Computing Business Cycles with Endogenous Risk[1]

James S. Costain
Universitat Pompeu Fabra
Michael Reiter
Universitat Pompeu Fabra

March 2001
VERY PRELIMINARY

Correspondence address:
Department of Economics and Business
Universitat Pompeu Fabra
Ramon Trias Fargas 25-27
08005 Barcelona Spain
james.costain@econ.upf.es
reiter@upf.es

[1]Thanks to Albert Marcet for many helpful suggestions, and to Chris Telmer and Kjetil Storesletten for sharing some related computer programs. Thanks also to Chris Telmer, Masako Ueda, Xiaohong Chen, Sevi Rodriguez, Wouter den Haan, and Eva Cárceles. Financial support from the Spanish Ministry of Education (DGES grant PB98-1065) is gratefully acknowledged. Remaining errors are the responsibility of the authors.

Abstract

We simulate the business cycle dynamics of an economy in which agents face a riskier environment during downturns. All agents act both as workers and as entrepreneurs, and each of these roles entails risks. As workers, they face (incompletely insured) risk of unemployment, while as entrepreneurs, they choose between riskless investment (goods storage) and risky investment (hiring for a small business). Risky investment acts as an input to the matching function which determines employment. Changes in employment are amplified because greater unemployment risk causes agents to decrease their investment in the risky asset; this fall in risky investment then further increases unemployment. This amplification is sufficient to generate large aggregate fluctuations from idiosyncratic shocks when the number of agents is finite.

We solve the model by a version of the parameterized expectations algorithm. In contrast to some previous papers which have found risk to be unimportant, we find that our economy generates strikingly different aggregate time series when we change the level of insurance of idiosyncratic shocks. When insurance is low, recessions are characterized by high unemployment, low consumption, low investment in the risky asset, and high investment in the riskless asset. Similar fluctuations remain even if we restrict the idiosyncratic shocks to have a constant cross-sectional mean. When insurance is low, our model also implies a highly dispersed distribution of wealth, because the richest agents invest more in the risky asset, which has a higher return.

JEL classification: E32, E21, E24, G11

Keywords: Business cycles, investment, risk, insurance, incomplete markets, portfolio choice, heterogeneity

1 Introduction

A great deal is known about the dynamics of real general equilibrium models under full insurance, but much remains to be learned about similar models with incomplete insurance. While full insurance, or complete markets, may be an interesting theoretical baseline, it is necessary to ask whether its dynamics are robust to the likely imperfections of real world insurance markets. More importantly, since investors appear seriously concerned about risk, and since investment behavior is a central component of the business cycle, there are *a priori* reasons to suspect that imperfectly insured risk may do more than slightly alter the economy's fluctuations: risk may be an underlying cause of the business cycle itself.

Costain (1999) proposes one intuitively plausible way in which the fluctuations of an imperfectly insured economy may differ— and be amplified— relative to those of a fully insured economy. That paper studies a two period, two asset economy in which recessions imply an increase in risk. Since recessions are risky, there are countercyclical reallocations from risky to riskless assets which may amplify the downturns and may, in fact, imply multiple equilibria. Some related mechanisms have been proposed by Acemoglu and Zilibotti (1997) and Chatterjee (1988). Unfortunately, these papers all present rather stylized models, which makes the empirical plausibility of their arguments difficult to assess. In particular, the volatility of the economy in Costain (1999) depended critically on initial asset holdings. This suggests that an infinite horizon model, in which asset holdings are a state variable, is the appropriate context in which to judge the quantitative relevance of the proposed effects.

The present paper computes equilibria of an infinite horizon version of the economy studied in Costain (1999). All agents are both workers and entrepreneurs. As workers, they face a risky income stream because they may be either employed or unemployed. As entrepreneurs, they allocate their savings between a risky asset and a safe asset. In general equilibrium, the probability that workers find employment is determined, via a matching technology, as a function of the current unemployment rate and of the quantity of *risky* investment. This assumption implies that the economy is characterized by positive feedbacks: an increase in unemployment means a riskier labor income stream, which gives agents an incentive to shift investment out of the risky asset (the input to the matching function) and into the riskless asset (a storage technology), further increasing unemployment. We explore computationally how much this positive feedback amplifies fluctuations.

This outline reminds us why so much remains unknown about imperfect insurance economies: their behavior is extremely difficult to characterize, not only analytically, but even numerically. The difficulty, in the context of this paper, is that we are interested in the *dynamics* of an economy in which the state variable is the *distribution* of asset holdings. Only recently have economists begun to develop algorithms for calculating distributional dynamics; see for instance Krusell and Smith (1997, 1998) and den Haan (1997). We characterize the equilibrium of our economy using a version of the parameterized expectations algorithm, under the assumption that the aggregate state can be adequately summarized by a finite list of moments of the distribution. Our algorithm allows for choice between multiple assets, with occasionally binding portfolio constraints, and for a large but finite number of agents.

1.1 Related literature

Precautionary saving:

Deaton (Econometrica, 1991)

Aiyagari (QJE, 1994)

Carroll (QJE, 1997)

Matching:

Pissarides (1991)

Macroeconomics under financial market imperfections:

Bernanke and Gertler (AER, 1989)

Greenwald and Stiglitz (QJE, 1993)

Kiyotaki and Moore (JPE, 1997)

Positive feedbacks based on risk:

Chatterjee (1988)

Acemoglu and Zilibotti (JPE 1998)

Costain (1999)

Computational models of idiosyncratic risk:

Marcet and Singleton (Macroeconomic Dynamics 1999)

Krusell and Smith (Macroeconomic Dynamics 1997, JPE 1998)

Den Haan (Macroeconomic Dynamics 1997, 1999)

Storesletten, Telmer, and Yaron (1999)

2 The model

There is a large but finite number of agents I. Each agent in this job matching economy plays two roles: she is both a worker and an entrepreneur. As a worker, she supplies labor inelastically, but can do so only if she finds a job. As an entrepreneur, she chooses between two types of assets: a riskless one which can be thought of as a storage technology, and a risky one which can be interpreted as small business formation. Two types of risk thus arise naturally for all agents: risk of unemployment, and risky returns to investment. The feedbacks in our model result from the fact that changes in unemployment risk affect agents' willingness to invest in the risky asset, together with the fact that risky investment is an input to the matching function.

2.1 The individual's problem

Each agent cares only about her utility from consumption over her own lifetime. Instantaneous utility exhibits constant relative risk aversion (CRRA), and lifetime utility takes the form:

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \mathbf{1}(\epsilon_t^z = 1) v(c_t) \quad \text{where} \;\; v(c_t) \;\; = \;\; \frac{c_t^{1-\alpha} - 1}{1-\alpha}$$

Here $\alpha > 0$, and $\rho > 0$ is a pure time discount rate. The function $\mathbf{1}(.)$ is an indicator function showing that the individual is still alive as long as the random variable ϵ_t^z equals one.

An agent with wealth $a_t > 0$ at time t can split this wealth between current consumption c_t, a riskless asset b_{t+1} with price 1, and a risky asset n_{t+1} with price $P_t > 0$, so that

$$c_t \; + \; b_{t+1} \; + \; P_t n_{t+1} \;\; \leq \;\; a_t \tag{1}$$

Consumption and asset choices are constrained to be non-negative.

Wealth in the following period consists of two components. Part comes from holdings of consumable goods $q_{t+1} > 0$, which are derived from labor income $w_{t+1}^* > 0$, the gross payoff of the safe asset $Rb_{t+1} > 0$, the dividends from risky investment $\pi_{t+1}^* n_{t+1} > 0$, and inheritances of goods $Z_{t+1}^b > 0$:

$$q_{t+1} \;\; = \;\; w_{t+1}^* \; + \; Rb_{t+1} \; + \; \pi_{t+1}^* n_{t+1} \; +$$

indiosyncratic productivity state ϵ^π_{t+1}, as well as on the aggregate state Φ_{t+1}, so that the profit rate is $\pi^*_{t+1} = \pi^*(\epsilon^\pi_{t+1};\Omega_{t+1})$. Second, the aggregate state itself varies randomly over time, which implies that prices and inheritances are random. These variables depend on the aggregate state only: $P_{t+1} = P(\Phi_{t+1})$, $Z^b_{t+1} = Z^b(\Phi_{t+1})$, $Z^n_{t+1} = Z^n(\Phi_{t+1})$, and hence also for the total value of inheritances: $Z_{t+1} \equiv Z^b_{t+1} + P_{t+1}Z^n_{t+1} = Z(\Phi_{t+1})$.

The individual state variables $\epsilon \equiv (\epsilon^e, \epsilon^\pi, \epsilon^z)$ have a Markovian structure over time. Employment transition probabilities are governed by a variable job-finding probability M_t; simplifying slightly, the transitions are as follows.[1] An individual fully employed at t loses her job with probability δ at the end of period t. Those who have just lost their jobs, together with those unemployed at t, are eligible for reemployment at $t+1$; they find jobs with probability M_t. Thus the transition probabilities are:

$$\begin{array}{rcl} \text{prob}(\epsilon^e_{t+1} = 0|\epsilon^e_t = 0) & = & 1 - M_t \\ \text{prob}(\epsilon^e_{t+1} = 1|\epsilon^e_t = 0) & = & M_t \\ \text{prob}(\epsilon^e_{t+1} = 0|\epsilon^e_t = 1) & = & \delta(1 - M_t) \\ \text{prob}(\epsilon^e_{t+1} = 1|\epsilon^e_t = 1) & = & 1 - \delta(1 - M_t) \end{array} \quad (4)$$

The profitability shock ϵ^π is an *iid* binary variable, satisfying

$$\begin{array}{rcl} \text{prob}(\epsilon^\pi_{t+1} = 0) & = & 1 - p \\ \text{prob}(\epsilon^\pi_{t+1} = 1) & = & p \end{array} \quad (5)$$

Finally, we assume that an individual has probability ζ of dying in each period, so that ϵ^z follows

$$\begin{array}{rcl} \text{prob}(\epsilon^z_{t+1} = 0|\epsilon^z_t = 0) & = & 1 \\ \text{prob}(\epsilon^z_{t+1} = 0|\epsilon^z_t = 1) & = & \zeta \\ \text{prob}(\epsilon^z_{t+1} = 1|\epsilon^z_t = 1) & = & 1 - \zeta \end{array} \quad (6)$$

Note that the transitions described in (4)-(6) are all independent. They are all exogenous and constant except for the job finding probability M_t, which we assume depends on the aggregate state: $M_t = M(\Phi_t)$. Thus, conditional on

[1] The Markov transition probabilities stated in (4) are only an approximation. The exact dynamics must take into account the fact that there are only a finite number of agents. To ensure exact labor market clearing, we must allow exactly one agent to work part time at each t. The exact specification is described in the appendix, equations (35) - (37).

the aggregate state, equations (4)-(6) define a transition function T^{ϵ} for the individual state, of the form

$$T^{\epsilon}(\epsilon'; \epsilon, \Phi) \;\; = \;\; \text{prob}[\epsilon_{t+1} \leq \epsilon' | (\epsilon_t, \Phi_t) = (\epsilon, \Phi)] \tag{7}$$

Optimal behavior in this dynamic context requires that the individual use available information to predict future payoff-relevant variables. We assume that agents behave competitively, in the sense that they ignore any impact of their own choices on the dynamics of the variables P, M, Z, w^*, and π^*, and of the aggregate state Φ.[2] Since the aggregate state Φ (via M) determines the transition probabilities of the idiosyncratic states ϵ, these assumptions imply that the individual takes as given the joint transition function T governing the probability of future idiosyncratic/aggregate states (ϵ, Φ):

$$T(\epsilon', \phi'; \epsilon, \Phi) \;\; = \;\; \text{prob}[(\epsilon_{t+1}, \Phi_{t+1}) \leq (\epsilon', \Phi') | (\epsilon_t, \Phi_t) = (\epsilon, \Phi)] \tag{8}$$

With this information in hand, the individual can calculate the conditional distribution of all payoff-relevant quantities at all times in the future; in particular, she knows the probability distribution over the aggregate variables P, M, Z, and over their idiosyncratic shocks w^*, π^*, and ϵ^z, at all future times.

We are now ready to state the individual's decision problem in recursive form. Notice that once the aggregate state Φ_t and the price $P(\Phi_t)$ are realized, the agent no longer cares how her assets are split into consumable goods and risky asset stocks; only her wealth a_t enters her budget constraint. She must also take into account her current employment status ϵ_t^e, since it affects her future employment probabilities, but she can ignore the *iid* technology shock ϵ_t^π once she learns a_t. Thus her idiosyncratic state at t can be summarized as $\hat{\phi} \equiv (a_t, \epsilon_t^e)$. Her decision problem is described by the following Bellman equation:

[2] There are two possible justifications for this assumption. The more technically correct justification is that we actually have a finite number of *types*, and a continuum of agents of each type, rather than a finite number of agents. A technically sloppier, but probably more economically appealing interpretation is that the impact of individual behavior on payoff-relevant aggregate variables is small when there are many agents, so that boundedly rational agents would be likely to ignore these effects.

Individual's Bellman equation:

$$V(\hat{\phi}_t;\Phi_t) \;\equiv\; V(a_t,\epsilon^e_t;\Phi_t) \;= \tag{9}$$

$$\underset{c_t,\, b_{t+1},\, n_{t+1} \geq 0}{Max} \left\{ \frac{1}{1-\alpha} c_t^{1-\alpha} \;+\; \frac{1-\zeta}{1+\rho} E_T \left[V(\hat{\phi}_{t+1};\Phi_{t+1}) | \epsilon^e_t;\Phi_t \right] \right\}$$

$$\text{where } \; c_t \;=\; a_t \;-\; P(\Phi_t)n_{t+1} \;-\; b_{t+1}\,, \quad \text{and}$$

$$a_{t+1} \;=\; \left[P(\Phi_{t+1})(1-\delta) + \pi^*(\epsilon^\pi_{t+1};\Phi_{t+1})\right] n_{t+1} + w^*(\epsilon^e_{t+1};\Phi_{t+1}) + Rb_{t+1} + Z(\Phi_{t+1})$$

The value function V here refers to maximal intertemporal utility conditional on being alive at t, and E_T refers to an expectation given the transition law T. This Bellman equation suffices to determine the choices of c_t, b_{t+1} and n_{t+1} for each possible individual/aggregate state $(\hat{\phi}_t;\Phi_t)$ as long as the functions P, Z, w^*, and π^*, and the transition law T are known. That is, it determines policy functions $c_t = C(\hat{\phi}_t;\Phi_t)$, $b_{t+1} = B(\hat{\phi}_t;\Phi_t)$ and $n_{t+1} = N(\hat{\phi}_t;\Phi_t)$.

With CRRA utility and positive assets a, consumption will be chosen to be strictly positive. What we cannot rule out in general is zero investment in one or both assets, so that the Euler conditions for investment can take the form of inequalities rather than equations. From the first-order and envelope conditions associated with (9), we find that consumption, conditional on survival, must satisfy the following pair of Euler inequalities:

$$c_t^{-\alpha} \;\geq\; \tfrac{1-\zeta}{1+\rho} E_T \left[R c_{t+1}^{-\alpha} | \hat{\phi}_t;\Phi_t \right] \;\text{ and }\; b_{t+1} \;\geq\; 0 \tag{10}$$

with at least one equality, and

$$P(\Phi_t)c_t^{-\alpha} \;\geq\; \tfrac{1-\zeta}{1+\rho} E_T \left[(\pi^*(\epsilon^\pi_{t+1};\Phi_t) + P(\Phi_{t+1})(1-\delta)) c_{t+1}^{-\alpha} | \hat{\phi}_t;\Phi_t \right]$$

$$\text{and } \; n_{t+1} \;\geq\; 0 \tag{11}$$

with at least one equality.

2.2 General equilibrium.

Prior to knowing the price level, an individual's state can be completely described by the variables $\phi_t \equiv (b_t, n_t, \epsilon_t)$. Thus a possible description of the aggregate state of the economy is the cross-sectional distribution function

$$\Phi_t(\phi) \equiv \frac{1}{I}\#(\phi_{it} \leq \phi) \tag{12}$$

(Here subscript i denotes individuals, and the function $\#(x)$ denotes the number of individuals for whom statement x is true.) For the purposes of this paper, we will make the mild assumption that an equilibrium exists in which P, and all other payoff-relevant aggregate variables, depend only on the aggregate state as defined in (12). That is, we assume there exists an equilibrium in which sunspots play no role, and individual names are irrelevant.

Our goal in this paper is to understand the possible interactions between risky investment decisions and the endogenously changing risks arising from the business cycle itself. Here this endogenous risk takes one simple form: risk of unemployment. We link this risk to the state of economic activity by defining risky investment as "hiring", broadly interpreted as the sum of recruitment expenditures plus purchases of capital complementary to labor. We intend this as a relatively simple specification for studying the effects of entrepreneurial risk-taking on employment. The risky asset holdings n_t of a given agent at time t are therefore interpreted as the number of employees contracted by that agent to work at time t. For simplicity, n_t is not required to be an integer; but it must be consistent with aggregate unemployment:

$$N_t = 1 - U_t \tag{13}$$

where

$$N_t \equiv N(\Phi_t) = \int n_t d\Phi_t \quad \text{and} \quad U_t \equiv U(\Phi_t) = \int (1 - \epsilon_t^e) d\Phi_t \tag{14}$$

This interpretation requires that we specify some relationships between the labor income that individuals receive as workers, and the profit income that they receive as entrepreneurs. The gross return $R > 1$ to riskless storage (which might also be interpreted as the returns to foreign bonds) is exogenous. The output $\underline{w}$ produced during unemployment ($\epsilon^e = 0$) is also exogenous. The firm operated by an entrepreneur experiences an idiosyncratic productivity shock $\tilde{y}$, which is *iid* across agents and over time, taking the values $\overline{y}$ with probability p and $\underline{y}$ with probability $1 - p$. Production requires both capital and labor, in fixed proportions; output is

$$\tilde{y}\min(n, k/\bar{k})$$

Thus firms must purchase $\bar{k}$ units of capital goods for each worker hired. Capital is assumed specific to the worker hired, and it does not depreciate until the worker separates from the firm. This means that the cost P of hiring a new worker can be broken into two parts:

$$P \;= \bar{k} \;+\; P_H \tag{15}$$

where in addition to the cost of capital purchases, P_H is the actual cost of recruiting the worker.

We assume that the wage must be paid at the beginning of the period, prior to the realization of the firm-specific productivity shock $\tilde{y}$. Abstracting from optimal contracting issues, we assume that workers are paid a fraction β of their expected productivity; that is, the wage for a fully employed worker is the non-random quantity $\overline{w} = \beta\mu_y$, where $\mu_y \equiv p\overline{y} + (1-p)\underline{y}$. We pick parameters so that $0 < \underline{w} < \overline{w}$; note that therefore each individual receives a labor income shock $\tilde{w}$, which takes values in $[\underline{w}, \overline{w}]$, and which is a function of the idiosyncratic labor market state ϵ^e.[3] The entrepreneur is the residual claimant for the output of her firm, so profits per employee are the random quantity $\tilde{\pi} \equiv \tilde{y} - \overline{w}$.

To understand the role of idiosyncratic risk in this economy, we will compare the behavior of the economy with and without insurance for the idiosyncratic risks $\tilde{w}$ and $\tilde{\pi}$. Note that average (pre-insurance) labor income at time

[3] Part time employees, which are necessary for exact labor market clearing, receive the wage $\overline{w}$ for the time worked, and $\underline{w}$ for the time not worked, so in general $\tilde{w} = \epsilon^e\overline{w} + (1-\epsilon^e)\underline{w}$. See the appendix.

t is $\mu_w(\Phi_t) \equiv N(\Phi_t)\overline{w} + U(\Phi_t)\underline{w}$. If we define $\theta^u \in [0,1]$ as the degree of unemployment insurance, then fair (balanced budget) insurance transfers make it possible to reduce idiosyncratic fluctuation in labor income by paying each worker the wage $w_t^* \equiv (1-\theta^u)\tilde{w}_t + \theta^u \mu_w(\Phi_t)$. Notice that if there is no unemployment insurance ($\theta^u = 0$), then each worker simply receives $w_t^* = \tilde{w}_t$, while if there is full unemployment insurance ($\theta^u = 1$), all workers share total time t labor income equally: $w_t^* = \mu_w(\Phi_t)$, so that labor income only varies with aggregate unemployment. Note that the spirit of our exercise will be to ask how the economy differs depending on the exogenously specified level of insurance θ^u; modelling the information problems that underlie the degree of insurance is beyond the scope of this paper.

Similarly, we can consider insurance (or equivalently, diversification) of the idiosyncratic profitability shock $\tilde{\pi}$. Notice that average profitability per unit of investment in the economy at time t is

$$\mu_\pi(\Phi_t) \quad = \quad N_t^{-1} \int \tilde{\pi}_t n_t d\Phi_t \tag{16}$$

Fair insurance transfers then make it possible to pay each entrepreneur profits of $\pi_t^* \equiv (1-\theta^\pi)\tilde{\pi}_t + \theta^\pi \mu_\pi(\Phi_t)$, where $\theta^\pi \in [0,1]$ is the exogenous level of investment insurance.

We must still ensure aggregate consistency in our labor market dynamics. A valid initial distribution of assets and employment must satisfy the labor market clearing condition (13). Thereafter, in line with the (approximate) transition probabilities described in (4), we define the following accounting relations.[4] Define aggregate job seekers at the end of period t as

$$S_t \quad \equiv \quad S(\Phi_t) \quad = \quad U_t + \delta(1-U_t) \tag{17}$$

Assume that the fraction of job seekers who find work for $t+1$ is $M_t = M(\Phi_t)$, so that the total formation of new jobs is

$$J_t \quad \equiv \quad J(\Phi_t) \quad = \quad M_t S_t \tag{18}$$

[4] Although (4) is only an approximation, the dynamics in (17) - (20) and elsewhere are exactly correct.

If the recruitment cost of hiring a new worker is $P_{Ht} = P_H(\Phi_t)$, then total expenditures on hiring must be

$$H_t \equiv H(\Phi_t) = P_{Ht} J_t \tag{19}$$

Given new job formation J_t, we must have

$$(1 - U_{t+1}) = (1-\delta)(1-U_t) + J_t = (1 - \delta(1 - M_t))(1 - U_t) + M_t U_t \tag{20}$$

or equivalently

$$M_t = \frac{(1 - U_{t+1}) - (1-\delta)(1-U_t)}{U_t + \delta(1 - U_t)} \tag{21}$$

and the new level of unemployment U_{t+1} must still satisfy the labor market consistency condition (13).

The most common way to define the matching rate M_t and the recruitment cost P_{Ht} would be to explicitly define an aggregate matching function $J(H_t, S_t)$ and then derive M_t and P_{Ht} from (18) and (19). We find it helpful instead to directly define the relationship between M_t and P_{Ht}, because this makes it easier for us to calibrate the relationship between these variables; the aggregate matching function is then defined implicitly. We assume that the recruitment cost of hiring workers is an increasing function of the matching rate:

$$P_{Ht} = m \left(\frac{M_t}{1 - M_t} \right)^{\gamma} \tag{22}$$

Notice that when workers have difficulty finding jobs (slack labor market conditions where M_t approaches zero), the cost of recruitment goes to zero; likewise when workers easily find jobs (a tight labor market where M_t approaches one), the cost goes to infinity. Substituting, (22) implies

$$H_t = J_t m \left(\frac{J_t / S_t}{1 - J_t / S_t} \right)^{\gamma} \tag{23}$$

which implicitly defines a matching function $J_t = J(H_t, S_t)$ which can be shown to be increasing and convex in both arguments.

In practice, to calculate an equilibrium price in a given period, we start by calculating the unemployment rate and eligible job seekers from (14) and (17).

Then we guess a matching rate M_t and calculate the resulting unemployment rate for $t+1$ from (20). However, we can also calculate the associated price P_t from (22) and (15), and then solve the individual's problem for period t (using the parameterized expectations for $t+1$) to calculate risky investment n_{t+1} for all individuals. We then use labor market clearing (13) to convert this into an unemployment rate. If individuals are hiring too little, we guess a lower M_t; if individuals are hiring too much, we raise M_t. A fixed point of this one dimensional search clears markets for the given period, and allows us to calculate P_t and N_{t+1}.[5] Once we know time $t+1$ employment, we can then calculate actual idiosyncratic employment transitions from (35) - (37).

We should make two additional comments about our labor market specification. First, note that we have not restricted individual hiring to be positive; that is, we assumed that $n_{t+1} \geq 0$ but not $n_{t+1} - (1-\delta)n_t \geq 0$. Implicitly, this means that workers, once hired, can be traded to other entrepreneurs. With tradeable employees, arbitrage requires that the price P_t arising from the matching function also governs trades of workers between entrepreneurs. We allow for trading in employees in order to avoid irreversibility in risky investment at the individual level. Individual irreversibility is a potentially interesting form of risk which we plan to investigate in future work, but would distract from the focus of the current paper. Second, our assumption that the entrepreneur's hiring process is deterministic is made for simplicity only. Implicitly, this means workers who "find jobs" are shared across firms. It would undoubtedly seem more natural to assume that the number of workers actually hired by any given entrepreneur were an exponential random variable. However, this would merely introduce an extra element of risk into the

[5] If there insufficient demand for n_{t+1} when $P_t = \bar{k}$, we set $M_t = 0$ and search for a $P_t < \bar{k}$ that clears markets. It can then be shown that under the parameterized expectations algorithm described in Section 4 there always exists at least one (M_t,P_t) pair which equates risky asset demand with labor supply, thus satisfying labor market consistency. First, as M goes from 0 to 1, the corresponding P goes from 0 to ∞. Second, demand for risky assets, under our parameterized expectations setup, is a continuous function of P, which goes to infinity as $P \to 0$, and to a quantity less than the outstanding undepreciated supply as $P \to \infty$.

entrepreneur's problem without altering the intuition of the precautionary saving and portfolio choice incentives which are the focus of this paper. We doubt this type of risk plays a large role in real world investment decisions.

Finally, we should discuss death and inheritance. Agents who die at the end of period t ($\epsilon^z_{t+1} = 0$) are replaced by new agents at the beginnig of $t+1$, so that population is constant. The newborns inherit the employment status ϵ^e_{t+1} of their deceased predecessors, but start with assets $b_{t+1} = n_{t+1} = 0$. Thus the wealth of newborns at $t+1$ is given by $a_{t+1} = w^*(\epsilon^e_{t+1}; \Phi_{t+1}) + Z(\Phi_{t+1})$. Inheritances are spread evenly among all agents, so they satisfy the aggregate consistency condition

$$Z(\Phi_{t+1}) \;\; = \;\; \int (b_{t+1} + P(\Phi_{t+1}) n_{t+1})(1 - \epsilon^z_{t+1}) d\Phi_t \tag{24}$$

We are at last ready to define equilibrium. Eliminating some superfluous variables defined earlier, an equilibrium consists of the following objects:

A law of motion T for the idiosyncratic and aggregate state $(\epsilon; \Phi)$
a value function $V(\hat{\phi}; \Phi)$,
policy functions $B(\hat{\phi}; \Phi)$, $N(\hat{\phi}; \Phi)$, and $C(\hat{\phi}; \Phi)$,
a pricing function $P(\Phi)$, a matching rate function $M(\Phi)$, an inheritance function $Z(\Phi)$, earnings functions $w^*(\epsilon^e, \Phi)$ and $\pi^*(\epsilon^\pi, \Phi)$, and an unemployment rate $U(\Phi)$.

These objects must satisfy the following conditions:

Given T, P, Z, w^* and π^* the value function V and the policy functions B, N, and C result from the individual's maximization problem (9).

The probability distribution over future ϵ implied by transition law T is consistent with (33)-(37).

The probability distribution over future Φ implied by transition law T is consistent with the policy functions B and N, as well as the dynamics of ϵ given in (33)-(37).

Earnings w^* and π^* have the distributions described in the text. Inheritances satisfy (24). Labor markets clear, as stated in (13). The matching rate is related to the rate of change of unemployment via (21), and the functions P and M satisfy (22) and (15).

3 A parameterized expectations algorithm

The parameterized expectations algorithm approximates optimal behavior by finding functions of time t variables that can approximate the expectations appearing in the complementary slackness conditions (10) and (11). These approximate expectations are then used to calculate individual choices at t. The approximations can be found by fitting the expectations to current variables in simulation.

For computational speed, it is essential to write the complementary slackness conditions in an invertible way, so that choice variables can be calculated explicitly when the expectations are known.[6] Thus rather than writing the complementary slackness conditions in the previous form (10) - (11), we will rewrite them in terms of the following variables:

$$\tilde{f}_{t+1} \equiv f(\hat{\phi}_{t+1};\Phi_{t+1}) \equiv \frac{\bar{R}(1-\zeta)}{1+\rho}\left[\frac{C(\hat{\phi}_{t+1};\Phi_{t+1})}{a_t}\right]^{-\alpha}$$

and

$$\tilde{g}_{t+1} = g(\hat{\phi}_{t+1};\Phi_{t+1})$$
$$\equiv \frac{1-\zeta}{\exp(b_{t+1})}\left(\frac{\pi^*(\epsilon^{\pi}_{t+1};\Phi_{t+1})+P(\Phi_{t+1})(1-\delta)}{1+\rho}\right)\left[\frac{C(\hat{\phi}_{t+1};\Phi_{t+1})}{a_t}\right]^{-\alpha}$$

Note that a_t, $b_{t+1} = B(\hat{\phi}_t;\Phi_t)$ and $n_{t+1} = \bar{N}(\hat{\phi}_t;\Phi_t)$ are all functions of the time t individual state $(\hat{\phi}_t;\Phi_t)$. Furthermore, the transition law T, together with the policy functions $b_{t+1} = B(\hat{\phi}_t;\Phi_t)$ and $n_{t+1} = \bar{N}(\hat{\phi}_t;\Phi_t)$, gives the

[6]Marcet and Singleton (1998) discuss this point, and provide helpful examples.

distributions of $\hat{\phi}_{t+1}$ and Φ_{t+1} in terms of $\hat{\phi}_t$ and Φ_t. Hence we can write the time t conditional expectations of $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$ as functions of $(\hat{\phi}_t; \Phi_t)$:

$$F(\hat{\phi}_t; \Phi_t) \equiv E_T\{\tilde{f}_{t+1} | \hat{\phi}_t; \Phi_t\}$$

$$G(\hat{\phi}_t; \Phi_t) \equiv E_T\{\tilde{g}_{t+1} | \hat{\phi}_t; \Phi_t\}$$

Note that the functions F and G are scaled versions of the expected marginal utilities of investment in the two assets. F represents the expected marginal utility of investment in the safe asset, multiplied by $(a_t)^\alpha$, a quantity known at time t. Similarly, G is the expected marginal utility of investment in the risky asset, multiplied by $(a_t)^\alpha / \exp(b_{t+1})$, which is also known at t. We will see in a moment that this rescaling makes the complementary slackness conditions explicitly invertible to solve for current choices when F, G, and the price level are known.

If we knew functions F and G as described above, then when $P_t = P(\Phi_t)$ we could solve for optimal behavior b_{t+1} and n_{t+1} from the following version of the complementary slackness conditions (10) and (11):

$$\left(\frac{c_t}{a_t}\right)^{-\alpha} \geq F(\hat{\phi}_t; \Phi_t) \text{ and } b_{t+1} \geq 0 \tag{25}$$

with at least one equality, and

$$\left(\frac{c_t}{a_t}\right)^{-\alpha} \geq P(\Phi_t)^{-1} \exp(b_{t+1}) G(\hat{\phi}_t; \Phi_t) \text{ and } n_{t+1} \geq 0 \tag{26}$$

with at least one equality. There are four cases, corresponding to binding or non-binding constraints on each asset choice. Given any positive and finite values for a_t, P_t, F_t, and G_t, we have the following four possibilities.

Case I. $b_{t+1} > 0$ and $n_{t+1} > 0$ requires:

$$\left(\frac{c_t}{a_t}\right)^{-\alpha} = F_t\,, \quad b_{t+1} = \log(F_t P_t / G_t)\,, \quad \text{and } n_{t+1} = (a_t - c_t - b_{t+1})/P_t$$

Case II. $b_{t+1} > 0$ and $n_{t+1} = 0$ requires:

$$\left(\frac{c_t}{a_t}\right)^{-\alpha} = F_t \quad \text{and} \quad b_{t+1} = a_t - c_t$$

Case III. $b_{t+1} = 0$ and $n_{t+1} > 0$ requires:

$$\left(\frac{c_t}{a_t}\right)^{-\alpha} = G_t/P_t \quad \text{and} \quad P_t n_{t+1} = a_t - c_t$$

Case IV. $b_{t+1} = 0$ and $n_{t+1} = 0$ requires $c_t = a_t$.

Given the four sets of formulas above, we can distinguish the ranges of P_t, a_t, F_t, and G_t for which the four cases are valid. In particular, the condition

$$F_t^{-1/\alpha} < 1 \quad \text{or equivalently} \quad F_t > 1 \tag{27}$$

is necessary for cases I and II, and is inconsistent with case IV (it has no implication regarding case III.) Secondly, the condition

$$\log(P_t F_t / G_t) > 0 \quad \text{or equivalently} \quad P_t F_t > G_t \tag{28}$$

is also necessary for cases I and II, and is inconsistent with case III (it has no implication regarding case IV.)[7] Hence if both (27) and (28) are satisfied, then we know that $b_{t+1} > 0$ strictly, and we are either in case I or case II. To distinguish between these two cases, we then check whether

$$\log\left(\frac{P_t F_t}{G_t}\right) < a_t\left(1 - F_t^{-1/\alpha}\right)$$

$$\text{or equivalently} \quad G_t > \frac{P_t F_t}{\exp(a_t(1 - F_t^{-1/\alpha}))} \tag{29}$$

which is necessary for case I and inconsistent with case II. On the other hand if either (27) or (28) is false, then we must be in case III or case IV. These two cases can be distinguished by checking whether

$$\left(\frac{G_t}{P_t}\right)^{-1/\alpha} < 1 \quad \text{or equivalently} \quad G_t > P_t \tag{30}$$

which is necessary for case III and inconsistent with case IV. Hence there exists a unique solution to the complementary slackness conditions associated with each positive, finite set of values for P_t, a_t, F_t, and G_t.

[7] To see that (28) is necessary for case II, note that when $n_{t+1} = 0$ and $b_{t+1} > 0$ the slackness conditions (25) and (26) imply $F_t \geq \exp(b_{t+1}) G_t / P_t > G_t / P_t$.

To simulate our economy using the parameterized expectations approach, we must find functions $\mathcal{F}(\hat{\phi}_t;\Phi_t) \approx F(\hat{\phi}_t;\Phi_t)$ and $\mathcal{G}(\hat{\phi}_t;\Phi_t) \approx G(\hat{\phi}_t;\Phi_t)$ which approximate the true expectations. We must choose some information vector $x_t = x(\hat{\phi}_t;\Phi_t)$, of length $\#_x$, of which each element is a function of the idiosyncratic/aggregate state, which can hopefully serve to summarize the information on which individuals base their decisions at t. Here we will consider approximating functions of the form $\mathcal{F}(\hat{\phi}_t,\Phi_t) \equiv \exp(\beta_F' x_t)$ and $\mathcal{G}(\hat{\phi}_t,\Phi_t) \equiv \exp(\beta_G' x_t)$, where β_F and β_G are vectors of the same length $\#_x$. The prediction errors associated with this functional form are given by

$$\tilde{f}_{t+1} - \mathcal{F}_t = f(\hat{\phi}_{t+1};\Phi_{t+1}) - \exp(\beta_F' x(\hat{\phi}_t;\Phi_t))$$

and

$$\tilde{g}_{t+1} - \mathcal{G}_t = g(\hat{\phi}_{t+1};\Phi_{t+1}) - \exp(\beta_G' x(\hat{\phi}_t;\Phi_t))$$

Note that the prediction errors are functions of the parameter choices β_F and β_G as well as the explanatory variables x_t and the dependent variables $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$.

The algorithm works as follows. Given any β_F and β_G, we simulate many histories of the economy, starting from a wide variety of arbitrarily chosen initial distributions. Each history consists of a sequence of distributions of states $x^i_{j,t}$ (where i indexes individuals, j indexes histories, and t indexes time) and random outcomes $\tilde{f}^i_{j,t+1}$ and $\tilde{g}^i_{j,t+1}$. Since our parameterized expectations represent expectations conditional on survival, we delete the explanatory variables x_t and the realizations $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$ recorded for individuals with $\epsilon^z_{t+1} = 0$: that is, we neither record marginal utility of newborns, nor the explanatory variables of those who are about to die. After many simulations, we update the parameter vectors β_F and β_G by a nonlinear least squares regression, that is, by solving the problems

$$\underset{\beta_F}{Min} \sum_{i,j,t} \left(\tilde{f}^i_{j,t+1} - \exp(\beta_F' x^i_{j,t})\right)^2 \tag{31}$$

and

$$\underset{\beta_G}{Min} \sum_{i,j,t} \left(\tilde{g}^i_{j,t+1} - \exp(\beta_G' x^i_{j,t})\right)^2 \tag{32}$$

Table 1: Parameterized expectations algorithm

Step 1. Choose explanatory variables x.

Step 2. Initialize β_F, β_G.

Step 3. Simulate $\mathcal{J}$ histories of the economy of length $\mathcal{T}$ periods each under expectations $\mathcal{F}$ and $\mathcal{G}$ parameterized by β_F and β_G.

3a. Define $j = 1$.

3b. Define $t = 1$.

3c. Initialize Φ_t to satisfy the market clearing condition (13).

3d. Calculate $U_{j,t}$ from definition (14).

3e. Choose $M_{j,t}$.

3f. Calculate $P_{j,t}$ from (22) and (15) and $U_{j,t+1}$ from (20).

3g. Calculate $x^i_{j,t}$, $\mathcal{F}^i_{j,t}$, and $\mathcal{G}^i_{j,t}$ for all individuals i.

3h. Calculate asset demands $b^i_{j,t+1}$ and $n^i_{j,t+1}$ for all individuals i from the complementary slackness conditions (Cases I to IV.)

3i. Check market clearing for risky assets, equation (13). If markets clear, go to (3j). If not, return to (3e).

3j. Calculate shocks $\epsilon^i_{j,t+1}$ for all agents, consistent with the dynamics (33)-(37).

3k. Record $\tilde{f}^i_{j,t}$, $\tilde{g}^i_{j,t}$ (unless i is newborn) and $x^i_{j,t}$ (unless i is about to die).

3l. If $t < \mathcal{T} + 1$, increase t by one and go to (3d). Otherwise, increase j by one and go to (3m).

3m. If $j \leq \mathcal{J}$, return to (3b). Otherwise, go to (4).

Step 4. Solve for new β_F and β_G by nonlinear least squares regression of $\tilde{f}^i_{j,t+1}$ on $x^i_{j,t}$ and $\tilde{g}^i_{j,t+1}$ on $x^i_{j,t}$ for all i, j, and t. That is, solve (31) and (32).

Step 5. Check whether β_F and β_G have converged. If converged, go to (6). If not, go to (3).

Step 6. Check whether current predictions of $\tilde{f}$ and $\tilde{g}$ are accurate. If accurate, STOP. Otherwise, return to (1), and use previous simulations to search for more useful explanatory variables x. Alternatively, consider a different functional form for the parameterized expectations $\mathcal{F}$ and $\mathcal{G}$ and return to (1).

If the minimizers β_F and β_G have changed substantially from their previous values, then we must perform more simulations. On the other hand, if β_F and β_G are roughly unchanged, then we have a candidate approximate equilibrium. We must still check the quality of the approximation, that is, we must check whether the predictions are sufficiently accurate. If not, we must try a different information vector x or a different functional form for $\hat{F}$ and $\hat{G}$. In practice, we consider a large set of possible explanatory variables x, and go regression fishing to choose those which most improve the explanatory power of the parameterized expectations. This algorithm is spelled out in greater detail in Table 1.

4 Preliminary results

These results are generated by the parameterized expectations algorithm spelled out in section 3. The parameters used are stated in Table 2, and the random variables considered for inclusion in the parameterized expectations are described in Table 3. (The variables actually used in the parameterized expectations are those which turned out to have highest explanatory power in practice. For these variables, we report descriptive statistics; those not included are shown with a blank line in the table.) The parameters are selected to give quite a high degree of risk, in the absence of insurance; they are also normalized for simplicity to target a price level around one.

The simulations reported here are based on the assumption, considerably stronger than that stated earlier, that prices and other equilibrium quantities can be written as a function of the distribution $\hat{\Phi}$, defined as the distribution of idiosyncratic states $\hat{\phi} \equiv (a, \epsilon^e)$. Thus we form the parameterized expectations entirely as functions of moments of this distribution. The moments used are shown in Table 3. Six moments were exogenously imposed; fifteen more were selected by searching for the most powerful explanatory variables. (Variables not selected are also shown in the table.)

We compare typical time series in economies under three insurance environments: Case I: near full insurance ($\theta^u = \theta^\pi = 0.98$); Case U: near full insurance

Table 2: Baseline parameters

Preferences	α	2	*Matching technology*	m	1.0316
	ρ	0.05		γ	0.1
	ζ	0		$\bar{k}$	0
Output technology	R	1.03		δ	0.1
	$\underline{w}$	0.003061		β	0.5
	$\overline{y}$	0.3367	*Computational parameters*	$\mathcal{J}$	810
	$\underline{y}$	0.003061		$\mathcal{T}$	5
	p	0.8			
Policies	θ^u	0 or 0.98			
	θ^π	0 or 0.98			

of unemployment, but no insurance of investment ($\theta^u = 0.98, \theta^\pi = 0$); and Case N: no insurance ($\theta^u = \theta^\pi = 0$). Figures 1 and 2 show the dynamics of Cases I and U. Both are quite similar. With a return of only 3% on the riskless asset, many agents choose a corner solution with investment in the riskless asset only, so that riskless investment is very low. While this is expected in Case I, it is somewhat surprising in Case U. The large aggregate shocks to technology are therefore buffered only by accumulation and decumulation of the risky asset, which implies that unemployment fluctuates quite a lot. However, this buffer stock behavior in the risky asset is reasonably effective in protecting consumption: we see that average consumption fluctuates little.

The remaining figures relate to Case N, (no insurance) which is substantially different. Figure 3 shows a time series comparable to those in Figures 1 and 2. (In fact, all three series are generated from the same initial conditions, under the same sequence of distributions of shocks Ψ_t.) While the variability of unemployment is not much greater in Case N than in the insured cases, the unemployment rate varies much more sharply, being concentrated in sharp peaks. There is much more investment in the riskless asset than in previous cases, and investment in the riskless asset increases sharply at the times of peak unemployment. At the same peak, the price of the risky asset drops, and consumption falls.

Table 3: Explanatory variables in parameterized expectations

Variable (x)	μ_x	β_F	$\mu_x\beta_F$	$\sigma_x\beta_F$	β_G	$\mu_x\beta_G$	$\sigma_x\beta_G$
1^*	1.0000	3.4964	3.4964	0.0000	3.5945	3.5945	0.0000
U	0.1086	0.1379	.01497	.00118	-.05491	-.00596	-.00047
A^*	.01840	-0.2390	-.00440	-.00610	-0.6337	-.01166	-.01617
AA	0.2803	-0.1530	-.04289	-.01443	-0.4157	-0.1165	-.03919
UU							
UA							
AAA	-0.1986	-.09487	.01884	-.04283	-.07627	.01514	-.03443
AAU							
AUU							
UUU							
$AAAA$							
$AAAU$							
u^*	0.1086	0.1592	.00556	.01579	0.1240	.01347	.03825
a^*	.01840	1.2405	.02283	0.6574	1.2068	.02221	0.6396
ua							
aa^*	0.2813	-.06214	-.01748	-.06529	-0.2670	-.07510	-0.2806
Au^*	.00192	-.07323	-.00014	-.00074	-0.1398	-.00027	-.00141
Aa	.00099	.08102	.00008	.00133	-.01619	-.00058	-.00968
Uu	.01186	0.4178	.00495	.01413	.09915	.00118	.00335
Ua							
aaa	-0.1879	.00557	-.00104	.02825	-.04504	.00846	-0.2284
aau	.08797	0.1037	.00912	0.1024	0.1756	.01544	0.1734
$aaaa$	1.1831	-.00402	-.00475	-0.1114	-.00676	-.00800	-0.1873
$aaau$	-0.2261	-.02549	.00576	-0.1281	.00274	-.00062	.01381
Aaa	.00358	-.00339	-.00001	-.00012	-0.1239	-.00044	-.00439
Aau							
Uau	-.00416	-0.6263	.00261	-.02078	-0.5877	.00244	-.01949
AAa	.00355	.09988	.00035	.01762	-0.2916	-.00104	-.05144
AAu	.03076	-0.1019	-.00313	-.00948	-0.2079	-.00639	-.01934

NOTES:

1. μ and σ are means and standard deviations of explanatory variables; β_F, β_G are coefficients.

2. Asterisks mark explanatory variables exogenously imposed. Other variables for which results are reported were chosen by regression fishing.

3. a represents log wealth; u represents unemployment; capital letters represent aggregates.

4. Composite variables represent higher moments. For example, if $\mu_t(x)$ represents the cross-sectional mean of variable x at time t, then

$$(AAu)_{it} \; = \; u_{it}\mu_t[(a_{it} - \mu_t(a_{it}))^2]$$

First entry in last row shows mean of $(AAu)_{it}$ across 100 individuals and 200 periods.

This is exactly the type of fluctuation we expected to find, given the positive feedback mechanism discussed in the introduction. We also find that these sharp fluctuations are not caused by the relatively large aggregate technology shocks we imposed. In Figure 16, we fix the aggregate technology shock at its mean by giving exactly 80 out of 100 agents a positive idiosyncratic technology shock each period: qualitatively similar fluctuations remain. What does turn out to be crucial is that the law of large numbers does not apply in our model. If we recalculate the equilibrium with a much larger number of agents (1000 or 10000), fluctuations have a similar shape, but smaller amplitude. However, since our main intention is to model the investment behavior of firms (rather than individual consumers), we believe that an assumption of finite numbers is probably much more realistic than assuming the law of large numbers holds.

Next we show more diagrams to help characterize the behavior of Case N. Figure 4 shows the distribution of wealth and consumption at time $t = 200$ in our simulation. We distinguish idiosyncratic states: those with two good shocks are shown with a star, the unemployed with a good technology shock are shown with a plus sign, the employed with a bad technology shock are shown with a letter "x", and those who have both bad shocks are shown with a letter "o". Dots show the consumption levels of the same agent. The distribution of wealth is highly dispersed, with a standard deviation comparable to its mean. (By contrast, in Case I, the distribution almost completely converges to a unique wealth level.) Moreover, we found in our simulations that the distribution is highly non-stationary. Once a given agent reaches a wealth level around 5, she tends not to fall back into the mass of other agents; here we find one agent with substantially more wealth than the median. In the next two figures we report time series showing the highest wealth level in the population. Clearly, certain aspects of the wealth distribution are extremely persistent. However, the basic shape of "recessions" in our simulations remains similar under very different wealth distributions.

The next four figures show the expectations. Figures 6 and 7 plot our parameterized expectation approximation against the realized average value of the random variables $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$ (we fix a distribution Φ_t and then randomly

Table 4: Apparent errors in cross section, $t = 200$

	consumption	*hiring*	*storage*	*utility*
Avg log difference	-0.000549	0.0104	-0.0951	0.000413
Avg abs log difference	0.00319	0.0105	0.0956	0.000919
Max abs log difference	0.0616	0.0316	0.199	0.0252

NOTES:

1. 100 individuals: change in behavior if simulated expectation used instead of parameterized expectation.
2. Utility change is expressed as certainty equivalent consumption loss.

draw 300 possible distributions Ψ_{t+1}, then average over realized $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$.) Overall, the parameterized expectations are quite accurate, except at the very lowest wealth level. However, closer inspection shows some nontrivial errors. In Figures 11-13 we show how behavior would change if individuals made their decisions on the basis of the simulated expectations shown in Figures 6 and 7 instead of on the basis of the parameterized expectations. Consumption and hiring behavior appears to change very little, but one can clearly see that storage should drop by about ten percent, on the basis of the simulated expectations, in Figure 13. We calculate some statistics summarizing these errors in Table 4. On average, at time $t = 200$, agents' policy errors result in a utility loss worth roughly 0.1% of their current consumption. The worst error in the cross section, however, is worth approximately 2.5% of current consumption. Hence for the time being, the results must be regarded as preliminary— they can be seen as an equilibrium of boundedly rational agents, but we can not yet conclude that they are similar to the behavior in the true rational expectations equilibrium.

Figures 8 to 10 report the three policy functions for c_t, n_{t+1}, and b_{t+1}. Notice that, as expected, the consumption function shifts down for the unemployed (as before, represented by "+" or "o"). The consumption function, as demonstrated by Carroll and Kimball (1996), is concave in wealth. Investment behavior is quite complicated, but readily interpreted. The very poorest choose a corner solution with safe asset holding only. Since the risky asset has a higher return, a small increase in wealth brings individuals to an interior solution, and then to a corner solution in which only the risky asset is held. However, once agents become sufficiently rich, investment income plays an ever larger part in their total human wealth. Therefore, the richest agents begin to invest in both assets again. However, in the limit, the proportion of wealth invested in the riskless asset is decreasing. It is this final interval of investment behavior which makes the wealth distribution so unstable: among the very richest agents, greater wealth is associated with higher average returns, so it takes a really exceptional string of bad luck to bring the richest back down to median wealth levels. In our last diagrams, we again graph the distribution of wealth (and various categories of asset holding) at time $t = 200$, and show the Lorenz curves. While the distribution depicted is highly unequal, it is by no means as unequal as the actual US wealth distribution.

REFERENCES

Aiyagari, S. Rao (1994), "Uninsured Risk and Aggregate Saving." *Q.J.E.* 109 (3) pp. 239-54.

Carroll, Christopher D. (1997), "Buffer Stock Saving and the Life Cycle / Permanent Income Hypothesis" *Q.J.E.* 112 (1) pp. 1-55.

Chatterjee, Satyajit (1988), "Participation Externality as a Source of Coordination Failure in a Competitive Model of Centralized Markets." Manuscript, Univ. of Iowa.

Cocco, João F.; Francisco J. Gomes; and Pascal J. Maenhout (1998), "Consumption and Portfolio Choice over the Life-Cycle." Manuscript.

Cooley, Thomas F., and Vincenzo Quadrini (1998), "Monetary Policy and the Financial Decisions of Firms." Manuscript.

Costain, James S. (1999), "A Simple Model of Multiple Equilibria Based on Risk." Univ. Pompeu Fabra Economics Working Paper #407.

Deaton, Angus (1991), "Saving and Liquidity Constraints." *Econometrica* 59.

Den Haan, Wouter (1997), "Solving Dynamic Models with Aggregate Shocks and Heterogeneous Agents." *Macroeconomic Dynamics*, 1 (2) pp. 355-86.

Den Haan, Wouter (1999), "The Importance of the Number of Different Agents in a Heterogeneous Asset-Pricing Model." Manuscript.

Duffie, D.; J. Geanakoplos; A. Mas-Colell; and A. McLennan (1994), "Stationary Markov Equilibria." *Econometrica* 62 (4) pp. 745-81.

Elmendorf, Douglas W., and Miles S. Kimball (1991), "Taxation of Labor Income and the Demand for Risky Assets." N.B.E.R. Working Paper 3904.

Farmer, Roger, and Jang Ting Guo (1994), "Real Business Cycles and the Animal Spirits Hypothesis." *J.E.T.* 63 (1) pp. 42-72.

Krusell, Per, and Anthony Smith (1998), "Income and Wealth Heterogeneity in the Macroeconomy." *J.P.E* 106 (5) pp. 867-96.

Krusell, Per, and Anthony Smith (1997), "Income and Wealth Heterogeneity, Portfolio choice, and Equilibrium Asset Returns." *Macroeconomic Dynamics* 1 (2) pp. 387-422.

Marcet, Albert, and Kenneth J. Singleton (1999), "Equilibrium Asset Prices and Savings of Heterogeneous Agents in the Presence of Incomplete Markets and Portfolio Constraints." *Macroeconomic Dynamics* 3 (2) pp. 243-77.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron (1998), "Asset Pricing with Idiosyncratic Risk and Overlapping Generations." Manuscript.

Appendix

Since we have not constrained aggregate employment IN_t to be an integer, we will impose labor market clearing by assuming that exactly one worker has a part time job at each t. We now show how this affects the idiosyncratic labor market transitions in our economy, while respecting as closely as possible the simple approximate transition dynamics stated in (4). We define the shocks $\psi_t \equiv (\psi_t^f, \psi_t^h, \psi_t^y, \psi_t^z)$, which we assume are *iid* across time and between individuals, and all uniformly distributed in $[0,1]$. (Defining this notation is particularly helpful since it mirrors our actual computations.) The distribution of these shocks across individuals at t is $\Psi_t \equiv (\Psi_t^f, \Psi_t^h, \Psi_t^y, \Psi_t^z)$.

The shock ψ^y controls the idiosyncratic productivity process, as follows:

$$\begin{array}{ll} \epsilon_{t+1}^y = 1 & \text{if} \quad \psi_{t+1}^y \leq p \\ \epsilon_{t+1}^y = 0 & \text{if} \quad \psi_{t+1}^y > p \end{array} \tag{33}$$

The shock ψ^z governs death and rebirth:

$$\begin{array}{ll} \epsilon_{t+1}^z = 0 & \text{if} \quad \psi_{t+1}^z \leq \zeta \\ \epsilon_{t+1}^z = 1 & \text{if} \quad \psi_{t+1}^z > \zeta \end{array} \tag{34}$$

Since our parameterized expectations represent expectations conditional on survival, neither the explanatory variables x_t nor the realizations $\tilde{f}_{t+1}$ and $\tilde{g}_{t+1}$ are recorded for individuals with $\epsilon_{t+1}^z = 0$.

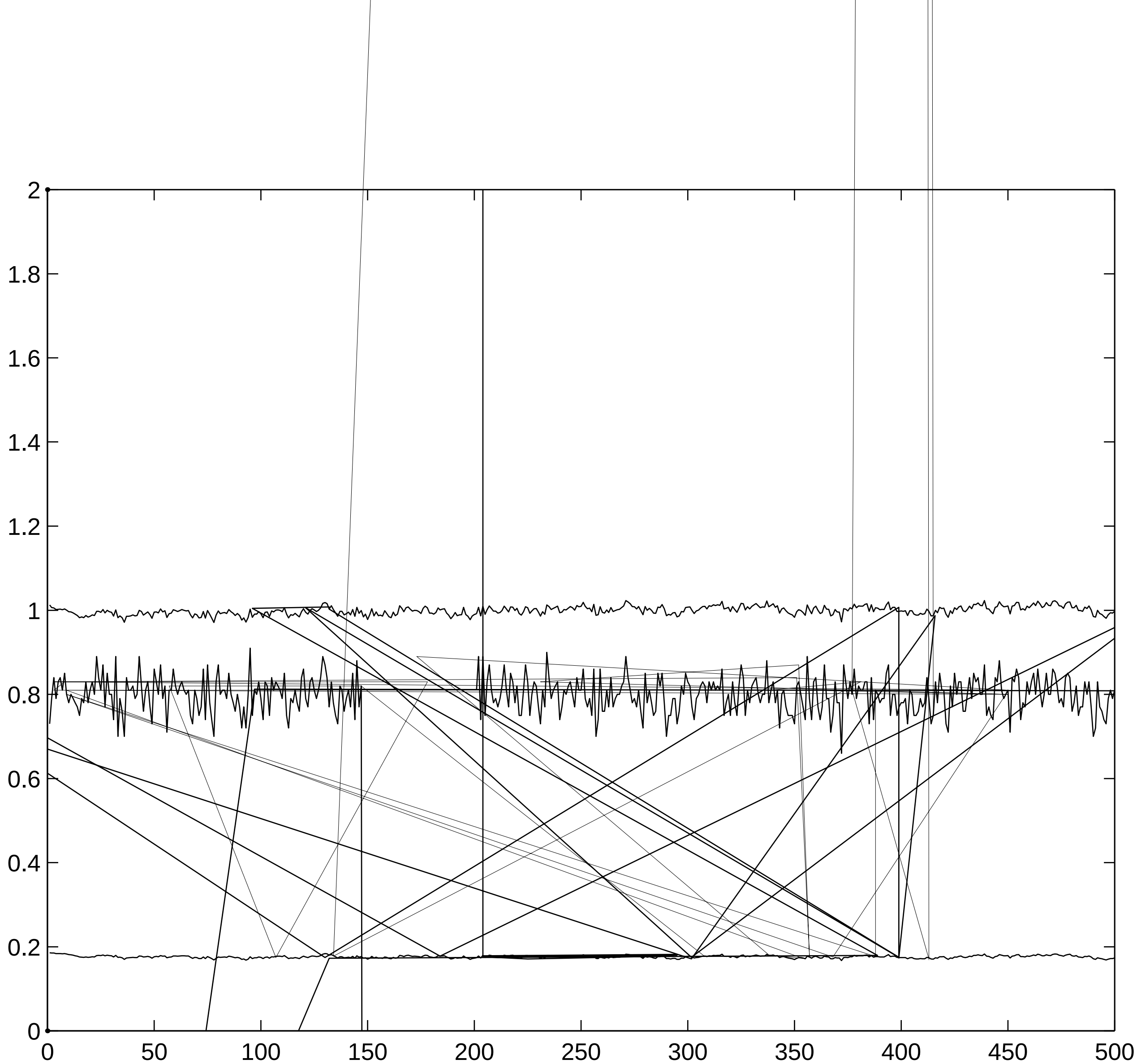
In the labor market, the shock ψ^f controls "firing" and ψ^h controls "hiring". Define the idiosyncratic labor market state ϵ_t^s, where $\epsilon_t^s = 1$ means that the individual is in the pool of job seekers at the end of t for jobs at $t+1$ while $\epsilon_t^s = 0$ means that the individual is not seeking a job at the end of t, which only applies if they were employed at t and have not been fired at the end of t. Define also ceil(x) to be the smallest integer at least as large as x; floor(x) to be the largest integer no greater than x; $\#(z)$ to be the number of agents for whom statement z is true; and let $\mathcal{R}(\psi, \Psi|z)$ be the rank of the shock ψ out of the distribution Ψ among those agents for whom statement z is true. Then we can write the labor market dynamics according to the following scheme:

$$\epsilon_t^e = 1 \quad \rightarrow \quad \left\{ \begin{array}{ll} \epsilon_t^s = 1 & \text{if} \quad \mathcal{R}(\psi_t^f, \Psi_t^f | \epsilon_t^e = 1) \leq \text{floor}(\delta \#(\epsilon_t^e = 1)) \\ \epsilon_t^s = 0 & \text{if} \quad \mathcal{R}(\psi_t^f, \Psi_t^f | \epsilon_t^e = 1) > \text{floor}(\delta \#(\epsilon_t^e = 1)) \end{array} \right. \tag{35}$$

$$\epsilon^e_t < 1 \quad \rightarrow \quad \epsilon^s_t = 1 \tag{36}$$

$$\epsilon^s_t = 1 \quad \rightarrow \left\{ \begin{array}{rl} \epsilon^e_{t+1} = 1 & \text{if} \quad \mathcal{R}(\psi^h_t, \Psi^h_t | \epsilon^s_t = 1) \leq \text{floor}[IN_{t+1} - \#(\epsilon^s_t = 0)] \\ \epsilon^e_{t+1} = IN_{t+1} - \text{floor}(IN_{t+1}) & \text{if} \quad \mathcal{R}(\psi^h_t, \Psi^h_t | \epsilon^s_t = 1) = \text{ceil}[IN_{t+1} - \#(\epsilon^s_t = 0)] \\ \epsilon^e_{t+1} = 0 & \text{if} \quad \mathcal{R}(\psi^h_t, \Psi^h_t | \epsilon^s_t = 1) > \text{ceil}[IN_{t+1} - \#(\epsilon^s_t = 0)] \end{array} \right. \tag{37}$$

Equation (35) says that slightly more than fraction δ of full time workers are fired. All those fired, as well as the unemployed and the part time worker, enter the search pool. The number of new full time hires is IN_{t+1} minus the continuing full time workers from the last period. It is important to notice that we have treated aggregate uncertainty differently in the case of technology shocks and employment shocks. By drawing each ϵ^y independently on the basis of ψ^y, there remains aggregate uncertainty about the average technology shock in the economy; hence our model displays an aggregate productivity shock. However, we have chosen to suppress any aggregate shocks to employment. Since n_{t+1} is known for each individual at t, N_{t+1} is known at t, and our use of the ranking function $\mathcal{R}$ to derive ϵ^e from ψ^f and ψ^h ensures that total full time and part time employment equals N_{t+1}.



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
350
400
450
500

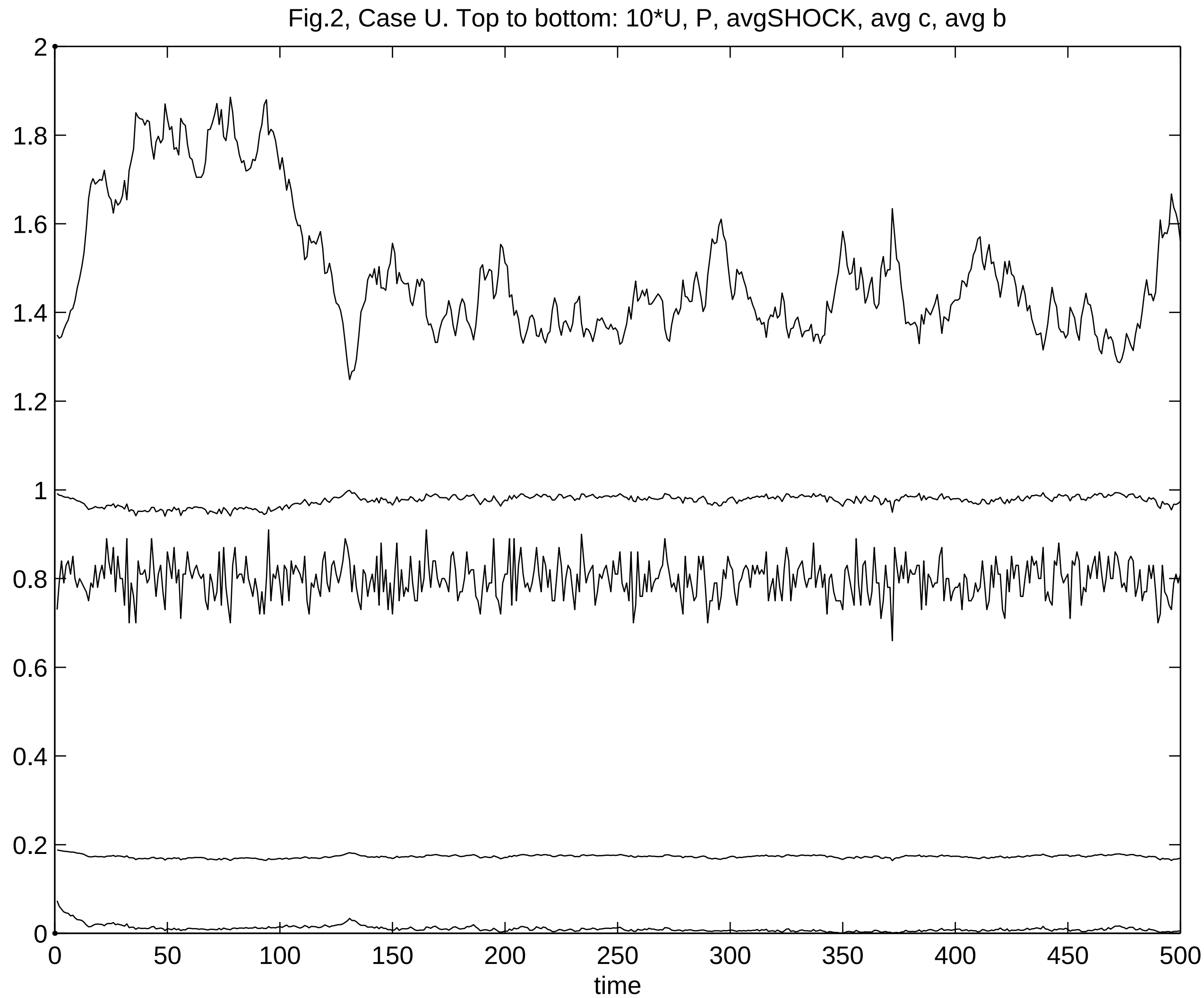


Fig.2, Case U. Top to bottom: 10*U, P, avgSHOCK, avg c, avg b
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
350
400
450
500
time

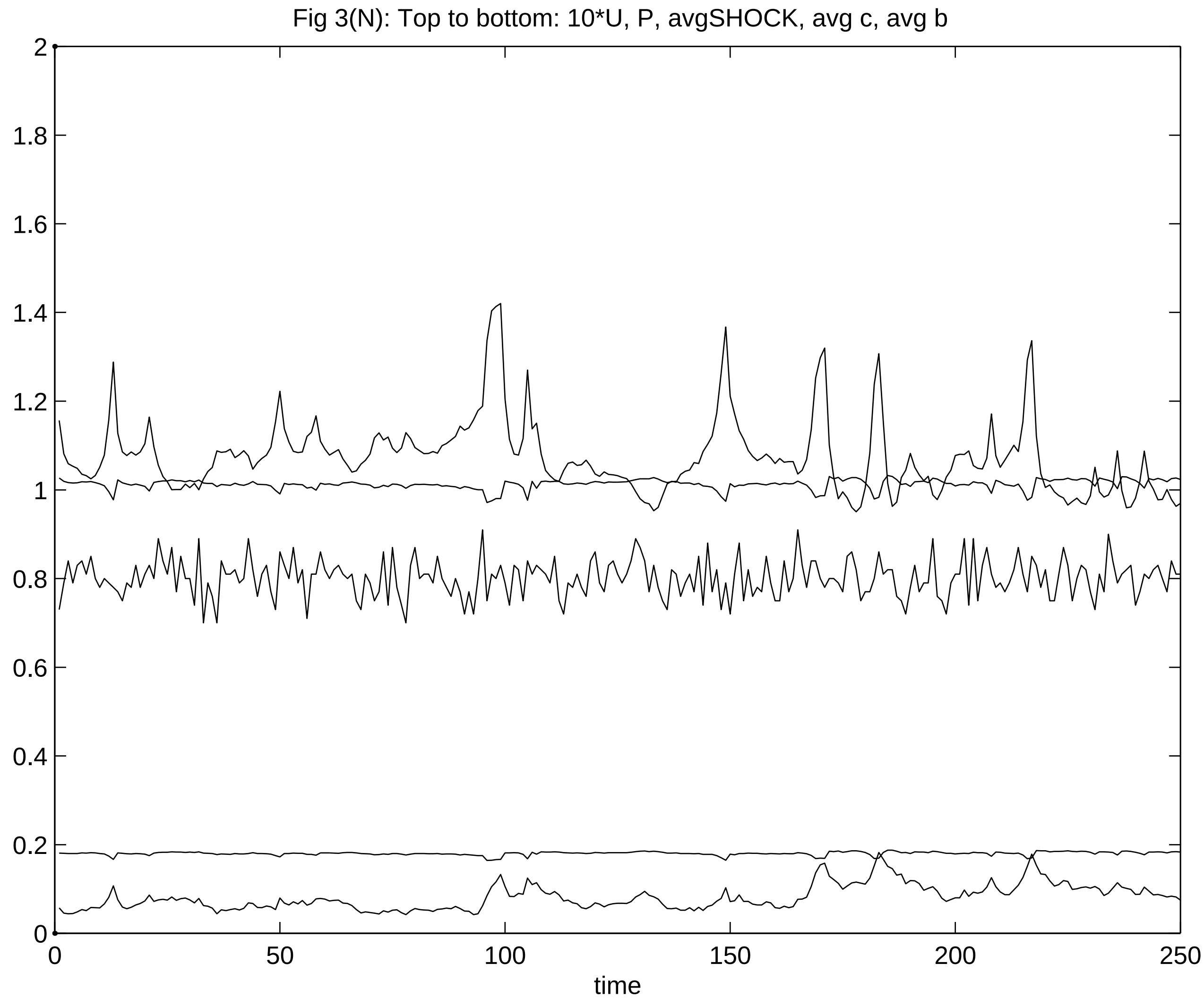
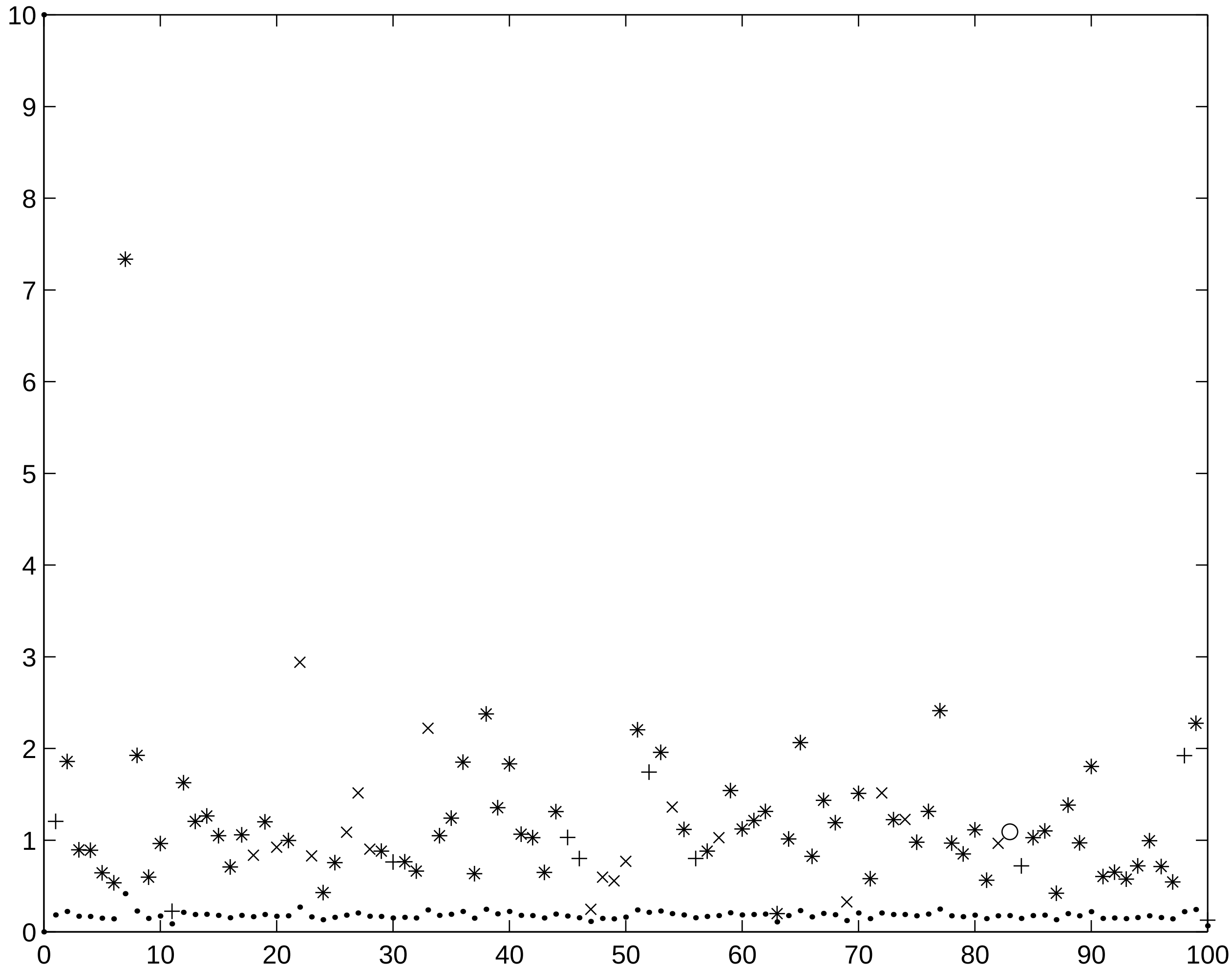


Fig 3(N): Top to bottom: 10*U, P, avgSHOCK, avg c, avg b
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
time

Fig.4(N): Wealth distribution (*,x,+,o) and consumption (dots)



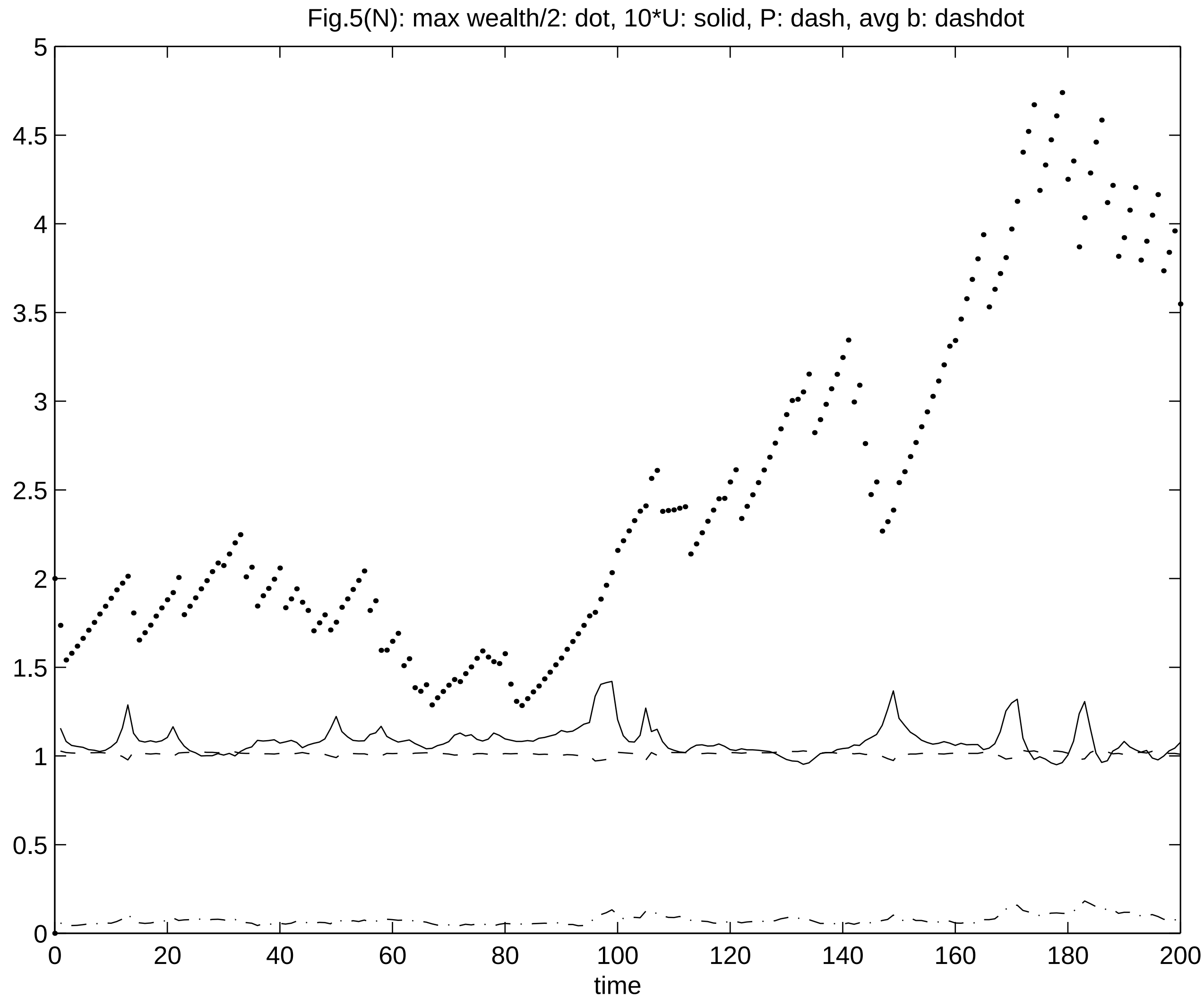


Fig.5(N): max wealth/2: dot, 10*U: solid, P: dash, avg b: dashdot
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
120
140
160
180
200
time

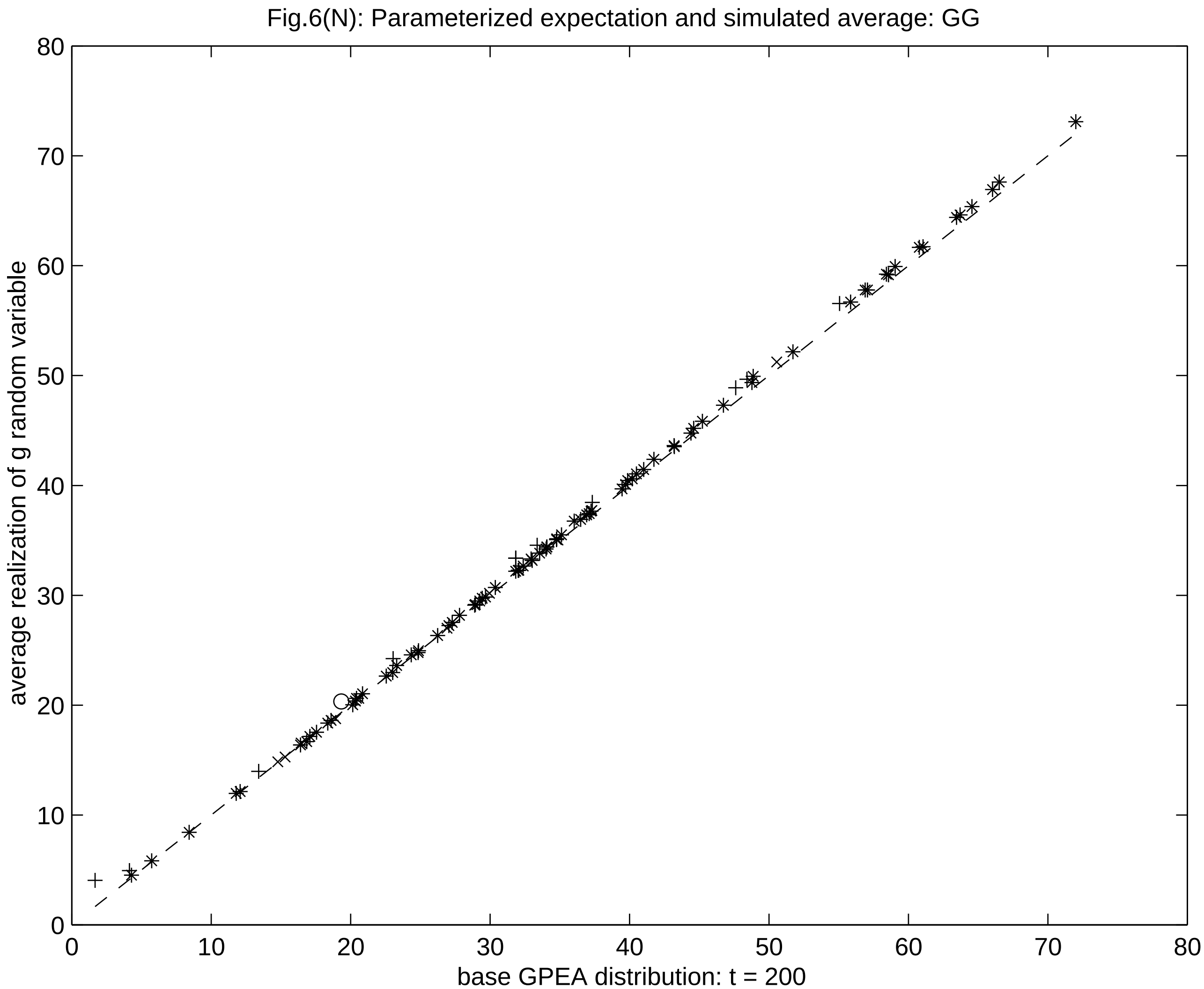
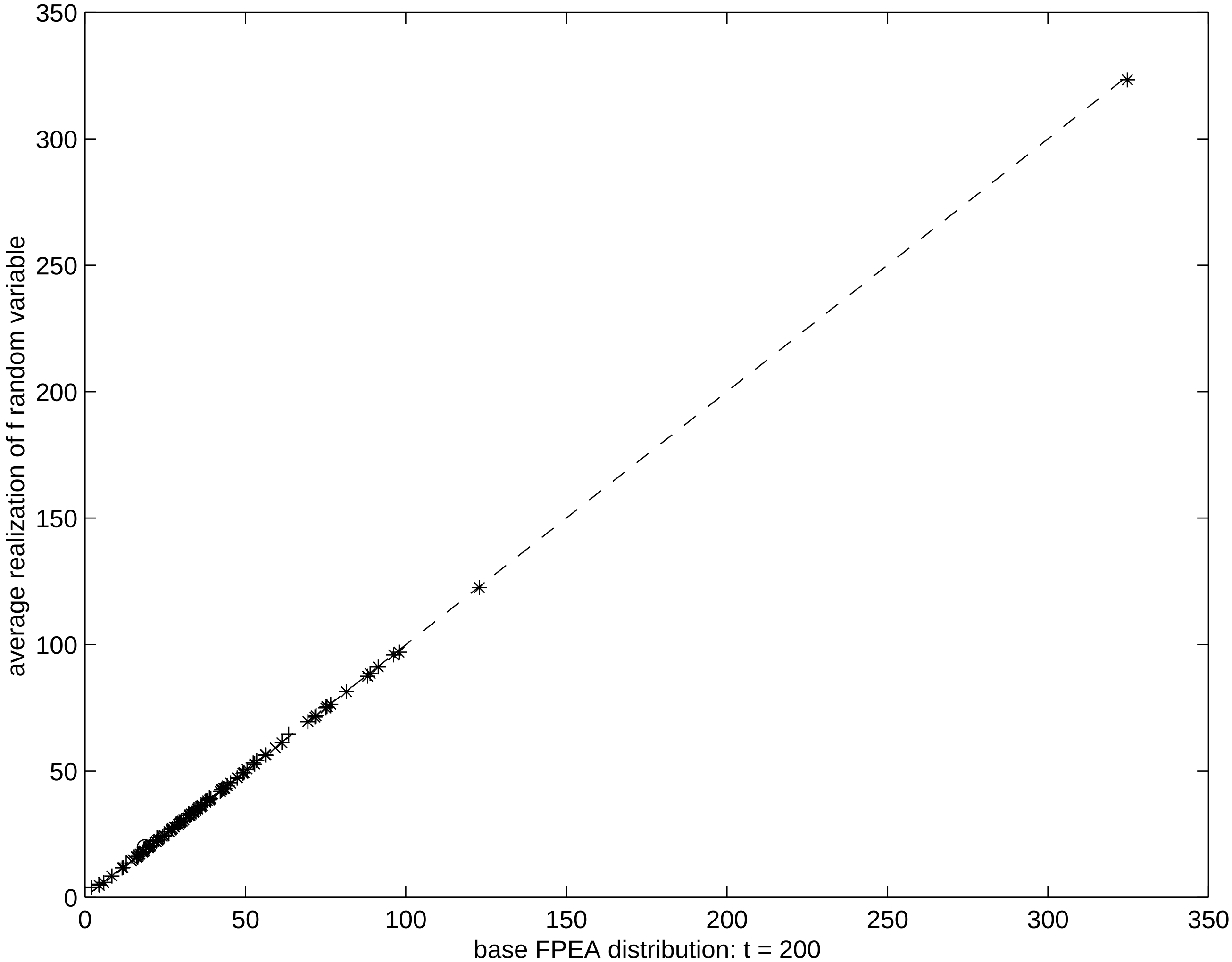


Fig.6(N): Parameterized expectation and simulated average: GG
average realization of g random variable
base GPEA distribution: t = 200
0
10
20
30
40
50
60
70
80

Fig.7(N): Parameterized expectation and simulated average: FF



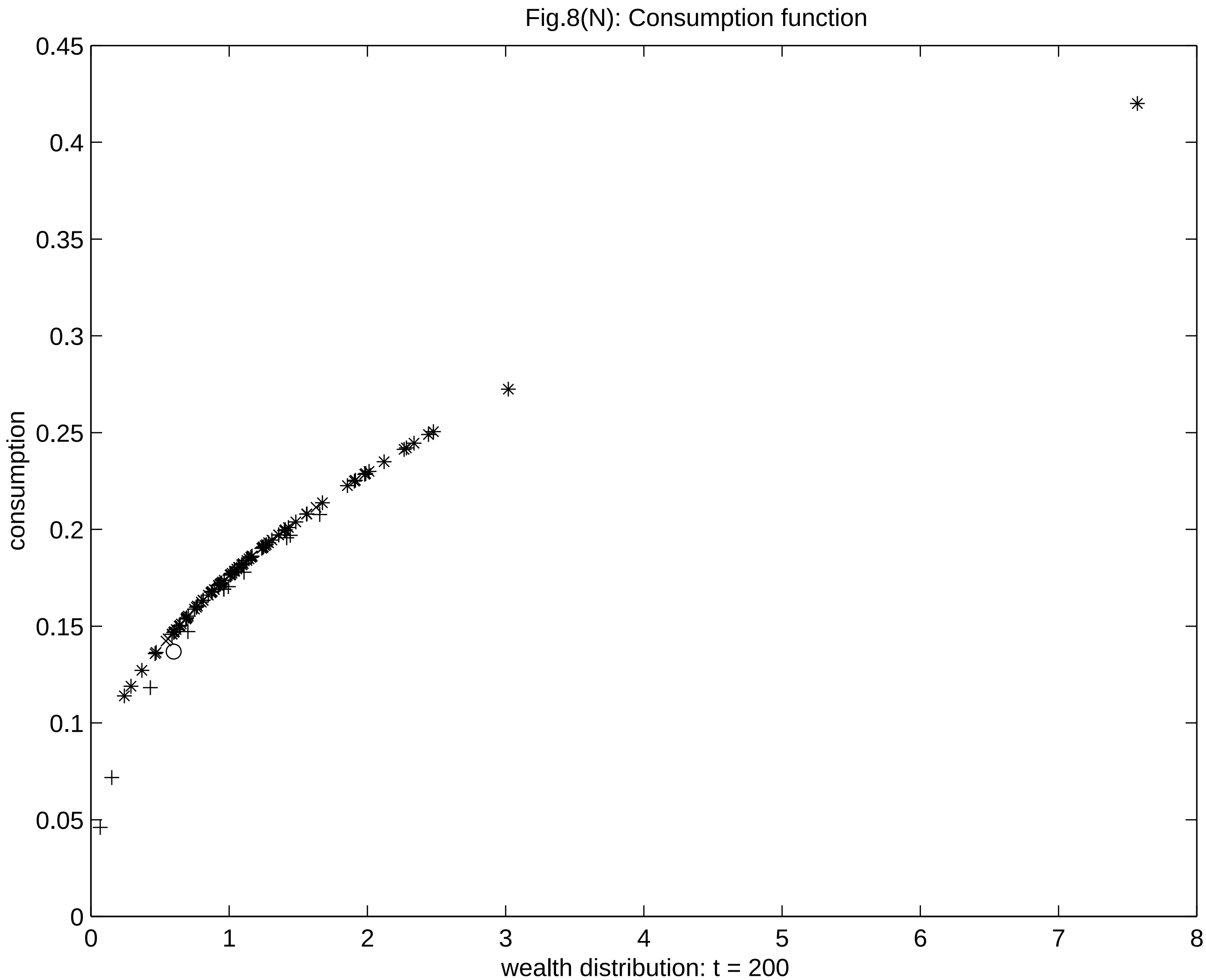


Fig.8(N): Consumption function
consumption
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8
wealth distribution: t = 200

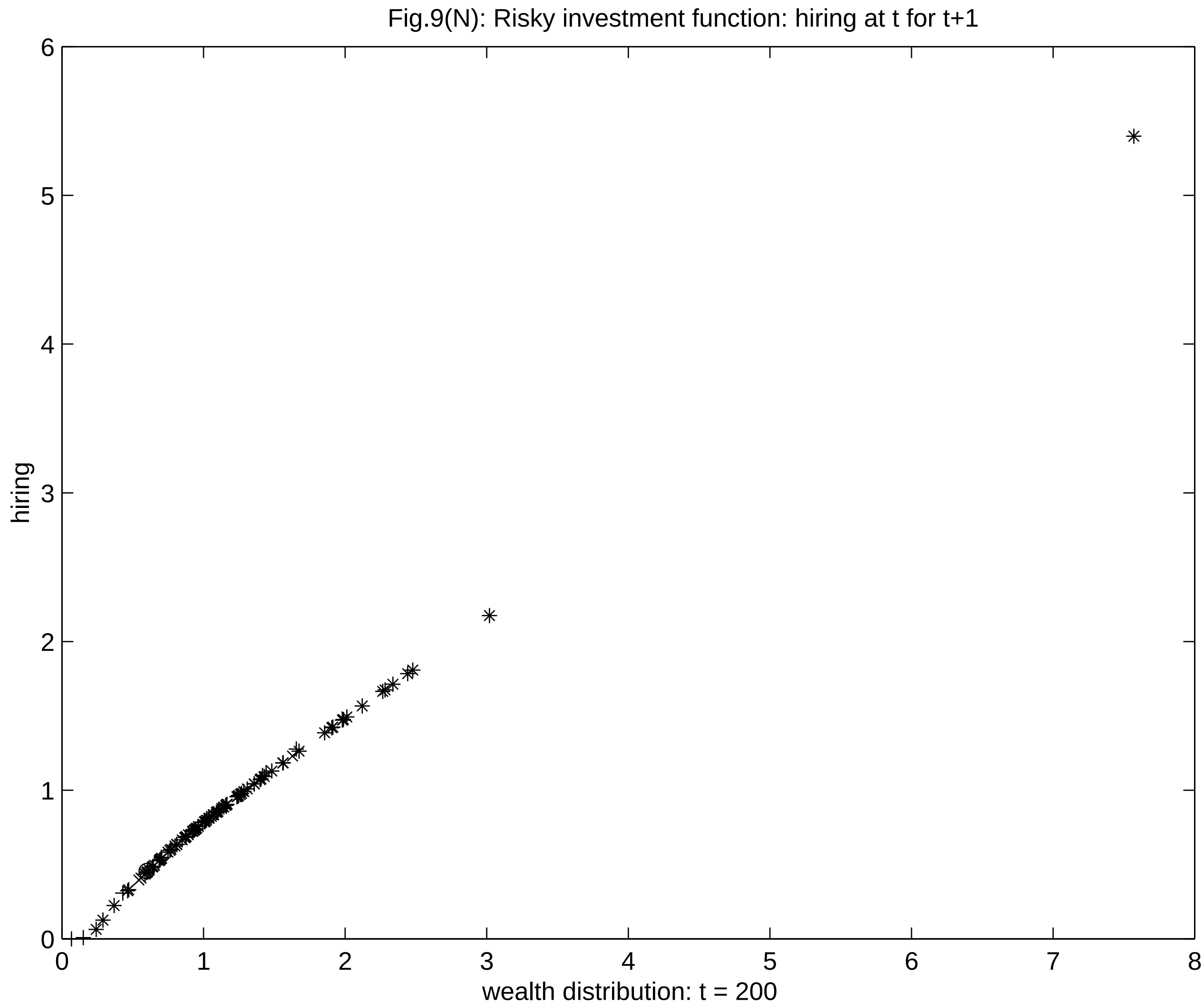


Fig.9(N): Risky investment function: hiring at t for t+1
hiring
wealth distribution: t = 200
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5
6

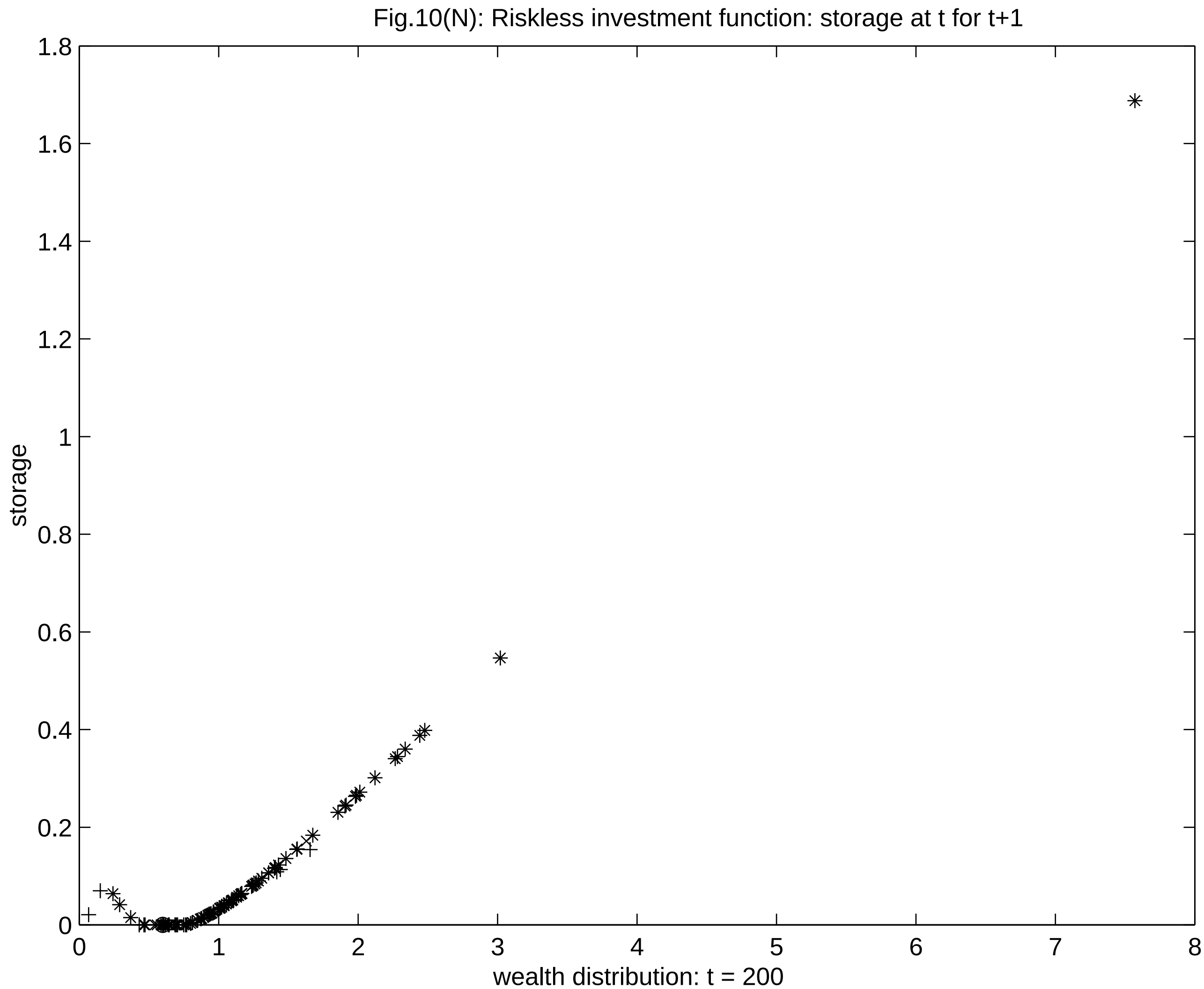


Fig.10(N): Riskless investment function: storage at t for t+1
storage
wealth distribution: t = 200
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
1
2
3
4
5
6
7
8

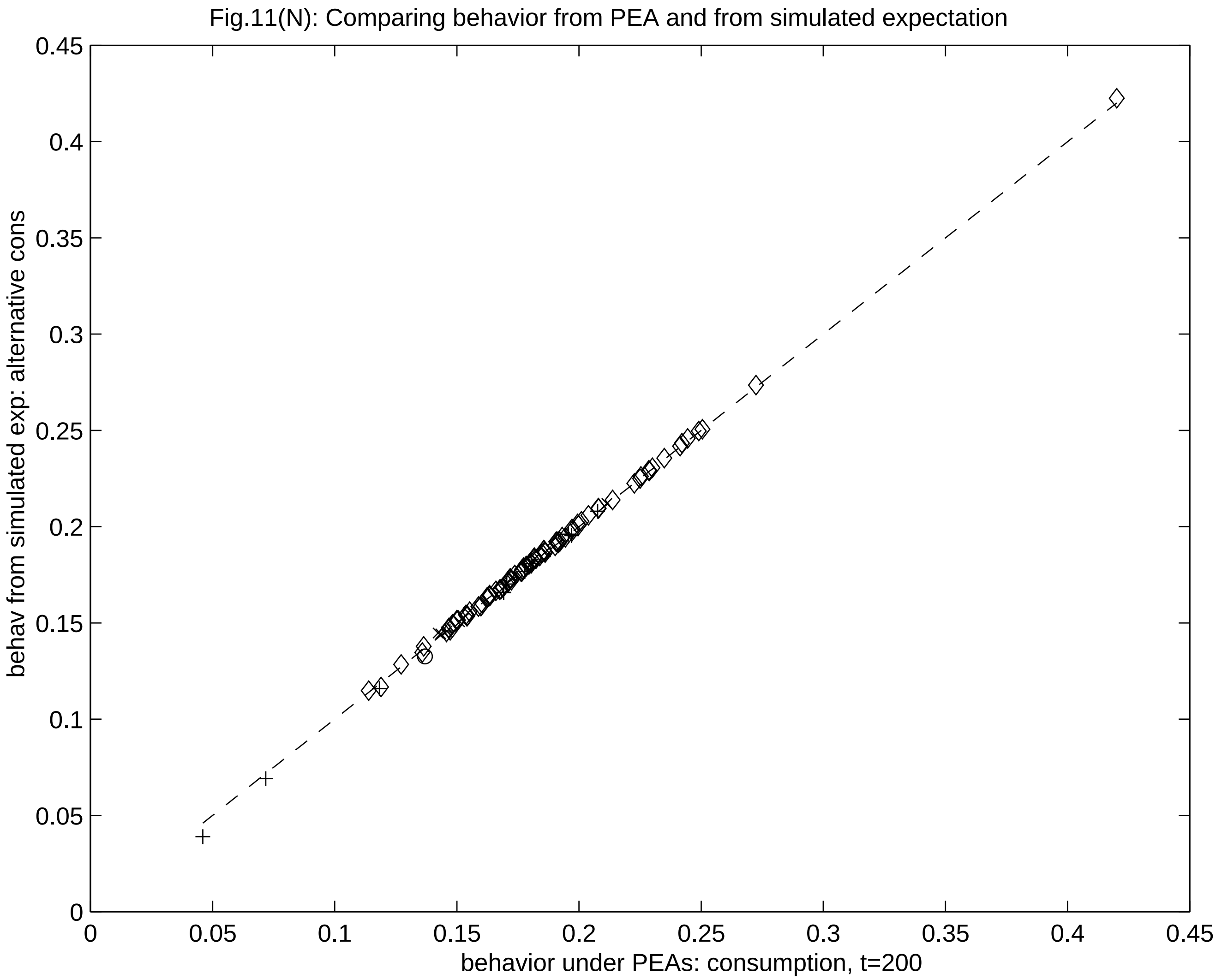


Fig.11(N): Comparing behavior from PEA and from simulated expectation
behav from simulated exp: alternative cons
behavior under PEAs: consumption, t=200
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

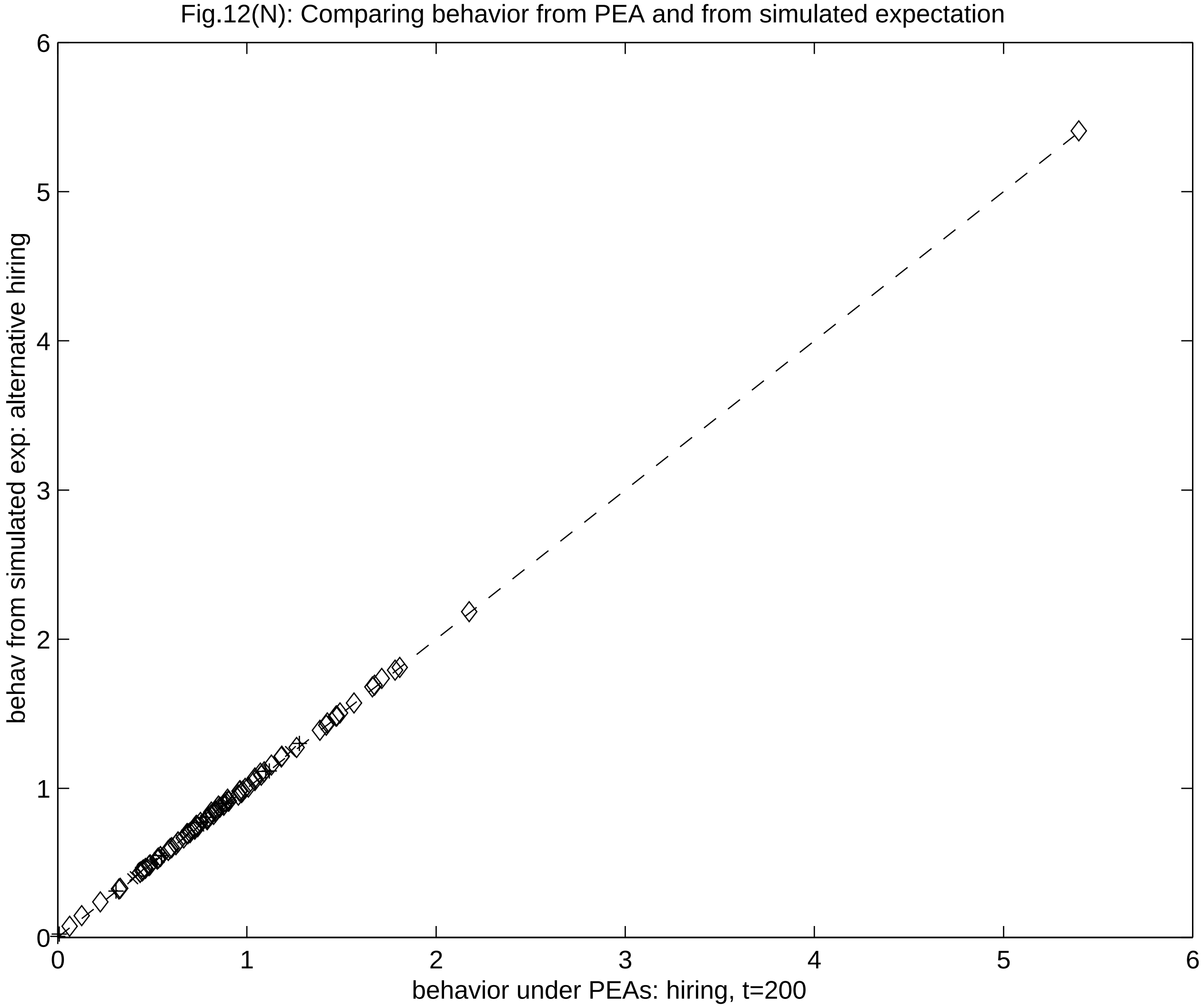


Fig.12(N): Comparing behavior from PEA and from simulated expectation
behav from simulated exp: alternative hiring
behavior under PEAs: hiring, t=200
0
1
2
3
4
5
6

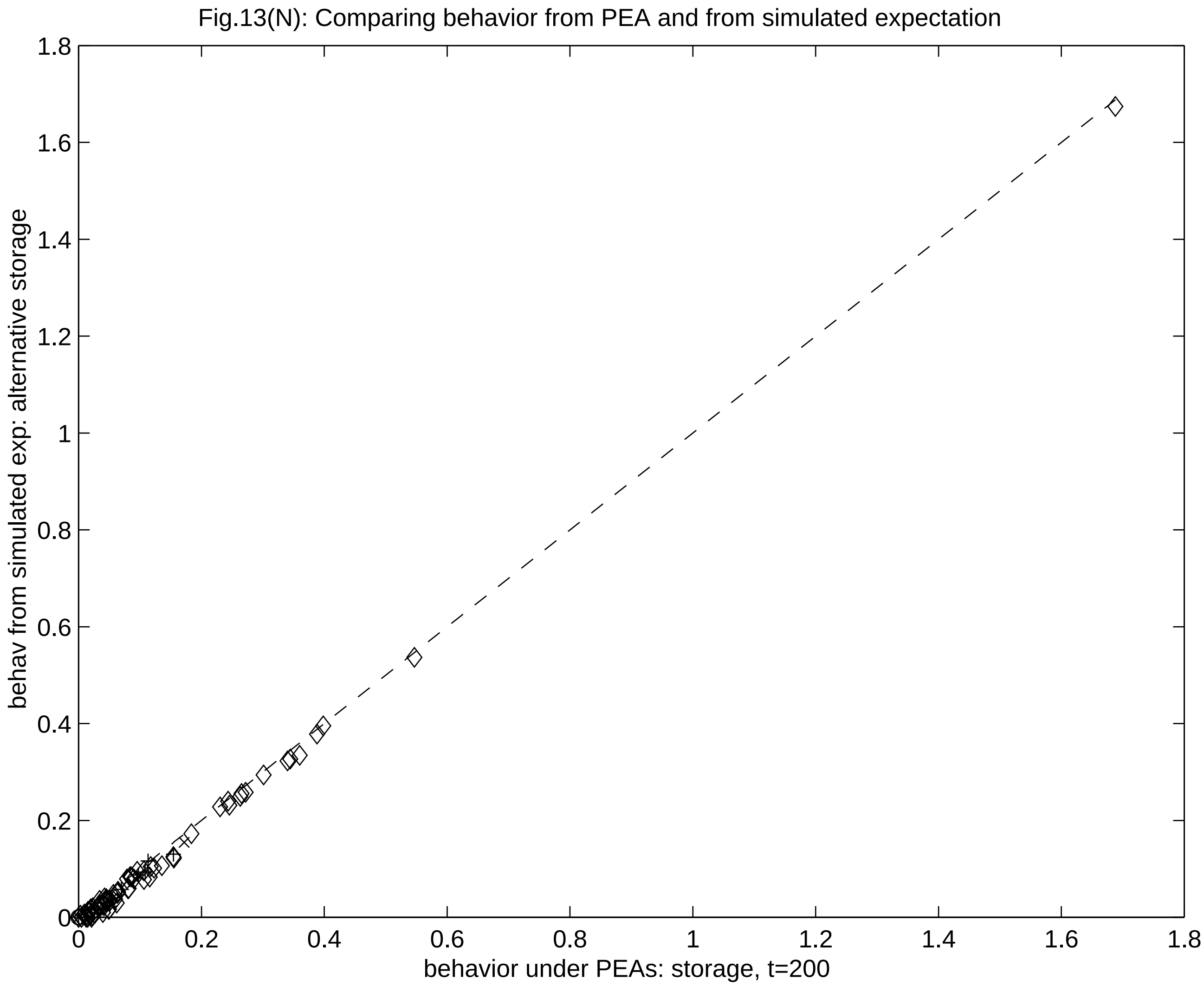


Fig.13(N): Comparing behavior from PEA and from simulated expectation
behav from simulated exp: alternative storage
behavior under PEAs: storage, t=200
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

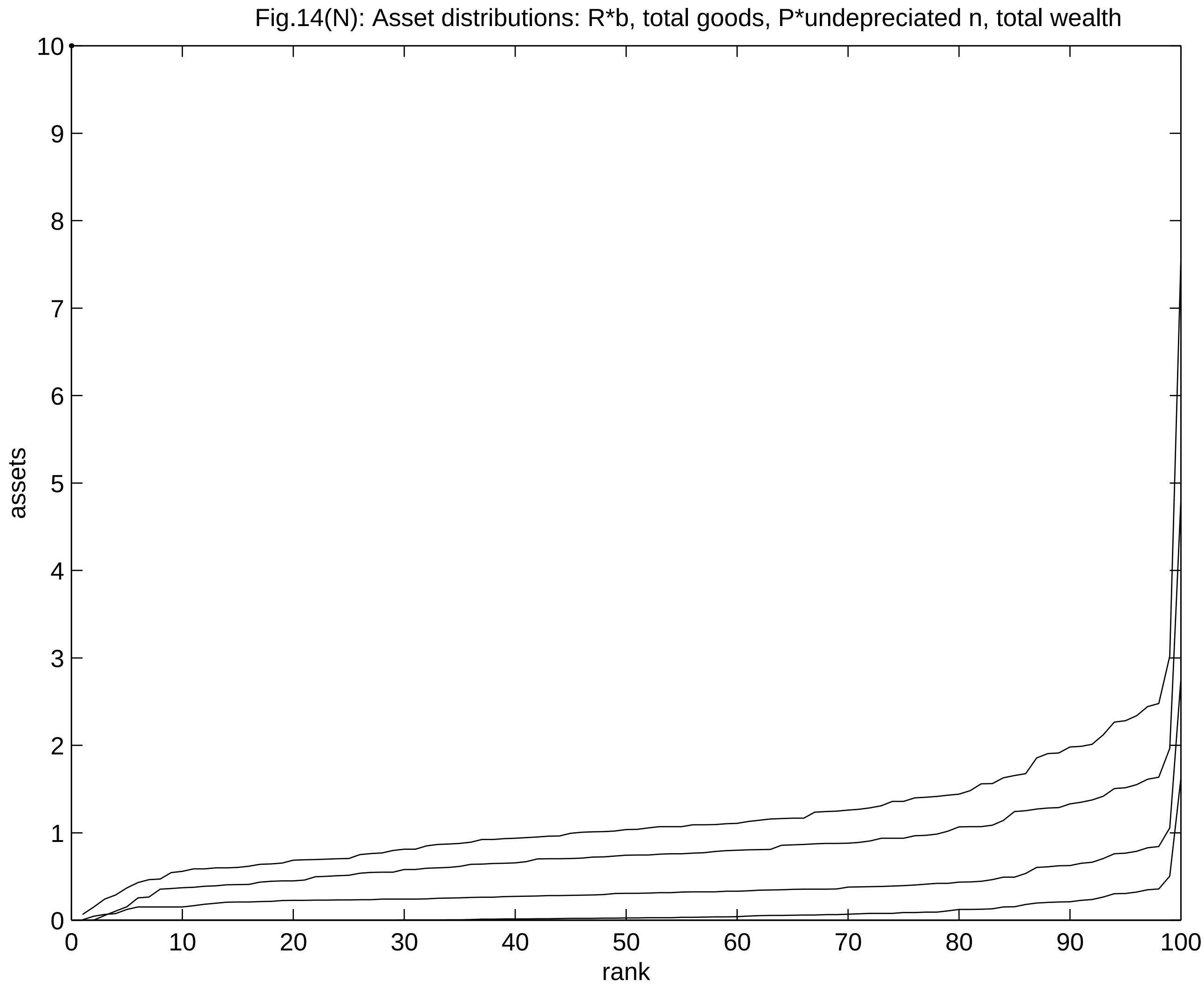


Fig.14(N): Asset distributions: R*b, total goods, P*undepreciated n, total wealth
assets
rank
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60
70
80
90
100

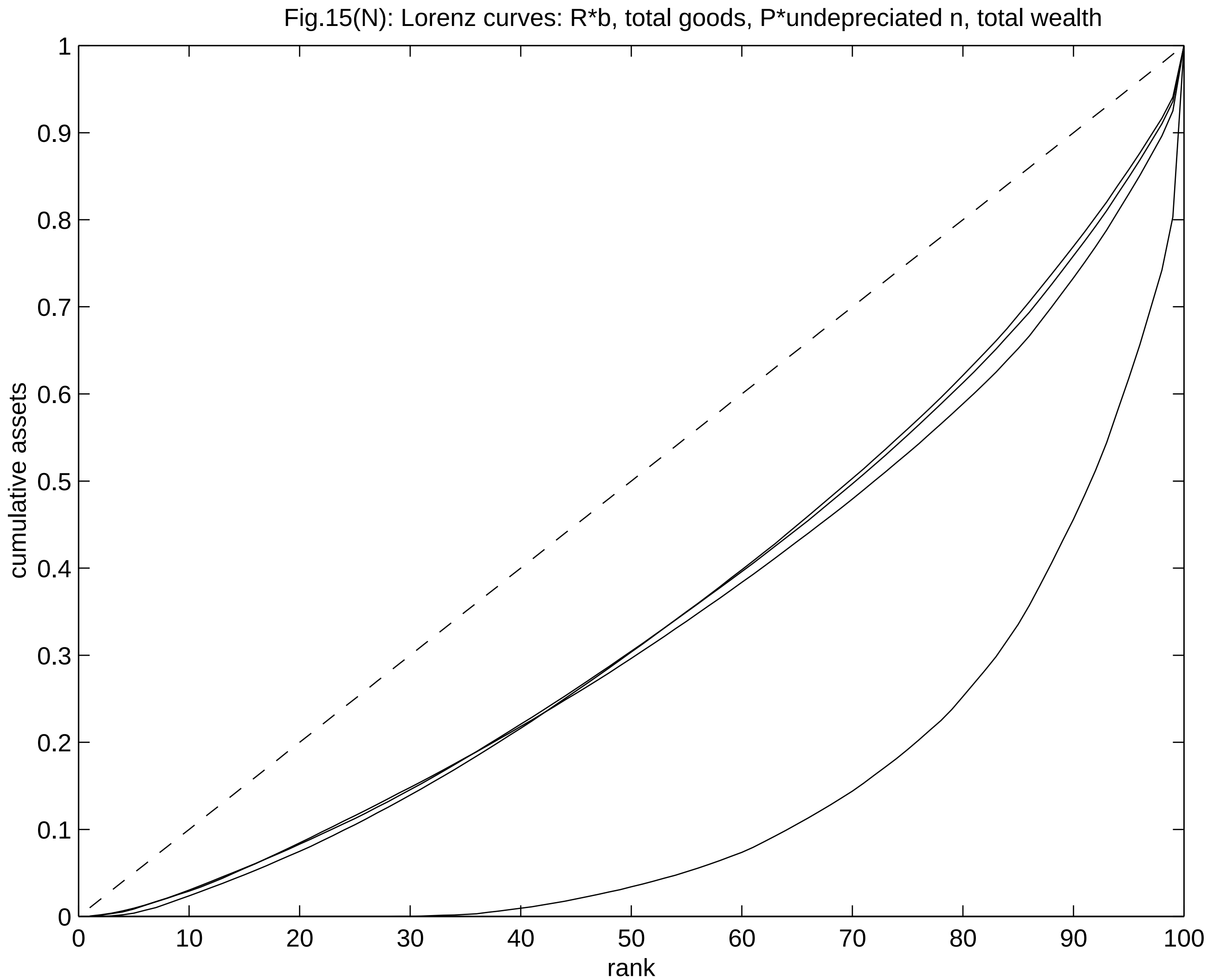


Fig.15(N): Lorenz curves: R*b, total goods, P*undepreciated n, total wealth
cumulative assets
rank
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90
100

Fig.16(N): Case N: NO AGGREGATE SHOCK: 10*U, P, avg c, avg b