Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank[¤]

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Abstract

We model systemic risk in an interbank market. Banks face liquidity needs as consumers are uncertain about where they need to consume. Interbank credit lines allow to cope with these liquidity shocks while reducing the cost of maintaining reserves. However, the interbank market exposes the system to a coordination failure (gridlock equilibrium) even if all banks are solvent. When one bank is insolvent, the stability of the banking system is a¤ected in various ways depending on the patterns of payments across locations. We investigate the ability of the banking industry to withstand the insolvency of one bank and whether the closure of one bank generates a chain reaction on the rest of the system. We analyze the coordinating role of the Central Bank in preventing payments systemic repercussions and we examine the justi…cation of the Too-big-to-fail-policy.

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NON-TECHNICAL ABSTRACT

In the last years, di¤erent events around the world, ranging from the Saving and Loans crisis in the US to the recent Long Term Capital Management hedge fund crisis have given rise to legitimate concern on the emergency of a systemic crisis and on the adequacy of the existing mechanisms that are supposed to prevent it. Directly related to that issue, a debate has emerged on the role of Central Banks, because excessive concern for …nancial stability will presumably lead to a lack of market discipline with the consequence that banks will be eager to take more risks with the understanding that they will be bailed out, and investors will reduce their monitoring e¤ort for the same reason. Since money markets in well developed economies are liquid and well functioning, the question arises as to whether a central bank should limit its interventions to open market operations, thus leaving to other institutions the responsibility to decide whether a bank that is asking for a loan is illiquid or insolvent (peer monitoring).

In contrast to the importance of these issues, theory has not succeeded yet in providing a convenient framework to analyze systemic risk so as to determine how the interbank markets and the payment system should be structured and what the Lender of Last Resort role should be.

These considerations have motivated our explicit modeling of a banking industry where banks face liquidity needs. These payment needs across locations arise because consumers are uncertain about where they need to consume. Long term investment opportunities make it costly for banks to maintain liquid reserves. For this reason, an interbank credit market where banks can obtain liquidity allows to reduce the opportunity cost of liquid reserves. However, in the presence of illiquid investments, interbank relations expose the system to the possibility that a coordination failure arise (gridlock equilibrium) even if all banks are solvent: if consumers wishing to consume in other locations believe that there will not be enough good left for their consumption at the location of destination, their best response is to liquidate early their investment at the home location, which by backward induction, makes it optimal for consumers in other locations to do the same.

When, instead, the poor return of its investment leads one bank to insolvency, the existence of the interbank market will a¤ect the stability of banking system and may lead to an ine¢ cient allocation for two completely di¤erent reasons. First, the losses of the insolvent bank may be absorbed by the rest of the system without making it optimal for any depositor to withdraw; so that market discipline disappears; hence the resiliency of the banking system with respect to

insolvency shocks may lead to forbearance.. Second, the closure of one insolvent bank may generate a chain reaction on the rest of the system and cause the liquidation of solvent banks in a contagion fashion. We analyze these issues under different structures of the banking industry, reflecting the intensity of the interbank payment flows (the percentage of depositors wishing to consume in other locations) and the structure of these flows across locations (the degree of connectivity between the banks).

The predictions of the model are consistent, for example, with the consequences of the Bank of Credit and Commerce International crisis. The crisis did not threaten financial stability in any way. Still, it forced the Bank of England to intervene because depositors in small, sometimes ethnic minorities as well as municipalities decided to transfer their deposits to larger banks. Although the effect on global liquidity was null, for some reason, the interbank market did not provide the small banks with the liquidity they needed, and so the Bank of England had to intervene. This imperfection of the interbank market is consistent with our set up, where for some architectures of the interbank payment flows it may be impossible for solvent institution to obtain liquidity.

The model has implications on the role of the Central Bank. First, the bank has a role as a "crisis manager" because it allows to coordinate the agents actions thus preventing a speculative gridlock. By guaranteeing the credit lines of all banks, the Central Bank eliminates any incentive for early liquidation. This entails no cost for the Central Bank since the guarantees are never used in equilibrium. Second, the Central Bank cannot play this role just by "lending to the market" through open market operations, since if this was the case the banks would have to hold a large amount of liquid tradeable low-return assets instead of making illiquid investments. Third, the Central Bank has a role in the orderly closure of inefficient banks. When a bank is to be liquidated, the Central Bank has to organize the bypass of the defaulting bank and provide liquidity to the banks that depend on the defaulting bank. Finally, because the existence of an interbank market may loosen market discipline, there is a role for monitoring and supervision with the regulatory agency having the right to close down a bank in spite of the absence of any liquidity crisis at that bank.

Our model can be extended to consider the spreading of financial crisis from one country to another. When depositors belong to different countries instead of different locations within the same country, their consumption needs in other locations have the natural interpretation of demand of goods of other countries, i.e. import demand. Goods of the other country can be purchased through currency or through a credit line system whereby the imports of a country are …nanced by its exports. Although our results remain valid, the lending capacity of the monetary authorities is limited by the amount of its capital.

1. INTRODUCTION

The possibility of a systemic crisis affecting the major financial markets has raised regulatory concern all over the world. Whatever the origin of a financial crisis, it is the responsibility of the regulatory body to provide adequate fire walls for the crisis not to spill over other institutions. In this paper we explore the possibilities of contagion from one institution to another that can stem from the existence of a network of financial contracts. These contracts are essentially generated from three types of operations: the payments system, the interbank market and the market for derivatives.[1] Since these contracts are essential to the financial intermediaries' function of providing liquidity and risk sharing to their clients, the regulating authorities have to set patterns for Central Bank intervention when confronted with a systemic shock. In recent years, the 1987 stock market crash, the Saving and Loans crisis, the Mexican, Asian and Russian crises and the crisis of the Long Term Capital Management hedge fund have all shown the importance of the intervention of the Central Banks and of the international financial institutions in affecting the extent, contagion, patterns and consequences of the crises.[2]

In contrast to the importance of these issues, theory has not succeeded yet in providing a convenient framework to analyze systemic risk so as to derive how the

[1] There is ample empirical evidence on financial contagion. Kaufman (1994) reviews empirical studies that measure the adverse effects on banks' equity returns of default of a major bank and of a sovereign borrower or unexpected increases in loan-loss provisions announced by major banks. Others have studied contagion through the flow of deposits (Saunders and Wilson 1996), and using historical data (Gorton 1988, Schoenmaker 1996 and Calomiris and Mason 1997). Whatever the methodology, these studies support the view that pure panic contagion is rare. Far more common is contagion through perceived correlations in bank asset returns (particularly among banks of similar size and/or geographical location).

[2] A well known episode of near financial gridlock where a coordinating role was plaid by the Central Bank is represented by the series of events the day after the stock crash of 1987. Brimmer (1989 pp.14-15) writes that "On the morning of October 20, 1987, when stock and commodity markets opened, dozens of brokerage firms and their banks had extended credit on behalf of customers to meet margin calls, and they had not received balancing payments through the clearing and settlement systems. [...] As margin calls mounted, money center banks (especially those in New York, Chicago, and San Francisco) were faced with greatly increased demand for loans by securities firms. With an eye on their capital ratios and given their diminished taste for risk, a number of these banks became increasingly reluctant to lend, even to clearly creditworthy individual investors and brokerage firms.[...] To forestall a freeze in the clearing and settlement systems, Federal Reserve officials (particularly those from the Board and the Federal Reserve Bank of New York) urged key money center banks to maintain and to expand loans to their creditworthy brokerage firm customers."

interbank markets and the payments system should be structured and what the Lender of Last Resort (LOLR) role should be.

A good illustration of the wedge between theory and reality is provided by the deposits shift that followed the distress of Bank of Credit and Commerce International (BCCI). In July 1991, the closure of BCCI in the UK made depositors with smaller banks switch their funds to the safe haven of the big banks, the so-called "‡ight to quality" (Reid 1991). Theoretically this should not have had any e¤ect, because big banks should have immediately lent again these funds in the interbank market and the small banks could have borrowed them. Yet the reality was di¤erent: the Bank of England had to step in, to encourage the large clearers to help those hit by the trend. Some packages had to be agreed (as the £200m. to the National Home Loans mortgage lender), thus supplementing the failing invisible hand of the market. So far theory has not been able to explain why the intervention of the LOLR in this type of events was important.

Our motivation to analyze a model of systemic risk stems from both the lack of a theoretical set up, and the lack of consensus on the way the LOLR should intervene. In this paper we analyze interbank networks, focusing on possible liquidity shortages and on the coordinating role of the Central Bank in avoiding and solving them. To do so we construct a model of the payment ‡ows that allows us to capture in a simple fashion the propagation of …nancial crises in an environment where both liquidity shocks and solvency shocks a¤ect …nancial intermediaries that fund long term investments with demand deposits.

We introduce liquidity demand endogenously by assuming that depositors are uncertain about where they have to consume. This provides the need for a payments system or an interbank market.[3] In this way we extend the model of Freixas and Parigi (1998) to more than two banks, to di¤erent speci…cations of travel patterns and consumers' preferences. The focus of the two papers is different. Freixas and Parigi consider the trade-o¤ between gross and net payments systems. In the current paper we concentrate instead on system-wide …nancial fragility and Central Bank policy issues. This paper is also related to Freeman (1996a,b). In Freeman, demand for liquidity is driven by the mismatch between supply and demand of goods by spatially separated agents that want to consume

[3] Payment needs arising from agents' spatial separation with limited commitment and default possibilities were …rst analyzed in Townsend (1987). For the main theoretical issues related to systemic risk in payment systems see Berger, Hancock and Marquardt (1996) and Flannery (1996), for an analysis of peer monitoring on the interbank market see Rochet and Tirole (1996) and for an analysis of the main institutional aspects see Summers (1994).

the good of the other location, at different times. If agents' travel patterns are not perfectly synchronized, a centrally accessible institution (e.g. a clearing house) may arise to provide means of payments. This allows to clear the debt issued by the agents to back their demand. In our paper, instead, liquidity demand arises from the strategies of agents with respect to the coordination of their actions.

Our main findings are, first, that, under normal conditions, a system of interbank credit lines reduces the cost of holding liquid assets. However, the combination of interbank credit and the payments system make the banking system prone to experience (speculative) gridlocks, even if all banks are solvent. If the depositors in one location wishing to consume in other locations believe that there will not be enough resources for their consumption at the location of destination, their best response is to withdraw their deposits at the home location. This triggers the early liquidation of the investment at the home location, which, by backward induction, makes it optimal for the depositors in other locations to do the same.

Second, the structure of financial flows affects the stability of the banking system with respect to solvency shocks. On the one hand, interbank connections enhance the "resiliency" of the system to withstand the insolvency of a particular bank, because a proportion of the losses on one bank's portfolio is transferred to other banks through the interbank agreements. On the other hand, this network of cross liabilities may allow an insolvent bank to continue operating through the implicit subsidy generated by the interbank credit lines, thus weakening the incentives to close inefficient banks.

Third, the Central Bank has a role to play as a "crisis manager". When all banks are solvent, the Central Banks's role to prevent a speculative gridlock is simply to act as a coordinating device. By guaranteeing the credit lines of all banks, the Central Bank eliminates any incentive for early liquidation. This entails no cost for the Central Bank since its guarantees are never used in equilibrium. When instead one bank is insolvent because of poor returns on its investment, the Central Bank has a role in the orderly closure of this bank. When a bank is to be liquidated, the Central Bank has to organize the bypass of this defaulting bank in the payment network and provide liquidity to the banks that depend on this defaulting bank. Furthermore, since the interbank market may loosen market discipline, there is a role for supervision with the regulatory agency having the right to close down a bank even if this bank is not confronted with any liquidity problem.

Fourth, when depositors have asymmetric payments needs across space, the role of the locations where many depositors want to access their wealth (money center locations) becomes crucial for the stability of the entire banking system. We characterize the too-big-to-fail (TBTF) approach often followed by Central Banks in dealing with the …nancial distress of money center banks, i.e. banks occupying key positions in the interbank network system.

The results of our paper are closely related to those of Allen and Gale (1998a) where …nancial connections arise endogenously between banks located in di¤erent regions. In our work inter-regional …nancial connections arise because depositors face uncertainty about the location where they need to consume. In Allen and Gale, instead, …nancial connections arise as a form of insurance: when liquidity preference shocks are imperfectly correlated across regions, cross holdings of deposits by banks redistribute the liquidity in the economy. These links, however, expose the system to the possibility that a small liquidity shock in one location spread to the rest of the economy. Despite the apparent similarities between the two models and the related conclusions pointing at the relevance of the structure of …nancial ‡ows, it is worth noticing that in our paper instead we focus on the implications for the stability of the system when one bank may be insolvent.

This paper is organized as follows. In section 2 we set up our basic model of an interbank network. In section 3 we describe the coordination problems that may arise even when all banks are solvent. In Section 4 we analyze the "resiliency" of the system when one bank is insolvent. In Section 5 we investigate whether the closure of one bank triggers the liquidation of others, and we show under which conditions the intervention of the Central Bank is needed to prevent a domino or contagion e¤ect. Section 6 provides an example of asymmetric travel patterns and its implications for Central Bank intervention. Section 7 discusses the policy implications, o¤ers some concluding remarks and points to possible extensions.

2. THE MODEL

2.1. Basic Set Up

We consider an economy with 1 good and N locations with exactly one bank[4] in each location. There is a continuum of consumers of equal mass (normalized to one) in each location. There are three periods: $t = 0; 1; 2$. The good can be either stored from one period to the next or invested: Each consumer is endowed with one unit of the good at $t = 0$. Consumers cannot invest directly but must deposit their

[4] That can be interpreted as a mutual bank, in the sense that it doe not have any capital and acts in the best interest of its customers.

endowment in the bank of their location, which stores it or invests it for future consumption. Consumption takes place at $t = 2$ only. The storage technology yields the riskless interest rate which we normalize at 0: The investment of bank i yields a gross return R_i at $t = 2$, for each unit invested at $t = 0$; and not liquidated at $t = 1$. At $t = 0$ the bank optimally chooses the fraction of deposits to store or invest. The deposits contract specifies the amount c_1 received by depositors if they withdraw at $t = 1$, and their bank is solvent. At $t = 2$; remaining depositors equally share the returns of the remaining assets. To finance withdrawals at $t = 1$ the bank uses the stored good, and for the part in excess, liquidates a fraction of the investment. Each unit of investment liquidated at $t = 1$ gives only α units of the good (with $\alpha \leq 1$).

We extend this model by introducing a spatial dimension: a fraction λ of the consumers who need the good at date $t = 2$ (we call them the travellers), must consume in other locations. The remaining $(1 - \lambda)$ depositors (the non travellers) consume at $t = 2$ in the home location. So in our model, consumers are uncertain about where they need to consume.

Our model is in the spirit of Diamond and Dybvig's (1983) (hereafter D-D) but with a different interpretation. In D-D, risk averse consumers are subject to a preference shock as to when they need to consume. The bank provides insurance by allowing them to withdraw at $t = 1$ but exposes itself to the risk of bank runs since it funds an illiquid investment with demand deposits. Our model corresponds to a simplified version of D-D where the patient consumers must consume at home or in the other location(s) and the proportion of impatient consumers is arbitrarily small. This allows us to concentrate on the issue of payments across locations without analyzing intertemporal insurance. Our focus is on the coordination of the consumers of the various locations, and not on the time-coordination of the consumers at the same location.[5]

Since we analyze interbank credit, the good should be interpreted as cash (i.e. Central Bank money). Cash is a liability of the Central Bank that can be moved at no cost, but only by the Central Bank.[6]

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[5] The demandable deposit feature of the contract in this model does not rely necessarily on intertemporal insurance but may have alternative rationales. For example Calomiris and Khan (1991) suggest that the right to withdraw on demand, accompanied by a sequential service constraint, gives informed depositors a credible threat in case of misuse of funds by the bank.

[6] Models in the tradition of Diamond Dybvig have typically left the characteristics of the one good in the economy in the mist. This is all right in a microeconomic set-up, but the model has monetary implications that lead to a different interpretation depending on the fact that the good is money or not. In particular, if the good is not money (wheat) then Wallace (1988) criticism

If we interpret our model in terms of payment systems the sequence of events takes place within a 24-hour period. Then we could interpret $t = 0$ as the beginning of the day, $t = 1$ as intraday, $t = 2$ as overnight, and the liquidation cost $(1 - \alpha)$ as the cost of (fire) selling monetary instruments in an illiquid intraday market.[7]

We assume that R_i is publicly observable at $t = 1$. In a multi-period version of our model, R_i would be interpreted as a signal on bank i's solvency that could provoke withdrawals by depositors or liquidation by the central bank at $t = 1$ (intraday). For simplicity, we adopt a two period model, and we assume here that the bank is liquidated anyway, either at $t = 1$, or at $t = 2$. Notice that even if R_i is publicly observed at $t = 1$ (we make this assumption to abstract from asymmetric information problems) it is not verifiable by a third party at $t = 1$ (only ex-post, at $t = 2$). Therefore the deposit contract cannot be fully conditioned on R_i. More specifically, the amount c_1 received for a withdrawal at $t = 1$ can just depend on the only verifiable information at $t = 1$, namely the closure decision. We denote by D_0 this contractual amount[8] in the case where the bank is not closed at $t = 1$. On the other hand, whenever the bank is closed (whether at $t = 1$ or at $t = 2$) its depositors equally share its assets (see Assumptions 1 and 2 below).

2.2. General formulation of consumption across space

Travel patterns, that is which depositor travels and to which location, are exogenously determined by nature at $t = 1$ and privately revealed to each depositor. They result from depositors' payment needs arising from other aspects of their economic activities. For each depositor initially at location i, nature determines whether he or she travels and in which location j he or she will consume at $t = 2$.

applies. In our model, if the good was wheat we would have to justify why the Central Bank was endowed with a superior transportation technology. As we assume the good to be money, it is the fact that commercial banks use central bank money to settle their transactions that gives the Central Bank the monopoly of issuing cash, and therefore the possibility to transfer money from one location to another corresponds to the ability to create and destroy money. Notice, also that interpreting the good as cash implies that currency crises, which are often associated with systemic risk, are left out of our analysis. This is so because "cash" is then limited by the level of reserves of the Central Bank.

[7] Since banks specialize in lending to information-sensitive customers, α can also be interpreted as the cost of selling loans in the presence of lemons problem.

[8] This amount results from ex-ante optimal contracting decisions that could be solved explicitly. For conciseness, we take D_0 as given. See Appendix 1 for an illustration. Notice that if R_i was verifiable, D_0 could be contingent on it and the risk of contagion could be fully eliminated.

To consume at $t = 2$ at location j ($i \neq j$) the travellers at location i can withdraw at $t = 1$ and carry the cash by themselves from location i to location j. The implicit cost of transferring the cash across space is the foregone investment return.[9] This motivates the introduction of credit lines between banks to minimize the amount of good not invested. The credit line granted by bank j to bank i gives the depositors of bank i going to bank j the right to have their deposits transferred to location j and obtain their consumption at $t = 2$ as a share of the assets at bank j at date $t = 2$.

A way to visualize the credit line granted by bank j to bank i; is to think that consumers located at i arrive in location j at $t = 2$ with a check written on bank i and credited in an account at bank j. Bank i, in turn, gives credit lines to one or more banks as specified below.[10] At $t = 2$ the banks compensate their claims and transfer the corresponding amount of the good across space. The technology to transfer the good at $t = 2$ is available for trades between banks only.

We take the view that credit lines are agreements between banks on behalf of their depositors that cannot be revised in light of new information. This is motivated by the prevalence of this feature in many settlement schemes. Basically, banks receiving orders to pay do not have the time necessary to continuously assess the solvency of the sending banks and consequently, cannot adjust their credit rates or credit limits. This may be justified by the cost for the participants to a settlement scheme that involves a large volume of transactions to monitor each other continuously and adjust intraday interest rates or credit limits to changing conditions.

To make explicit the values of the assets and the liabilities resulting from interbank relations we adopt the simplest sharing rule, namely:

Assumption 1. All the liabilities of a bank have the same priority at t=2.

This rule defines how to divide bank's assets at $t = 2$ among the claim holders. It implies that credit lines are honored in proportion to the amount of the assets of the bank at date $t = 2$. In particular if D_i is the ex post value of a (unit of) deposit in bank i, then $D_i = \frac{\text{Bank}_i \text{ Total Assets}}{\text{Bank}_i \text{ Total Liabilities}}$: Notice that more complex priority rules could be more efficient in the resolution of liquidity crises. However, we assume that they are not feasible in our context: this is a reduced form assumption aiming at capturing the limitations of the information that is instantaneously available in

[9] We could also add an explicit cost of "travelling with the cash" (i.e. bypassing the payments system). It would not affect our results.

[10] For a similar characterization of credit chains in the context of trading arrangements, see Kiyotaki and Moore (1997).

interbank networks. An additional assumption is needed to describe what happens in case a bank is closed at $t = 1$.

Assumption 2. If a bank is closed at time 1 its assets are shared between its own depositors only.

Assumption 2 simply means that when the bank is closed at time $t = 1$, it has not entered into contractual relationships with the other banks, and therefore only its depositors have a claim on its assets. Bank closure at time 1 may come from a decision of the regulator or from the withdrawals of all depositors. Assumption 2 implies that when a bank is closed at time 1, it is deleted from the interbank network.

Let π_{ij} be the measure of depositors from location i consuming at location j; where i can take any value including j, and let t_{ij} be the proportion of travellers going from location i to j; $j \neq i$ (by definition, $t_{ii} = 0$): The matrix Π that defines where consumers go and in which proportions is related to the matrix T of travel patterns by:

$$\Pi \equiv (1 - \lambda) I + \lambda T \tag{2.1}$$

where $\Pi = (\pi_{ij})_{ij}$ and $T = (t_{ij})_{ij}$. This specification allows us to parameterize independently two features of the payment system: λ captures the intensity of interbank flows and the matrix T captures the structure of these flows. By definition, we have for all i; $\sum_j \pi_{ij} = 1$: For the sake of simplicity, unless otherwise specified (see Section 6), we will impose the following additional restrictions:

Assumption 3. For all j; $\sum_i \pi_{ij} = 1$:

In this way we discard the supply and demand imbalances at a specific location as the cause of a disruption in the payments system or in the interbank market. Because of the complexity of the transfers involved in an arbitrary matrix Π, we will illustrate our findings in two symptomatic cases:

- In the first one $t_{ij} = 1$ if $j = i + 1$ and $t_{ij} = 0$ otherwise, with the notational convention that $N + 1 \equiv 1$: To visualize this case it is convenient to think that the consumers are located around a circle as in Salop's (1979) model. All travellers from i go to location $i + 1$, the clockwise adjacent location, where they must consume at $t = 2$: The payments structure implied by this travel pattern generates what we define as credit chain interbank funding, when the bank at location $i + 1$ provides credit to the incoming depositors from location i.

- In the second travel pattern $t_{ij} = \frac{1}{N-1}$ with $i \neq j$: Each two banks swap $\frac{\lambda}{N-1}$ customers so that at time $t = 2$ at location j there are $\frac{\lambda}{N-1}$ travellers from each of the other $(N-1)$ locations. We will refer to this perfectly isotropic case as the diversified lending case.[11]

With credit chain interbank funding, credit flows in the direction opposite to travel. With diversified lending every bank gives credit lines uniformly to all other $N-1$ banks. In terms of payments mechanisms, the interbank credit described above can be interpreted as a compensation scheme (net system) or a Real Time Gross System (RTGS) with multilateral credit lines.

Let us now introduce the players of the game, namely the N banks and their depositors. At $t = 0$ banks decide whether to extend each other credit lines. In the absence of credit lines, all travellers have to withdraw at $t = 1$, which reduces the quantity that each bank can invest: this is what we call the autarkic situation. On the other hand, in the general case with credit lines, the value of final consumption at $t = 2$ is determined by a non-cooperative game played by the banks' depositors. At $t = 1$ each depositor located at i and consuming at location j simultaneously and without coordination determines the fraction x_{ij} of his or her deposit to maintain in the bank. Accordingly, the percentage of investment remaining at location j where he or she must consume is

$$X_j \equiv \max\left[1 - \sum_k \pi_{jk}(1-x_{jk})\frac{D_0}{\alpha}; 0\right]; \tag{2.2}$$

Because of Assumption 1, the final consumption of depositors $(i; j)$ results from a combination of a withdrawal at time $t = 1$ in bank i (i.e. $(1-x_{ij})D_0$) plus a proportion x_{ij} of the value at $t = 2$ of a deposit D_j in bank j. To determine the possible equilibria of the depositors' game, we have to compare D_0 with the (endogenous) values of the deposits $D_1; \ldots; D_N$ in all the banks at $t = 2$. Now, to determine D_i, consider the balance sheet equation for bank i at time $t = 2$:

$$X_i R_i + \sum_j \pi_{ji} x_{ji} D_j = \left(\sum_j \pi_{ji} x_{ji} + \sum_j \pi_{ij} x_{ij}\right) D_i \tag{2.3}$$

where the LHS (RHS) represents the assets (liabilities) of bank i, $X_i R_i$ is the return on its investment, $\sum_j \pi_{ji} x_{ji} D_j$ are the credits due from other banks, $\sum_j \pi_{ji} x_{ji} D_i$ are the debts with other banks, and $\sum_j \pi_{ji} x_{ji} D_i$ are its deposits. Notice that Assumption 2 implies that the above equation does not apply when bank i is closed at $t = 1$: In this case $X_i = D_i = 0$.

[11] The structure of the payment flows implied by credit chain interbank funding and diversified lending is very similar to that studied in Allen and Gale (1998a).

The optimal behavior of each depositor (i, j) is $x_{ij} = 1 \Leftrightarrow D_j \geq D_0; \forall i, j$. Since it depends only on j, we denote by x^j; the common value of the x_{ij} where $x^j = 1$ if $D_j > D_0$ and $x^j = 0$ otherwise. This allows a simplification of (2.3):

$$X_i R_i + \left(\sum_j \pi_{ji} D_j\right) x^i = \left[\left(\sum_j \pi_{ji}\right) x^i + \sum_j \pi_{ij} x^j\right] D_i. \tag{2.4}$$

We establish the following notation: $D = (D_1, \ldots, D_N)'$; $R = (R_1, \ldots, R_N)'$, and Π' is the transpose of $\Pi = (\pi_{ij})_{i,j}$. For a given strategy vector $(x_{ij})_{i,j}$ one can compute the assets in place at bank i (X_i) and the return on a deposit at bank i (D_i). Then we check whether the strategies are optimal:

$$x^*_{ij} = \begin{cases} 1 \text{ if } D_j > D_0 \\ 0 \text{ if } D_j < D_0 \end{cases}. \tag{2.5}$$

Any fixed point of this algorithm (i.e., $x^*_{ij} \equiv x_{ij}$) is an equilibrium of our game.

When the mechanism of interbank credit functions smoothly, $x_{ij} \equiv 1$ for all (i, j) and depositors' welfare is greater than in the autarkic situation. This is because interbank credit lines allow each bank to keep a lower amount of liquid reserves and to invest more. However, the system is also more fragile. As we show in the next Sections, the non cooperative game played by depositors has other equilibria than $x_{ij} \equiv 1$.

3. PURE LIQUIDITY SHOCKS

We first analyze the equilibria of the game when all deposits are invested at $t = 1$ investment returns are certain and all banks are solvent so that the only issue is the coordination among depositors. Disregarding the mixed strategy equilibria where depositors are indifferent between withdrawing their deposits and transferring them to the recipient banks, we obtain our first result:

Proposition 3.1. : We assume $R_i > D_0$ for all i (which implies that all banks are solvent). There are at least two pure strategy equilibria: (i) the inefficient bank run allocation where $x^* = 0$ (Speculative Gridlock Equilibrium) and (ii) the efficient allocation where $x^* = 1$ (Credit Line Equilibrium).

Proof. See Appendix 2.

Several comments are in order. In the Credit Line Equilibrium there is no liquidation while in the Speculative Gridlock Equilibrium all the banks' assets are liquidated. Since liquidation is costly and all banks are solvent, the Credit Line Equilibrium dominates the Speculative Gridlock Equilibrium as well as any other equilibrium where some liquidation takes place. The Speculative Gridlock Equilibrium arises as a result of a coordination failure like in D-D. If depositors rationally anticipated at $t = 0$ a Speculative Gridlock Equilibrium, they would prefer the autarkic situation.

In the Credit Line Equilibrium with diversified lending, bank i extends credit lines to all the other banks and receives credit lines from them. In equilibrium the debt arising from bank i's depositors at $t = 2$ using bank i's credit lines with the other banks, is repaid at $t = 2$ by bank i serving the depositors from the other banks. It is precisely because the behavior of one bank's depositors is affected by the expectation of what the depositors going to the same location will do, that this equilibrium is vulnerable to a coordination failure. If the depositors in a sufficiently large number of banks believe that they will be denied consumption at the location where they have to consume, it is optimal for them to liquidate their investment, which makes it optimal for the depositors in all other banks to do the same. The Speculative Gridlock Equilibrium is related to the notion of Domino Effect that may arise in payments systems as a result of the settlement failure of some participant. Still, it may occur here even if all banks are solvent. Notice, that banks do not play any strategic role: only depositors play strategically.

From the efficiency viewpoint, when all the banks are solvent the Credit Line Equilibrium dominates autarky which in turn dominates the Speculative Gridlock Equilibrium.[12] Hence there is a trade-off between a risky interbank market based on interbank credit and a safe payment mechanism which foregoes investment opportunities.[13]

Both the Gridlock and the Credit Line Equilibria involve the use of credit

[12] When $\alpha = 1$ the last two are equivalent. The cost of the Gridlock Equilibrium is proportional to $1-\alpha$. Notice that autarky is equivalent to a payment system with fully collateralized credit lines like TARGET (Trans-European Automated Real-Time Gross Settlement Express Transfer), the payment system designed to handle transactions in the Euro area.

[13] For an analysis of this trade off in a related setting see Freixas and Parigi (1998). However, even a Real Time Gross System like the European system TARGET is not immune to a systemic crisis. As Garber (1998) points out if there is a risk that a currency will leave the Euro currency area, the very infrastructure of TARGET where National Central Banks of the participating countries extend to each other unlimited daily credit, provides the perfect mechanism to mount speculative attacks on the system.

lines. In both equilibria banks extend and honor credit lines up to the amount of their $t = 2$ resources. In the Speculative Gridlock Equilibrium it is not the banks that do not honor the credit lines, rather are the depositors that, by forcing the liquidation of the investment, reduce the amount of resources available at $t = 2$.

There is a clear parallel between these two equilibria in our economy with N locations and the equilibria in a one-location D-D model. These results are also related to the papers by Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994) that consider N-location D-D economies without geographic risks.

The Credit Line Equilibrium can be implemented in several ways: through a Compensation System where credits are netted, by a RTGS (Real Time Gross Settlement) system with multilateral or bilateral credit lines, through lending by the Central Bank and through Deposit Insurance.

In this basic version of the model, in the event of a gridlock, every bank is solvent although illiquid. Thus no difficulty in distinguishing between insolvent and illiquid banks arises for the Central Bank.[14] The Central Bank has a simple coordinating role as a LOLR in guaranteeing private-sector credit lines or in providing fiat money, both backed by the authority of the Treasury to tax the return on the investment. The Central Bank cannot play this role just by "lending to the market" through open market operations, since if this was the case the banks would have to hold a large amount of liquid tradeable low-return assets instead of making illiquid investments with a higher return.

Similarly, by guaranteeing the value of deposits at the consumption locations, Deposit Insurance eliminates any incentive for the depositors to protect themselves by liquidating the investment, thus making it optimal for banks to extend credit to each other.

Like Deposit Insurance which is never used in equilibrium in the D-D model, the coordination role of the Central Bank costs no resources (excluding moral hazard issues), since in equilibrium it will not be necessary for the Central Bank to intervene.[15]

[14] For an analysis of this issue see the companion paper by Freixas, Parigi, and Rochet (1998).

[15] The Federal Reserve's role in facilitating the private-sector rescue of the hedge fund Long Term Capital Management (LTCM) in 1998 offers an example of the coordinating role of the Central Bank. The Federal Reserve Bank of New York organized rescue loans by private institutions to LTCM for fear that a default of the fund on the $80 plus billion that it had borrowed from some key international banks and securities firms could jeopardize the stability of the entire financial system (Wall Street Journal 1998). Greenspan (1998) argues that in the refinancing of LTCM "no Federal Reserve funds were put at risk, no promises were made by the Federal

4. RESILIENCY AND MARKET DISCIPLINE IN THE INTERBANK SYSTEM

In the next two Sections we tackle the issue of the impact of the insolvency of one bank on the rest of the system. In this Section we investigate under which conditions the losses of one bank can be absorbed by the other banks without provoking withdrawals by depositors (resiliency) and what are the implications in terms of market discipline. In the next Section we consider the issue of contagion. That is we investigate whether the closure of an insolvent bank generates a chain reaction causing the liquidation of solvent banks.

In order to model the possibility of insolvency in a simple way, we make the extreme assumption that the return R_i on the investment at location i can be either $R \geq D_0$, or 0: If $R = 0$; bank i is insolvent, in which case it is efficient to liquidate it, absent contagion issues.[16] For the remainder of this paper we assume that the probability of $R = 0$ is so low that it is optimal for the banks to invest all deposits at $t = 0$. Returns are publicly observable at $t = 1$ but verifiable only at $t = 2$ so that no contract can be made contingent on these returns. The efficient allocation of resources requires that banks be liquidated if and only if they are insolvent:

$$X_i = \begin{cases} 0 \text{ if } R_i = 0 \\ 1 \text{ if } R_i = R \end{cases} : \qquad (4.1)$$

Whether this efficient closure rule is a Nash Equilibrium of the non-cooperative game between depositors, will depend on the structure of the interbank payment system. To illustrate this, we focus on the case in which one bank (say, bank 1) is insolvent, and we investigate under which conditions $x = (1, \ldots, 1)$ is still an equilibrium, i.e. under which conditions $D_i \geq D_0$ for all i. When $x = (1, \ldots, 1)$

Reserve, and no individual firms were pressured to participate. Officials of the Federal Reserve Bank of New York facilitated discussions in which the private parties arrived at an agreement that both served their mutual self interest and avoided possible serious market dislocations. Financial market participants were already unsettled by recent global events. Had the failure of LTCM triggered the seizing up of markets, substantial damage could have been inflicted on many market participants, including some not directly involved with the firm, and could have potentially impaired the economies of many nations, including our own."

[16] For an analysis of the distinction between fundamental and speculative bank runs see Jacklin and Bhattacharya (1988). For a model of fundamental runs and Central Bank intervention see Allen and Gale (1998b).

and $R_1 = 0$ the balance sheet equations (2:4) give

$$D = (2I - \Pi')^{-1} \begin{pmatrix} 0 \\ R \\ \vdots \\ R \end{pmatrix} = R\,(2I - \Pi')^{-1} \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{4.2}$$

From (4:2) we define by γ the minimum of the components of the vector

$$(2I - \Pi')^{-1} \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

We establish the following proposition:

Proposition 4.1. (Resiliency and Market discipline) When $R_1 = 0$, necessary condition for $x = (1, \ldots, 1)$ to be an equilibrium is that the smallest value of time $t = 2$ deposits $(R\gamma)$, which depends on the structure of interbank payment flows, exceeds D_0.

Proof. From (4:2) and the definition of γ we see that $D_i \geq D_0$ for all i if and only if $R\gamma \geq D_0$. ■

Several comments are in order. Proposition 4.1 highlights an important aspect of the tension between efficiency and stability of the interbank system. One the one hand it establishes the conditions under which the system can absorb the losses of one bank without any deposit withdrawal. Resiliency, however, entails the cost of forbearance of the insolvent bank. On the other hand it establishes the conditions for the existence of the opposite equilibrium where the insolvent bank is closed down (i.e. $x = (1, \ldots, 1)$ is no longer an equilibrium). If a bank is known at $t = 1$ to be insolvent, depositors may withdraw and withdrawals may not be confined to the insolvent bank, hence market discipline entails the cost of possibly excessive liquidation.

We interpret γ as a measure of the exposure of the interbank system as a whole to market discipline when one bank is insolvent. The lower γ, the more exposed is the system to market discipline. In Appendix 1 we compute D_0 in the case where it corresponds to the limit of the D-D optimal contract when the proportion of early diers tends to zero and we show that $\frac{D_0}{R}$, the threshold of γ, can be interpreted as the critical value of the exposure of the banking system to market discipline.

We now study how ° varies with ¸ (the proportion of travellers) and N (the number of locations) in the two cases of credit chain and diversi…ed lending.

Proposition 4.2. Both in the credit chain case and in the diversi…ed lending case, ° increases with ¸ and N; i.e. when the proportion of travellers increases or the number of banks increases, the system becomes less exposed to market discipline.

Proof. See Appendix 2.
As it is intuitive, when the number of banks increases, the insolvency of one bank has a lower impact on the value of the deposits in the other banks. Similarly an increase in the fraction of travellers spreads on the other banks a larger fraction of the loss due to the insolvency of one bank. We now compare the two systems for given values of ¸ and N. We then compare the exposure to market discipline of the credit chain and the diversi…ed lending structures..

Proposition 4.3. In case of the insolvency of one bank, the system is more exposed to market discipline under diversi…ed lending than under credit chains; i.e. $\circ^{CRE} > \circ^{DIV}$.

Proof. See Appendix 2.
Proposition 4.3 may appear counterintutive since diversi…cation is usually associated to the ability to spread losses. The result depends on the proportion of the losses on its own portfolio that the insolvent bank is able to transfer to other banks through the payments system. In a diversi…ed lending there is more diversi…cation so that solvent banks exchange a larger fraction of their claims. As a consequence in a diversi…ed lending the insolvent bank is able to pass over to the solvent banks a smaller fraction of its losses. The case with three banks (N = 3) and everybody travels (¸ = 1) provides a good illustration.

In a diversi…ed lending system the balance sheet equations (2:4) become:

$$D_i = \frac{1}{2}\left[R_i + \frac{1}{2}(D_{i-1} + D_{i+1})\right] \qquad i = 1; 2; 3:$$

This means that if bank 1 is insolvent (i.e., $R_1 = 0$), depositors at banks 2 and 3 get an equal share of total surplus, while bank 1 depositors receive 50% less. After easy computations, we …nd that bank 1 depositors receive $\frac{2}{5}R$, or equivalently bank 1 is able to pass $\frac{3}{5}$ of its losses to the solvent banks whose depositors end up receiving $\frac{4}{5}R$.

Consider now the case of credit chains. Still assuming $\lambda = 1$, the balance sheet equations give:

$$D_i = \frac{1}{2}[R_i + D_{i+1}] \qquad i = 1; 2; 3:$$

We can compute the losses experienced by each bank (with respect to the promised returns R) and it is a simple exercise to check that the only solution is:

$$D_1 = \frac{3}{7}R; D_3 = \frac{5}{7}R; D_2 = \frac{6}{7}R:$$

Therefore, bank 1 is able to pass on a higher share of its losses than in the diversified lending case, which explains the lower exposure of the interbank system to market discipline in the credit chain system.

The results of this Section highlight another side of interbank markets in addition to their role in redistributing liquidity efficiently studied by Bhattacharya and Gale (1987). Interbank connections enhance the "resiliency" of the system to withstand the insolvency of a particular bank. However, this network of cross liabilities may loosen market discipline and allow an insolvent bank to continue operating through the implicit subsidy generated by the interbank credit lines. This loosening of market discipline is the rationale for a more active role for monitoring and supervision with the regulatory agency having the right to close down a bank in spite of the absence of any liquidity crisis at that bank.

The effect of a Central Bank's guarantee on interbank credit lines would be that $x = (1; \ldots; 1)$ is always an equilibrium, even if one bank is insolvent. The stability of the banking system would be preserved at the cost of forbearance of inefficient banks.

5. CLOSURE-TRIGGERED CONTAGION RISK

5.1. Efficiency vs. Contagion Risk

We now turn to the other side of the relationship between efficiency and stability of the banking system, and investigate under which conditions the closure at time $t = 1$ of an insolvent bank does not trigger the liquidation of solvent banks in a contagion fashion. Suppose indeed that bank k is closed at $t = 1$: Assumption

2 implies that $X_k = 0$ and $D_k = 0$. Two are the implications of closing bank k at $t = 1$. First, we have an unwininding of the positions of bank k since $\pi_{ki}D_k$ assets and $\pi_{ki}D_i$ liabilities disappear from the balance sheet of bank k. Second, a proportion π_{ik} of travelers going to location k will be forced to withdraw early the amount $\pi_{ik}D_0$ and bank i will have to liquidate the amount $\pi_{ik}\frac{D_0}{\alpha}$. If $\pi_{ik}\frac{D_0}{\alpha}$ is sufficiently large bank i is closed at $t = 1$, otherwise the cost at, $t = 2$, of the early liquidation is $\pi_{ik}\left(\frac{D_0}{\alpha}R - D_i\right)$.

Notice that if $\pi_{ik}\frac{D_0}{\alpha} \geq 1$ then $X_i = 0$, i.e. bank i is liquidated simply because there are too many depositors going from location i to location k; the insolvent bank closed at $t = 1$. The type of contagion that takes place here is of a purely mechanical nature stemming simply from a direct effect. Since this case is straightforward let us instead concentrate on the other case, namely $\pi_{ik}\frac{D_0}{\alpha} < 1$. Because of unwinding and forced early withdrawal, the full general case is more complex. Since $x^k = 0$, we have to suppress all that concerns bank k from the equations (2.4). We obtain:

$$X_{i(k)}R_i + \sum_{j \neq k} \pi_{ji} D_j x^i = \left(\sum_{j \neq k} \pi_{ij} x^j + \sum_{j \neq k} \pi_{ji} x^i \right) D_i, \tag{5.1}$$

where

$$X_{i(k)} = \max \left[1 - \pi_{ik}\frac{D_0}{\alpha} - \sum_{j \neq k} \pi_{ji}(1 - x^j)\frac{D_0}{\alpha}, 0 \right].$$

We now have to check whether $x_{ij} \equiv 1$ for all $i, j \neq k$, can correspond to an equilibrium. In this case, $X_{i(k)} = \max[1 - \pi_{ik}\frac{D_0}{\alpha}, 0]$ and system (5.1) becomes:

$$R_i = \left(\sum_{j \neq k} \frac{\pi_{ij} + \pi_{ji}}{X_{i(k)}} \right) D_i - \sum_{j \neq k} \frac{\pi_{ji}}{X_{i(k)}} D_j, \tag{5.2}$$

Since by assumption $R_i \equiv R$ for all $i \neq k$, (5.2) becomes

$$\left(1 - \pi_{ik}\frac{D_0}{\alpha}\right)R + \sum_{j \neq k} \pi_{ji} D_j = (2 - \pi_{ik} - \pi_{ki}) D_i \tag{5.3}$$

This allows us to establish a result analogous to Proposition 4.1.

Proposition 5.1. (Contagion Risk) There is a critical value of the smallest time $t = 2$ deposits below which the closure of a bank causes the liquidation of at least another bank. This critical value is lower in the credit chain case than in the diversified lending case. The diversified lending structure is always stable when the number N of banks is large enough whereas N has no impact on the stability of the credit chain structure.

Proof. It is analogous to that of Proposition 4.1. Denoting by M_k the inverse of the matrix defined by system (5.3), stability is equivalent to:

$$\begin{pmatrix} D_1 \\ \ldots \\ D_N \end{pmatrix} = RM_k \begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix} > D_0 \begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix}$$

One can see that all the elements of M_k are non negative[17], thus stability obtains iff $\frac{D_0}{R} \leq \psi_k$, where ψ_k denotes the minimum of the components $M_k \begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix}$:
The computation of ψ_k is cumbersome in the general case but easy in our benchmark examples (where, because of symmetry, k does play any role). One finds:

$$\Psi_{cre} = 1 - \lambda\left(\frac{D_0}{\alpha} - 1\right); \; \Psi_{div} = 1 - \lambda\left(\frac{\frac{D_0}{\alpha} - 1}{N - 1 - \lambda}\right)$$

in the credit chain example, and in the diversified lending case, respectively. It is immediate from these formulas that $\Psi_{cre} < \Psi_{div}$ (for $N > 2$) and that Ψ_{div} tends to 1 when N tends to infinity while Ψ_{cre} is independent of N. ■

5.2. Comparison with Allen and Gale (1998a)

It is useful to compare our results with those of Allen and Gale (1998a). Proposition 4.1 establishes that systemic crises may arise for fundamental reasons, like in Allen and Gale. However, the focus of the two papers is different. Allen and Gale are concerned with the stability of the system with respect to liquidity shocks arising from the random number of consumers that need liquidity early in the absence of aggregate uncertainty. They show that the system is less stable when the interbank market is incomplete (in the sense that banks are allowed to cross hold deposits only in a credit chain fashion) than when the interbank market is

[17] The fact that the matrix M_k has non negative elements follows from a property of diagonal dominant matrices (See e.g. Takayama 1985 p.385).

complete (in the sense that banks are allowed to cross hold deposits in a diversified lending fashion).

In our paper interbank links arise, instead, from consumers geographic uncertainty and we focus on the implications of the insolvency of one bank in terms of market discipline and the stability of the system. In particular in Proposition 4.3 we show how the structure of interbank links allow to spread over other banks the losses of one bank. We show that a diversified lending system is more exposed to market discipline (i.e. less resilient) than a credit chain system because in the latter the insolvent bank is able to transfer a larger fraction of its losses to other banks thus reducing the incentives for its own depositors to withdraw. In Proposition 5.1 we are concerned with the stability of the system with respect to contagion risk triggered by the efficient liquidation at time $t = 1$ of the insolvent bank.

6. TOO-BIG-TO-FAIL AND MONEY CENTER BANKS

Regulators have often adopted a too-big-to-fail approach (TBTF) in dealing with financially distressed money center banks and large financial institutions.[18] One of the reasons is the fear of the repercussions that the liquidation of a money center bank might have on the corresponding banks that channel payments through it. Our general formulation of the payments needs where the flow of depositors going to the various locations is asymmetric offers a simple way to model this case and to capture some of the features of the TBTF policy. We interpret the TBTF policy as designed to rescue banks which occupy key positions in the interbank network, rather than banks simply with large size.[19]

Consider for example the case where there are three locations ($N = 3$). Locations 2 and 3 are peripheral locations and location 1 is a money center location. All the travellers of locations 2 and 3 must consume at location 1, and one half of the travellers of location 1 consume at location 2 and the other half at location

[18] See for example the intervention of the monetary authorities in the Continental Illinois debacle in 1984 and, to some extent, in arranging the private-sector rescue of Long Term Capital Management.

[19] The Barings' failure of 1996 is an example of the crisis of a large financial institution that did not create systemic risk.

3. That is $t_{12} = t_{13} = \frac{1}{2}$ and $t_{21} = t_{31} = 1$, $t_{23} = t_{32} = 0$.[20] This implies that

$$X_1 = \max\left\{1 - \frac{D_0}{\alpha}\left[1 - (1-\lambda)x^1 - \lambda\left(\frac{x^2 + x^3}{2}\right)\right]; 0\right\};$$

and

$$X_2 = \max\left\{1 - \tfrac{D_0}{\alpha}[1 - (1-\lambda)x^2 - \lambda x^1]; 0\right\}$$
$$X_3 = \max\left\{1 - \tfrac{D_0}{\alpha}[1 - (1-\lambda)x^3 - \lambda x^1]; 0\right\}.$$

Suppose now that one of these banks (and only one) is insolvent (this is known at $t = 1$). The next proposition illustrates how the closure of a bank with a key position in the interbank market may trigger a systemic crisis.

Proposition 6.1. (i) If $\lambda > \mu = \alpha(\frac{1}{D_0} - \frac{1}{R})$ the liquidation of bank 1 triggers the liquidation of all other banks (Too-big-to-fail); (ii) If $\lambda > \frac{2\alpha}{D_0}$, liquidation of banks 2 or 3 does not trigger the liquidation of any of the other two banks.

Proof. To prove (i) notice that if bank 1 is closed then $X_1 = 0$, and $x^1 = 0$. Then $D_2 = X_2 R = (1 - \frac{D_0}{\alpha}\lambda)R$. Thus $x^2 = 0$ if $(1 - \frac{D_0}{\alpha}\lambda)R < D_0$, $\lambda > \alpha(\frac{1}{D_0} - \frac{1}{R})$. To prove (ii) notice that if bank 2 is closed then $x^2 = 0$. If $(1; 0; 1)$ is an equilibrium the balance sheet equations become, when $\frac{D_0\lambda}{\alpha} < 2$:

$$D_1\left(1 - \frac{\lambda}{2}\frac{D_0}{\alpha} + \lambda\right) = \left(1 - \frac{D_0}{\alpha}\frac{\lambda}{2}\right)R_1 + \lambda D_3$$
$$D_3\left(1 + \frac{\lambda}{2}\right) = R_3 + \frac{\lambda}{2}D_1.$$

If $R_3 = R_1 = R$ this yields $D_3 = D_1 = R$. This implies that $x = (1; 0; 1)$ is an equilibrium whenever $\frac{D_0\lambda}{\alpha} < 2$. ■

In Appendix 1, we prove that μ can be interpreted as another measure of bank exposure to market discipline: the larger μ, the more withdrawals it can accommodate..

Our last result concerns the optimal attitude of the Central Bank when the money center bank becomes insolvent ($R_1 = 0$). When $\frac{D_0}{R}$ is low, no intervention is needed. When $\frac{D_0}{R}$ is large, the Central Bank has to inject liquidity. More precisely we have:

[20] Notice that we now abandon Assumption 3 (the symmetry assumption).

Proposition 6.2. When $R_1 = 0; x = (1;1;1)$ is an equilibrium if $\frac{D_0}{R}$ is su¢ciently low (no Central Bank intervention is needed). In the other case, the cost of bailout increases with $\frac{D_0}{R}$.

Proof. When $R_1 = 0; x = (1;1;1)$ can be an equilibrium if $D > D_0 \begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix}:$ When $x = (1;1;1)$; the balance sheet equations (2.4) become

$$R_1 + (D_2 + D_3) = 3D_1 \quad (6.1)$$

$$R_2 + \frac{1}{2}D_1 = \frac{3}{2}D_2; R_3 + \frac{1}{2} = \frac{3}{2}D_3: \quad (6.2)$$

Solving (6:1) and (6.2) when $R_1 = 0; R_2 = R_3 = R$ yields $D_1 = \frac{4}{7}R; D_2 = D_3 = \frac{6}{7}R$, which is an equilibrium i¤ $\frac{D_0}{R} < \frac{4}{7}$: The cost of bailout is 0 i¤ $\frac{D_0}{R} < \frac{4}{7}$; it is D_0 ¡ $\frac{4}{7}R$ i¤ $\frac{4}{7} < \frac{D_0}{R} < \frac{6}{7}$: When $\frac{D_0}{R} > \frac{6}{7}$, the Central Bank also has to inject liquidity in the solvent banks. The total cost to the Central Bank becomes $3D_0$ ¡ $\frac{16}{7}R$. ■

7. DISCUSSIONS AND CONCLUSIONS

We have constructed a model of the banking industry where liquidity needs arise from consumers' uncertainty about where they need to consume. Our basic insight is that the interbank market allows to minimize the amount of resources held in low-return liquid assets. However, interbank links expose the system to the possibility that a number of ine¢cient outcomes arise: the excessive liquidation of productive investment as a result of coordination failures among depositors; the reduced incentive to liquidate insolvent banks because of the implicit subsidies o¤ered by the payments networks; the ine¢cient liquidation of solvent banks because of the contagion e¤ect stemming from one insolvent bank.

7.1. Policy implications

We use this rich set-up to derive a set of policy implications (summarized in Table 1) with respect to the interventions of the Financial Authorities, which we refer to as the Central Bank for short.

First, the interbank market may not yield the e¢cient allocation of resources because of possible coordination failures that may generate a "gridlock" equilibrium. The Central Bank has thus a natural coordination role to play which consists of implicitly guaranteeing the access to liquidity of individual banks. If the banking system as a whole is solvent the costs of this intervention is negligible and its distortionary effects may stem only from moral hazard issues (Proposition 3.1).

Second, if one bank is insolvent, the Central Bank faces a much more complex trade-off between efficiency and stability. Market forces will not necessarily force the closure of insolvent banks. Indeed the resiliency of the interbank market allows to cope with liquidity shocks of individual banks by providing implicit insurance and weakens market discipline (Proposition 4.1). Therefore the Central bank has the responsibility to provide ex ante monitoring of individual banks. However, the closure of insolvent banks may cause systemic repercussions (Proposition 5.1) which is the responsibility of the Central Bank to handle. In this case two courses of actions are available: orderly closure or bailout of insolvent banks. Given the interbank links, the closure of an insolvent bank must be accompanied by the provision of Central Bank liquidity to the counterparts of the closed bank.[21] This is what we called orderly closure. Assuming that this is possible, theoretically it entails no costs apart from moral hazard. However, the orderly closure might simply not be feasible for Money Center Banks (Proposition 6.1) in which case the Central Bank has not option but to bailout the insolvent institution, with the obvious moral hazard implications of the TBTF policy.

Our model can be extended in various directions some which are discussed below.

7.2. Imperfect Information on Banks' Returns

In reality, both the Central Bank and the depositors have only imperfect signals on the solvency of commercial banks (although the Central Bank' signals are hopefully more precise). Therefore, the Central Bank will have to act knowing that with some probability it will be lending to (guaranteeing the credit lines of) insolvent institutions and with some probability it will be denying credit to solvent institutions. Also, depositors may run on all the banks which have generated a bad signal.

The consequences are different depending on the structure of the interbank market. In the credit chain case, the Central Bank will have to intervene to provide

[21] For instance, in the credit chain case, if bank k is closed the Central Bank can borrow from bank $k-1$ and lend to bank $k+1$, thus allowing the interbank arrangements to function smoothly.

credit with a higher probability than in the diversi…ed lending case. Therefore in the credit chain case the Central Bank has a higher probability of ending up …nancing insolvent banks. Ex ante, therefore, the Central Bank intervention is much more expensive in the credit chain case, so that in this case a fully collateralized payments system may be preferred.

7.3. Payments among di¤erent countries

Systemic risk is often related to the spreading of …nancial crisis from one country to another. Our basic model can be extended to consider various countries instead of locations within the same country. When depositors belong to di¤erent countries, travel patterns that generate a consumption need in another location have the natural interpretation of demand of goods of other countries, i.e. import demand. Goods of the other country can be purchased through currency (like in autarchy in the basic model) or through a credit line system whereby the imports of a country are …nanced by its exports.

Our results extend to the model with di¤erent countries but the role of the monetary authority is somewhat di¤erent. While in our set-up the lending ability of the domestic monetary authority was backed by its taxation power, the lending ability of an international …nancial organization is ultimately backed by its capital. Hence the resources at its disposal are limited and in case of aggregate uncertainty its ability to guarantee banks' credit lines is limited.[22]

[22] See the role of the I.M.F. in the 1997 Asian crises and the 1998 Russian crisis.

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Appendix 1
Measures of exposure to market discipline in the case of a unique bank

In this appendix, we briefly study the properties of the optimal deposit contract in the D-D model when the proportion of early diers tends to zero. This provides a useful benchmark for measuring the exposure of the interbank system to market discipline in our multi-bank model.

Let θ denote the proportion of early diers and u be the Von Neumann Morgenstern utility function of depositors. The optimal deposit contract $(c_1; c_2)$ maximizes $\theta u(c_1) + (1 - \theta)u(c_2)$ under the constraint $\theta c_1 + (1 - \theta)c_2 / R = 1$:

Together with the budget constraint, this optimal contract is characterized by the first order condition: $u'(c_1) = Ru'(c_2)$: When θ tends to zero, it is easy to see that c_2 tends to R and that c_1 tends to $D_0 = u'^{-1}(Ru'(R))$. Since $R > 1$ and u' is decreasing, we see immediately that $D_0 < R$: Therefore if the bank is known to be solvent no depositor has interest to withdraw unilaterally before he actually needs his money.

However, suppose that bad news on the bank's assets arrive at date $t=1$: future returns are only $R_1 = \gamma R$ where $\gamma < 1$ is a public signal. Early withdrawals will rationally take place whenever $\gamma < \frac{D_0}{R}$: Thus $\frac{D_0}{R}$ gives a natural threshold of the exposure to market discipline for the bank: the higher this threshold, the more exposed the bank is. Notice that $\frac{D_0}{R}$ differs from the threshold $\frac{\alpha}{R}$. When $u(c) = c^{1-a}/(1 - a)$ (CRRA utility), $\frac{D_0}{R} = R^{-a}$; which is decreasing in R : therefore more profitable assets decrease the exposure of the bank to market discipline.

A second measure of bank exposure to market discipline is defined as the maximum proportion μ of early withdrawals that the bank can cope with when it is solvent. It is defined implicitly by

$$(1 - \mu\frac{D_0}{\alpha})R = D_0;$$

which gives

$$\mu = \alpha(\frac{1}{D_0} - \frac{1}{R}):$$

The bigger μ; the more withdrawals the bank can accommodate and thus the less exposed the bank is. When withdrawals are not costly $(\alpha = D_0)$, μ is just equal to $1 - \frac{D_0}{R}$; so that the two measures of coincide.

Appendix 2

Notation. Define

$$M(\lambda) \equiv [2I - \Pi']^{-1} = [(1+\lambda)I - \lambda T']^{-1} = \tag{7.1}$$

$$= \frac{1}{1+\lambda}\left[I - \frac{\lambda}{1+\lambda}T'\right]^{-1}$$

where I is the identity matrix. We first need a technical lemma:

Lemma 7.1. All the elements of $M(\lambda)$ are non negative: $m_{ij}(\lambda) \geq 0$ for all i, j. Moreover for all i, $\sum_j m_{ij}(\lambda) = 1$. As a consequence, if $R_i > D_0$ for all i then $M(\lambda)R > D_0 \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$.

Proof. $M(\lambda) = (2I - \Pi')^{-1}$. Since Π' is a Markov matrix (because of assumption 3), all its eigen values are in the unit disk and $M(\lambda)$ can be developed into a power series:

$$M(\lambda) = \frac{1}{2}\left(I - \frac{\Pi'}{2}\right)^{-1} = \sum_{k=0}^{+\infty} \frac{\Pi'^k}{2^{k+1}}.$$

This implies that $M(\lambda)$ has positive elements. Moreover $\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ being an eigen vector of Π' (for the eigen value 1), it is also an eigen vector for $M(\lambda)$. ■

Proof of Proposition 3.1.

(i) Because of assumption 2, $D_i = 0$ when $x_{ij} = 0$ for all j. Therefore $x^*_{ij} \equiv 0$ is always an equilibrium.

(ii) $x^j = \infty \Rightarrow X_j = \infty$. Using the assumption that $\sum_j \pi_{ji} = 1$ equation (2.4) becomes

$$2D = R + \Pi' D.$$

For $x^j = \infty$ to be an equilibrium for all j, it must be

$$D = [2I - \Pi']^{-1}R = M(\lambda)R \geq D_0 \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

This is an immediate consequence of the above lemma, which implies that $x = (1; \dots; 1)$ is always an equilibrium when all banks are solvent. There are no other equilibria when $\alpha = D_0$. Indeed if $x^i = 0$ then equation (2.4) implies that $X_i = 0$ or $D_i = R_i$. But X_i cannot be zero (unless all x^j are also zero) and $D_i = R_i > D_0$ contradicts the equilibrium condition. Notice, however, than when $\alpha < D_0; X_i$ can be zero even if some of the x^j are positive, which implies that other equilibria may exist. ■

Before establishing Proposition 4.2, we have to compute the expression of matrix $M(\lambda)$ in the two cases of credit chain and diversified lending.

Consider the credit chain case first, where the matrix T is given by:

$$T = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ & & \dots & & \\ & & & \dots & \\ 0 & \dots & & 0 & 1 \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix} \tag{7.2}$$

Therefore $T'^N = I$, so that $T'^k = T'^{k+N} = T'^{k+2N} \dots$. Now

$$M(\lambda) = \left(\frac{1}{1+\lambda}\right) \sum_{k=0}^{\infty} (\theta T')^k;$$

where $\frac{\lambda}{1+\lambda} \equiv \theta$: Let $\Theta \equiv \left\{1 + \theta + \theta^N + \theta^{2N} \dots\right\}$: Thus

$$M(\lambda) \equiv \frac{\Theta}{1+\lambda}\left[I + \theta T' + (\theta T')^2 + \dots + (\theta T')^{N-1}\right] =$$

$$= \frac{1-\theta}{1-\theta^N} A \tag{7.3}$$

where

$$A \equiv \left[I + \theta T' + \dots + (\theta T')^{N-1}\right] = \begin{pmatrix} 1 & \theta^{N-1} & \dots & \dots & \theta^2 & \theta \\ \theta & 1 & \theta^{N-1} & \dots & \dots & \theta^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 1 & \theta^{N-1} \\ \theta^{N-1} & \dots & \dots & \theta^2 & \theta & 1 \end{pmatrix} \tag{7.4}$$

Consider now the diversified lending case, where the matrix T is given by:

$$T = \frac{1}{N-1}\begin{pmatrix} 0 & 1 & \cdots & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cdots & 1 & 0 & 1 \\ 1 & \cdots & \cdots & 1 & 0 \end{pmatrix}. \tag{7.5}$$

It follows that $T = T'$. Now

$$M(\lambda) = \frac{1}{1+\lambda}\left[I - \frac{\lambda}{1+\lambda}T'\right]^{-1} = (1-\theta)\sum_{k=0}^{\infty}(\theta T')^k.$$

Notice that

$$T'^2 = \frac{1}{N-1}I + \frac{N-2}{N-1}T';$$

$$T'^3 = \frac{1}{N-1}T' + \frac{N-2}{N-1}T'^2 = \frac{1}{N-1}T' + \frac{N-2}{N-1}\left(\frac{1}{N-1}I + \frac{N-2}{N-1}T'\right).$$

Finally,

$$T'^3 = \frac{N-2}{(N-1)^2}I + \left[1 - \frac{N-2}{(N-1)^2}\right]T'.$$

Recursively we obtain

$$T'^k = \beta_k I + (1-\beta_k)T'$$

where

$$\beta_k = \frac{1}{N}\left[1 - \left(\frac{-1}{N-1}\right)^{k-1}\right] \tag{7.6}$$

Therefore

$$M(\lambda) = (1-\theta)\sum_{k=0}^{\infty}(\theta T')^k = (1-\theta)\sum_{k=0}^{\infty}\left[\theta^k\beta_k I + \theta^k(1-\beta_k)T'\right] \tag{7.7}$$

Proof of Proposition 4.2. If $R = \begin{pmatrix} 0 \\ R \\ \cdots \\ R \end{pmatrix}$ the necessary condition for $x = (1; \ldots; 1)$ to be an equilibrium becomes

$$D = M(\lambda)R = M(\lambda)\begin{pmatrix} 0 \\ R \\ \cdots \\ R \end{pmatrix} \geq D_0. \tag{7.8}$$

In the credit chain case equation $(7{:}4)$ implies that the first row of condition $(7{:}8)$ becomes

$$\frac{1-\theta}{1-\theta^N}(\theta^{N-1}+\dots+\theta)R \geq D_0$$

or

$$\frac{D_0}{R} \leq 1-\frac{1}{1+\theta+\dots+\theta^{N-1}} \equiv \gamma_N^{CRE}$$

It is easy to see that γ_N^{CRE} increases in N and in θ (and therefore in γ). Notice that $\gamma_1^{CRE}=\theta$.

Under diversified lending, $M(\lambda)$ is given by $(7{:}7)$. Checking the first row of $(7{:}8)$ and dividing by R yields

$$\frac{D_1}{R}=(1-\theta)\sum_{k=1}\theta^k\left[(1-\beta_k)\frac{N-1}{N-1}\right]\equiv\gamma_N^{DIV}\geq\frac{D_0}{R}. \tag{7.9}$$

Using

$$\beta_k=\frac{1}{N}\left[1-\left(\frac{-1}{N-1}\right)^{k-1}\right];$$

equation $(7{:}9)$ becomes

$$\gamma_N^{DIV}=(1-\theta)\sum_{k=1}\theta^k\left(1-\frac{1}{N}\left[1-\left(\frac{-1}{N-1}\right)^{k-1}\right]\right);$$

or

$$N\gamma_N^{DIV}=(1-\theta)\left[(N-1)\sum_{k=1}\theta^k+\sum_{k=1}\theta^k\left(\frac{-1}{N-1}\right)^{k-1}\right]. \tag{7.10}$$

Since

$$(1-\theta)\sum_{k=1}\theta^k=\frac{(1-\theta)\theta}{(1-\theta)}=\theta;$$

and

$$\begin{aligned}(1-\theta)\sum_{k=1}\theta^k\left(\frac{-1}{N-1}\right)^{k-1} &= \theta(1-\theta)\sum_{k=0}\theta^k\left(\frac{-1}{N-1}\right)^{k}\\ &= \frac{\theta(1-\theta)}{1+\frac{\theta}{N-1}}=\frac{(N-1)\theta(1-\theta)}{N-1+\theta};\end{aligned}$$

equation (7:10) becomes

$$N \circ^{DIV}_{N} = (N ¡ 1)µ + \frac{(N ¡ 1)µ(1 ¡ µ)}{N ¡ 1 + µ} = \frac{(N ¡ 1)µ[N ¡ 1 + µ + 1 ¡ µ]}{N ¡ 1 + µ}$$

$$\circ^{DIV}_{N} = \frac{(N ¡ 1)µ}{N ¡ 1 + µ} = \frac{1}{\frac{1}{µ} + \frac{1}{N ¡ 1}}:$$

Recalling that $µ = \frac{¸}{1+¸}$, we see that $\circ^{DIV}_{N}$ increases with ¸ and N, and that $\circ^{DIV}_{1} = µ$: ■

Proof of Proposition 4.3.Comparing $\circ^{DIV}_{N}$ and $\circ^{CRE}_{N}$ we obtain

$$\frac{\circ^{DIV}_{N}}{µ} = \frac{(N ¡ 1)}{N ¡ 1 + µ} = \frac{1}{1 + \frac{µ}{N ¡ 1}};$$

and

$$\frac{\circ^{CRE}_{N}}{µ} = \frac{1 ¡ µ^{N ¡ 1}}{1 ¡ µ^{N}} = \frac{1 + µ + µ^2 + µ^3 ::: + µ^{N ¡ 2}}{1 + µ + µ^2 + µ^3 ::: + µ^{N ¡ 1}} = \frac{1}{1 + \frac{µ^{N ¡ 1}}{1 + µ + µ^2 + µ^3 ::: + µ^{N ¡ 2}}}$$

Since $µ^{N ¡ 2} < µ^{N ¡ 3} < µ^{N ¡ 4} < :::$, then

$$\frac{µ^{N ¡ 2}}{1 + µ + µ^2 + µ^3 ::: + µ^{N ¡ 2}} < \frac{1}{N ¡ 1}: \qquad (7.11)$$

Thus

$$\frac{µ^{N ¡ 1}}{1 + µ + µ^2 + µ^3 ::: + µ^{N ¡ 2}} < \frac{µ}{N ¡ 1} \;) \; \frac{\circ^{CRE}_{N}}{µ} > \frac{\circ^{DIV}_{N}}{µ} \; ■$$