Organizational Obsolescence, Drifting Tastes, and Age Dependence in Organizational Life Chances

Gaël Le Mens
Universitat Pompeu Fabra and Barcelona School of Management, 08005 Barcelona, Spain, gael.le-mens@upf.edu

Michael T. Hannan
Stanford University, Stanford, California 94305, hannan@stanford.edu

László Pólos
Durham University, Durham DH1 3LE, United Kingdom, laszlo.polos@durham.ac.uk

Various patterns of age dependence in hazards of organizational failure have been documented: liabilities of newness, adolescence, and obsolescence. Prior efforts at providing a unified theory that can accommodate these patterns as special cases have not dealt properly with obsolescence. We tackle this problem by proposing a new model that builds on the most recent unification attempt while integrating the core intuition behind obsolescence: organizations have trouble adapting to drifting environments, which leads to declining performance and, in turn, to decreasing viability. In doing so, we develop a comprehensive representational framework to precisely characterize obsolescence. Our perspective builds on recent theory and research that treats categories as constructions by audiences. We characterize environmental drift as changing audience tastes in a multidimensional feature space and organizational inertia as a decreasing ability for producers to move quickly in that space. This combination creates obsolescence with aging. We then integrate this perspective with prior theory to make novel predictions regarding the age dependence in life chances over the life courses of organizations. We also show how the predictions of our theory can be tested empirically by adapting Levinthal’s random walk model [Levinthal DA (1991) Random walks and organizational mortality. Admin. Sci. Quart. 36(3):397–420] to incorporate the possibility of organizational obsolescence.

Keywords: organizational failure; age dependence; organizational adaptation; drifting tastes; random walk

1. Introduction
Why do organizations generally become obsolete as they age? Two key concepts lie at the core of current theoretical thinking about this question: (1) environmental drift and (2) organizational inertia (Ranger-Moore 1997, Hannan et al. 2007). The standard perspective holds that environments drift because (new) rivals introduce superior technologies, designs, and strategies (Utterback and Abernathy 1975, Tushman and Anderson 1986) and because audience tastes change (Simmel 1904), but aging organizations cannot adapt well to such changes because of increasing inertial pressures. As a result, performance starts to decline with age at some point. Organizational performance declines with further aging. This phenomenon has been documented empirically in analyses of the hazard of organizational mortality (Barnett 1990, Barron et al. 1994) and rates of innovation (Sørensen and Stuart 2000). This kind of pattern of decline in organizational performance at old age signals what is commonly called a liability of obsolescence.

This perspective on organizational obsolescence makes sense, but it suffers important limitations. First, it does not clearly specify what environmental drift means. Drift could mean that competitive conditions are changing, that audience tastes are moving away from what the organization offers, or that audience tastes are changing in an absolute way (defined independently of the offer of any specific organization); it could be a combination of these phenomena. Second, it does not specify the mechanism by which environmental drift and organizational inertia affect organizational performance. This lack of theoretical clarity has a cost: current theory can make only coarse empirical predictions, such as “the hazard of organizational failure will increase with age for old organizations,” or “technological innovations by old organizations will tend to be less significant than technological innovations by young ones.” More refined predictions, such as those concerning the timing of the onset of obsolescence, require that the analysis of organizational obsolescence be embedded in a model of organizational evolution that specifies how the relevant aspects of the organization–environment interface affect organizational outcomes such as the rate of resource accumulation and the hazard of organizational failure.

We propose a theory of organizational obsolescence that addresses these shortcomings. We build on a model of organizational evolution first proposed by Hannan et al. (2007) and further refined by Le Mens et al. (2011) (hereafter, LHP). This model proposes that organizations
obtain the resources that matter for organizational survival from the audiences that they cater to, such as potential customers, clients, employees, or political constituents. A producer obtains resources from the relevant audiences when what it offers them is more appealing than what its competitors offer. And an offer tends to be appealing to the extent that it fits the tastes of the audience. Our theory emphasizes the role of the fit between the taste of the audience and the offer of the producer. Specifically, it examines how the drift of the audience’s taste affects the appeal of the offer for the audience and, in turn, the dynamics of resource accumulation. The central idea holds that when tastes drift, an aging producer has more and more trouble adapting its offer to maintain an acceptable fit with the audience’s taste. At some point the offer becomes so inconsistent with the audience’s taste that the producer can no longer capture enough resources from its audience to overcome its fixed cost structure. When this happens, an organization’s stock of resources starts to deplete, and the hazard of organizational failure starts to increase.

To characterize what we mean by drifting taste, we build on recent theory and research on the role of categories in structuring markets (for reviews, see Hannan 2010, Negro et al. 2010). This work treats categories as constructions by audiences. Audience members sometimes label certain sets of producers/products and come to agree about what these labels mean. These shared meanings shape tastes and, therefore, the appeal of producers and their offers to the audience. We define environmental drift in terms of changes in the meanings that audience members associate with category labels.

To capture the idea that producers have more trouble adapting their offers to drifting tastes as they age, we have to revisit one feature of the model of organizational evolution proposed in LHP: the assumption that organizational performance (the rate of resource accumulation) continuously improves thanks to learning (Stinchcombe 1965). This assumption was useful for keeping the model simple and tractable, but it is too strong for our purpose here because it obviates the possibility of dealing properly with obsolescence. Besides, this assumption contradicts the best empirical evidence (Barron et al. 1994, Ranger-Moore 1997) and current thinking about the processes underlying organizational obsolescence (Hannan 1998, Carroll and Hannan 2000, Sorensen and Stuart 2000). Here, we drop the assumption that organizational performance continuously improves. Instead, we assume that age-related inertia creates increasingly more constraining barriers to adaptation. Early on, producers have a high adaptive capacity, but this capacity declines with age. We use nonmonotonic logic to integrate this new assumption with the framework developed in LHP and the predictions of that model which concerned age dependence in failure hazards for low and medium organizational ages.

Our theory differs from existing approaches to organizational obsolescence on at least two dimensions. First, our perspective focuses on intraorganizational processes rather than on selection processes that imply that older organizations are less robust to changing environmental conditions (Hannan and Freeman 1989, Hannan et al. 2004, Pelí et al. 2000). Second, we emphasize the role of changing audience tastes rather than changing competitive conditions. This distinction is important. The release of innovative products or services by new entrants can affect incumbents through two related but distinct mechanisms. First, offers by new entrants might alter the tastes of the audience, influencing their expectations regarding the characteristics of the appealing offers in the category. Second, their offers might fit existing audience tastes better than the offers of incumbents. This creates an increased competitive pressure on incumbents and older producers (Schumpeter 1934, Nelson and Winter 1982, Tushman and Anderson 1986, Christensen et al. 1998). This innovation-driven increased competitive pressure can occur even if the offers of the new entrants do not systematically alter the taste of the audience. Our theory does not focus this kind of Schumpetarian competition. It instead focuses on the effects of changing audience tastes. By contrast to existing perspectives, it does not require that the competitive pressure on incumbents increase over time. In fact, our theory makes the distinctive prediction that organizational obsolescence can emerge even if competitive conditions remain stable.

Our focus on changing tastes is all the more relevant at the time of this writing given the converging evidence that people’s tastes are changing increasingly rapidly (Lieberson 2000). Albums stay on the charts for a shorter time than they used to (Bhattacharjee et al. 2007), automakers change their models more frequently (Volpato and Stocchetti 2008), and casual observations of undergraduate students suggest that the popular social networking platforms change at a staggering speed. We do not focus on the causes of changing tastes but on their consequences. Nonetheless, this general acceleration in the pace of change suggests that our attempt at explaining organizational obsolescence on the basis of changing audience tastes will gain increasing relevance as time passes.

The next section lays out the essential elements of the theoretical framework on which we build. We also explain how we revisit and revise the assumption of continually increasing performance in the LHP model. Section 3 introduces a novel formalism to model the interface between producers and their audiences built on geometric representations of audiences’ schemas and producers’ offers in a multidimensional metric space that has enough structure to allow for the precise formulation of the dynamics of tastes and organizational adaptation. Section 4 builds on this formalism to state and derive our main obsolescence theorem: under conditions of decreasing
competitive pressure and drift in audience tastes, older organizations will be subject to positive age dependence of their failure rates. Section 5 explores some theoretical implications of this result. In particular, we develop an integration of the predictions of our obsolescence theorem with the age-dependent patterns predicted by LHP. With the use of nonmonotonic logic and specificity considerations, we can make our theory a refinement of the earlier one rather than an attack. In §6, we discuss the relations between our theory and prior empirical findings concerning the age dependence in failure hazards for old organizations. After discussing issues of level of analysis, we build on Levinthal’s (1991) random walk model and its evolution in LHP to propose a new model that allows for the possibility of organizational obsolescence. We estimate the model on a data set of American microbreweries and brewpubs. We find that brewpubs are subject to organizational obsolescence. Finally, §7 sketches some possible empirical implications of the new theory.

2. Background: Learning and Patterns of Age Dependence

2.1. Theoretical Context and Assumptions

To facilitate theoretical integration and theory building, our analysis relies on the basic analytical framework used by LHP. Specifically, we consider a generic producer that operates in an (unspecified) category and tries to capture resources controlled by members of the audience for that category. The amount of resources devoted by the audience to the category does not vary over time. The relevant audience is external to the producer. It consists of actual and potential customers; actual and potential organizational members; and, more generally, any individual, organization, or governmental agency that operates in an (unspecified) category and tries to capture resources controlled by members of the audience for that category. It is important to note that, in this framework, actual appeal depends on the actual appeal of the offer to the targeted audience.

A producer’s level of engagement increases with age (Levinthal 1991). An ample stock of (financial, material, and social) resources buffers the organization from failure, but a small stock provides little buffer causing failure hazards to be elevated. This was formalized in the following postulate.

**POSTULATE L1. (ORGANIZATIONAL CAPITAL AND THE FAILURE HAZARD).** A producer’s failure hazard, \( h_x(t) \), normally decreases with its stock of organizational capital, \( \kappa_x(t) \).

Given such negative monotonic relation between organizational capital and viability, the hazard of failure falls (rises) when the producer experiences a net inflow (outflow) of resources. Whether the hazard of failure increases or decreases with aging therefore depends on the sign of the net flow of resources from the audience.

Seeing the failure hazard as a decreasing function of the stock of organizational capital implies that, in our framework, organizational viability depends on the history of the events that affected the inflow and outflow of resources to and from the organization. As such, it recognizes that some organizational episodes can have lasting consequences. This idea is consistent with prior work on the effect of density delays on organizational viability (Carroll and Hannan 1989). For example, a decrease in organizational performance (e.g., as a result of organizational change) will have long-term effects on the trajectory of organizational capital and thus organizational viability. This is consistent with existing empirical evidence (Amburgey et al. 1993).

2.2. Learning, Engagement, and Resource Flows

The theory on which we build emphasized the effect of learning on the accumulation of organizational capital. It translated a general line of argument, initially proposed by Stinchcombe (1965), that performance increases with aging as an organization gains experience and its members learn to operate with one another and with the institutional environment.

In this framework, performance depends primarily on the actual appeal of the offer to the targeted audience. High actual appeal translates into an accumulation of organizational capital to the extent that appeal is high enough relative to the cost structure of the focal producer and the appeals of the competitors (the producers that target the same audience). It is important to note that, in this framework, actual appeal depends on two separate constructs (Hannan et al. 2007): (1) the intrinsic appeal of the offer to the target audience and (2) the level of engagement of the producer with that audience. An offer has intrinsic appeal to the extent it fits the taste of the relevant audience. However, even those offers that do fit tastes will generally not gain actual appeal unless their producers engage the audience and make their offers available in an appropriate way.

The learning argument was formalized in a postulate that claims that engagement increases with age. We restate it here informally. Appendix B give the formal rendering. Appendix A describes the most important notational symbols used in the paper.

**POSTULATE L3.** A producer’s level/quality of engagement normally rises with age.

Under these conditions, the actual appeal of an offer presumably increases with its producer’s age, as formulated in the following proposition.

**PROPOSITION L2.** The actual appeal of an offer presumably rises with the age of its producer.

This means that a producer normally gets increasingly better at addressing the audience’s demands, thanks to the positive effects of learning. But the link to the dynamics of the hazard of failure depends on actual appeal relative to competitors, called fitness in the line of theory we follow.
DEFINITION 1. An organization’s fitness, relative to the other producers in the category, is its share of the total actual appeal among the offers in the category at the unique position it targets:

\[ \varphi_x(t) = \frac{A_x(t)}{A_x(t) + C_x(t)}, \]

where \(A_x(t)\) denotes the producer \(x\)'s (actual) appeal at the focal social position and \(C_x(t)\) denotes the sum of the actual appeals of all of its competitors. (This is Definition 9.1 in Hannan et al. 2007 specialized to one social position.)

When fitness exceeds the threshold, the hazard of failure declines.

PROPOSITION L1C. A producer’s failure hazard presumably decreases with age if its fitness exceeds its cost-structure threshold \(\hat{\tau}_x\) and increases with age if its fitness falls below \(\hat{\tau}_x\).

Under specific conditions (outlined in Appendix B), Proposition L2 implies that an organization’s fitness becomes higher than the crucial cost-structure threshold and remains above it. Proposition L1C implies that the failure hazard declines with age for old organizations. In other words, the life chances of old organizations improve with age. (See Theorems L1A and L1B in Appendix B.)

2.3. Revisiting the Relation Between Engagement and Actual Appeal

The positive relationship between age and actual appeal (Proposition L2) was instrumental in keeping the LHP model simple and tractable. But it has the detrimental consequence that the model, as originally stated, cannot properly deal with obsolescence. The basic idea of obsolescence holds that performance tends to decline with age for old organizations. This happens because older organizations are subject to structural inertia that prevents them from adapting to changing environments (Hannan and Freeman 1984). Such a pattern of declining performance at older ages contradicts Proposition L2, which claims that actual appeal increases with age at all ages.

We address this contradiction by revisiting the relation between increased engagement and actual appeal (Proposition L2). We keep all the other elements of the LHP theory unchanged. A crucial feature of that argument is that it does not specify how intrinsic appeal varies with age:

We see no need here to try to model variations over time in intrinsic appeal. . . . The capability story works with increasing engagement as long as intrinsic appeal is positive for the producer according to the construction introduced by Hannan et al. (2007: Post 8.3B), which they introduced as a way to fill in gaps in arguments when the available information is partial, e.g., when the analyst knows about engagement but knows less about intrinsic appeal (namely, only that it is positive). (Le Mens et al. 2011, p. 109)

As this quote makes clear, the first stage of the theory relies on a relatively weak relationship of actual appeal with intrinsic appeal and engagement. It assumes simply that an offer gains actual appeal when its producer increases engagement. In particular, it remains silent about the role of possible changes in intrinsic appeal.

Here, we introduce a more specific relationship that will support our analysis. We propose that (1) the actual appeal of an offer equals some portion of its intrinsic appeal, and (2) the ratio of the actual appeal to intrinsic appeal normally increases with the producer’s engagement (with respect to that offer).

We represent this imagery as follows. Let \(\epsilon_x(t)\) denote a nonnegative real-valued function that records the level/quality of a producer’s engagement with respect to its current offer at the target social position, let \(g\) denote an increasing mapping from the nonnegative real numbers into the interval \([0, 1]\), and let \(\alpha_x(y, t)\) be a real-valued function that tells the (actual) appeal of the offer of the producer in the market for the category to the audience member \(y\) at time \(t\).

MEANING POSTULATE 1. Increased engagement normally raises the ratio of actual appeal to intrinsic appeal:

\[ \forall x, y \forall t \left[ (\tau_x \leq t) \rightarrow \alpha_x(y, t) = \tilde{\alpha}_x(y, t)g(\epsilon_x(t)) \right]. \]

Here and elsewhere in the paper, \(\forall\) is a nonmonotonic quantifier. Formulas quantified by \(\forall\) provide formal representations of generic rules (rules with possible exceptions). Such rules tell what is normally the case.4

This meaning postulate crucially imposes the restriction that low intrinsic appeal guarantees low actual appeal, no matter the engagement. This restriction will prove crucial to the theoretical developments delineated below: in what follows, we develop a formalism that leads to the prediction that intrinsic appeal declines with age for old organizations. This, in turn, will imply that actual appeal declines with the consequence that the hazard of failure increases with age for old organizations. Even if older organizations become increasingly better at what they do (increasing engagement), they lose touch with the audience’s tastes (declining intrinsic appeal).

In the theoretical developments that follow, we use the flexibility afforded by nonmonotonic logic to integrate the novel predictions of our model together with the predictions of the first stage of the theory. Instead of contradicting that formulation, the new theory refines it by making more specific claims. When we lack any more information about the evolution of intrinsic appeal than what was assumed in the LHP model, our theory makes the same predictions because Meaning Postulate 1 is not constraining in this case. But under more specific
conditions (i.e., when we have information about intrinsic appeal), this postulate is constraining and it allows the novel theory to make distinct predictions about the dynamics of organizational failure (see Appendix C for further discussion).

3. The Geometry of Tastes and Offerings

We now build a formal representation with enough structure to relate ideas about organizational adaptation to issues pertaining to the evolution of audience tastes. This will allow us to define precisely what we mean by obsolescence. In developing the new theory, we build partly on the framework delineated by Hannan et al. (2007) and on subsequent research.

3.1. Multidimensional Feature Space, Schemas, and Offerings

A concept is a cognitive representation. Those concepts that have social relevance are attached to labels, because we have trouble communicating about cognitive representations without (shared) labels. So we concentrate on cases in which concepts are paired with labels. For instance, if “Sushi restaurant” refers to a concept (to an American audience), then members of this audience largely agree on what it means for a restaurant to qualify as a typical instance of this category. For this example, the relevant features likely include various aspects of the menu offers and mode of service, e.g., raw fish prepared in sight of the clientele by a skilled artisan and served with specific kinds of rice served on lacquered plates in minimalist “Japanese-style” room (Carroll and Wheaton 2009). The relevant features of “beer” likely include color, transparency, several taste dimensions, country of origin, type of producer (e.g., large brewery, microbrewery), and so forth (Carroll and Swaminathan 2000).

The cognitive sciences generally use the notion of schema to represent this kind of cognitive-cultural model of meaning. A schema pertains to attributes, which can be regarded as dimensions (those that the focal audience member regards as relevant). Let \( f_n = \{f_1, f_2, \ldots, f_n\} \) be the indexed set of \( n \) relevant features for a concept, and let \( \Gamma = r_1 \times \cdots \times r_n \) denote the set of \( n \)-tuples of the values of the relevant features; \( \Gamma \) denotes the multidimensional feature space used in our analysis.

An agent’s schema for a label may be represented as a nonempty, closed, and bounded subset of \( \Gamma \) that contains exactly the patterns (\( n \)-tuples) of feature values that conform to her meaning. Let \( \mathcal{U}(\Gamma) \) denote a set of nonempty subsets of \( \Gamma \), let \( p \) denote the set of audience members at the focal social position, and let \( t \) be a set of time points. In formal terms, a schema maps labels, audience members, and time points to a nonempty subset of the Cartesian product of the ranges of the set of relevant features: \( \sigma: p \times t \to \mathcal{U}(\Gamma) \neq \emptyset \). This function takes a triplet of a label, an audience member \( y \), and a time point \( t \), and it returns \( \sigma(y, t) = s_{yt} \). Note that \( \sigma(y, t) \) is a subset—not an element—of \( \Gamma \), because a schema need not be a single vector of feature values. Social schemas often allow several combinations. For example, the schema for a family sedan will generally allow cars with four or five doors but will likely exclude cars with just two doors (or even one).

Producers frequently make multiple offers in a given market (e.g., automobile models) or a menu of options (e.g., degree programs in a university), which makes the offer a set. However, the models get very complex if we allow this kind of realism. In the interest of clarity, we restrict our attention to producers making single offers, e.g., one car model or one curriculum.

**Notation 1 (Schemas and Offerings).** We use an informal notation for schemas in the interest of simplicity. Whenever we refer to a set \( s_{yt} \), we intend that this be understood to mean that the agent \( y \) associates a schema with the focal label at the time point \( t \), and \( \sigma(y, t) = s_{yt} \). We denote the (schematized) offer of the producer \( x \) at time \( t \) as \( o_{xt} \). As we noted above, we treat the schema-relevant offer of a producer as a point in the \( n \)-dimensional space supporting the schema of the audience member: \( o_{y1t5} \in \Gamma \).

Research in cognitive psychology and sociology reveals that agents often perceive that familiar natural-language concepts have a graded internal structure: objects vary in the degree to which they fit a concept. Rosch (1973, 1975), who initiated this line of thinking, used the term *typicality* to refer to the “goodness of representation” of an object (more precisely, of its cognitive representation) as an instance of a concept. For instance, she asked experimental subjects to rate how typical “apple,” “pineapple,” and “olive” (among other types) are as “fruit.” They reported that “apple” is highly typical, “olive” is very atypical, and “pineapple” lies in a middle range.

This research and the work it inspired elicited what are called direct typicality assessments. This research demonstrated that such measures yielded regular patterns. For instance, high typicality speeds cognition and productivity (ease of recall). Some subsequent work explored the implications of recasting typicality as similarity to a concept’s prototype. As for typicality, similarity can be elicited directly (by asking respondents “how similar are A and B?” on a chosen scale) or by using explicit measures of similarity of an object’s feature values to the feature values of the prototype. Dry and Storms (2009) contrasted the predictive value of the two approaches for relevant criteria using a large number of natural-language categories for which they had collected assessments of both types of similarity. Their research shows that the feature-based approach is superior. We follow this lead and focus on fit in feature-value terms. But we recast the analysis in terms of distance, the inverse of similarity (Shepard 1987).
3.2. Distance in a Cognitive Space

The mathematical description of schemas and objects (offers in our analysis) is not yet rich enough for modeling the fit of offers to schemas and how schemas change over time. We must introduce a distance measure. The combination of the multidimensional feature space $\Gamma$ and a distance measure creates a metric space, a mathematical construct with enough structure to allow for a precise specification of the dynamics of schemas and offers.

The choice of an appropriate distance measure depends on the type of features considered by audience members when they make comparisons between their schemas and offers or between two offers. The features might be real-valued, e.g., engine displacement, horsepower, miles per gallon, and so forth. In that context, it is natural to use the Euclidean distance. Alternatively, they might be qualitative, as in the example of “Sushi restaurant.” In some cases the relevant features are binary, as in the distinction between “public university” and “private university.” In other cases, qualitative features have multiple ranges; e.g., a restaurant might serve any combination of breakfast, lunch, and dinner or a bank might offer any subset of a list of services.

Schemas for qualitative features change by elaboration of possible values of a feature (e.g., changes in technology often provide new possibilities), deletion of features that are no longer relevant, and addition of features. Such offerings can be regarded as $(n$-long) strings of discrete feature values, and schemas can be regarded as sets of such strings. A conception of distance that applies to comparisons of such strings with possibly different lengths is the Levenshtein distance. This measure counts the number of replacements, deletions, and insertions needed to transform one string to another. It satisfies the properties of identity, symmetry, and the triangle inequality (which is important in our derivations). It is therefore a metric, just like the Euclidean distance. This allows the qualitative and quantitative cases to be given a uniform treatment, as seen below.

3.2.1. Distances Between Offers and Schemas. Let $o$ denote an offer, and let $s$ denote a schema. To clarify what we mean by the distance between $o$ and $s$, we utilize the standard definition of the distance between a point and a set as the smallest distance between the point and any element of the set: $d(o, s) = \inf_{s' \in s} d(o, s')$, where $d(\cdot, \cdot)$ denotes the chosen distance metric, either Euclidean distance or Levenshtein distance in this paper.

3.2.2. Distances Between Schemas. Because a schema is a set of $n$-tuples of feature values and not just one particular $n$-tuple, characterizing the distance between schemas requires a generalized measure of distance. The directed Hausdorff distance, $D$, generalizes the distance between points to a distance between sets in a metric space (Burago et al. 2001). In particular, the directed Hausdorff distance from one set to another is a function between $\mathcal{C}(\Gamma') \times \mathcal{C}(\Gamma')$ and $\mathbb{R}^+$ such that for all $s, s' \in \mathcal{C}(\Gamma')$,

$$
\tilde{D}(s, s') = \sup_{s \in s} \{d(s, s')\}.
$$

This formula tells that the directed distance identifies the point in $s$ that lies furthest from $s'$ and computes the distance of this point from the nearest point in $s'$. Because this measure can be asymmetric, it does not satisfy the metric properties.

3.3. Distance and Intrinsic Appeal

This geometric construction can be used to draw implications about intrinsic appeal. Offerings that lie close to the audience’s schema(s) in this space can be said to fit their aesthetics and to have high intrinsic appeal. Fitting an agent’s schema provides benefits only when the agent regards the category positively. Hence we analyze situations in which the agents use the label and attach positive valuation to it. In other words, we assume that fitting the schema for a focal concept makes an offer intrinsically appealing. Instead of conditioning every formula with complicated constructions that instantiate these restrictions, we state (as an auxiliary assumption) that the focal agents associate a (shared) schema with the label at all time points and attach positive valuation to the label at all time points.

**Auxiliary Assumption 1.** The focal concept has a positive valuation for all members of the audience at all time points.

Hannan et al. (2007) built their theory on the intuition that producers/offers that fit closely to audience members’ schemas (lie close in conceptual space to the schema) are more intrinsically appealing than those that fit less well. Much of the recent sociological work on categories in markets has retained this assumption. It turns out that a vibrant line of psychological research on cognitive fluency points in the same direction (Alter and Oppenheimer 2009 provide an overview). Fluency refers to ease of perception (perceptual fluency) and ease of cognition (processing fluency). Oppenheimer and Frank (2008) found that closeness of an exemplar to a prototype increases fluency. Over a range of domains, experimental research shows that fluency, in turn, affects emotional reaction and aesthetic appeal (Reber et al. 2004) and valuation (Alter and Oppenheimer 2008). Most directly relevant to our argument is that prototypicality increases aesthetic appeal (Reber et al. 2004). So we are on firm ground in maintaining the postulate that exemplars that are close to prototype/schema are more intrinsically appealing.

With these considerations in hand, we can relate distance and the intrinsic appeal. We let $\tilde{a}(y, t)$ denote a function with values in $[0, 1]$ that tells the intrinsic appeal of the offer of producer $x$ to the audience member $y$ at time $t$ (at the unspecified social position to which the
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elapsed. This definition concerns only schemas; it does the label becomes arbitrarily large once enough time has elapsed. This definition concerns only schemas; it does not refer to any producer's offer.

4. Drift and Inertia Yield Positive Age Dependence

4.1. Drifting Tastes

We have developed this geometric representation of a space of schemas and offers to specify precisely what it means for tastes to drift. Changing tastes presumably entail changes in meanings, in schemas. What used to be pleasing about a full-fledged "automobile" no longer pleases critics and consumers, this undoubtedly signals that the meaning of "automobile" has changed. What it took to be regarded as an acceptable instance during the time of the hegemony of the Ford Motor Company’s Model T and Model A would no longer qualify. Consumers have come to expect that automobiles possess many features that they lacked in that earlier era. The now prevailing schema refers to many characteristics that were unknown previously. Tastes and associated schemas in this domain have changed over time.

Analyses of the effect of drift and inertia must consider two clocks: one records the passage of time for the audience and the category (historical time) and the other tells the time elapsed since an organization's founding (organizational age). Throughout we denote the historical clock by \( t \), and we denote the time of an organization's founding by \( \tau \). With this notation, the age of organization \( x \) at historical time \( t \) is given by \( a_x(t) = t - \tau \). We condition formulas as holding for time points beginning at time \( \tau \) and evaluate functions and predicates at various time points \( t \geq \tau \).

Although changes in tastes might be cyclical, the main case for modeling obsolescence involves what Hannan (1998) called drift. In this section, we examine the consequences of drifting tastes. We say that the taste of the members of an audience drifts over time when the distance between their current and past schemas for the label becomes arbitrarily large once enough time has elapsed. This definition concerns only schemas; it does not refer to any producer's offer.

**Definition 2 (Drift).** A category's meaning to an audience drifts if and only if the directed Hausdorff distance between the schemas of the audience members for a label at an earlier point in time and their schemas at a later point in time normally increases monotonically with the length of the interval separating the two time points:

\[
\text{drift} \iff \exists b \left( \forall y \forall t, t' \left[ (t < t') \rightarrow \text{dist}(s_{y,t}, s_{y,t'}) \geq b(t' - t) \right] \right).
\]

We recognize that audience tastes change in response to cultural trends, to demographic change, and even to changes of the offers of other producers in the category. However, we do not model how and why tastes change. Such an analysis would be interesting in itself, but it lies beyond the scope of this paper. Therefore, we will simply consider situations where tastes drift, without asking why.

4.2. Age-Related Inertia

Our basic intuition is that producers face limits on the speed of change of architectural features. Some can make extensive changes rapidly, and, as just noted, this ability likely changes over time. Adaptive capacity (over an interval) can be defined as a radius in the feature space that bounds an organization’s change during the interval. We suggest that feature values can be altered freely; producers can move in the architectural space. But the total distance of these moves cannot exceed the radius.

We build this construction by defining an upper bound on the possible distance between the characteristics of an offer at the beginning and end of the interval.

We define this notion explicitly by treating adaptive capacity as a time-varying state variable, a real-valued, positive function that records the speed with which a producer can reshape the features of its offer over a very short time interval (at the limit, of length 0):

\[ \psi: x \times t \rightarrow \mathbb{R}^+ \].

This function takes \( x \) and \( t \) and returns \( \psi_x(t) \). The following meaning postulate provides an inductive definition.

**Meaning Postulate 2.** A producer’s adaptive capacity normally creates a limit on the speed of change in its offer:

\[
\forall x \forall t, t' \left[ (\tau_x \leq t \leq t') \rightarrow d(a_{x,t}, a_{x,t'}) \leq \int_{t}^{t'} \psi_x(s) ds \right].
\]

With these preliminaries in hand, we can characterize some consequences of aging from this new perspective. Based on extensive prior theory and research (Hannan and Freeman 1984, Barron et al. 1994, Sørensen and Stuart 2000, Dobrev et al. 2001), we assume that organizations become more inert as they age. In our framework, strong inertia translates into low adaptive capacity. The fundamental postulate that characterizes the effect of aging thus claims that adaptive capacity declines with age and ultimately becomes very low as producers get old.
Postulate 2. An organization’s adaptive capacity normally declines with age and approaches the limiting value of zero at very old age:

\[
\exists x \forall t, t' \left( t < t' \rightarrow (\psi_x(t - \tau_x) > \psi_x(t' - \tau_x)) \land \lim_{(t - \tau_x) \rightarrow \infty} \psi_x(t - \tau_x) = 0 \right). 
\]

This postulate replaces the frequently made, very strong, but unrealistic assumptions about imprinting and inertia in prior work. According to the standard perspective on organizational obsolescence, organizations get preselected at the time of founding to fit to prevailing environmental conditions, but have little ability to adapt to changing conditions (Hannan and Freeman 1984). Postulate 2 improves this aspect of obsolescence explanations by assuming that organizations can adapt to changing conditions but that their adapting capacity declines with age.

4.3. Positive Age Dependence at Old Ages

Now we incorporate considerations of drift and declining adaptive capacity into the LHP model of organizational dynamics summarized in §2 and Appendix B. Doing so leads to some different conclusions. These differences come from using specificity relations to control inferences when available argument chains lead to opposing conclusions. The integration follows the basic principle of nonmonotonic logic that specificity considerations control inferences when different arguments point in different directions.

The first step in the new argument examines actual appeal. The combination of drift and declining adaptive capacity implies that the distance between the audience’s schema and the producer’s offer becomes large after some time. This and Postulate 1 in turn imply that intrinsic appeal becomes very low after some time. Recall that no amount of engagement can compensate for negligible intrinsic appeal (Meaning Postulate 1). Therefore, drift drives actual appeal to an arbitrarily low level at old age, as formulated in the following proposition.

Proposition 1. If the meaning of a (positively valued) label drifts, then the actual appeal of an organization’s offer presumably approaches the limit of zero at very old age:

\[
\exists x, y \forall t \left[ \text{drift} \rightarrow \lim_{(t - \tau_x) \rightarrow \infty} \alpha_x(y, t) = 0 \right]. 
\]

The proofs of all the new propositions and theorems can be found in Appendix D.

In classical first-order logic, the claim that actual appeal converges to zero under drift in audience tastes would stand in contradiction with the claim expressed by Proposition L2: actual appeal increases with age. By working outside the classical first-order logical environment, we can avoid this contradiction. In nonmonotonic logic, when two or more rule chains of comparable specificity have opposing implications and one chain is more specific than the rest, then this most specific chain serves as the basis for inference (Veltman 1996, Antonelli 2012). In other words, the claim represented by this most specific rule chain is proven. The premises that lead to Proposition 1 are specific to the scenarios involving drift. Therefore, the minimal rule chain leading to this corollary is more specific than the one that warrants Proposition L2.

We now want to formulate an obsolescence theorem by connecting Proposition 1 with viability. However, this proposition does not guarantee that fitness becomes low at old ages, because fitness depends on the total appeal of competitors. We follow LHP and assume that the aggregate appeal of the competitors’ offers remains constant. This assumption of stable competitive pressure will be instantiated by the use of the predicate $cs(x)$ (see Definition 5 in Appendix B). This assumption is stronger than needed for this theorem to hold. It would be enough to assume that the aggregate appeal of the competitors, $C_i(t)$, remains above some positive constant. This would guarantee that the focal firm’s fitness would fall below its cost-structure threshold at some age, which in turn would drive a downward spiral in organizational capital and a persistent rise in the hazard of failure. This weaker assumption, however, would make the integration efforts undertaken in the subsequent section much more difficult. We would need information on the time path of the aggregate appeal of competitors to reach sharp conclusions.

We analyze viability in terms of the hazard of organizational failure. Let the hazard of failure for the producer at time $t$ be denoted by $h_i(t)$. We can now state our main theoretical result.

Theorem 1 (Obsolescence Due to Categorical Drift). Given categorical drift for a positively valued category and stable competitive pressure of competitors, an organization’s hazard of failure increases with its age after some age:

\[
\exists x \exists q \forall t, t' \left[ (\tau_x < q \leq t < t') \land \text{drift} \land cs(x) \implies h_i(t) < h_i(t') \right]. 
\]

4.4. Relations to Other Explanations of Organizational Obsolescence

A distinctive feature of Theorem 1 is that it predicts the emergence of organizational obsolescence even if competitive conditions remain stable. This differs from the predictions of the literature on technological and business-processes innovations (Schumpeterian competition). This perspective emphasizes the fact that innovations by new entrants allow them to produce offers with more appeal to customers. Because incumbents have trouble adopting these innovations or coming up with appropriate responses, they become obsolete. For example,
innovations in manufacturing by Toyota, in supply chain management by Walmart, and so forth, made obsolete those firms that could not match the improvements (e.g., Sears, Kmart, General Motors, Chrysler Corporation). Similarly, in the disk drive industry, many firms that had engaged in innovative activity before the establishment of dominant design had trouble adapting their production to the new design and ultimately failed (Christensen et al. 1998). This perspective suggests that superior new rivals can use their advantages to reduce costs and improve quality, thereby increasing the appeal of their offers to the audience even if tastes do not drift.

Our theory also assumes implicitly that organizations continue to enter the population. To see why, note that the competitive stability assumption implies that, under conditions of drifting audience tastes, there are new entrants with offers that have an aggregate appeal that compensates for the decline in aggregate appeal of the incumbents. Our model does not predict the occurrence of organizational obsolescence in settings where there are no entries at all. In fact, such a setting would be incompatible with our assumption of competitive stability.10 But, in contrast to the perspective that relies on Schumpeterian competition, our theory does not assume that there is true innovation or technological progress. Even if the offers of the new entrants are not more appealing than the offers of the incumbents at the time they entered, organizational obsolescence will nevertheless occur under conditions of drifting tastes.

Despite these differences, our model can easily be adapted to incorporate the idea of Schumpeterian competition. Innovations by new entrants can be captured by giving up the assumption of competitive stability and adjusting the aggregate appeal of competitors. If innovators enter the population, then $C_x(t)$ increases. Then Definition 1 implies that organizational fitness, $\phi_x(t)$, decreases. This has the effect of accelerating the trajectory of the fitness level to the point at which it will go below and stay below the cost-structure threshold. Now, consider a continuous flow of innovations by new entrants so that $C_x(t)$ increases steadily. In this case, fitness will ultimately decline. And this will happen even if tastes do not drift. This discussion shows that the alternative mechanisms operate at different levels of analysis. Whereas drifting tastes negatively affect appeal to the audience and thus the numerator of the fitness, Schumpeterian competition mainly affects the fitness of the producer through an increase of the denominator in the ratio that defines fitness (see Definition 1).

Our emphasis on the role of the audience does not preclude the possibility that innovations by competitors also have an indirect effect where they affect the tastes of the audience. For example, the iPhone and iOS ecosystem have contributed to durably and profoundly changing customers’ expectations regarding what a phone should be. Such events could be integrated in our theoretical framework by positing that tastes drift in the direction of the new offers by competitors. The process by which new offers affect the audience’s tastes, however, is complex, and this influence is highly unpredictable. Therefore, we leave the attempt at modeling this mechanism for future research. More generally, although the mechanism that invokes increasing competition resulting from innovations by other producers certainly plays a role in many empirical settings, our theory points to a complementary mechanism that implies that organizational obsolescence can also occur in settings where that explanation does not apply, but audience tastes are nevertheless drifting because of general cultural changes and the seemingly stochastic character of fads and fashions.

Our theory also differs from explanations that rely on selection processes (Hannan and Freeman 1989, Péli et al. 2000, Hannan et al. 2004). These theories proposed that selection forces favor organizations that are well adapted to the conditions prevailing at founding and that have reliable production processes. Selection leads to the survival of reliable producers, which are those who have trouble adapting to changing conditions. The very reliability that once gave a survival advantage becomes a liability when the environment changes. Overall, this implies that older organizations have a higher failure hazard. Our explanation is very different, because it emphasizes intraorganizational processes rather than population-level processes of selection. We return to this issue of levels of analysis in §6.

5. Theoretical Implications

Now we integrate our obsolescence argument with the predictions of the model in LHP regarding age dependence in failure hazards over lifetimes of organizations. In effect, that model implies a liability of newness (Carroll and Delacroix 1982, Carroll 1983, Freeman et al. 1983), a liability of adolescence (Carroll and Huo 1988, Brüderl and Schüssler 1990, Fichman and Levinthal 1991), or a liability of aging, depending on the conditions prevailing at time of founding. As discussed in §1, however, that model could not deal with obsolescence because it assumed that organizational performance systematically increased over time. Here, we show how the obsolescence theorem (Theorem 1) amends the patterns predicted by the earlier theory stage.

5.1. Onset of Obsolescence

Prior attempts at theoretical integration postulated a priori the existence of qualitatively different age periods—marked by a common age of the onset of obsolescence across organizations in the population—with distinct dynamics (Hannan 1998; Pólos and Hannan 2002, 2004; Hannan et al. 2007). By contrast, we need not make such an assumption. As noted in §1, our theory conceptualizes and models the underlying processes as varying continuously over the organizational life course. Because the
new theory operates strictly at the organizational level, it can accommodate heterogeneity among producers in the population.

In our framework, the speed at which inertial forces come into play depends on the speed of drift and the organization’s adaptive capacity (its initial level and the rate of decay). But how can we define the onset of obsolescence? Our answer compares the speeds of adaptation and drift. Once adaptation speed falls below the drift speed and stays there, the producer has lost its alignment with the audience. Then it is only a matter of time until the intrinsic appeal of its offer starts to fall, followed by the decline of actual appeal, fitness, and finally organizational capital and the corresponding rise in the failure hazard.

**Definition 3 (Onset of Obsolescence).** The age of onset of obsolescence for the producer \( x \) is the minimal time, \( \omega_x \), such that the speed of drift in taste exceeds its adaptive capacity for all \( t > \omega_x \):

\[
\omega_x = \inf \{ q \mid (\tau_x \leq q \leq t) \rightarrow \psi_x(t) < \beta \}.
\]

Definition 2 and Postulate 2 imply that \( \omega_x \) is well defined and finite under conditions of drift. With this definition we know what inevitably comes after adaptive capacity becomes lower than the speed of drift as a consequence of increasing inertial constrains. This definition has the potential weakness of not telling exactly when these misfortunes materialize.

An alternative definition would set the time of the onset of obsolescence to the beginning of the period of monotonically increasing failure hazard. Such a definition is sharper, but it loses the desirable feature of expressing inevitability. Moreover, it would make the onset of obsolescence contingent on the level of competitive pressure exerted by other producers. This would be inconsistent with our desire to emphasize the role of the audience in explaining organizational obsolescence. This reasoning motivates our choice of Definition 3.

This specific definition allows us to draw an interesting connection with the theory of Red Queen evolution (Barnett and Hansen 1996, Barnett and Sorenson 2002). On the basis of (mostly) a selection-based argument, this theory makes the prediction that organizations that have survived competition have evolved routines that make them adapted to their competitive environment. This makes them subject to a “competency trap” that renders them all the more vulnerable to changes in environmental conditions in future periods. Adapted to our setting, this theory predicts that organizations that have survived a history of competition will tend to have a lower adaptive capacity than organizations that did not go through that kind of episode (Barnett and Pontikes 2008). This implies that the onset of obsolescence as defined above will likely occur earlier for those organizations that survived a history of intense competition. It is difficult, however, to formulate a clear prediction about the beginning of the period of the monotonically increasing failure hazard, because of complications induced by selection. Survivors of competitive episodes are likely to have offers that are more appealing, and thus closer to the audience’s schema, than producers of a similar age that would have somehow avoided the competitive pressure. This implies it might take a while for tastes to drift so far away from the offers of survivors of a history of competition that their failure hazard increases. All in all, the beginning of the period of monotonically increasing failure hazards does not have to occur earlier for survivors of a history of competition than for other producers, despite the clear prediction in that direction regarding the onset of obsolescence.

### 5.2. Theoretical Integration: Age Dependence in Failure Hazards

We naturally want to integrate the obsolescence theorem together with the LHP predictions regarding age dependence. However, we lack the knowledge required to do so in a completely general way. The full set of premises gives rise to lines of argument leading to opposing conclusions even for the period before obsolescence, and these opposing arguments do not have a clear specificity order. Two issues need to be addressed to understand the theoretical implications of this apparent impasse.

First, before an audience’s taste drifts beyond what a producer’s adaptive capacity can accommodate, a drifting schema might move toward the offer, before drifting away. Our construction does not rule out the possibility that a producer adapts its offer to “track” the drift; it also does not require this. This feature of our modeling strategy rules out a claim that intrinsic appeal necessarily does not decrease over the early life course. This means that we cannot derive predictions about the complete time paths of fitness and organizational capital.

Second, even if we introduce a resolution to the first problem (as we will do below), we face another complication. According to the first stage of the theory, it makes a crucial difference whether long-run fitness surpasses the cost-structure threshold \( (f_\tau) \). In that context, the long-run view appears justified because the argument implies that actual appeal increases with age up to a limit (on the basis of increasing engagement, given that the variations of intrinsic appeal are unspecified in that theory stage). Bringing obsolescence and organizational inertia into the picture makes this long-run approach uninformative. This is because intrinsic appeal approaches zero in the limit, which means that actual appeal also falls to zero in the limit. And it makes a decisive difference in the time path of capital (and thus of the failure hazard) whether engagement had time to increase to a level high enough to bring fitness above the threshold before obsolescence kicks in.

These uncertainties arise because we made very weak assumptions about the functional relations between the
constructs of the theory. (This is because we believe that the current state of knowledge does not support more precise assumptions about the matters under study.) Lacking assumptions about functional relations and associated parameters, we cannot make predictions about the predicted time paths of change. Nonetheless, we can make progress and gain new predictions by narrowing the scope of the argument. We do so in a way that identifies a condition that, if satisfied, allows the two theory stages to be integrated.

The condition that separates the cases concerns the alignment of the focal offer with the drifting schemas. We introduce a predicate that tells that a producer tracks changing tastes over the period before obsolescence gains sway, and we invoke this predicate in the antecedents of formulas to limit the scope of the argument to this well-behaved case.

**Definition 4 (Alignment).** A producer experiences preobsolescence alignment with the relevant audience if and only if the distance between its offer and the schemas of audience members does not increase with age before the onset of obsolescence:

\[
\text{AL}(x) \leftrightarrow \forall t, t', y \left[ (\tau_x \leq t < t' \leq \omega_x) \rightarrow \bar{d}(a_{st}, s_{yt}) \geq \bar{d}(a_{st'}, s_{yt'}) \right].
\]

With the assumption of alignment and increasing engagement as a result of organizational learning, we can show that fitness rises over the early life course. Depending on the conditions at founding, this leads to different patterns of age dependence. The following theorems summarize our theoretical integration. Specifically, they integrate the predictions about early aging (consistent with Theorem L1) and about old age (consistent with Theorem L1A) under the constrained scenario involving preobsolescence alignment by relying on the integrative capability afforded by nonmonotonic logic (and specificity considerations).

Unfortunately, none of these theorems makes predictions for the full lifetimes of organizations. The gap in predictions concerns a period immediately following the onset of obsolescence, for an age interval \([\omega_x, \omega_x + q]\), where \(q\) is a positive constant. Fitness can still increase after the onset of obsolescence, even though it will ultimately decline. This is because actual appeal can still increase for a time after \(\omega_x\) if engagement rises fast enough to offset the initial decline in intrinsic appeal. Also, we cannot show that intrinsic appeal declines for sure after the onset of obsolescence due to complexities associated to the fact that schemas are sets—not points—in the sociocultural space.

In what follows, we use a shorthand for the scope conditions of drift, initial alignment, and stable competitive pressure. We let \(\Theta_x \leftrightarrow \text{DRIFT} \land \text{AL}(x) \land \text{CS}(x)\).

Consider first the scenario with fitness at founding above the threshold (in parallel with Theorem L1A). Introducing considerations of drift yields a pattern not seen in the first theory stage: U-shaped age dependence in the hazard.

**Theorem 2 (Constrained Unification: Case A).** If a producer’s fitness at founding exceeds the threshold, then its hazard of failure presumably initially decreases with age up to a point and then (perhaps after some gap) increases with age:

\[
\forall x \exists q_4^x \forall t_1, t_2, t_3, t_4 \left[ \Theta_x \land (\varphi_x(t_x) > t_x) \land (\tau_x \leq t_1 < t_2 \leq \omega_x \leq q_4^x < t_3 < t_4) \rightarrow (h_x(t_1) > h_x(t_2)) \land (h_x(t_3) < h_x(t_4)) \right].
\]

Next consider what happens when fitness at founding lies below the threshold. Recall that Theorem L1 holds that long-run fitness relative to the threshold decisively shapes the pattern of age dependence in failure hazards. The first theory stage, which warrants that provisional theorem, does not consider variations in intrinsic appeal, and it assumes that that engagement rises with age. Because this construction implies that actual appeal increases with age up to a limit, it is natural to define long-run fitness as a limiting construction in that setting. In the current theory stage, such a limiting construction does not work, as we noted above. With drift and declining adaptive capacity in the picture, fitness falls in the long run for all producers. So we must proceed in another way.

Our analysis reveals that we can consider two distinct cases depending on fitness at the onset of obsolescence, \(\varphi_x(\omega_x)\). We first consider a producer whose fitness begins below the threshold but then surpasses it before obsolescence rules, which parallels the case addressed by Theorem L1B.

**Theorem 3 (Constrained Unification: Case B).** If a producer’s fitness at founding lies below the threshold but its fitness at the onset of obsolescence exceeds it, then its hazard of failure first increases with age to a point, then declines with age until the onset of obsolescence, and then (perhaps after some gap) increases with further aging:

\[
\forall x \exists q_5^x \exists q_6^x \forall t_1, t_2, t_3, t_4, t_5, t_6 \left[ \Theta_x \land (\varphi_x(t_x) < t_x) \land (\varphi_x(\omega_x) > t_x) \land (\tau_x \leq t_1 < t_2 \leq q_5^x < t_3 < t_4 \leq q_6^x < t_5 < t_6) \rightarrow (h_x(t_1) < h_x(t_2)) \land (h_x(t_3) > h_x(t_4)) \land (h_x(t_5) < h_x(t_6)) \right].
\]

In the final case, fitness never passes the threshold. Introducing considerations of drift and declining adaptive capacity yields a weaker version of the prediction from Theorem L1C: the hazard of failure increases with age during a youthful period and in old age, but there is no prediction for some intermediate age range.

**Theorem 4 (Constrained Unification: Case C).** If both fitness at founding and fitness at the onset of obsolescence lie below the fitness threshold, then the
hazard of failure presumably rises with age over the early age and over a later age range:

\[ \exists x \ (q_x^C \forall t_1, t_2, t_3, t_4 \\bigg[ (\Theta_x \land (\varphi_x(\tau_x) < \tau_x) \land (\varphi_x(\omega_x) < \omega_x) \land (\tau_x \leq t_1 < t_2 \leq \omega_x \leq q_x^C \leq t_3 < t_4) \rightarrow (h_x(t_1) < h_x(t_2)) \land (h_x(t_3) < h_x(t_4)) \bigg] \].

Figure 1 provides a summary of the patterns predicted by the three theorems that form part of the theoretical integration. As illustrated in the left panel, Theorem 2 predicts that when initial fitness is high (higher than the threshold), there is a liability of newness followed by a liability of obsolescence. Theorems 3 and 4 deal with the cases where initial fitness is low. When performance has time to increase to a level high enough before obsolescence kicks in, there is a liability of adolescence followed by a liability of obsolescence. Finally, if obsolescence kicks in before the producer gains a positive rate of resource accumulation, we have a liability of aging.

6. Relations to Prior Empirical Analyses of Organizational Obsolescence

6.1. Levels of Analysis and Empirical Estimation of Age Dependence

How does the new theory relate to prior empirical studies? This question is hard to answer because of the difference in levels of analysis. As discussed above, the new theory operates at the level of the individual organization. But the pattern of age dependence that will prevail in a given organizational population depends on both the individual-level age-dependent processes and the population-level selection forces that likely vary over time.

The standard empirical estimation strategy of prior studies does not reliably account for selection. This standard approach relies on the estimation of proportional hazard models that include effects of one or several age terms (or, possibly, constants for various age ranges). The estimated coefficients for the age terms imply a pattern of age dependence in the population. But the pattern of age dependence obtained by this approach does not have to be the pattern of age dependence that operates at the level of the individual organization. This problem was first
pointed out by Levinthal (1991). He demonstrated that standard analysis of a population of organizations that are not subject to any systematic organizational-level age-dependent processes can yield systematic age-dependent patterns (such as a liability of newness or a liability of adolescence) at the level of the population.

Levinthal (1991) proposed a solution to this selection problem by modeling organizational evolution with a random walk with an absorbing barrier. In his model, the stock of organizational capital follows a random walk. The parameters of the random walk model capture the organizational-level dynamics of organizational capital. More precisely, the “drift” of the random walk (not to be confused with the environmental drift that plays a crucial role in the new theory) directly corresponds to the rate of resource accumulation by the focal organization.

Moreover, Levinthal (1991) empirically estimated the “drift” of the random walk on a population data set and demonstrated the age-dependent pattern implied by the estimated parameters can be incompatible with the age-dependent pattern implied by estimations of a standard hazard rate model. More precisely, Levinthal’s estimations of a random walk suggest a liability of aging, whereas estimations of a Makeham model (a type of hazard rate model) suggest a liability of newness; that is, the pattern of age dependence obtained after properly accounting for the effect of selection was opposite in this case to the pattern of age dependence obtained by the standard approach. (See LHP for a more detailed discussion.)

This discussion of selection and its impact on the estimation of hazard models might cast some light on the empirical findings regarding the age dependence in failure hazards for old organizations and its interactions with organizational size (for reviews, see Baum 1996, Carroll and Hannan 2000). Although the findings vary across studies, an overall picture emerges from the reviews just cited. The age dependence of failure hazards for old organizations tends to be negative when the analysis does not control for age-varying organizational size. Once such controls are added, the estimated effect of age on the hazard of mortality tends to change from negative to positive. There is also evidence that a larger size tends to be associated with a lower failure hazard. A possible interpretation of this pattern, in light of the foregoing discussion, is that including organizational size in the regression equation provides an approximate way of controlling for selection. If smaller organizations tend to fail early (because of their low level of organizational capital), the average organizational size in the population will increase over time. When this is the case, this almost surely implies that the average level of organizational capital in the population will increase, even if it is decreasing for every organization because of organizational obsolescence. In terms of hazard model estimations in settings where obsolescence processes operate, coefficient estimates will likely suggest a negative age dependence in failure hazards when the analysis does not control for age-varying size but a positive age dependence when it does.

In what follows, we extend Levinthal’s (1991) estimation strategy and demonstrate how we can estimate the parameters of a random walk model that incorporates the possibility of organizational obsolescence.

6.2. Empirical Illustration Using a Random Walk Model

In LHP, we built on Levinthal’s (1991) random walk model to propose a more sophisticated model that could account for time-varying organizational performance (his original model could not). We estimated the parameters of this model on the data on microbreweries and brewpubs assembled by Carroll and Swaminathan (2000). Here, we go one step further and adapt the random walk model used by LHP to incorporate the possibility of organizational obsolescence. We estimate the new model on the same microbrewery and brewpub data. The estimations in LHP will provide a baseline for comparison.

The model on which we build assumes that organizational capital can be modeled as a random walk. The stock of organizational capital of producer \( x \) at the end of period \( t \), \( \kappa_x(t) \), is defined recursively as follows:

\[
\kappa_x(t) = \kappa_x(t-1) + \mu_x(t) + \eta_x(t), \quad \eta_x(t) \sim N(0, \sigma^2).
\]

The capital at the end of a period equals the capital at the start of the period plus the flow of resources, which, in turn, is the sum of the systematic component of organizational performance, \( \mu_x(t) \), and of a random component, \( \eta_x(t) \), that follows a normal distribution with mean 0 and standard deviation 1. A producer fails when its stock of organizational capital reaches the absorbing barrier (arbitrarily set at 0). In terms of the constructs of our theory, \( \mu_x(t) \) is positive when the fitness level of producer \( x \) at time \( t \), \( \varphi_x(t) \), exceeds the fitness threshold \( f_x \). LHP modeled the dynamics of organizational performance by assuming that \( \mu_x(t) \) could monotonically increase (decrease) up (down) to a limiting value:

\[
\mu_x(t) = \mu_x(\tau_x) + \beta \arctan(\gamma(t - \tau_x)), \quad t \geq \tau_x, \tag{2}
\]

where \( \mu_x(\tau_x) \) is the (systematic component of) organizational performance at founding. \( \beta \) is such that \( \mu_x(\tau_x) + \beta \pi/2 \) is an upper limit on \( \mu_x(t) \), and \( \gamma \) is the rate of systematic performance improvement. In the first theory stage, \( \mu_x(t) \) is increasing over time. In terms of Equation (2), this amounts to assuming \( \beta > 0 \) and \( \gamma > 0 \).

We now modify this equation to account for the possibility of organizational obsolescence. The obsolescence theorem (Theorem 1) characterizes obsolescence in terms of a mortality hazard increasing with age. In the random walk framework, this occurs when \( \mu_x(t) \) is negative;
see the proof of Theorem 1 for details. So we rewrite the systematic component of organizational performance as follows:

\[
\mu_x(t) = \mu_x(t_s) + (\beta + \Omega t) \arctan(\gamma(t - t_s)),
\]

\(t \geq t_s\), (3)

where \(\mu_x(t_s) + (\beta + \Omega t_s) \pi/2\) is a time-varying upper bound on the systematic component of organizational performance. In cases where organizations are subject to organizational obsolescence, we should find that \(\Omega\) is negative. When this happens, \(\mu_x(t)\) becomes negative when \(t\) becomes large. It is worth noting that if \(\Omega\) is positive, organizational performance can increase indefinitely. Our model can thus accommodate cases where organizational obsolescence does not occur.

We estimate a version of this model that includes controls for unobserved heterogeneity using the simulated maximum likelihood technique outlined in LHP.\(^1\) The results are reported in Table 1. Columns (1) and (3) report the estimation results for the baseline model that does not allow for obsolescence (see Equation (2)). According to the earlier findings, both microbreweries and brewpubs were subject to a liability of adolescence: the rate of resource accumulation is initially negative and becomes positive after some time. In terms of mortality hazards, the mortality hazard increases initially and then decreases. Columns (2) and (4) report the estimation results for the new model (see Equation (3)). For the microbreweries, the results concerning obsolescence are inconclusive. In this case the estimated value for \(\Omega\) is very close to zero, and the standard error is too large to allow us to derive any clear conclusion about the occurrence of obsolescence. For brewpubs, however, the results are clear: The coefficient for \(\Omega\) is negative, and the new model provides a better fit than the model that did not allow for the possibility of obsolescence. This suggests that brewpubs are subject to organizational obsolescence. This result is not surprising: brewpubs in our data operated mostly between 1980 and 1997, a period of profound changes in beer consumption habits (and thus, likely, underlying taste) by American consumers (for detailed discussion, see Carroll and Swaminathan 2000).

Our parameter estimates suggest that the rate of resource accumulation becomes positive after 3.9 years and becomes negative again after 12.6 years; that is, in terms of our unification theorems, brewpubs fall in Case B (Theorem 3), and we have \(q_0^R = 3.9\) years and \(d_0^R = 12.6\) years. In terms of failure hazards, we thus have a liability of adolescence followed by a liability of obsolescence. Interestingly, we find that brewpubs start to accumulate resources earlier than what was suggested by the analyses that do not allow for the possibility of obsolescence (column (3) suggests that this occurs after 4.6 years). This suggests that appropriately accounting for the possibility of organizational obsolescence has the potential to improve the estimation of parameters that also characterize organizational dynamics in earlier stages of organizational life.

### Table 1: Maximum Simulated Likelihood Estimates of Random Walk Models of Organizational Failure of American Brewpubs and Microbreweries

<table>
<thead>
<tr>
<th></th>
<th>Microbreweries ((N = 553))</th>
<th>Brewpubs ((N = 929))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) LHP</td>
<td>(2) Obsolescence</td>
</tr>
<tr>
<td>Heterogeneity:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial capital: (\kappa_x(t_s))</td>
<td>5.74* (0.05)</td>
<td>5.72* (0.06)</td>
</tr>
<tr>
<td>(\mu_x(t_s))</td>
<td>−3.00* (0.02)</td>
<td>−2.99* (0.06)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>2.10* (0.21)</td>
<td>2.07* (0.24)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>1.99* (0.01)</td>
<td>1.98* (0.06)</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>0.02 (6.77)</td>
<td>−0.02 (6.77)</td>
</tr>
<tr>
<td>Heterogeneity: (k)</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>0.52 (1.24)</td>
<td>0.52 (1.24)</td>
</tr>
<tr>
<td>(\ln L)</td>
<td>−434.0 (1.24)</td>
<td>−434.0 (1.24)</td>
</tr>
<tr>
<td>BIC</td>
<td>899.5 (1.24)</td>
<td>909.8 (1.24)</td>
</tr>
</tbody>
</table>

*Notes. Standard errors are in parentheses. For the heterogeneity parameter, \(k\), the null hypothesis is \(k = 1\). BIJC stands for Bayesian Information Criterion.

\(p < 0.01\) (one-sided \(t\)-tests).
whether a producer will fall in Case A or Case B depends on founding conditions. If initial fitness is higher than the cost-structure threshold, our theory predicts the dynamics of Case A. Otherwise, it predicts the dynamics of Case B. Although organizational fitness, as we defined it, is very difficult to measure empirically, empirical tests of our theory might rely on more easily measurable constructs such as the intrinsic appeal of the offer of the focal firm or the competitive pressure by other producers. It might also attempt to measure factors likely to affect the appeal of the early offer (and the cost structure) and the composition of the founding team (see LHP for a discussion of this idea and its relations to empirical research on founding teams and entrepreneurial success). Column (1) in Table 1 summarizes the predicted effect of the various constructs of our model as implied by the relations between these constructs.

A potentially promising approach would try to measure cost-structure parameters for groups of organizations in a population that plausibly are founded with similar levels of initial fitness (and thus with offers that have similar levels of appeal to the audience). For instance, consider subpopulations of exploiters and explorers (March 1991). Explorers divert some resources to experimentation, which imposes additional costs without leading to a corresponding increase in appeal at the time of exploration. So explorers will tend to have higher cost structures than exploiters that have similarly appealing offers. Although it is not part of the theory we proposed, standard arguments claim that exploration can yield the benefit of higher adaptive capacity.

What happens in this kind of application? Figure 1 helps answer this question. Explorers are more likely to have an initial fitness above their cost-structure parameter and thus to display over the early life course the pattern claimed by Theorem 2: declining mortality hazards before the onset of obsolescence. Because explorers are likely to have an initial fitness below their cost-structure threshold, however, the early pattern for explorers would be that given by Theorem 3 (for successful exploration within that age interval) or Theorem 4 (for not-yet-successful exploration).

Another approach would measure the competitive pressure at founding, as in existing empirical tests of the density delay hypothesis (Carroll and Hannan 1989). One could analyze populations of organizations characterized by local competition (such as restaurants, brewpubs, or garment shops) and select empirical settings that vary in terms of local density at founding. Our theory predicts that Case A is more likely when the local density at founding is low, whereas Case B is more likely when the local density at founding is high.

Although the possible testing strategies are very similar to those relevant to testing the predictions of the first theory stage (see LHP), the theory of the current paper adds one more dimension: drift in tastes. In settings where we know that tastes are drifting (as in many technological settings and industries driven by fads and fashions), the predicted patterns are those of this paper. In settings where tastes are not (or very slowly) drifting, the predictions are those of LHP. In these settings, obsolescence will be less likely (e.g., churches, bakeries in European cities, local bank branches).

7.2. Timing of Monotonically Increasing Failure Hazards

The second set of empirical predictions made by our model concerns the age at which an organization’s failure hazard starts its monotonic increase (\( q^*_C \)), which is most relevant in Cases A (Theorem 2) and B (Theorem 3). In our framework, this happens when fitness goes from above to below the cost-structure threshold and remains there. How fast does this happen over an organization’s lifetime? Although it is difficult to make formal predictions about this because of the period of indeterminacy between \( \omega_x \),

<table>
<thead>
<tr>
<th>Construct</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial fitness ( \psi_x (\tau_x) )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Initial actual appeal ( A_x (\tau_x) )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Initial intrinsic appeal ( \overline{A}_x(\tau_x) )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Aggregate actual appeal of competitors ( C_x )</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Cost-structure parameter ( f_x )</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Initial level of adaptive capacity ( \psi_x (\tau_x) )</td>
<td>NA</td>
<td>Later</td>
<td>+</td>
</tr>
<tr>
<td>Rate of decrease in adaptive capacity</td>
<td>NA</td>
<td>Later</td>
<td>+</td>
</tr>
<tr>
<td>( -\psi_x (t)/\delta t )</td>
<td>NA</td>
<td>Later</td>
<td>+</td>
</tr>
<tr>
<td>Rate of increase in engagement ( \delta x/\delta t )</td>
<td>NA</td>
<td>Earlier</td>
<td>–</td>
</tr>
</tbody>
</table>

* Column (1) shows the effect on the likelihood of Case A versus Case B or C; column (2) the effect on the timing of monotonic increasing hazard given Case A or B; and column (3) the effect on the likelihood of Case B given that Case B or C prevails.
and $q_c$, our model can be used to develop an intuition about the effect of the various constructs. Two classes of factors matter: the conditions at founding and the subsequent organizational evolution. Regarding the former, the model implies that what matters is an organization’s initial fitness level relative to its cost-structure threshold. Obsolescence will thus occur at later ages in settings where producers have a low cost structure, their offers have a high initial appeal, and they face little competition (low level of aggregate appeal of the competitors). A higher initial fitness has the effect of moving the fitness curve up in the graphs of Figure 1, which implies that $q_c$ moves to the right (to a later age).

Regarding the features of organizational evolution and adaptation, our theory suggests that obsolescence will occur at a later age in settings where the learning curve is steep (the producer improves its engagement quickly), when the adaptive capacity is high or decreases slowly, or when the drift in tastes is slow. In terms of the graphs of Figure 1, a steep learning curve implies that actual appeal will quickly reach the level of intrinsic appeal (which, in our model, provides a upper bound on the level of actual appeal). A high initial adaptive capacity or slowly decreasing adaptive capacity and a slow drift imply that the decreasing part of the intrinsic appeal curve is less steep. This, in turns, implies a slower decline of the fitness level.

Again, a number of these constructs are difficult or impossible to measure directly. Nonetheless, investigators could rely on proxies to test these predictions empirically. For example, a novel prediction from the current theory is that the age of the onset of monotonically rising mortality hazards ($q_c$) will generally occur later for successful explorers because of their (presumed) higher adaptive capacity. The decline in intrinsic appeal (and, later, of actual appeal) will be shifted to the right in these subfigures. This could be tested in a within-population design where parameter estimates would be estimated separately for subpopulations of exploiters and explorers.

To test for the prediction regarding the effect of the speed of drift, one could look for variations of that construct (relative to the typical speeds of adaptive capacity) in two or more populations. One possibility would be to rely on population properties alone to make the distinctions. For instance, one might expect that the taste of the relevant audience for organizational populations whose offers have a strong component of fad or fashion (e.g., popular music, teenage clothing) would have higher-velocity drift than those whose offers are highly institutionalized (e.g., law, higher education). Alternatively, one might try to obtain measures of the speed of drift in schemas. Schemas are generally tied to labels, and turnover in labels for product classes can be measured (for instance, Carroll and Wheaton 2009 documented the turnover in genre labels applied to high-quality restaurants in Chicago over several decades). So turnover in such labels ought to be roughly proportional to the speed of drift in schemas.

Finally, to test the predictions regarding differences in the evolution of adaptive capacity, one could look at differences among organizations in turnover rates of personnel. In a follow-up paper, we develop an argument that proposes that the level of adaptive capacity is driven by the distribution of the tenure of organizational members. And when there are more long-tenured members, adaptive capacity is posited to be lower than when there are few long-tenured employees because of processes of institutionalization (Le Mens et al. 2013). One could classify the organizations in a population by turnover rate and check whether the onset of monotonically rising mortality hazards occurs at later ages for low-turnover producers.

### 7.3. Liability of Aging

The final set of empirical predictions concerns the likelihood of finding a liability of adolescence followed by a liability of obsolescence or a liability of aging. These two patterns correspond to Case B (Theorem 3) and Case C (Theorem 4). Our theory predicts that a liability of aging occurs when fitness starts low (below the cost-structure threshold) and obsolescence kicks in before fitness has had time to reach a level high enough so that the producer starts accumulating resources and building its organizational capital. From the perspective of our theory, this is a case where obsolescence starts at a very early age. Hence all the factors that lead to an earlier onset of monotonically rising hazards in the setting of the previous subsection will also increase the likelihood of a liability of aging. These include parameters that affect the conditions at founding as well as organizational evolution. Regarding the conditions at founding, a low initial fitness relative to the cost-structure threshold is the main factor that will increase the likelihood of liability of aging. This setting is more likely to occur when the initial appeal of the offer is low and when the competitive pressure is high (i.e., when the aggregate actual appeal of competitors is high). In more concrete terms, we expect firms that start with a poor product positioning in an environment with intense competitive pressure to show a liability of aging. Regarding the parameters controlling organizational evolution, we expect that a liability of aging will be more likely when the speed of drift is high, the rate of increase in engagement is slow, the initial adaptive capacity is low and its rate of decrease is high. We therefore expect that a liability of aging will be more likely to be observed in particularly turbulent industries that are also characterized by high fixed costs and high investments in specialized knowledge that potentially impede adaptation. Some industries that seem to fit these conditions particularly well are video-game design and production, semiconductor/microprocessor production, and hospitals.
8. Conclusion
Our formulation of the interface between producers and audience members in a multidimensional feature space allows us to specify precisely what it means for tastes to drift and for a producer to lose the ability to adapt. These ideas have received only a sketchy treatment in earlier work, which limited the potential for clear empirical predictions.

Of course, a price must be paid for gaining such precision. We had to limit the analysis in a number of ways. We treat the case of drifting tastes at a single homogeneous social position. An obvious next step would be to generalize the model to allow heterogeneity on the audience side, to take multiple positions into account. Our model can, when suitably adapted, precisely identify the conditions favoring growing homogeneity of tastes within an audience (Koçak et al. 2014) and favoring divergence (Berger and Le Mens 2009, Smith 2011, Pontikes 2012).

Finally, perhaps the most interesting avenue for future research concerns integrating in our model the possibility that actors take actions that aim to alter audience tastes. For example, recent research has focused on the effect of commercial success, but it also influences the experience of actors of growing homogeneity of tastes within an audience. 

Acknowledgments
The authors are grateful for discussions with and comments by participants at the 2010 and 2011 Annual Organizational Ecology Workshops and the 2012 European Organizational Ecology Workshop. G. Le Mens benefited from financial support from the Barcelona School of Management, a Juan de la Cierva Fellowship from the Spanish Ministry of Science and Innovation, and the Spanish Ministry of Science and Innovation [Grant ECO2010-17145]. M. T. Hannan was supported by the Stanford Graduate School of Business, the Graduate School of Business Trust, and the Durham Business School. L. Pólos was supported by the Durham Business School.

Appendix A. Notation
This appendix supplies the formulas that convey the technical details of the theory. The functions and predicates, mostly defined in terms of the unspecified focal audience member y and the unspecified focal producer x, are listed in Table A.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL(x)</td>
<td>Producer x experiences preobsolescence alignment</td>
</tr>
<tr>
<td>α(x,y,t)</td>
<td>Actual appeal of producer x to audience member y at t</td>
</tr>
<tr>
<td>A(x,t)</td>
<td>Actual appeal of x to the targeted audience at targeted social positions</td>
</tr>
<tr>
<td>S(x,y,t)</td>
<td>Intrinsic appeal of producer x to y at t</td>
</tr>
<tr>
<td>LCS(x)</td>
<td>Intrinsic appeal of producer x to the targeted position at t</td>
</tr>
<tr>
<td>CS(x)</td>
<td>Total appeal of all of the focal producer’s competitors</td>
</tr>
<tr>
<td>CS(x)</td>
<td>The aggregate actual appeal of x’s competitors remains constant</td>
</tr>
<tr>
<td>n</td>
<td>n-dimensional feature space of feature values</td>
</tr>
<tr>
<td>(x,y,t)</td>
<td>Set of nonempty subsets of (x,y,t) (i.e., space of schemas)</td>
</tr>
<tr>
<td>δ(o, o’)</td>
<td>Distance between offer o and offer o’ in space (x,y,t)</td>
</tr>
<tr>
<td>δ(o, s)</td>
<td>Distance between offer o and schema s in space (x,y,t)</td>
</tr>
<tr>
<td>D(s, s’)</td>
<td>Directed Hausdorff distance from schema s to schema s’ in space (x,y,t)</td>
</tr>
<tr>
<td>DRIFT</td>
<td>The category’s meaning to the audience drifts</td>
</tr>
<tr>
<td>e, (t)</td>
<td>Engagement of producer x with the audience at targeted position at time t</td>
</tr>
<tr>
<td>t</td>
<td>Fitness threshold of producer x</td>
</tr>
<tr>
<td>κ(x,t)</td>
<td>Fitness level of producer x at time t</td>
</tr>
<tr>
<td>κ(x)</td>
<td>Limiting fitness of producer x</td>
</tr>
<tr>
<td>h(x,t)</td>
<td>Hazard of failure of producer x at time t</td>
</tr>
<tr>
<td>h(x)</td>
<td>Limiting fitness of producer x</td>
</tr>
<tr>
<td>k</td>
<td>Nonmonotonic “normally” quantifier</td>
</tr>
<tr>
<td>n</td>
<td>Nonmonotonic “presumably” quantifier</td>
</tr>
<tr>
<td>s</td>
<td>Adaptive capacity of producer x at time t</td>
</tr>
<tr>
<td>s</td>
<td>Stock of organizational capital of producer x at time t</td>
</tr>
<tr>
<td>5</td>
<td>Nonmonotonic “normally” quantifier</td>
</tr>
<tr>
<td>6</td>
<td>Nonmonotonic “presumably” quantifier</td>
</tr>
<tr>
<td>7</td>
<td>Hazard of failure of producer x</td>
</tr>
<tr>
<td>8</td>
<td>Date of inception of producer x</td>
</tr>
<tr>
<td>9</td>
<td>Onset of obsolescence for producer x</td>
</tr>
</tbody>
</table>

Appendix B. Formal Details of the LHP Argument
Here, we provide formal renderings of the definitions, postulates, propositions, and theorems from LHP that are mentioned in the text.

Proposition L1C. A producer’s failure hazard presumably decreases with age if its fitness exceeds its cost-structure threshold 1 and increases with age if its fitness is below 1:

\[ \forall x \exists \tau, t, r, u \left[ (\tau \leq t \leq u < t^*) \rightarrow (\varphi(x,u) > \tau \rightarrow h(x,t) > h(x,t^*)) \right] \]

Postulate L3. A producer’s level/quality of engagement normally rises with age:

\[ \forall x \forall t_1, t_2 \left[ (\tau \leq t_1 < t_2) \rightarrow \varphi(x,t_1) < \varphi(x,t_2) \right] \]

Proposition L2. The actual appeal of an offer presumably rises with the producer’s age:

\[ \forall x \forall t_1, t_2 \left[ (\tau \leq t_1 < t_2) \rightarrow A(x,t_1) < A(x,t_2) \right] \]

Definition 5. A producer faces stable competitive pressure if the sum of the actual appeals of its competitors in the focal category remains constant and positive (Definition 3 in LHP):

\[ \forall x \exists A \forall t \left[ (A(x) > 0) \land (\tau \leq t) \rightarrow C(x,t) = C(x) \right] \]
PROPOSITION L3. A producer’s fitness presumably 
(a) increases with age: 
\[ \forall x \forall t_1, t_2 \left[ \text{cs}(x) \land (t_2 \leq t_1 < t_2) \rightarrow \varphi_x(t_1) < \varphi_x(t_2) \right] \; \]
(b) converges toward a limiting value \( \hat{\varphi}_x \) (which we call long-run fitness): 
\[ \forall x \left[ \text{cs}(x) \rightarrow \lim_{(t-\tau_x)\to\infty} \varphi_x(t) = \hat{\varphi}_x \right] \; \]

THEOREM L1. (a) If a producer’s initial fitness exceeds the threshold, then the failure hazard presumably decreases with age—a liability of newness: 
\[ \forall x \forall t_1, t_2 \left[ \text{cs}(x) \land (\varphi_x(t_1) > \Sigma_x) \land (\tau_x \leq t_1 < t_2) \rightarrow h_x(t_1) > h_x(t_2) \right] \; \]
(b) If a producer’s initial fitness falls below the threshold but its long-run fitness exceeds it, the failure hazard presumably first increases and then decreases with age—a liability of adolescence: 
\[ \forall x \exists q \forall t_1, t_2, t_1, t_4 \left[ \text{cs}(x) \land (\varphi_x(t_1) < \Sigma_x) \land (\varphi_x \leq \Sigma_x) \rightarrow (h_x(t_1) < h_x(t_2)) \land (h_x(t_3) > h_x(t_4)) \right] \; \]
(c) If both a producer’s initial fitness and its long-run fitness fall below the threshold, then the failure hazard presumably increases with age at all ages—a liability of aging: 
\[ \forall x \forall t_1, t_2 \left[ (\varphi_x(t_1) < \Sigma_x) \land (\varphi_x(t_2) < \Sigma_x) \rightarrow h_x(t_1) < h_x(t_2) \right] \; \]

Appendix C. Specificity Considerations
When no information about intrinsic appeal is available, Proposition L2 holds. But when intrinsic appeal is known, the relation given by this proposition can be overridden. In the context of nonmonotonic logic, this is fine, because this formalism allows a proposition to be overridden by another proposition whose antecedent is more specific. To see how this works here, consider the formal rendering of Proposition L2: \[ \forall x \forall t_1, t_2 \left[ (t_1 \leq t_1 < t_2) \rightarrow A_x(t_1) < A_x(t_2) \right] \; \]
The antecedent of Proposition L2 \( t_1 \leq t_1 < t_2 \) is a part of the antecedent of the following proposition, implied by Meaning Postulate 1:
\[ \forall x \forall t_1, t_2 \left[ (t_1 \leq t_1 < t_2) \land (A_x(t_1) \neq 0) \land (A_x(t_2) \neq 0) \rightarrow (A_x(t_1) = g(e)A_x(t_1)) \land (A_x(t_2) = g(e)A_x(t_2)) \right] \; \]
In classical logic, if \( \phi \Rightarrow \gamma \), then \( \phi \land \xi \Rightarrow \gamma \) holds too. In nonmonotonic logic, this needs not be the case, and even \( \phi \land \xi \Rightarrow \neg \gamma \) is consistent with \( \phi \Rightarrow \gamma \), and in situations where both \( \phi \) and \( \xi \) hold, \( \neg \gamma \) overrides \( \gamma \).
Suppose we have \( g(e(t_1))A_x(t_1) > g(e(t_2))A_x(t_2) \). This implies that presumably \( A_x(t_1) > A_x(t_2) \), which runs counter to Proposition L2. But when intrinsic appeal is known, as it is in the case here, this proposition gets overridden and does not apply.

Appendix D. Proofs of the New Propositions/Theorems

PROOF OF PROPOSITION 1. We first relate drift with the evolution of the distance between the producer’s offer and the schema of audience members (Lemma 1), and then we use Postulate 1 to relate age and intrinsic appeal (Lemma 2). Then we build on these results to prove the proposition. Q.E.D.

LEMMA 1. If the meaning to an audience of a (positively valued) category drifts, then the distance between a producer’s offer and the audience members’ schemas presumably becomes arbitrarily large:
\[ \forall x, y \left[ \text{drift} \rightarrow \lim_{(t-\tau_x)\to\infty} \delta(o_x, s_y) = \infty \right] \; \]

Proof. Consider a producer facing age-related inertia and drifting tastes.14 We want to show that \( \delta(o_x, s_y) \) is no smaller than a quantity that becomes arbitrarily large when \( t - \tau_x \) becomes large.

As a preliminary, choose a value of \( \psi \) such that \( 0 < \psi < \beta \).
Postulate 2 implies that normally there exists \( q \) such that if \( t > q \), then \( \psi(t) < \psi \). This, in turn, implies that for all \( t > q \) it presumably holds that \( \delta(o_x, o_y) \leq (t - q)\psi \).

The assumption of drifting tastes implies \( \delta(o_x, s_y) \geq \beta(t - \tau_x) \).

Using the definition of the directed Hausdorff distance and invoking the triangle inequality for the distance measure \( d(\cdot, \cdot) \), we have 
\[ \delta(o_x, s_y) = \sup_{s_y \in \mathcal{A}_x} \inf_{s_y \in \mathcal{A}_x} d(s_y, s_y) \leq \sup_{s_y \in \mathcal{A}_x} \inf_{s_y \in \mathcal{A}_x} \left[ d(s_y, o_y) + d(o_y, o_0) + d(o_0, s_y) \right] \leq \sup_{s_y \in \mathcal{A}_x} d(s_y, o_y) + d(o_y, o_0) + \inf_{s_y \in \mathcal{A}_x} d(o_0, s_y). \]

Then, 
\[ \delta(o_x, s_y) \geq \inf_{s_y \in \mathcal{A}_x} \sup_{s_y \in \mathcal{A}_x} \delta(s_y, s_y) - \sup_{s_y \in \mathcal{A}_x} d(s_y, o_y) - d(o_y, o_0) \geq \sup_{s_y \in \mathcal{A}_x} \inf_{s_y \in \mathcal{A}_x} d(s_y, o_y) - d(o_y, o_0) \geq \sup_{s_y \in \mathcal{A}_x} d(s_y, o_y) - \psi(t - q) \geq (\beta - \psi)(t - q) - \sup_{s_y \in \mathcal{A}_x} d(s_y, o_y). \]

Because \( \beta - \psi > 0 \), the right-hand side in the above formula becomes arbitrarily large as \( t \) becomes large (the first term becomes arbitrarily large and the second term does not vary with \( t \)). Q.E.D.

We have a powerful corollary about the evolution of intrinsic appeal.

LEMMA 2. If the meaning of a (positively valued) label drifts, then the intrinsic appeal of its offer goes to zero as it becomes old:
\[ \forall x, y \left[ \text{drift} \rightarrow \lim_{(t-\tau_x)\to\infty} A_x(y, t) = 0 \right] \; \]

Proof. The lemma follows from Lemma 1 and Postulate 1. Q.E.D.

With these lemmas in hand, we can now derive Proposition 1: Meaning Postulate 1 implies that the intrinsic appeal is an
upper bound to the actual appeal. Lemma 2 implies that the intrinsic appeal presumably converges to 0 when time becomes large. Since the actual appeal is nonnegative, it also presumably converges to 0 when the age becomes large. Q.E.D.

**Proof of Theorem 1.** Specificity considerations make an important difference. Parts a and b of Theorem L1 tell that failure hazards decline with age at older ages. The formula in this theorem states the opposite. As in the proof of Proposition 1, we rely on the fact that definitions and premises are stated as generic ("normally") statements (rules with exceptions) and use specificity considerations to control the clash between the two arguments. The rule chains supporting the three subtheorems of Theorem L1 are relatively nonspecific (once we take into account that the antecedents in the subtheorems include terms about fitness relative to the threshold that, when taken together, include all of the relevant possibilities). The rule chain that supports the current theorem (sketched below) begins with \( \text{DRIFT} \land \text{CS}(x) \). The set of situations that satisfy these conditions is a proper subset of those that satisfy the conditions given by the first term in the rule chain for Theorem L1. It is, therefore, more specific, and, as such, overrides the argument of Theorem L1 when schemas drift.

The rule chain that yields this theorem goes as follows. Considerations of stable competitive pressure, \( \text{CS}(x) \), do not override the rule chain behind Proposition 1; therefore, its claim applies unchanged in the current context. This proposition implies that actual appeal presumably becomes arbitrarily small when enough time has elapsed since the producer's founding. Moreover, Definition 5 in Appendix B guarantees that \( C \) remains above a positive floor. These two facts imply that fitness presumably shrinks toward zero. Therefore, there exists a time \( q \) such that \( \varphi_q(t) < 1 \) if \( t \geq q \).

Considerations of drift and stable competitive pressure do not override the rule chain behind Proposition L1C; therefore, its claim applies unchanged in the current context. This implies that the hazard of failure presumably rises for \( t \geq q \). Q.E.D.

**Proof of Theorem 2.** This theorem (and the two that follow) applies to situations where \( \text{DRIFT} \land \text{AL}(x) \land \text{CS}(x) \) holds. Because those situations are more specific than the situations of applicability of Proposition L2 (actual appeal increases with age), any argument whose supporting rule chain makes use of that proposition cannot hold. In particular, Proposition L3 (which was crucial in deriving Theorem L1) does not hold for all ages in the setting we consider. Therefore, we cannot directly rely on Theorem L1 to derive results about what happens during the early phase of organizational lifetimes. The situations of application are also more specific than those invoked in the general drift and inertia theory (which forms a key part of the rule chains behind Proposition 1 and Theorem 1). Therefore, we cannot use those results unchanged either.

The condition of drift invoked in the antecedent guarantees that the onset of obsolescence, \( \omega_x \), is well defined (\( \omega_x < \infty \)). The rule chain that links the antecedent with the first term in the consequent applies to times before \( \omega_x \). Postulate L3 holds that engagement increases at all ages within the range being considered, and the definition of \( \text{AL}(x) \) (Definition 4) and Postulate 1 jointly imply that intrinsic appeal does not decrease in this age range. If engagement increases and intrinsic appeal does not decrease, then actual appeal rises (Meaning Postulate 1). Given the restriction to stable competitive pressure \( \text{CS}(x) \), Definition 5), this rule chain warrants the claim that fitness rises with age. This implies that fitness remains above the threshold and thereafter remains above it for the remainder of the preobsolescence period. This time point is uniquely defined, which we call \( q^B_x \). Fitness lies under the threshold at all ages before \( q^B_x \) and above the threshold at all ages after \( q^B_x \) and before \( \omega_x \). By this construction, the hazard of failure presumably rises with age before \( q^B_x \) and declines with age after \( q^B_x \) (as implied by the chain rule underlying Proposition L1C).

The rule chain linking the antecedent with the last term in the consequent is the same as for Theorem 2, because the antecedent states that \( q^B_x \) is not earlier than \( \omega_x \). Q.E.D.

**Proof of Theorem 4.** As in the proofs of the previous theorem, the claim of Proposition L1C still applies under conditions of drift, initial alignment, and stable competitive pressure. The antecedent supplies that initial fitness lies below the threshold and that fitness exceeds the threshold at \( \omega_x \). The restriction to initial alignment ensures that intrinsic appeal does not decrease before \( \omega_x \) (according to Definition 4). Then the assumption that engagement increases at all ages implies that fitness increases monotonically with age before \( \omega_x \). Therefore, there must be a time point such that fitness first reaches the threshold and thereafter remains above it for the remainder of the preobsolescence period. This time point is uniquely defined, which we call \( q^C_x \). Fitness lies under the threshold at all ages before \( q^C_x \) and above the threshold at all ages after \( q^C_x \) and before \( \omega_x \). By this construction, the hazard of failure presumably rises with age before \( q^C_x \) and declines with age after \( q^C_x \) (as implied by the chain rule underlying Proposition L1C).

The rule chain linking the antecedent with the last term in the consequent is the same as for Theorem 2, because the antecedent states that \( q^C_x \) is not earlier than \( \omega_x \). Q.E.D.

**Endnotes**

1 In the sense of Pólos and Hannan (2002), we treat that paper as the “first stage” of the developing theory.

2 We make two major simplifications to keep the argument and the notation as simple as possible. First, we assume that each producer specializes in the space of categories—that it bears only the focal category label. Among other advantages, this restriction lets us avoid the complicated matter of aggregating fitness in multiple categories to come up with an overall measure of fitness. Second, we assume that each producer specializes in resource space—that it operates at a single (unspecified) social position by targeting a relatively homogeneous subaudience.
(We do not represent positions formally.) Following standard sociological arguments, we assume that the audience members at a social position have similar tastes.

3 We number some of the postulates, propositions, and theorems as “L1,” “L2,” etc., to make clear that these are postulates and implications of the stage of the theory presented by LHP. For instance, the label “Postulate L3” means that the stated postulate is Postulate 3 in LHP. For the new postulates, propositions, and theorems, we use standard numerals.

4 The formal semantics of this quantifier are spelled out in Pólos and Hannan (2004).

5 If s is included in s′ (in the sense of set inclusion), then D(s, s′) = 0, even if the inclusion is strict.

6 This assumption is to be understood as made for analytical convenience rather than as a substantive claim about the world.

7 The implications of a set of rules with exceptions are the logical consequences of a stage of a theory. Such provisional theorems have a haphazard existence: what can be derived at one stage might not be derivable in a later stage. So the status of a provisional theorem differs from that of a causal story. The syntax of the language codes this difference. It introduces a “presumably” quantifier, denoted by ¯ψ. Sentences (formulas) quantified by ¯ψ are provisional theorems at a stage of a theory (if they follow from the premises at that stage).

8 The contradiction arises because, according to the rule chain supporting this proposition, the producer’s offer has positive actual appeal for at least one time point. Let t₁ be such a time point. We have Aᵢ(t₁) > 0. Because Aᵢ converges to 0 when t becomes large, there must also exist some t* such that for all t₂, [(t₂ > t*) → Aᵢ(t₂) < Aᵢ(t₁)]. Then, for all t₁, [max{t₁, t*} < t₁ → Aᵢ(t₁) < Aᵢ(t₁)]. But this result contradicts the claim of Proposition L2: for all t₁, t₂, [t₁ < t₂ → Aᵢ(t₁) < Aᵢ(t₂)].

9 As with previous work on the subject, we do not regard other types of exits, such as voluntary acquisitions and mergers, as failure events.

10 Imagine that a cohort enters at a given time and then there are no subsequent entries. This setting is incompatible with our assumptions of stable aggregate appeal of competitors. If tastes drift and the organizations are subject to structural inertia, the appeal of the offers of all the organizations should decline. But because the total appeal declines as well, the fitness level of a given producer does not have to decline. In fact, because we can expect that some producers will fail, the fitness of the survivors will likely increase even as the appeal of their offers declines.

11 This discussion presumes that size is neither a perfect control for selection, nor offers a perfect proxy for organizational capital. In our framework, if organizational capital were exactly equal to size, the prediction would be an absence of age dependence once size is controlled for.

12 The supremum of the arctangent function is π/2.

13 We perform a grid search around the parameter estimates obtained in LHP. We do a grid search by varying all the parameters from their original value −0.5 to their original value +0.5 in steps of 0.5. We let Ω vary from −3 to +3 in steps of 0.05, and we let k, the parameter that controls for unobserved heterogeneity, vary from 0 to 20 in steps of 0.5. After having identified the estimates with the highest log-likelihood in this grid search, we use these estimates as a starting point to run a Matlab R14 optimization routine (fminsearch) to refine our results.

References


Gaël Le Mens is an associate professor of economics and business at Universitat Pompeu Fabra (Spain), with appointments at the Barcelona School of Management and the Graduate School of Economics. His research interests include theoretical and empirical analysis of learning mechanisms by individuals and organizations, the development of the “sampling approach” to human judgment, and formalization of sociological theory.

Michael T. Hannan is the StrataCom Professor of Management and a professor of sociology at Stanford University and a professor of organisation theory at Durham University (UK). His research interests include theoretical and empirical analysis of categories in markets and formalization of sociological theory.

László Pólos is a professor of organization theory at Durham University (UK). His research interests include theoretical and empirical analysis of categories in markets, status reputation and authenticity, and formalization of sociological theory.