd*	*de*	*ee*
a	1200	1140	1000	880	800	1080	940	820	740	800	680	600	560	480	400
b	1311	1242	1081	943	851	1173	1012	874	782	851	713	621	575	483	391
c	1500	1410	1200	1020	900	1320	1110	930	810	900	720	600	540	420	300
d	1584	1476	1224	1008	864	1368	1116	900	756	864	648	504	432	288	144
e	1600	1480	1200	960	800	1360	1080	840	680	800	560	400	320	160	0

Note: The order in which the actions of the other group members is displayed does not matter.

three *groups*, such that always one *X*-player is matched with one of the *Y*-players and one of the *Z*-players. Subjects are informed about this interaction technology. One might wonder why we introduce roles to study behavior in a symmetric game. The answer is twofold. First, it is exactly this "trick" that allows us to disentangle the effects of imitation rules and information. Second, we will be able to use the identical setup for studying asymmetric games in follow-up projects. One might also wonder why we randomly rematch subjects in every round. The answer to this is that we want to minimize potential repeated-game effects.

While subjects know that they are randomly matched each period, they are not told with whom they are matched and there are no subject-specific labels. In each experimental session, two independent populations of nine subjects participate to increase anonymity. After each period, subjects learn their own payoff. Additional feedback information depends on the treatment. There are three treatments altogether.

Treatment ROLE: In treatment ROLE a player is informed, after each period *t*, of the actions and payoffs in *t* of players who have the same role as himself but play in different groups.

Treatment GROUP: In treatment GROUP a player is informed, after each period *t*, of the actions and payoffs in *t* of players in his own group.

Treatment FULL: In treatment FULL a player can observe all the information given in treatments ROLE and GROUP and learn the average payoff in the entire population.[4]

The payoff table (unknown to subjects) is displayed in Table 1 . The payoffs are compatible with a linear Cournot market with inverse demand, $p = 120 - X$, and zero costs. In this case, the strategies a, b, c, d, and e correspond to the output quantities 20, 23, 30, 36, and 40, respectively.[5] That is, a corresponds to the symmetric joint profit maximizing output, c to the Cournot output, and e to the symmetric Walrasian output, where price equals marginal cost (of zero). Subjects are told that the experimental payoffs are converted to Euros using an exchange rate of 3000:1.[6]

The computerized experiments[7] were carried out in the Laboratory for Experimental Research in Economics in Bonn. Subjects were recruited via posters on campus. For each treatment we

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[4] Notice that in FULL a player cannot observe the choices and payoffs of players that are neither in his group nor in his role.

[5] Note, however, that any positive transformation of these quantities, together with an appropriate transformation of the payoff function, would also yield the payoffs in Table 1.

[6] In the first session of treatment FULL we used an exchange rate of 4000:1.

[7] The program was written with z-tree of Fischbacher [8].

carried out three sessions—each with two independent populations of nine subjects, which gives us six independent observations per treatment. Accordingly, the total number of subjects was 162 (=$9 \times 6 \times 3$). The experiments lasted on average 70 min, and average payments were 15.25 Euros.[8] After the 60 rounds subjects were presented with a questionnaire in which they were asked for the motivation of their decisions.

3. Imitation models

3.1. Theory

In this section we develop a generalized framework to study imitation that contains, both, Vega-Redondo's [23] and Schlag's [18] model as special cases. We shall then apply the theoretical model to the experimental environment described in the previous section. Recall that the treatments vary with respect to the information subjects receive about actions and/or payoffs in the previous round. Following Schlag [19] we call a behavioral rule *imitating* if it prescribes for each individual to choose an observed action from the previous round. A *noisy* imitating rule is a rule that is imitating with probability $1 - \varepsilon$ and allows for mistakes with probability $\varepsilon > 0$. (In case of a mistake any other action is chosen with positive probability.) A behavioral rule with *inertia* allows an individual to change his action only with probability $\theta \in (0, 1)$ in each round. In the following we shall first characterize different imitation rules according to their properties without noise and inertia. Predictions for the Cournot game will then be derived by adding noise and inertia.

A popular and plausible rule is "imitate the best" (see e.g. Vega-Redondo [23]), which simply prescribes to choose the strategy that in the previous period performed best among the observed actions. In our setting it is possible that an action yields different payoffs in different groups. This implies that it is a priori not clear how an agent should evaluate the actions he observes. An *evaluation rule* assigns a value to each action in a player's set of observed actions. When an action yields the same payoff everywhere in his reference group, there is no ambiguity and the action is evaluated with this observed payoff.[9] When different payoffs occur for the same action, various rules might be applied. Below we will focus on two evaluation rules that appear particularly natural in a simple imitation setting with boundedly rational agents: the *max rule* where each strategy is evaluated according to the highest payoff it received and the *average rule* where each strategy is evaluated according to the average payoff observed in the reference group.[10] Of course, other rules, such as a "pessimistic" min rule, might also have some good justification. Nevertheless, we shall follow the previous literature and focus on the max and the average rules.[11]

An imitating rule is called "imitate the best" if it satisfies the property that (without noise and inertia) an agent switches to a new action only if this action has been played by an agent in his reference group in the previous round, and was evaluated as at least as good as that of any other action played in his reference group. When several actions satisfy this, each is chosen with positive probability. "Imitate the best" combined with the average rule is called IBA. "Imitate the best" combined with the max rule is called IBM.

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[8] At the time of the experiment one Euro was worth about one US dollar.

[9] This is always the case in treatment GROUP.

[10] Although it will turn out below that in our experimental data the evaluation rule matters only in very few cases, it is important to realize that theoretically it might matter.

[11] For "imitate the best average" (IBA) see, e.g. Ellison and Fudenberg [5] and Schlag [18]. For "imitate the best max" (IBM) see Selten and Ostmann [21].

Schlag [18] shows in the context of a decision problem in which agents can observe one other participant that "imitate the best" and many other plausible rules do not satisfy certain optimality conditions. Instead, Schlag [19] advocates the "proportional imitation rule" which prescribes to imitate an action with a probability proportional to the (positive part of the) payoff difference between that action's payoff from last period and the own payoff from last period. If the observed action yielded a lower payoff, it is never imitated. To extend this analysis to the case of agents observing two or more actions, we introduce broad *classes* of imitation rules.

Definition 1. An imitating rule is called a "weakly imitate the best average" (WIBA) rule if it satisfies (without noise and inertia) the following two properties:

(i) Never switch to an action with an average payoff lower than the average payoff of the own action.
(ii) Imitate the action with the highest average payoff in the sample with strictly positive probability (unless one already plays an action with the best average payoff).
If we replace "average payoff" by "maximal payoff" in Properties (i) and (ii), we obtain the class of "weakly imitate the best max' (WIBM) rules.

Both, WIBA and WIBM allow for a large variety of specific adjustment rules, including Vega-Redondo's imitate the best rule as well as some of the probabilistic imitation rules considered by Schlag [18] (e.g. his "double imitation rule").

Before we proceed with deriving theoretical predictions, we need to introduce some further notation. The imitation dynamics induce a Markov chain on a finite state space Ω. A state $\omega \in \Omega$ is characterized by three strategy profiles, one for each group. We shall refer to *uniform states* as states where all nine players use the same strategy. Two uniform states will be of particular interest. The state in which everybody plays the Cournot–Nash strategy c, to which we will refer as the *Cournot state* ω^c; and the state in which everybody plays the Walrasian strategy e, to which we shall refer as the *Walrasian state* ω^e.

To analyze the properties of the Markov processes induced by the various imitation rules discussed above, we shall now add (vanishing) noise and inertia. That is, whenever we refer in the following to some rule as, for example, "imitate the best" (or, in short, IBM), we shall imply that agents are subject to, both, inertia and (vanishing) noise. States that are in the support of the limit invariant distribution of the process (for $\varepsilon \to 0$) are called *stochastically stable* (Kandori et al. [14]; see e.g. Fudenberg and Levine [9], for a textbook treatment).

Proposition 1. *If agents follow either a WIBA ("weakly imitate the best average") or a WIBM ("weakly imitate the best max") rule and if the reference group is as in treatment GROUP, the Walrasian state ω^e is the unique stochastically stable state.*

The intuition for this result is analogous to the intuition in Vega-Redondo's original treatment of the imitate the best rule. In any given group, the agent with the highest output obtains the highest profit as long as prices are positive. This induces a push toward more competitive outcomes.[12] Insofar, Proposition 1 can be seen as generalization of Vega-Redondo's original result. As long as the informational structure is such that agents observe only their competitors (in the last period) the Walrasian outcome results.

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[12] Introducing constant positive marginal cost does not change the result. If price is below marginal cost, the agent with the lowest output is imitated which again pushes the process towards the Walrasian state.

Proposition 2. *If agents follow a WIBA or a WIBM rule and if the reference group is as in treatment ROLE, the Cournot state* ω^c *is the unique stochastically stable state.*

The intuition for Proposition 2 is that any deviation from the Cournot–Nash equilibrium play lowers the deviator's absolute payoff. Agents in the same role will observe this but will not imitate because they earn more using the equilibrium strategy. On the other hand, every non-equilibrium state can be left by a single mutation, namely by having an agent who is currently not playing his best reply switch to his best reply. This improves his payoff and will be observed by other agents in the same role who will follow suit. What remains to be shown is that one can construct sequences of one-shot mutations that lead into the Cournot state from any other state. To establish this claim we use the fact that the game at hand has a potential.

Comparing Propositions 1 and 2 establishes our earlier claim. While the specifics of an imitation rule do not matter as long as the rule falls in the rather large class of WIBA and WIBM rules, changing the informational structure has a profound effect on long-run behavior. Turning to treatment FULL one might expect that its richer informational structure (with agents having the combined information of treatments GROUP and ROLE) causes some tension between the Walrasian and the Cournot outcome. It turns out that this intuition is correct. In fact, for treatment FULL the Cournot state (where everybody plays c) and the Walrasian state ω^e can be stochastically stable depending on the evaluation rule.

Proposition 3. *If agents follow a WIBA (WIBM) rule and if the reference group is as in treatment FULL, then the Cournot state* ω^c *(Walrasian state* ω^e*) is the unique stochastically stable state.*

3.2. Some qualitative hypotheses and simulations

To sum up the theoretical predictions, we indicate here their implications for average profits in the Cournot games. All imitation rules suggest that profits in treatment GROUP (where Walrasian levels are expected in the long run) should be rather low, whereas in treatment ROLE profits around the Cournot outcome are expected. Finally, the theoretical results suggest for treatment FULL profits between GROUP and ROLE depending on whether one believes more in the average or the max rule.[13] Thus, we obtain the following qualitative hypothesis about the ordering of profits[14] :

$\mathscr{QH}$: GROUP$\preccurlyeq$FULL$\preccurlyeq$ROLE.

Hypothesis $\mathscr{QH}$ has, strictly speaking, two parts. First, it suggests that there is a difference between the experimental treatments (what many other theories would not predict). Second, it suggests a particular order that would be expected if imitation is an important force for subjects' adaptations.

The problem with long-run predictions derived from stochastic stability analysis is that they are just that: long-run predictions. Furthermore, in general they crucially depend on the assumption of vanishing noise. Thus, the issue arises how imitation processes behave in the short run and in the

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[13] Interestingly, the predictions of Selten and Ostmann's [21] *imitation equilibrium* agree with those for WIBA (see online Appendix B).

[14] Hypothesis $\mathscr{QH}$ provides a convenient summary of the predictions in one dimension. Formulating the hypothesis in terms of profits (rather than quantities) makes sense because profits are invariant to the transformations described in Footnote 5.

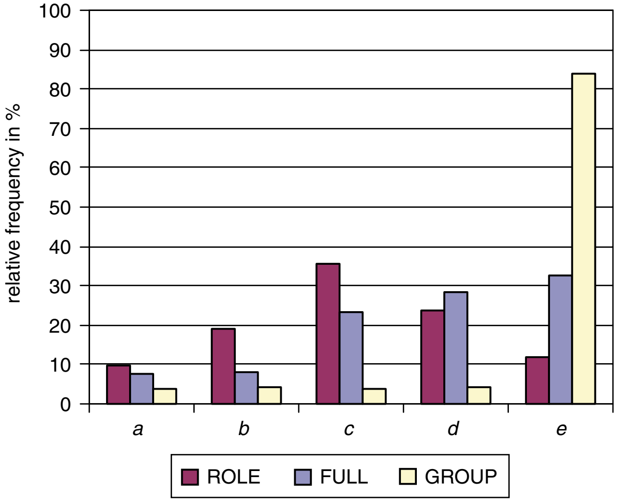

Fig. 1. Relative frequencies of actions, average of 100 simulations, rounds 31–60.

presence of non-vanishing noise. In order to address this issue, we run simulations for the different treatments. In particular, we simulate populations of nine players over 60 rounds when each player behaves according to the IBM rule (IBA yields almost identical results)[15] given the reference group defined by the respective treatment. The noise level we use is substantial: with probability 0.8 in each round a player follows IBM and with probability 0.2 a player chooses randomly one of the five actions (each then with equal probability). For each treatment we simulate 100 such populations with starting actions chosen from a uniform distribution.

Fig. 1 shows relative frequencies of actions in these simulations. Already after 20–30 rounds, behavior is fairly constant. Thus, we report frequencies aggregated over rounds 31–60. The prediction ω^e for treatment GROUP is clearly confirmed by the simulations. Apart from action e, all other actions survive only due to the relatively high noise level. In treatment FULL the predicted stochastically stable state ω^e is the modal choice while ω^c and ω^d are also relatively frequent in the simulations. Actions a and b survive only due to the noise. States ω^c and ω^d are relatively frequent because two mutations are required in order to leave them in favor of ω^e. In this sense, the simulations reinforce the qualitative conclusion that FULL is the intermediate treatment between ROLE and GROUP.

In treatment ROLE, the predicted action c is also the modal and median choice. However, convergence is relatively slow. The reason seems to be the following. In treatment ROLE the number of absorbing states (of the unperturbed imitation process) is higher than in the other treatments because besides uniform states, all states in which players in a given role play the same action are absorbing (see the proof of Proposition 2). A detailed look at the simulations reveals that indeed the process often gets stuck in such states which of course slows down the process.

Averaged over the last 30 periods, average profits in the simulations were 855.3 for ROLE, 591.1 for FULL, and 204.9 for GROUP, differences being significant at any conventional significance

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[15] The reason for this is that the two evaluation rules coincide in most cases. Thus it seems that the theoretical difference can play out only in the very long run and for vanishing noise levels.

level. Therefore, importantly, the theoretical predictions we obtained for the long run and with vanishing noise are robust also for the short run and in the presence of noise.

4. Experimental results

We now turn to the experimental analysis of the generalized imitation framework proposed above. We organize this section as follows. First, based on the qualitative hypotheses $\mathscr{Q}\mathscr{H}$, we evaluate the data on the aggregate level. This might show whether and, if so, how the different informational structures affect outcomes. While this will provide some indirect evidence for the relevance of imitation, a more thorough study of imitation must be based on data from individual adjustments. Thus, in Section 4.2 we analyze individual data by counting how often actual adjustments are in line with predicted adjustments. This is followed in Section 4.3 by a regression analysis that helps us to test whether the probability of imitating is indeed, as Schlag's models suggest, a function of the observed payoff differences. Finally, we conclude this section by analyzing the post-experimental questionnaire. This will provide additional insight into whether subjects are intentional imitators or whether it just looks *as if* they are.

4.1. Aggregate behavior

We begin by considering some summary statistics on the aggregate level. Table 2 shows average profits for all treatments, separately for the first round, all 60 rounds of the experiment, and the last 30 rounds. Standard errors of means for the six observations per treatment are shown in parentheses.

Considering Table 2 we find no significant difference between average profits in the first round according to MWU tests (see, e.g. Siegel and Castellan [22]) on the basis of the average profit per population. However, the differences in profits over all 60 and the last 30 rounds are highly significant. The p-values for (two-sided) MWU tests based on rounds 31–60 are as follows:

$$\text{GROUP} \prec_{0.037} \text{FULL} \prec_{0.006} \text{ROLE}.$$

This is exactly in line with the qualitative predictions of the generalized imitation model derived in the previous section. Profits in ROLE are higher than in FULL, and in FULL higher than in GROUP. Notice also that the differences are rather substantial in economic terms.

Table 2
Summary statistics

	Treatment		
	ROLE	GROUP	FULL
Avg. profits, round 1	974.1	1011.0	1001.7
	(66.7)	(56.2)	(57.0)
Avg. profits, rounds 1–60	824.1	634.9	731.0
	(10.2)	(24.8)	(22.8)
Avg. profits, rounds 31–60	804.6	604.6	691.1
	(13.3)	(30.2)	(25.1)

Note: Standard errors of mean profits for the six independent observations per treatment are given in parentheses.

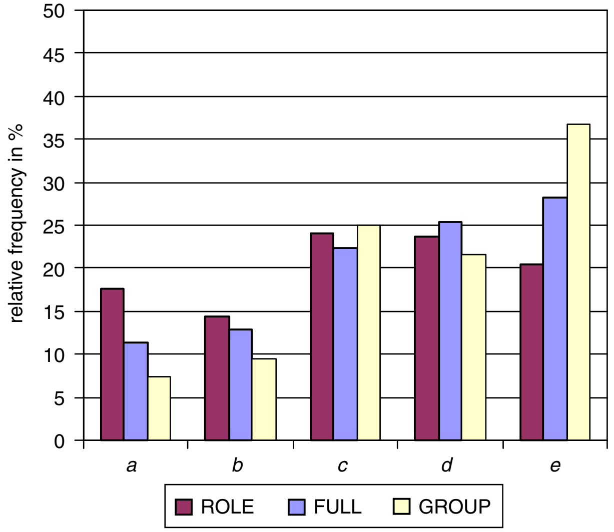

Fig. 2. Relative frequencies of actions, experimental data, rounds 31–60.

Fig. 2 shows relative frequencies of actions per treatment for the second half of the experiment. According to (two-sided) MWU tests, action *e* is chosen significantly more often in GROUP than in ROLE at the 1% level. On the other hand, action *a* is chosen significantly more often in ROLE than in GROUP at the 1% level. Furthermore, action *e* is chosen more often in GROUP than in FULL, and action *a* is chosen more often in ROLE than in FULL, both at the 5% level.

Both Table 2 and Fig. 2 clearly show that the seemingly innocent changes in information conditions have a systematic impact on behavior. However, the quantitative differences in average profits and the distribution of actions are less pronounced than predicted by imitation theory, which indicates that other factors play a role, too. For now, we summarize our findings in the following two statements.

Result 1. *The reference group has a significant impact on behavior.*

Result 2. *Profits are ordered as predicted by hypothesis* $\mathscr{QH}$.

Given the usual noise in experimental data from human subjects, Result 2 seems quite remarkable. However, before drawing more definite conclusions about the viability of imitation it is necessary to analyze individual adjustments which we shall do in the following section.

4.2. Individual behavior

In this section we evaluate the success of the imitation models by computing compliance rates of individual adjustment behavior with the predictions of the respective models. We begin by classifying individual behavior into the following categories: (i) “Best”: the subject played an action that was evaluated as best action in his reference group in the last period, (ii) “Better”: the subject switched to an action that was evaluated as better than his own action, but not as the best, (iii) “Same”: the subject did not change his action despite observing a better strategy in his

Table 3
Classification of individual behavior by type of change

	Best (%)	Better (%)	Same (%)	Worse (%)	Different (%)
ROLE	34.9 35.9	1.7 1.8	13.4 12.3	8.5 8.5	41.5
GROUP	41.2	2.3	18.1	5.3	33.1
FULL	32.6 32.8	7.0 7.1	22.8 22.7	11.1 10.9	26.5

Note: Reported are the percentages of subjects that switched to actions in the various categories. Upper values are calculated using the average rule, lower values by using the max rule.

reference group, (iv) "Worse": the subject changed to an action that was evaluated as worse than his own action, and (v) "Different": the subject changed to an action that was not observed in the reference group. Table 3 reports how many decisions fall into each of the categories (i)–(v) for each treatment and both evaluation rules. The differences between the max and the average rules are very small which is due to the fact that the two rules typically prescribe the same actions (because the strategy with the highest max is typically also the one with the highest average). Only in less than 2% of all cases do they diverge. Hence, for ease of presentation we will focus on the max rule from now on.

There are a couple of observations which are immediate from inspecting Table 3: (1) there is very little switching to worse or better (but not best) actions. Most subjects either repeat their previous choice, imitate the most successful action, or experiment by switching to a new action. (2) Imitation of the previously most successful action is most prevalent in treatment GROUP.

Recall that WIBA and WIBM predict that agents should not switch to actions evaluated as worse than the own action in the previous round. Table 3 shows that pooled over all treatments only 8.3% of choices violate this condition. To put this rate into perspective, we need a method that contrasts it with the corresponding rate that would obtain if there were no relation between behavior and imitation. We use the following method. We randomly simulate the behavior of 100 populations of nine players for 60 periods, and calculate the success of the hypothesis relative to this simulated data. In order to give random behavior the best shot, we take the experimentally observed frequencies of actions as the theoretical distribution from which random behavior is generated. That is, we generate i.i.d. behavior in each round from the aggregate experimentally observed frequencies. The simulations show that random behavior would violate the "never imitate worse than own" condition in 16% of cases, which is substantially and significantly higher than the actual rate at all standard significance levels according to an MWU test.

Result 3. *On average, the "never imitate worse than own" condition is violated in only* 8.3% *of cases which significantly outperforms random predictions.*

Another interesting way of slicing through the data shown in Table 3 is to compute how often subjects are in line with the predictions of a simple imitation rule like IBM. We classify behavior as compliant with IBM if either the best action was imitated or there was no change in action (inertia). Thus, by summing the values obtained for "Best" and "Same" in Table 3 we find a compliance rate of 58.3% pooled over all treatments. Given that there are many non-imitating choices, it is not surprising that this rate is not terribly high, although it is significantly higher than

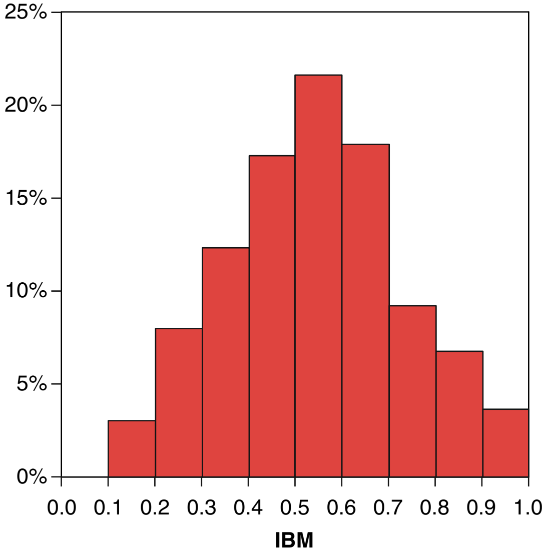

Fig. 3. Distribution of individual players on the basis of the compliance rates with IBM, all treatments pooled.

under random play, which would yield a compliance rate of 34.6% (using the method described above).[16] This further confirms that imitation is present in our data, and that, in particular, IBM and IBA play a significant role in explaining it. One can also compute a compliance rate for IBM given that subjects play an action they have previously observed.[17] In ROLE this yields a compliance rate of 82.9%, in GROUP 88.6%, and in FULL 75.5%. These rates are very high and indicate that *when* players imitate, they mostly imitate the best.

So far, we have only examined averages across subjects. But, as one would expect, there is substantial heterogeneity in subjects' propensity to imitate. Fig. 3 shows the distribution of individual players on the basis of the (unconditional) compliance rates for IBM (for all treatments pooled together). About 10% of the players show a percentage of unconditional compliance with IBM above 80%. This suggests that there is a sizeable number of almost pure imitators. It is also worth noting that more than 35% of the participants comply with IBM in more than 60% of all decisions.

Let us summarize this by stating a further result.

Result 4. *IBM and IBA do about equally well*, *and both outperform random predictions significantly*. *Moreover*, 10% *of subjects are almost pure imitators whose choices are in line with IBM/IBA in more than* 80% *of all decisions*.

Finally, let us briefly discuss the second observation we made after inspecting Table 3. There is more compliance with IBM (or IBA) in treatment GROUP than in ROLE. An MWU test yields significance at the 5% level (two-sided).[18] This is an interesting finding that will gain further

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[16] MWU tests on the basis of the average rates of compliance for the populations show that IBM outperforms random predictions at any conventional significance level.

[17] By dividing the sum of "Best" and "Same" through (100 minus "Different").

[18] All other pairwise comparisons are not statistically significant.

Table 4
Linear probability model: estimating the likelihood that subjects change actions

	ROLE		GROUP		FULL	
Constant	886***	997***	579***	730***	611***	756***
	(42.6)	(40.4)	(26.9)	(26.4)	(44.1)	(37.3)
Own payoff	−0.316***	−0.289***	−0.197***	−0.164***	−0.121***	−0.077***
	(0.033)	(0.033)	(0.024)	(0.024)	(0.029)	(0.029)
Payoff diff.	0.098***	0.100***	0.476***	0.454***	0.211***	0.208***
	(0.035)	(0.034)	(0.043)	(0.043)	(0.032)	(0.031)
Relative propensity	—	−387***	—	−418***	—	−467***
		(37.5)		(37.6)		(36.5)
R^2	0.075	0.131	0.077	0.146	0.042	0.174
# of obs.	3186	3186	3186	3186	3186	3186

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses.
*** denotes significance at the 1% level, ** denotes significance at the 5% level.

support below. Intuitively, one might expect that imitation of others who are in the same role as oneself is more appealing than imitation of a competitor who, after all, might have a different payoff function. Recall that, at least initially, our subjects do not know that they are playing a symmetric game. Also, subjects are randomly rematched every period and cannot expect to face the same opponents as last period.

Result 5. *Imitation is significantly more pronounced when subjects can observe their immediate competitors* (*as in treatment GROUP*) *than when they can observe others who have the same role in different groups* (*as in treatment ROLE*).

4.3. Estimating imitation rules

Schlag's [18] "proportional imitation" rule explicitly refers to the *probability* of imitating an action. Thus, in this section we analyze how subjects' decisions to change their action depends on their own payoff as well as the best payoff they observe. Furthermore, we also analyze how the likelihood of following IBM depends on a subject's own payoff and the best observed payoff.[19]

Table 4 shows regression results for the first question—what makes subject change their strategy. The first column for each treatment shows estimations for a simple linear probability model with random effects:

$$\Pr(s_i^t \neq s_i^{t+1}) = \alpha + \beta\pi_i^t + \gamma(\pi_{i\,\max}^t - \pi_i^t) + v_i + \varepsilon_i^t, \tag{1}$$

where s_i^t denotes subject i's strategy in period t, π_i^t the subject's own payoff, $\pi_{i\,\max}^t$ the maximal payoff the subject observed in his reference group, while v_i is the subject-specific random effect, and ε_i^t is the residual. Note that we include π_i^t directly and also in form of the payoff difference between max payoff and own payoff. This allows to test whether only the difference matters, as predicted e.g. by Schlag's proportional imitation rule, or whether own payoff and maximal payoff enter independently. If β is not significantly different from zero, then only the payoff difference matters.

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[19] Due to the high correlation of the best max and the best average, results for IBA are very similar and therefore omitted.

As a robustness check Table 4 also shows estimation results for a model that includes an additional term borrowed from the reinforcement learning literature. Reinforcement learning could be seen as the main rival to imitation in our experiment where subjects know very little about their environment.[20] But including a term capturing an element of reinforcement learning is here not so much a step toward a more complete model of what our subjects really do but rather a check whether imitation remains a significant force when one allows for other ways of learning. As in the basic model of Erev and Roth [6] the propensity of a strategy is simply the sum of all past payoffs a player obtained with that strategy. The *relative propensity* is the propensity of a strategy divided by the sum of the propensities of all strategies. The regressions in Table 4 include the relative propensity of the currently used strategy. Thus, the expected sign of the coefficient is negative. As a further robustness check the online Appendix D shows that the results for all regressions are essentially the same for linear fixed-effects models as well as for random-effects probit models.

The regressions consistently show that the coefficients for own payoffs are significantly negative while those for the observed payoff difference between own and best strategy are significantly positive, which is in line with the theoretical prediction. This holds for all treatments and the coefficients have the same order of magnitude. However, confirming what we have seen in other parts of the data analysis, the coefficients are largest in treatment GROUP. Moreover, the estimated coefficients turn out to be very robust to the inclusion of the propensity term, which is significant and has the expected sign in all treatments. Thus, reinforcement learning seems to be a factor and it helps to improve the explanation of the observed variance. Nevertheless, the inclusion of the propensity term does not diminish the significance of the variables related to imitation.

After analyzing *when* subjects switch to a different action, we shall now analyze what makes them switch to the action with the best payoffs *if* they switch at all. Table 5 reports subjects' likelihood of following IBM (contingent on switching to another action)[21] as a function of their own payoff and the observed payoff difference. As before the estimation results shown here are for linear probability models with random effects. The online Appendix D contains fixed effects and probit models. The first column for each treatment shows results for

$$\Pr(s_i^{t+1} = s_{i\ \max}^t \mid s_i^{t+1} \neq s_i^t) = \alpha + \beta\pi_i^t + \gamma(\pi_{i\ \max}^t - \pi_i^t) + v_i + \varepsilon_i^t, \tag{2}$$

where $s_{i\ \max}^t$ is the action that had the highest maximal payoff (IBM) in period t in subject i's reference group and all other variables are as defined before. The second column shows, as before, estimation results for a model that includes a propensity variable, this time the propensity of the action with the highest observed payoff (and thus, the expected sign of the coefficient is positive).

Table 5 shows that, as Schlag's models suggest, for IBM only the payoff difference matters. In all three treatments the coefficient of the difference variable has the expected sign and is significant at the 1% level. In contrast, the coefficient of own payoff is only (weakly) significant in treatment GROUP and not significantly different from zero in the other treatments. This is strong support for all rules that satisfy Property (ii) above, in particular for Schlag's proportional imitation rule. Moreover, the results are, as before, robust to the inclusion of the propensity term (although this time the propensity term does not improve the explanation of the observed variance, has an unexpected sign in treatment ROLE, and fails to be significant in treatment FULL). We briefly summarize in:

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[20] Similar to Erev and Roth [7] we may assume that imitation and reinforcement learning are just two of possibly many cognitive strategies that subjects may employ in different situations, whichever is more appropriate or successful.

[21] Since the theories allow for inertia, not switching is always in line with the prediction.

Table 5
Linear probability model: estimating the likelihood that subjects follow IBM

	ROLE		GROUP		FULL	
Constant	127***	146***	145***	113***	164***	166***
	(41.2)	(41.9)	(22.5)	(25.4)	(43.6)	(45.3)
Own payoff	−0.001	0.004	−0.043	−0.058*	0.056	0.056
	(0.038)	(0.038)	(0.030)	(0.030)	(0.038)	(0.039)
Payoff diff.	0.248***	0.246***	0.551***	0.586***	0.156***	0.156***
	(0.038)	(0.038)	(0.045)	(0.047)	(0.040)	(0.040)
Relative propensity	—	−90.1*	—	131***	—	−13.4
		(4.77)		(49.1)		(61.6)
R^2	0.038	0.038	0.080	0.087	0.009	0.009
# of obs.	2079	2079	1644	1644	1920	1920

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. Only cases with $s_i^{t+1} \neq s_i^t$ included.
*** denotes significance at the 1% level.
* significance at the 10% level.

Result 6. *In line with Schlag's imitation models, estimations show that the probability with which a subject changes his action decreases in his own payoff and increases in the maximal observed payoff. Further, the probability of imitating the best action is driven mainly by the difference between maximal observed and own payoff.*

4.4. Questionnaire results

While the choice data we collected clearly show that many of our subjects behave *as if* they imitate, one cannot be sure whether subjects are aware of what they are doing and imitate intentionally. But we have additional evidence in form of a post-experimental questionnaire. We asked subjects to "Please sketch in a few words how [they] arrived at [their] decisions" and to answer a multiple choice question regarding the variables they based their decisions on. To summarize the answers we have classified them into seven main categories which are shown in Table 6 together with selected typical answers. Some subjects argued exactly as assumed by the various imitation theories (classifications "group" and "role"). But other subjects simply chose at random, tried to differentiate themselves from the behavior of others, or followed obscure patterns. There were also subjects who were clever enough to find out the payoff structure of the game (but were often in despair about their opponents' play). Finally, some subjects reported to follow only their own past payoffs.

Table 7 lists by treatment the frequency of answers that fall into these eight categories. Imitation of others in the same group is again a frequently cited motivation in both, GROUP and FULL, whereas role imitation is less prevalent. Random behavior and own-payoff-driven behavior is frequent in all treatments. But there are also types that like to differentiate themselves, types that believe in pattern recognition, and there are some clever types that guessed the payoff structure correctly.

The key finding in this subsection is:

Result 7. *Subjects not only behave as if they imitate but many imitate intentionally. Other behavioral modes e.g. random choices, pattern-driven behavior, or behavior determined by own past payoffs can also be observed.*

Table 6
Classification of questionnaire answers

Classification	Typical answer
Role	"Answer with highest payoff of other players in previous round"
Group	"When I had the highest payoff, kept the action for the next round. Otherwise switched to the action that brought the highest payoff. Sometimes had the impression that convergent actions of all players yielded lower payoffs."
Random	"By chance since all attempts of a strategy failed!"
Contrarian	"Tried to act anti-cyclically, i.e. not to do what the other Z-players have done" (in treatment ROLE)
Pattern	"Tried to find out whether an action yielded high payoffs in a particular order—but pattern remained unknown" "...proceeded according to the scheme: ADBECADBEC..."
Clever	"My impression of the rule was that low letters correspond to low numbers. The sum of payoffs seemed to be correlated with the sum of the letters but those with higher letters got more. I attempted to reach AAA but my co-players liked to play E..."
Own	"Found out empirically where I got most points on average"

Note: These answers are typical because they are very descriptive of the categories not because they are typical for all answers in this category.

Table 7
Frequency of questionnaire answers

Classification	Treatment		
	ROLE	GROUP	FULL
Role	6	—	3
Group	—	10	12
Random	17	9	15
Contrarian	5	2	5
Pattern	2	—	6
Clever	—	8	2
Own	9	13	11

Note: A few answers were classified into two categories.

5. A robustness check

The quite abstract setting of our experiment was motivated by our interest in *comparing* alternative imitation rules. This can be achieved best in an environment that makes imitation salient. Having said this, it is nevertheless important to check for robustness with respect to the information provided to subjects and framing effects. We have therefore run two additional treatments, GROUP-π and ROLE-π, which are the same as GROUP and ROLE, respectively, except that in the instructions subjects were told that they represent firms, that they have to choose quantities, and most importantly, subjects were given the payoff table (see Table 1 and the online Appendix for the instructions).[22]

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[22] In these treatments 72 new subjects participated in the laboratory of the University of Mannheim. Average payments were 17.4 Euros.

Table 8
Summary statistics of additional treatments

	Treatment	
	ROLE-π	GROUP-π
Avg. profits, round 1	976.61	1009.47
	(55.03)	(92.24)
Avg. profits, rounds 1–60	887.03	850.6
	(50.75)	(16.59)
Avg. profits, rounds 31–60	904.91	828.93
	(31.92)	(23.76)

Note: Standard errors of mean profits for the four independent observations per treatment are given in parentheses.

It seems plausible that imitation is used particularly often in relatively opaque environments. Previous experiments on imitation (see e.g. Huck et al. [11] and Offerman et al. [16]) used games with very large strategy spaces that could not have easily been represented in a table format. The payoff table for the additional treatments, on the other hand, is small and can easily be grasped by subjects. Both the Nash equilibrium and the collusive outcome are salient. We would thus expect that imitation is less prevalent in this environment. But the important question is whether our treatment effect between ROLE and GROUP still exists or whether it is swamped by subjects' enlarged possibilities for playing more sophisticated strategies.

The results for the additional treatments are quite interesting. Table 8 reports the summary statistics. Profits in both treatments are substantially higher than in the respective treatments without information on the payoff matrix (average profits, rounds 31–60 are 828.0 in GROUP-π and 904.9 in ROLE-π). Yet, the treatment effect that is the main interest of the current paper remains: even if we conservatively treat each group of nine subjects as one observation, there is a significant difference between GROUP-π and ROLE-π at the 5% level according to a two-sided MWU test. As predicted by the imitation rules studied in this paper, profits in GROUP-π are below those in ROLE-π. Thus, although the provision of payoff tables most likely encourages subjects to pursue other activities than just imitation, the question *whom* one observes and imitates still matters. Note that this result is in line with the finding of Huck et al. [11] who show that more information about the demand and cost conditions yields less competitive behavior while more information about the quantities and profits of other firms yields more competitive behavior.

6. Conclusion

In contrast to traditional theories of rational behavior, imitation is a behavioral rule with very "soft" assumptions on the rationality of agents. Imitation is typically modelled by assuming that subjects react to the set of actions and payoffs observed in the last period, by choosing an action that was evaluated as successful. Recent theoretical results have increased economists' interest in imitation. Of particular importance are results due to Vega-Redondo [23] and Schlag [18]. Remarkably, the models make quite different predictions in many games, most notably in Cournot games, where the former predicts the Walrasian outcome while the latter predicts the Cournot–Nash equilibrium. In principle, these differences could be due to the different adjustment rules the models employ and/or the different informational conditions they assume. We study both rules in a generalized theoretical framework and show that the different predictions mainly depend on the different informational assumptions.

Comparatively slight changes in feedback information are, thus, predicted to affect behavior. Behavior is predicted to be more competitive if agents observe their immediate rivals than if they observe others who play in different groups against different opponents. From the vantage point of many other (learning) theories these differences appear surprising. Yet, in an experiment we provide clear evidence for the relevance of the information structure.

If agents only receive information about others with whom they interact, all rules that imitate successful actions imply the Walrasian outcome as the unique stochastically stable state. If agents only receive information about others who have the same role as they themselves but interact in other groups, Cournot–Nash play is the unique stochastically stable state. If agents have both types of information, the set of stochastically stable states depends on the specific form of the imitation rule. But, in general, stochastically stable states range from Cournot to Walrasian outcomes in such settings.

The experimental results provide clean evidence that changing feedback information in this manner significantly alters behavior. Learning models that do not take into account the observation of others' payoffs cannot explain this effect. Moreover, the differences between treatments are ordered as the generalized imitation model suggests. Direct support for the role of imitation is found by analyzing individual adjustments. We find that imitation can explain a substantial number of adjustments and that some subjects are almost pure imitators. Moreover, estimating subjects's choice functions we find support for Schlag's result that suggests that the likelihood of imitating a more successful action increases in the difference between own and other's payoff.

Finally, we observe that imitation of actions seems to be more prevalent when subjects observe others with whom they interact as opposed to others who have the same role but play in different groups. There is no theoretical model that would account for such a difference. Moreover, one might think that imitation of others who are identical to oneself is more meaningful than imitation of others with whom we play but who might be different. (After all, subjects in our experiment did not know that they were playing a symmetric game.) But this is not supported by the data. One conjecture that might explain the difference we observe is that imitation of more successful actions might be particularly appealing when one directly competes with those who are more successful. In environments where imitation prevents agents to do worse than their immediate competitors, there is an obvious "evolutionary" benefit from imitating. Thus, evolution might have primed us towards imitative behavior if we compete with others for the same resources. This would explain our data but more theoretical work is needed to study the evolutionary advantages and disadvantages of imitative behavior.[23]

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[23] For first steps in this direction see Matros [15].

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at 10.1016/j.jet.2006.07.006.

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