http://pubsonline.informs.org/journal/stsc/

STRATEGY SCIENCE
Vol. 2, No. 1, March 2017, pp. 39–63
ISSN 2333-2050 (print), ISSN 2333-2077 (online)

When More Selection Is Worse

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Received: March 22, 2016
Revised: January 9, 2017
Accepted: January 16, 2017
Published Online: March 14, 2017

http://dx.doi.org/10.1287/stsc.2017.0025

Abstract. We demonstrate a paradox of selection: the average level of skill among the survivors of selection may initially increase but eventually decrease. This result occurs in a simple model in which performance is not frequency dependent, there are no delayed effects, and skill is unrelated to risk-taking. The performance of an agent in any given period equals a skill component plus a noise term. We show that the average skill of survivors eventually decreases when the noise terms in consecutive periods are dependent and drawn from a distribution with a "long" tail—a sub-class of heavy-tailed distributions. This result occurs because only agents with extremely high level of performance survive many periods, and extreme performance is not diagnostic of high skill when the noise term is drawn from a long-tailed distribution.

History: This paper has been accepted for the *Strategy Science* Special Issue on Evolutionary Perspectives on Strategy.

Keywords: organizational evolution • evolutionary economics • organizational ecology

1. Introduction

Suppose you observe an industry with the purpose of identifying and imitating best practice. You know that firms do not change their capabilities much over time due to learning or forgetting. The population of firms changes over time as a result of entry and exit. Low performing firms tend to exit the industry whereas highly performing firms tend to stay in. When should you observe the industry? In the early stages of the industry? Later on, when many firms have exited? Or in the middle stages? A standard evolutionary argument ("survival of the fittest") suggests that you should observe the firms remaining in the later stages of the industry as these firms have survived selection for a longer time and are likely to be the most capable. In this paper, we demonstrate that there are conditions under which learning from those who have survived for the longest possible time is suboptimal and that one would be better off learning from those who have survived for a shorter time.

Of course, it is well known that selection does not necessarily increase the proportion of the most capable firms—those that would have the highest performance in the long run, had they survived (Wright 1931; Levins 1968; Holland 1975; Nelson and Winter 1982, 2002). When the "fitness" of a practice depends on how many others have adopted this practice or on the presence of specific organizational characteristics, an inferior practice could become dominant (Wright 1931, Maynard Smith 1982, Arthur 1989, Carroll and Harrison 1994, Levinthal 1997). Similarly, when selection operates on short-term performance, it might eliminate practices with positive long-term effects, especially in changing environments or when firms can adapt (Levins 1968, Elster 1979, Nelson and Winter 1982, Levinthal and March 1981, Levinthal and Posen 2007, Levinthal and Marino 2015). Finally, selection can be biased against practices that lead to highly variable performance even if mean performance is high (Cohen 1966, Denrell and March 2001, Levinthal and Posen 2007).

Here we identify another, in some sense more basic, reason for why firms that have survived for a longer time might not have capabilities superior to firms that have survived for a shorter time. We analyze a model in which we assume away all of the above complications. The key to our argument is that the relation between the organizational characteristics and selection is probabilistic. Selection operates on performance in the sense that the lowest performing firms exit the population while the others stay in. However, performance is determined not only by organizational characteristics but also factors unrelated to organizational characteristics, i.e., luck. We show that when the association between organizational characteristics and actual performance is highly variable (i.e., luck plays a large role), selection may initially lead to an increase in the prevalence of superior characteristics and later to a *decrease* in the prevalence of superior characteristics! Such inefficient selection happens when luck has persistent effects. In contrast, when luck has non-lasting effects (e.g., the contribution of luck to performance changes from period to

period), selection is efficient and leads to an increased prevalence of superior characteristics.

In this paper we focus on just one aspect of evolutionary explanations—selection—and leave out many other important aspects. In particular, the dynamics of skills and routines, which is central to the evolutionary theory of the firm (Nelson and Winter 1982), is not explored here. Nelson and Winter (1982) assume that profitable firms expand while unprofitable firms contract. Moreover, they assume that unprofitable firms are more likely than profitable firms to search for new routines. Their focus is on how routines evolve as a result of such market-driven search processes (Winter 1971). Our focus is instead on the process of elimination of relatively poorly performing units. Using the terminology of Hodgson and Knudsen (2010), we focus on "subset" selection (selecting a subset of units for survival, without changing their properties) while the evolutionary theory of the firm focuses on "successor" selection (in which the units being selected change through imperfect replication involving mutation but also, in social science applications, processes of learning and search).

Evolutionary theories and selection explanations are sometimes viewed as alternative forms of explanations distinct from theories that emphasize goal-directed and adaptive behavior by managers. We do not pursue this agenda here. Our focus is on the population level consequences of selection regardless of the level of intentionality and rationality of the agents involved. Most selection models in economics and management assume that agents are intentional and goal-directed (Nelson and Winter 1982, pp. 10–11). Additionally, while many evolutionary accounts assume that agents are boundedly rational instead of profit maximizing (Alchian 1950, Winter 1964), there are several well-known selection models in economics that assume that agents are rational. For example, selection in the model of Jovanovic (1982) (i.e., exit by a firm) occurs as the result of optimal stopping based on Bayesian updating.

In what follows, we first provide an illustration of our argument using simulations of a simple model. Then we clarify the positioning of our work within the literature on selection in strategy and organization theory. We describe our model and our formal analysis. We establish formally the conditions under which selection is efficient (i.e., selection leads to an increasing prevalence of superior characteristics over time) or inefficient (i.e., selection leads to a decreasing prevalence of superior characteristics over time). Then we show, using computer simulations, that our basic result holds when we relax some of the simplifying assumptions of our formal model. Finally, we discuss when our technical results are relevant and discuss the implications of our findings for empirical phenomena such as organizational obsolescence.

2. Illustration

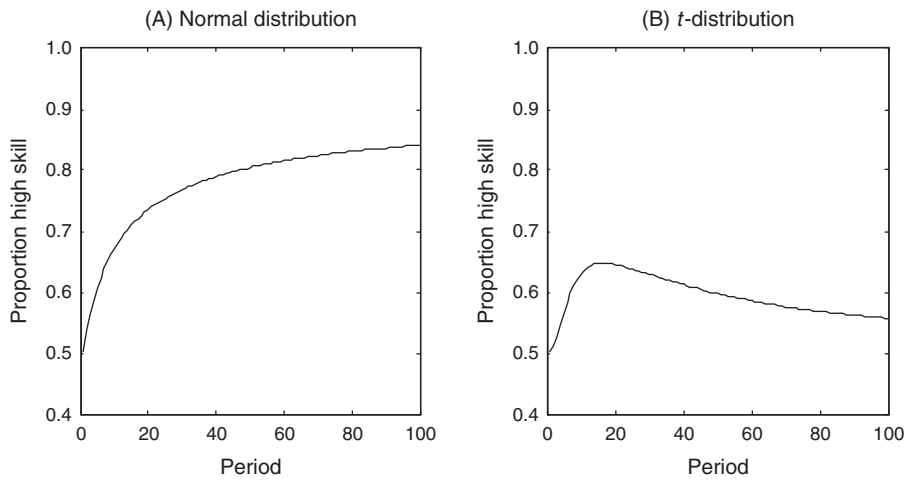
To illustrate the basic idea, consider the following simple model. There are $n = 50$ firms. The performance of firm i in period t, $p_{i,t}$, is equal to the sum of its skill u_i and a noise term ε_i: $p_{i,t} = u_i + \varepsilon_i$, where ε_i is drawn from density f_ε. There are two levels of skill: high ($u_i = 1$), and low ($u_i = 0$), each equally likely. The level of skill and the value of the noise term remain constant during the lifetime of a firm. It follows that the performance of firm i is the same in every period. This setup represents a situation when initial "luck" has strong long-term effects.

Selection works as follows. In each period t the 10% lowest performing firms (based on $p_{i,t}$) are removed from the population. Each eliminated firm is replaced by a new firm. Consider a new firm j. The probability that it has high skill ($u_j = 1$) is 50% and its performance is $p_{j,t} = u_j + \varepsilon_j$ where ε_j is drawn from the same density f_ε. Note that both u_j and ε_j remain constant during the lifetime of a firm. We are interested in how the proportion of high skill firms in the population evolves over time. Does it increase monotonically with time? Are older firms, which have survived more selections, more likely to have high skill?

Figure 1(A) shows that when the noise terms are drawn from a normal distribution, the proportion of high skill firms increases over time. The pattern differs, however, when the noise terms are drawn from a student's t-distribution. As shown in Figure 1(B), the proportion of high skill firms initially increases but eventually decreases and ultimately becomes close to 50%, the proportion of high skill firms at the beginning. What is noteworthy is that firms that have survived for a longer time (e.g., 100 periods) are *less* likely to have high skill ($u_i = 1$) than firms that survived for a shorter time (e.g., 20 periods). The mechanism underlying the dynamics depicted in Figure 1(B) thus leads to "inefficient" selection. Although firms with the highest level of performance are more likely to survive in every period, and firms with high skill have higher expected performance, selection based on performance does not increase the proportion of high skill firms. In contrast, after some time, it leads to a *decrease* in the proportion of high skill firms.

Why does the proportion of high skill firms decline over time when the noise term is drawn from a t-distribution? The precise mechanism is discussed in the formal analysis in Section 4 and the intuition is explained in more detail in Section 5. At this stage, note that (1) we know that survivors must have had a very high level of performance (because only firms with a very high level of performance survive many periods); and (2) it is known from prior research that when performance partly depends on luck and luck is drawn from a "long-tailed" distribution, such as the t-distribution, an extremely high-level performance is

Figure 1. Proportion of High Skill Firms as a Function of Time



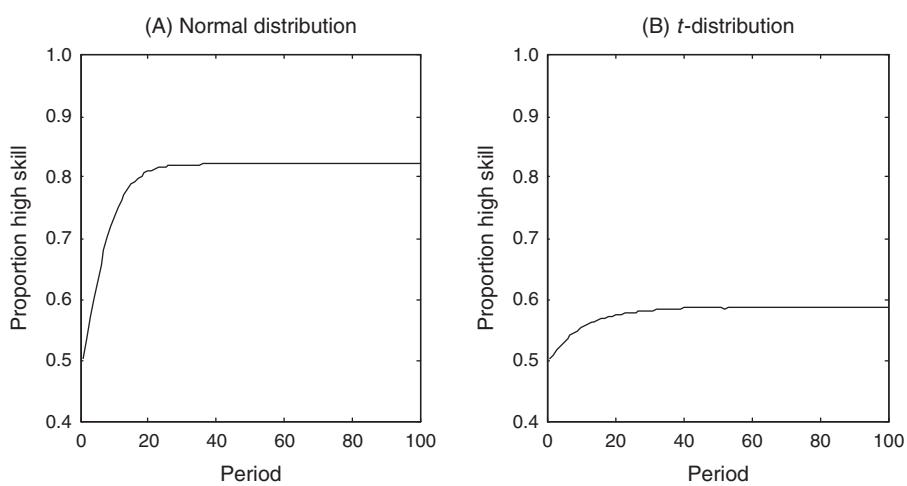
Notes. In each period, the firms with the 10% lowest performances are removed and replaced with new firms. The noise terms remain the same across periods and are drawn from (A) a normal distribution with mean 0 and variance 1; and (B) a t-distribution with mean 0 and 1 degree of freedom. Each graph is based on 10,000 simulations, each with $n = 50$ firms.

not diagnostic of high skill (Weibull et al. 2007, Denrell and Liu 2012). Hence, having survived many periods can in fact be less diagnostic of high skill than surviving fewer periods. This is because surviving many periods requires an extreme level of performance, whereas surviving fewer periods requires a high but not extreme level of performance.

Importantly, this mechanism only operates when the performance in period t is strongly dependent on the performance in prior periods (conditional on skill level). In particular, it does not apply when the noise terms are redrawn in every period. Consider the following variation of the model. As before, the performance of firm i in period t, $p_{i,t}$, equals its skill u_i plus a noise term. But here, we assume that the noise terms are not drawn just once at the time of entry but are redrawn in every period. Thus, we have a similar specification, but with a time index on the noise term $\varepsilon_{i,t}$: $p_{i,t} = u_i + \varepsilon_{i,t}$. We assume that the noise terms are *iid* for all t and i. Figure 2 displays the evolution of the proportion of high skill firms for the two noise distributions used before (Normal with mean 0 and variance 1 and t-distribution with mean 0 and 1 degree of freedom). Here, the proportion of high skill firms monotonically increases over time in both cases. In contrast to what happened in the previous setting (Figure 1(B)), selection is efficient even when the noise term follows a t-distribution (Figure 2(B)).

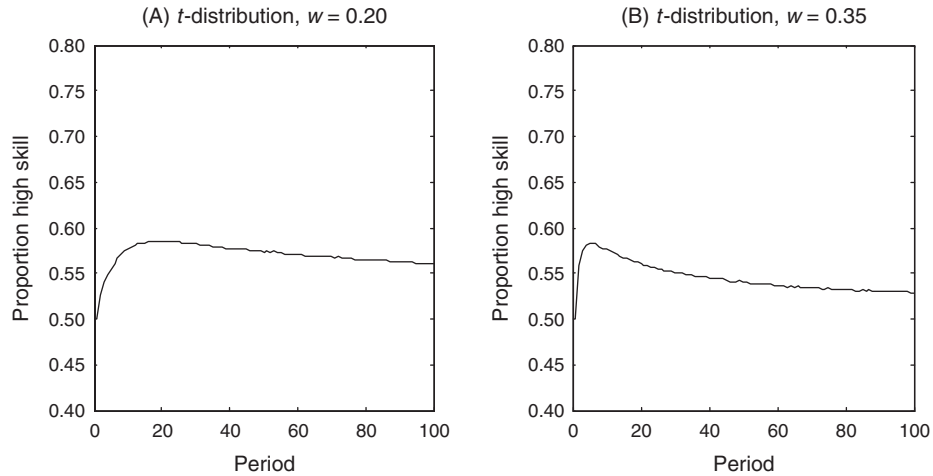
Why is the proportion of high skill firms monotonically increasing when noise terms are independently drawn in every period, but not when they are drawn just once at the time of entry (in the case of

Figure 2. Proportion of High Skill Firms as a Function of Time



Notes. In each period, the firms with the 10 % lowest performances are removed and replaced with new firms. The noise terms are redrawn in every period from (A) a normal distribution and (B) a t-distribution. Each graph is based on 10,000 simulations, each with $n = 50$ firms.

Figure 3. How the Proportion of High Skill Firms Changes Over Time When the Noise Term Follows a Random Walk and (A) $\varepsilon_{i,t}$ are Drawn from a t-Distribution with One Degree of Freedom and $w = 0.2$, and (B) $\varepsilon_{i,t}$ are Drawn from a t-Distribution with One Degree of Freedom and $w = 0.35$



Note. Each graph is based on 10,000 simulations, each with 50 firms, where the w percent firms with the lowest performances are replaced in every period.

a t-distribution)? When the noise terms are independently drawn in every period survival until the end of period t requires that a firm passes t distinct "tests." All these tests are independent *conditional on skill*. In this case, survival during early periods (the early "tests") is informative about skill, since survival during these early periods does not require extreme levels of performances. By contrast, when the noise term is drawn just at the time of entry, all t tests are dependent.

It is worth noting that inefficient selection can occur even when firm performance does not remain constant during the lifetime of a firm. For example, performance may follow a random walk (Levinthal 1991, Denrell 2004, Le Mens et al. 2011). Specifically, consider a firm that enters in period t. Its performance in period t follows the same specification as before: $\varepsilon_{i,t}$: $p_{i,t} = u_i + \varepsilon_{i,t}$. Its performance in period $t+1$ is $p_{i,t}$ plus an independent draw from the same noise term distribution: $p_{i,t+1} = u_i + \varepsilon_{i,t} + \varepsilon_{i,t+1}$. Similarly, $p_{i,t+2} = u_i + \varepsilon_{i,t} + \varepsilon_{i,t+1} + \varepsilon_{i,t+2}$, etc. In this setup, performance is not constant but changes in every period. Simulations show that if selection is strong enough (w, the proportion of firms that exit in each period, is high enough), the proportion of high skill firms will eventually decline (see Figure 3 for a depiction of the dynamics of this proportion for two levels of w). More generally, in Sections 6 and 7 we show that our basic result holds under a number of alternative assumptions about selection, replacement, and performance dynamics.

3. Prior Literature on the Efficiency of Selection

How do our results compare to prior work on selection in strategy and organization theory? Prior work has demonstrated that selection processes in markets and organizations are limited in how effectively they remove "inferior" organizations from a population. Some researchers have argued that selection processes may be only loosely coupled with performance (Meyer and Zucker 1989). The threshold for exit can depend on alternative employment opportunities of an entrepreneur in addition to firm performance (Gimeno et al. 1997). Well-connected and powerful firms can sometimes survive for a long time even if they are inefficient (Perrow 2002). Large firms can also become buffered from selection forces (Levinthal 1991, Barnett 1997). Other researchers have demonstrated that even if selection processes reliably select on the basis of economic performance, the most efficient organizations may fail to survive. As Levinthal and Posen write: "Even if selection is effective in removing inferior organizations at one point in time, it may be ineffective over time in that it may remove organizations that, had they survived, would have gone on to do well" (Levinthal and Posen 2007, p. 587). Two well-known mechanisms leading to such inefficient selection are time-dependent fitness and frequency-dependent fitness.

Consider time-dependent fitness. Fitness may change over time because firms or their environments change, but selection is often myopic and responds only to current levels of fitness (Elster 1979, Levinthal and March 1981, Levinthal and Posen 2007). Such myopic selection may eliminate units with high future potential but low current performance. As Nelson and Winter note: "[i]f firms are small in the early stages of industry growth, those that start with techniques that are efficient only after the firm has grown considerably may be defeated in the evolutionary struggle by firms whose techniques are better suited to low levels of output" (Nelson and Winter 1982, p. 159). Building

upon this insight, Denrell and March (2001) showed that practices that improve by learning-by-doing may be selected against because of their poor initial performance. In their simulation, firms were endowed with one of two possible technologies. The first technology generated a fixed payoff. The second technology generated a low initial payoff but its payoff increases over time. Even if the long-term payoff of the second technology is higher than the fixed payoff of the first technology, implying that the second technology is "superior," firms with the second technology were likely to fail before the potential of their technology was revealed. As a result, the proportion of firms with the second technology decreased over time. Selection is inefficient in this case because the proportion of firms with the second technology, which has a higher long-term payoff, is reduced over time. The inefficiency occurs because the fitness of the second technology changes systematically over time and the selection process is myopic in the sense that it reacts only to current payoffs and not to anticipated future payoffs.

Consider next frequency-dependent fitness. This refers to situations where the fitness of a gene depends on the relative abundance of that gene versus other genes in the environment. It is well known in evolutionary theory that when fitness is frequency dependent, a gene with lower potential fitness could become dominant (Wright 1931, Maynard Smith 1982). Evolutionary scholars in strategy and organization theory have similarly explored how the fitness of a particular practice may depend on the presence of other practices, creating a "rugged" fitness landscape with multiple local optima in which selection is unlikely to identify the global optima (Arthur 1989, Levinthal 1997). Carroll and Harrison (1994) showed, in an evolutionary model, how frequency dependence allows inefficient organizational forms to survive. Following past empirical work, they assume that both the founding and mortality rates are frequency (density) dependent. Organizational forms only differ in their competitive effects: the negative effect that *one* organization of form i exerts on organizations of form j. An organizational form i is superior to an organizational form j if the competitive effect of i is larger than the competitive effect of j. Their simulations show that whenever an inferior form emerges earlier than a superior form it can become dominant. The intuition is that the inferior form exerts a larger *total* competitive pressure on the superior form, than vice versa, because there are more organizations of the inferior form (higher density). Simulations of their model show that when the inferior form enters first, the density of the superior form initially increases but eventually declines to 0. The density of the superior form initially increases because, at this stage, the density of the inferior form is not yet very high. The decline occurs because when the inferior form becomes numerous, it exerts a high total competitive pressure on the superior form. This scenario also represents a kind of "inefficient" selection. The superior form is eliminated because there are more organizations of the inferior form and performance depends on the number (frequency) of organizations of the same form.

Variability is another reason why selection may not increase the proportion of a trait with the highest expected performance. It is well-known in evolutionary theory that a trait associated with the highest level of expected performance may be selected against if its performance is also highly variable (Cohen 1966, Yoshimura and Clark 1993, Cvijovic et al. 2015). Similarly, management theorists have argued that the proportion of an organizational practice with the highest expected performance may decrease over time as a result of selection if this form also has the highest variance in performance. In particular, Levinthal and Posen (2007) and Levinthal and Marino (2015) have shown that adaptive learning processes may be selected against because they imply increased variability in performance as a result of the adjustments the firm will go through. Adaptive learning may lead to superior average performance in the long run but may lead to a higher variability in performance in the short run. Higher variability, in turn, increases the chances of elimination in the short run, before the long-term advantages have been realized.

The mechanism we rely upon in this paper is different from past work because we do not assume that the performance effect of organizational characteristics is time- or frequency-dependent. Nor do we assume that variability in performance is systematically related to expected performance. We assume instead that variability in performance is the same across all units in the population (the noise terms of all firms are drawn from the same distribution, independent of skill). We show that even in this case there exist conditions under which selection will be inefficient in the sense that the proportion of agents with the superior trait does not necessarily increase over time. To the contrary, the proportion of agents with the superior trait ultimately *declines* over time.

It is important to note that our model does not imply that less "fit" firms are more likely to survive. In each period, selection removes firms with low performance levels, consistent with the idea of "survival of the fittest" if fitness is measured in terms of performance. Indeed, because poorly performing firms are replaced, average performance in the population increases over time, implying that the population becomes more "fit" over time. The fact that the population becomes more "fit" over time, however, is equally true for other mechanisms of inefficient selection, such as time-dependent fitness. Selection may reduce the proportion of the

technology with highest long-term performance, but average performance can nevertheless increase over time if the performance of the inferior technology also improves over time.

If selection leads to a monotonic performance increase over time in our model, why does it matter that the proportion of firms with high skill ($u_i = 1$) decreases over time? For an observer interested only in average performance, the systematic decline in u_i over time may not matter. The decline in u_i over time would matter for an observer interested in identifying practices and skills that contribute to high performance. If u_i represents the capability of a firm, while ε_i represents situational influences beyond the control of management, such an observer would be interested in learning from firms with high capabilities ($u_i = 1$). Our results imply that such an observer should not imitate firms that have been through many rounds of selection. In Section 8, we discuss in more detail when our results matter.

4. Formal Analysis: Selection and No Replacement

To analyze when and why selection can reduce the proportion of the type with the highest value of u_i, we first focus on a simple setting when there is selection but no replacement. That is, we analyze the effect of repeated selection on a cohort of agents. In Section 6.1, we show that the basic result continues to hold if there is also replacement.

4.1. Model

Consider a population of infinitely many agents (an agent can be an individual, a firm etc.). Assuming that a population is made of infinitely many agents is a standard assumption in many evolutionary models that allows for mathematical tractability. Our illustrative graphs in Section 2 showed that we get similar results in a population of 50 agents.

Agents are either high skill agents ($u_i = 1$) or low skill agents ($u_i = 0$). The level of skill of an agent remains the same during its lifetime. Initially, the proportion of high skill agents is 0.50. We use the label "skill" to denote a trait that contributes to the performance of agent i. The performance of agent i in period t, $p_{i,t}$ equals his or her skill u_i plus a noise term ε_i: $p_{i,t} = u_i + \varepsilon_{i,t}$. The noise term represents an aspect of performance beyond the control of the agent. We assume that the noise term is drawn from a density f_ε with positive variance. We assume that the support of f_ε is of the form $(a, +\infty)$ with $a \in \{-\infty, \mathbb{R}\}$.[1] Note that while we use the label "noise term" for ε_i, we do not assume that the expected value of f_ε has to be equal to 0.

Selection works as follows. In each period t, the w percent agents with the lowest performance levels (lowest values of $p_{i,t}$) are removed from the population.

We denote by π_t the proportion of high skill agents at the end of period t. We denote by π_0 the initial proportion of high skill agents ($\pi_0 = 0.50$). We are interested in how π_t evolves with t. Does it increase with t implying that selection increases the proportion of agents with high skill?

4.2. Independent Noise Terms

Suppose the noise terms are independent across agents and periods. That is, $p_{i,t} = u_i + \varepsilon_{i,t}$, and $\varepsilon_{i,t}$ are *iid* draws from the density f_ε for all t and i. We denote the density of the performance distribution of high skill agents by f_1 and the density of the performance distribution of low skill agents by f_0. Because the noise terms are redrawn in every period, the distribution of performance conditional on skill remains constant. Hence, we have the following:

$$f_0(p_{i,t+1}) = f_0(p_{i,t}),$$

and

$$f_1(p_{i,t+1}) = f_1(p_{i,t}).$$

It follows that, in every period, high skill agents (whose performances equal $p_{i,t} = 1 + \varepsilon_{i,t}$) are likely to have a higher performance than low skill agents (whose performances equal $p_{i,t} = \varepsilon_{i,t}$). As a result, the proportion of high skill agents increases over time. Theorem 1 demonstrates that this holds for any (continuous) distribution of the noise term

Theorem 1. *Suppose $p_{i,t} = u_i + \varepsilon_{i,t}$, where, for all t and i, $\varepsilon_{i,t}$ are iid draws from the continuous density f_ε. Whenever $w \in (0,1)$ expected skill increases over time: $\pi_{t+1} > \pi_t$ for all $t \geq 1$.*

Proof. See Appendix A. □

4.3. Constant Noise Terms

Suppose now that the noise terms remain the same in all periods. The noise term in the first period $\varepsilon_{i,1}$ is drawn from density f_ε. The noise terms in periods $2, 3, 4, \ldots$ are identical to the noise term drawn in period 1. The performance of agent i thus remains the same in all periods: For all t, $p_{i,t} = u_i + \varepsilon_{i,1}$. This setup represents a situation of extreme dependency across periods. Such dependency can occur when initial "luck" has long-term effects.

An important implication of this specification is that the distribution of performance conditional on skill and survival systematically changes over time. The reason is that selection in prior periods eliminates agents with low values of $p_{i,t}$ and thus indirectly eliminates agents with low values of $\varepsilon_{i,1}$. Whether selection increases average skill depends on the nature of the distribution of the noise term (f_ε).

The following theorem shows that whether selection leads to a monotonic increase in the proportion of high

skill agents depends on whether the hazard function of the noise distribution is an increasing function. The hazard function h of a distribution f is defined as the ratio of the density over 1 minus the cumulative density function F: $h(x) = f(x)/(1 - F(x))$.

Theorem 2. *Suppose $p_{i,t} = u_i + \varepsilon_{i,1}$ and $\varepsilon_{i,1}$ is drawn from density f_ε. Let h_ε denote the hazard function of distribution f_ε. In this case,*

(i) *The proportion of high skill agents increases as a result of selection during the first period: $\pi_1 > \pi_0 = 0.5$*

(ii) *The evolution of the proportion of high skill agents after the first period ($t > 1$) depends on the shape of the hazard function of the noise distribution:*

(a) *If h_ε is an increasing function, then π_t increases with t.*

(b) *If h_ε is a decreasing function, then π_t decreases with t for t large enough.*

(c) *If there exists c^* such that $h_\varepsilon(c-1) < h_\varepsilon(c)$ for all $c < c^*$ and $h_\varepsilon(c-1) > h_\varepsilon(c)$ for all $c > c^*$, π_t decreases with t for t large enough.*

(d) *If there exists c^* such that $h_\varepsilon(c-1) = h_\varepsilon(c)$ for all $c > c^*$, π_t remains constant for t large enough.*

Proof. See Appendix B. □

Table 1 lists several distributions and whether their hazard functions are increasing or decreasing (based on Bagnoli and Bergstrom 2005 and Glaser 1980). Table 1 shows that selection always increases average skill whenever the noise term is drawn from a normal distribution, the logistic distribution, the extreme value distribution, and the Weibull distribution (density $k\epsilon_i^{k-1}e^{-\epsilon_i^k}$) with parameter $k > 1$. Selection eventually decreases average skill, however, for several distributions with "fatter" tails than the normal distribution, including the t-distribution, Cauchy distribution, log-normal distribution, inverse Gaussian, Weibull distribution with parameter $k < 1$, and the Pareto distribution. There exists distributions, such as the Laplace distribution, which have fatter tails than the normal distribution but that nevertheless do not have a decreasing hazard function.

Table 1. Shape of the Hazard Functions of a Set of Distributions

Distribution	Shape of the hazard function
Uniform	Increasing
Normal	Increasing
Logistic	Increasing
Poisson	Increasing
Extreme value	Increasing
Exponential	Constant
Laplace	Initially increasing, eventually constant
Cauchy	Initially increasing, eventually decreasing
Log-normal	Initially increasing, eventually decreasing
Inverse Gaussian	Initially increasing, eventually decreasing
Weibull	Increasing when $k > 1$, decreasing when $k < 1$
Pareto	Decreasing

4.4. Long-Tailed Distributions

Theorem 2 shows that the proportion of high skill agents will eventually decline over time when the noise term is drawn at time of entry from a distribution with a hazard function which is (eventually) decreasing. This raises the question of which distributions have a declining hazard function. Can these be characterized in some intuitive way? In this section we show that a class of distributions, called "long-tailed" distributions, have exactly the properties we seek.

Formally, a random variable X has a long-tailed distribution if $(1 - F_x(c + y))/(1 - F_x(c)) \to 1$ as $c \to \infty$ for all $y > 0$ (Foss et al. 2013). Here, $F_x()$ denotes the cumulative distribution function of X. The property of having a long tail thus corresponds to the fact that if X is larger than some very large constant c (which occurs with probability $1 - F_x(c)$) then X is also likely to be larger than $c + y$ (which occurs with probability $1 - F_x(c + y)$). Examples of long-tailed distributions include the t-distribution with one degree of freedom (i.e, the Cauchy distribution) but also the Pareto distribution and the log-normal distribution.

The property of being a long-tailed distribution turns out to be the property that precisely characterizes the set of distributions for which survival during many periods eventually becomes uninformative about skill when the noise terms are constant. Formally, let p_t^* be the maximum level of performance among the agents that were eliminated in period t. All survivors during period t have a performance above p_t^*. When the noise terms remain constant the proportion of high skill agents among the survivors of t periods is $P(u_i = 1 \mid p_i > p_t^*)$, the proportion of high skill agents among agents that have a performance above p_t^*. The property of being long-tailed precisely captures the set of noise term distributions for which $P(u_i = 1 \mid p_i > p_t^*)$ becomes uninformative about skill as p_t^* becomes large. Formally,

Theorem 3. *Let $p_i = u_i + \varepsilon_i$, where $u_i = 1$ with probability 0.5 and $u_i = 0$ otherwise. Then,*

(i) $\lim_{c\to\infty} P(u_i = 1 \mid p_i > c) = 0.5$ *if and only if f_ε is the density of a long-tailed distribution.*

(ii) $\lim_{t\to\infty} P(u_i = 1 \mid p_i > p_t^*) = 0.5$ *if and only if f_ε is the density of a long-tailed distribution.*

Proof. See Appendix C. □

Theorem 3 implies that the proportion of high skill agents will converge to 50%, the initial proportion, when the noise term is drawn from a "long-tailed" distribution.

What about the condition regarding the hazard function? Theorem 2 states that average skill eventually declines if the hazard function of the noise term distribution is eventually declining. It turns out that the property of having a long tail also implies that the

hazard function is (eventually) declining. Formally, if a distribution is long-tailed then its hazard function $h(x)$ will eventually go to 0: $h(x) \to 0$ as $x \to \infty$ (Nair et al. 2013).

How does the property of being long-tailed relate to the more widely-known concept of "fat-tailed" distributions? A distribution is "fat-tailed" if the upper tail behaves as a power law, i.e., if $P(x > c) \approx c^{-x}$ as $c \to \infty$ (Foss et al. 2013). Fat-tailed distributions belong to the class of "heavy-tailed" distributions (all fat-tailed distributions are heavy-tailed, but there exists heavy-tailed distributions which are not fat-tailed). Informally, a distribution is heavy-tailed if its tail is heavier than the tail of an exponential distribution.[2] Long-tailed distributions are a sub-class of "heavy-tailed" distributions (Nair et al. 2013). Hence, all long-tailed distributions are heavy-tailed. There exist long-tailed distributions, however, which are heavy-tailed but not fat-tailed (an example is the log-normal distribution).

5. Intuition

Why does selection increase the proportion of high skill agents if the noise terms are redrawn, but can decrease the proportion of high skill agents if the noise terms are constant? And why does the effect of selection depend on the hazard function of the noise term? The basic result can be explained in two different ways.

5.1. Intuition 1: The Diagnosticity of Survival Decreases with Time

The first type of explanation focuses on the skill levels of the survivors. After several periods of selection, only a small fraction of the initial population remains. These survivors have had high performance during several periods. Is such high performance a reliable indication of high skill? This is not generally the case. It is well-known in statistics that a higher outcome may not indicate a higher expected value (Karlin and Rubin 1956). For some "heavy-tailed" noise term distributions, a very high outcome may indicate a lower expected value than a moderately high outcome does (Weibull et al. 2007, Denrell and Liu 2012). The reason is that extreme outcomes depend relatively more than moderately high outcomes on luck. Survival during many periods can, for similar reasons, be an unreliable indicator of high skill.

To explain this, consider the agents that have survived during all of the first t periods. What is the proportion of high skill agents among these survivors? Consider first the case when noise terms are constant. Performance in consecutive periods does not change (because the noise term remains constant). Moreover, because selection removes the agents with the lowest performances, the threshold for survival p_t^*, defined as the maximum level of performance among the agents that failed in period t, increases over time (see Lemma 4 in Appendix B). This implies that if an agent had a performance above the threshold in period t ($p_{i,t} > p_t^*$), his or her performance was also above the threshold in any previous period ($p_{i,t-1} > p_{t-1}^*$). The probability of surviving t periods for an agent with skill $u_i = k$ is thus simply the probability that the performance drawn in the first period is above the threshold in period t: $P(p_{i,t} > p_t^* \mid u_i = k)$. When the noise terms are constant, the proportion of high skill agents among the survivors is thus

$$\pi_t = P(u_i = 1 \mid p_{i,t} > p_t^*) = \frac{P(p_{i,t} > p_t^* \mid u_i = 1)P(u_i = 1)}{P(p_{i,t} > p_t^* \mid u_i = 1)P(u_i = 1) + P(p_{i,t} > p_t^* \mid u_i = 0)P(u_i = 0)}.$$

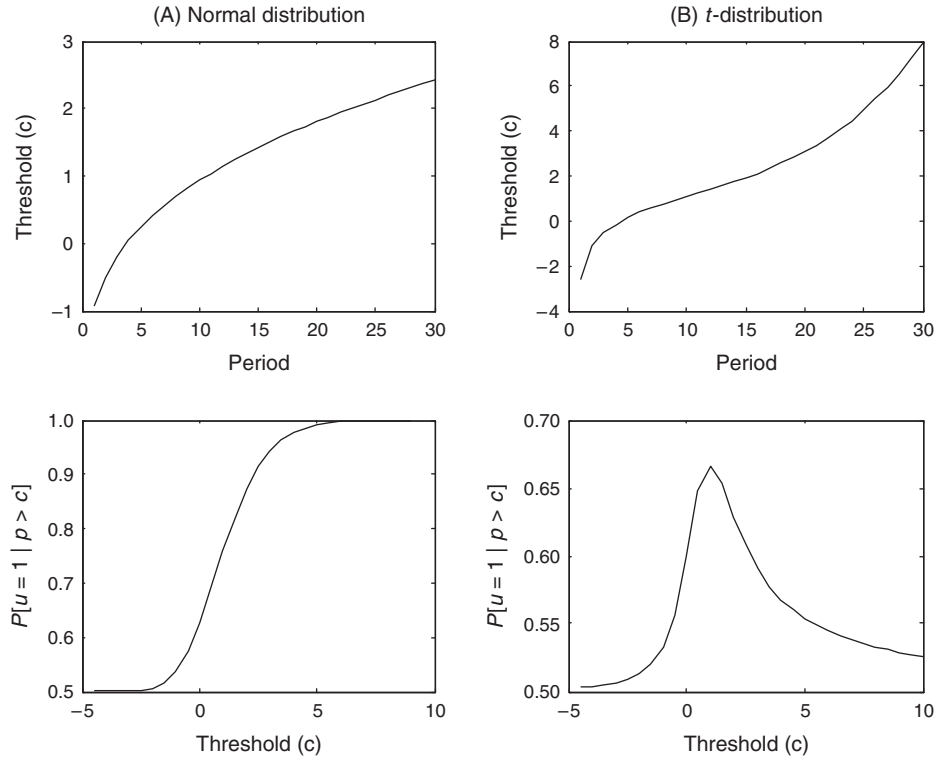
When the noise terms are constant, the only thing we know about the agents that have survived for t periods is that the performance they drew at the start was above p_t^*, the threshold for survival in period t. When t is large, the value of p_t^* will be high. Hence, we know that the performance levels of all the survivors are very high. But the fact that they all have a high level of performance does not imply that they are all high skill agents. It is not generally true that the proportion of high skill agents is higher among agents with higher levels of performance.

This is illustrated in Figure 4. The upper quadrants show that the threshold for survival increases over time (when $w = 0.1$) both when the noise term is drawn from a normal distribution and from a t-distribution. The lower quadrant shows how the proportion of high skill agents that have a performance above a threshold $P(u_i = 1 \mid p_{i,t} > c)$ varies with the threshold c. When the noise term is drawn from a normal distribution, $P(u_i = 1 \mid p_{i,t} > c)$ increases with c. When the noise term is drawn from a t-distribution, $P(u_i = 1 \mid p_{i,t} > c)$ eventually declines with c and converges to 0.5 as $c \to \infty$. As Theorem 3 shows, $\lim_{c\to\infty} P(u_i = 1 \mid p_{i,t} > c) = 0.5$ if and only if f_ε is the density of a long-tailed distribution.

The situation differs when the noise terms are independently drawn in each period. In this case, survival during many periods does not become uninformative. To explain why, note that in this case, performance changes every period because the noise terms are redrawn in every period. Moreover, conditional upon skill, performance levels in different periods are independent. The probability of survival during t periods for an agent with skill $u_i = k$ is thus the probability of surviving period 1, $P(p_{i,1} > p_1^* \mid u_i = k)$, multiplied by the probability of surviving period 2, $P(p_{i,2} > p_2^* \mid u_i = k)$, etc. The proportion of high skill agents among the survivors of t periods can thus be written as

$$\pi_t = [P(p_{i,1} > p_1^*, \ldots, p_{i,t} > p_t^* \mid u_i = 1)P(u_i = 1)] / [P(p_{i,1} > p_1^*, \ldots, p_{i,t} > p_t^* \mid u_i = 1)P(u_i = 1) + P(p_{i,1} > p_1^*, \ldots, p_{i,t} > p_t^* \mid u_i = 0)P(u_i = 0)].$$

Figure 4. Upper Panels: Survival Threshold as a Function of Time. Lower Panels: Proportion of High Skill Agents Among Those with a Performance Above the Threshold



Notes. Left panels: Noise term is drawn from a Normal distribution with mean 0 and variance 1. Right quadrants: Noise terms are drawn from a t-distribution with mean 0 and one degree of freedom.

Noting that $P(u_i = 1) = P(u_i = 0) = 0.5$ and that performance levels in successive periods are independent from each other conditional on skill, we can write

$$\pi_t = \frac{\prod_{j=1}^{t} P(p_{i,j} > p_j^* \mid u_i = 1)}{\prod_{j=1}^{t} P(p_{i,j} > p_j^* \mid u_i = 1) + \prod_{j=1}^{t} P(p_{i,j} > p_j^* \mid u_i = 0)}.$$

In contrast to the case of constant noise terms, this expression does not only depend on the probability that the final performance is above the final threshold, $P(p_{i,t} > p_t^* \mid u_i = k)$, but also on what happened in all prior periods. Moreover, because the performance in period t is independent from performance in earlier periods, conditional on skill, more information is available compared to the case when the noise terms are constant. Stated differently, when the noise terms are independent, an agent has to pass t independent tests to survive during t periods. While the test in the final period may be relatively uninformative about skill, because p_t^* is very high in that period, the early tests, when p_t^* is moderately high, remain informative.

5.2. Intuition 2: Failure Rates Conditional on Skill Change Over Time

While the first explanation that focused on the diagnosticity of survival provides an intuition for why the proportion of high skill agents may eventually decrease when noise terms are constant and drawn from a long-tailed distribution, it does not explain why the hazard function of the noise term matters. We now provide an intuition for this.

In every period, w percent of all agents "fail" and are removed from the population. The change in the proportion of high skill agents depends on the proportion of high skill and low skill agents among the failures. Suppose, for example, that almost all of the agents that fail have low skill. In this case, it is intuitively clear that the proportion of high skill agents among the survivors will increase. More generally, if the probability of failure is higher for low skill agents, the proportion of low skill agents decreases and hence, average skill increases (see Lemma 1 in Appendix A).

To understand when selection increases the proportion of low skill agents, we thus need to understand when low skill agents are, in every period, less likely to fail than high skill agents. An agent fails whenever its performance is among the w percent lowest performances. If p_{t+1}^* is defined as the maximum performance among the failures in period $t+1$, we can say that an agent fails in period $t+1$ whenever its performance in period $t+1$ is at or below p_{t+1}^*.

The difference between a high and low skill agent can then be stated as follows: (1) A low skill agent j

(with skill equal to $u_j = 0$) fails in period $t+1$ whenever $\varepsilon_{j,t+1} \le p^*_{t+1}$. The probability of failure in period $t+1$ for a low skill agent is thus the probability that $\varepsilon_{j,t+1}$ is at or below p^*_{t+1}. (2) A high skill agent i (with skill equal to $u_i = 1$) fails in period $t+1$ whenever $1 + \varepsilon_{i,t+1} \le p^*_{t+1}$. Formulated differently: a high skill agent i fails when $\varepsilon_{i,t+1}$ is at or below $p^*_{t+1} - 1$. The probability of failure in period $t+1$ for a high skill agent is thus the probability that $\varepsilon_{i,t+1}$ is at or below $p^*_{t+1} - 1$. This formulation holds the key to understanding the difference between independent and constant noise terms.

Consider the case with independent noise terms. The noise terms are, for all agents and all periods, drawn from the same distribution. Therefore, the noise term for a high skill agent i, $\varepsilon_{i,t+1}$, is drawn, in all periods $t+1$, from a distribution identical to that of $\varepsilon_{j,t+1}$, the noise term for a low skill agent j. Because $\varepsilon_{i,t+1}$ and $\varepsilon_{j,t+1}$ are independently drawn from the same distribution, it is clear that the probability of the low skill agent failing is higher than the probability of the high skill agent failing: $P(\varepsilon_{j,t+1} \le p^*_{t+1}) > P(\varepsilon_{i,t+1} \le p^*_{t+1} - 1)$. Because this holds for any period $t+1$, the proportion of high skill agents always increases (as long as the proportion of high skill agents is below 1).

Consider now the case with constant noise terms. Let us first focus on the probability that a low skill agent fails in period 2. A low skill agent j fails in period 2 (and not in period 1) if his or her performance is sufficiently high in period 1 and at or below the threshold for survival in period 2. Formally, a low skill agent j fails in period 2 if $p_{j,1} = \varepsilon_{j,1} > p^*_1$ and $p_{j,2} = \varepsilon_{j,1} \le p^*_2$. What matters for failure in period 2 is thus the distribution of performance in period 2 *conditional on survival during period* 1.

Suppose now that $p^*_2 = p^*_1 + y$, where $y > 0$. Agents with a performance above p^*_1 and at or below $p^*_1 + y$, for some $y > 0$, fail in period 2. The probability that a low skill agent j, who survived period 1, fails in period 2 is thus

$$P(p^*_1 + y \ge p_{j,1} > p^*_1 \mid p_{j,1} > p^*_1, u_j = 0) = \frac{P(p^*_1 + y \ge p_{j,1} > p^*_1 \mid u_j = 0)}{P(p_{j,1} > p^*_1, u_j = 0)}.$$

Because $p_{j,1} = \varepsilon_{j,1}$ when $u_j = 0$, this conditional probability equals

$$P(p^*_1 + y \ge p_{j,1} > p^*_1 \mid p_{j,1} > p^*_1, u_j = 0) = \frac{P(p^*_1 + y \ge \varepsilon_{j,1} > p^*_1)}{P(\varepsilon_{j,1} > p^*_1)}. \quad (1)$$

Imagine now that w is small. Only a small fraction of all agents are eliminated in each period. Thus, only agents with a performance just above p^*_1 (the threshold in period 1) are eliminated during period 2. That is, y is close to 0. Formally, let $y \to 0$ in Equation (1). As $y \to 0$, the right-hand side converges to $h_\varepsilon(p^*_1) = f_\varepsilon(p^*_1)/P(\varepsilon_{j,1} > p^*_1)$, the hazard function of the noise distribution at p^*_1 (Ross 2000, p. 220).

Now, let us focus on the probability that a high skill agent fails in period 2. A high skill agent i fails in period 2 if $p_{i,1} = 1 + \varepsilon_{i,1} > p^*_1$ and $p_{i,2} = 1 + \varepsilon_{i,1} \le p^*_2$. The probability that a high skill agent fails in period 2 thus depends on the distribution of performance, conditional upon survival during period 1: $f_{p_{i,1} \mid p_{i,1} > p^*_1}(p_{i,1} \mid p_{i,1} > p^*_1)$. Again, suppose that $p^*_2 = p^*_1 + y$. The probability that a high skill agent i, who survived period 1, fails in period 2 is

$$P(p^*_1 + y \ge p_{i,1} > p^*_1 \mid p_{i,1} > p^*_1, u_i = 1) = \frac{P(p^*_1 + y \ge p_{i,1} > p^*_1 \mid u_i = 1)}{P(p_{i,1} > p^*_1, u_i = 1)}.$$

Because $p_{i,1} = 1 + \varepsilon_{i,1}$, this conditional probability equals

$$P(p^*_1 + y \ge p_{i,1} > p^*_1 \mid p_{i,1} > p^*_1, u_i = 1) = \frac{P(p^*_1 + y - 1 \ge \varepsilon_{i,1} > p^*_1 - 1)}{P(\varepsilon_{i,1} > p^*_1 - 1)}.$$

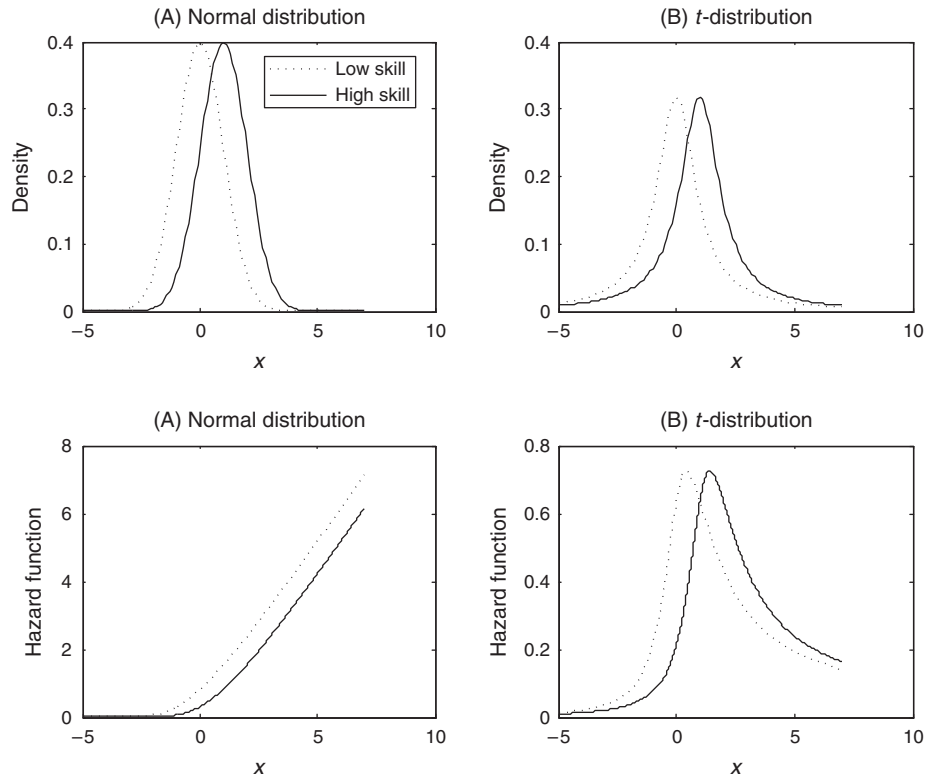
As $y \to 0$, this expression converges to $h_\varepsilon(p^*_1 - 1) = f_\varepsilon(p^*_1 - 1)/P(\varepsilon_{i,1} > p^*_1 - 1)$, the hazard function of the noise distribution at $p^*_1 - 1$.

In summary, whether low or high skill agents are more likely to fail during period 2 depends on the hazard function of the noise term distribution. Low skill agents are more likely to fail than high skill agents if the hazard function at p^*_1, $h_\varepsilon(p^*_1)$ is larger than the hazard function at $p^*_1 - 1$, $h_\varepsilon(p^*_1 - 1)$. It follows that low skill agents are more likely to fail than high skill agents whenever $h_\varepsilon(x) > h_\varepsilon(x-1)$, i.e., whenever the hazard function of the noise distribution is an increasing function.

This result can be restated in terms of the hazard functions for the performance distributions of low and high skill agents. The hazard function of the performance distribution for a low skill agent, $h_{p_{j,t},u=0}(x)$, is equal to the hazard function for the noise distribution $h_\varepsilon(x)$, since $p_{j,t} = \varepsilon_{j,1}$ for low skill agents. The hazard function of the performance distribution for a high skill agent, $h_{p_{i,t},u=1}(x)$, is equal to the hazard function for the noise distribution evaluated at $x-1$, $h_\varepsilon(x-1)$, since the equation $\varepsilon_{j,1} = p_{i,t} - 1$ holds for high skill agents. It follows that the inequality $h_\varepsilon(x) > h_\varepsilon(x-1)$ can be restated as $h_{p_{j,t},u=0}(z) > h_{p_{i,t},u=1}(z)$, i.e., the hazard function for low skill agents lies above the hazard rate function for high skill agents.

As Figure 5(A) shows, the hazard function is an increasing function when the noise term is drawn from a normal distribution. Moreover, the hazard rate function for low skill agents always lies above the hazard rate function for high skill agents. As a result, a low

Figure 5. Upper Panels: Density of Performance for Low and High Skill Agents When the Noise Distribution Is Drawn from (A) a Normal Distribution and (B) a t-Distribution. Lower Panels: Hazard Functions for High and Low Skill Agents



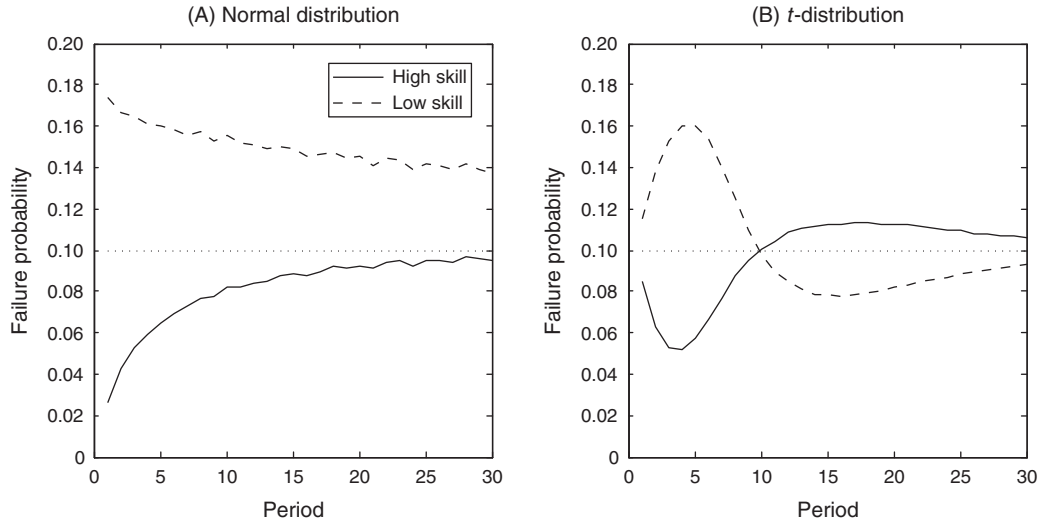
skill agent is, in every period, more likely to fail than a high skill agent. To demonstrate this, Figure 6 plots how the probability of failure varies over time for high and low skill agents (for the case when there is only selection and no replacement). Specifically, Figure 6 plots the probability of failure conditional on survival, $P(p_{i,t} \leq p_t^* \mid \forall j < t: p_{i,j} > p_j^*, u_i = k)$ for high and low skill agents. The probability of failure is always higher for low skill agents when the noise terms are drawn from a normal distribution (Figure 6(A)). When the noise term is drawn from a t-distribution, however, the hazard function for the high skill agents eventually becomes higher than the hazard function for the low skill agents (Figure 5(B)). The fact that the hazard functions cross implies that the probability of failure will eventually become higher for high skill agents than for low skill agents (Figure 6(B)).

Why do high skill agents eventually become more likely to fail than low skill agents when the noise term is drawn from a t-distribution and remains constant? The reason is that, when the noise term is drawn from such a "long-tailed" distribution, low skill agents who have survived tend to have very high performances. Of course, low skill agents initially tend to have lower performances than high skill agents. As a result, low skill agents are initially more likely to fail. Once these poor performers are removed from the population, however, the remaining low skill agents, who have survived the early periods, tend to have very high performance. Because there has been less selection operating on high skill agents—their high skill buffers them from selection—there may in fact be more high skill agents with a medium level of performance. Because the performance threshold increases over time, it becomes more likely that high skill agents have a performance level just above the threshold. Eventually, when many high skill agents have been removed, the surviving high and low skill agents have almost identical performance distributions and hence, are equally likely to fail.

5.3. Relation to Prior Work on Luck and Performance

The mechanism that drives our main result is related to the mechanisms discussed in prior work on the role of luck in explaining performance patterns. Specifically, it is known from prior work that an extremely high level of performance may not be diagnostic of high skill when the noise term is drawn from a fat-tailed distribution such as the t-distribution (Weibull et al. 2007, Denrell and Liu 2012). In this sense, our results follow from prior work. Nevertheless, our results are not a direct consequence of past work. For one thing, our analytical results (Theorems 1 and 2) enable us to go beyond prior work in precisely characterizing the

Figure 6. How the Failure Probability (When There is No Replacement) Varies Over Time for High and Low Skill Agents When the Noise Terms are Constant and Drawn from (A) a Normal Distribution and (B) a t-Distribution



Note. Minor oscillations in the graphs are due to numerical imprecision in the computations.

noise distributions for which average skill may decline over time. Note also that our results in this paper focus on expected skill conditional on performance *above* a given level (the performance threshold), while past work has focused on expected skill conditional on performance *equal to* a given level; a subtle difference but important for the formal proofs. More important, this paper shows that past work about the conditions under which extreme performance may not be diagnostic of high skill is only relevant in settings where the noise terms are dependent (noise terms do not change or are added up, like in a random walk). Whenever the noise terms are redrawn, survival during many periods, even if it requires extremely high levels of performance, is still diagnostic of high skill.

6. Alternative Assumptions About Selection and Replacement

The model on which our formal analysis has focused is admittedly a toy model that hardly maps onto any naturally occurring settings. Do our results also hold in more realistic settings? Here we explore this issue by simulating models with different assumptions about how selection and replacement operate.

6.1. Replacement

Our main result about the emergence of a non-monotonic pattern in the proportion of high-skill agents also holds when the agents who exit the population are replaced by new agents. More specifically, consider the setting of Section 4.3, where the noise terms are constant. Here, we assume that everything remains the same, except that each eliminated agent i is replaced by a new agent j with performance $p_j = u_i + \varepsilon_j$, where ε_j is independently drawn from density $f_\varepsilon(\varepsilon)$. The probability that a new agent has high skill ($u_j = 1$) is 50%. We have:

Theorem 4. *Theorem 2 holds in this setting as well.*

Proof. See Appendix D. □

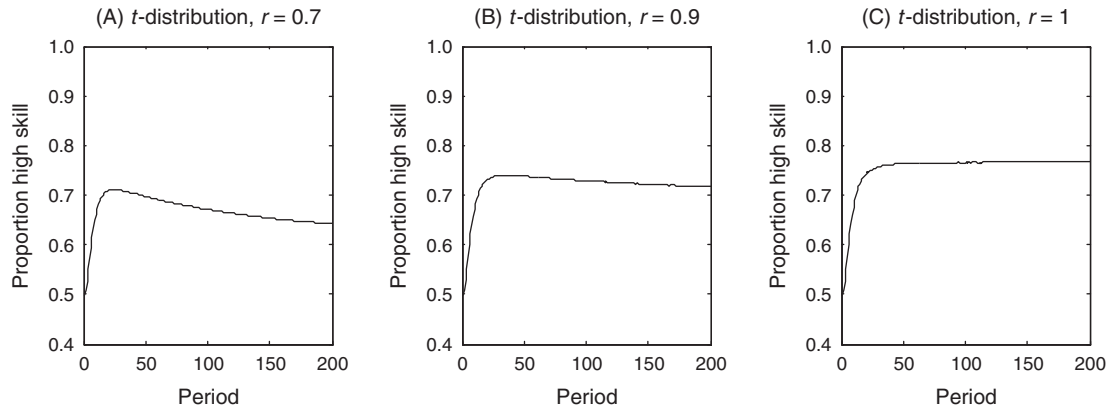
6.2. Replacement from a Changing Skill Pool

We have assumed that the probability that a new agent has high skill is 50% in all periods. Entrants may learn from survivors, however. If so, the probability that an entrant has high skill may increase over time. To model this, suppose the probability that a new agent entering in period t has high skill equals $q_t = r\pi_t + (1-r)0.5$. The probability that a new agent entering in period t has high skill is thus a weighted average between the proportion of agents with high skill among the survivors (π_t) and 50%. The weight on π_t is $r \in [0,1]$. It can be interpreted as the probability that an entrant is able to copy a survivor. Simulations show that the basic result holds even if $r > 0$ as long as r is not too close to 1. Figure 7 shows how the proportion of high skill agents changes over time in a simulation with $n = 50$ firms, the noise terms are drawn from a t-distribution with one degree of freedom, $w = 0.1$, and (A) $r = 0.7$, (B) $r = 0.9$ and (C) $r = 1$. The proportion of high skill agents eventually declines unless $r = 1$.

6.3. Probabilistic Selection

The models examined so far assumed that all agents with the lowest w percent performances were eliminated and replaced in each period. It may be more realistic to assume that selection operates in a probabilistic

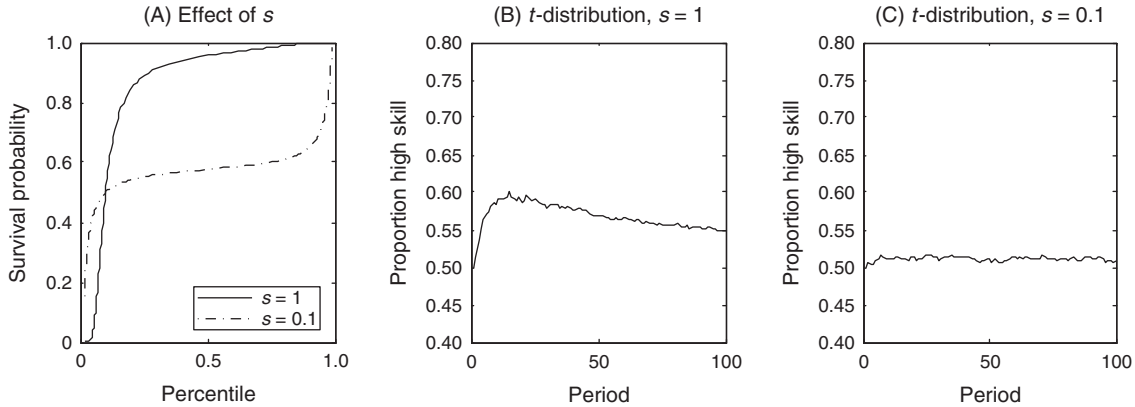
Figure 7. How the Proportion of High Skill Agents Changes Over Time in a Simulation When There is Replacement from a Changing Skill Pool, the Noise Terms are Drawn from a t-Distribution with One Degree of Freedom, $w = 0.1$, and (A) $r = 0.7$, (B) $r = 0.9$, and (C) $r = 1$ (Based on 10,000 Simulations, Each with 50 Agents)



fashion: agents with relatively low performance are more likely than agents with relatively good performance to fail and be replaced. To model this, let p_t^w be the level of performance such that w percent of all agents have a lower level of performance in period t. Previously we assumed that all agents with a performance below p_t^w were eliminated, and all agents with a performance above p_t^w survived. To model probabilistic selection, we assume instead that the probability that agent i survives period t is $1/(1 + \exp(-s(p_{i,t} - p_t^w))$. Here $s \geq 0$ is a parameter that regulates the extent to which selection is sensitive to relative performance. A larger value of s implies that the probability of survival is more sensitive to relative performance $(p_{i,t} - p_t^w)$. When $s \to \infty$ all agents with a performance above p_t^w survive and all agents with a performance below that threshold are replaced. When $s = 0$, the probability of survival equals 0.5 for all agents and is thus unrelated to relative performance.

Simulations show that the basic result holds even for probabilistic selection unless the value of s is low. To illustrate this, suppose $w = 0.1$. Here, p_t^w is equal to the tenth percentile of the performance distribution. Figure 8(A) shows the survival probability, in the first period, for different percentiles of the performance distribution when $s = 1$ and $s = 0.1$. The survival probability is 0.5 for a performance level equal to the tenth percentile (i.e., $p_{i,1} = p_1^{0.1}$). If $s = 1$, the survival probability quickly increases towards one for higher performance levels, while if $s = 0.1$, the survival probability remains at a moderate level (around 55%) unless performance is very high. Panels (B) and (C) in Figure 8 show how the proportion of high skill agents changes over time when $s = 1$ and $s = 0.1$. In each simulation, there are $n = 50$ agents, high and low skill are initially equally likely, the noise terms are constant during the lifetime of an agent, and each eliminated agent is replaced with a new agent. The proportion of high skill agents initially increases, but eventually declines when $s = 1$. When $s = 0.1$, and even high performing agents may fail to survive, the proportion initially increases but then reaches a plateau.

Figure 8. (A) The Impact of the Value of s on the Probability of Survival. (B) and (C) How the Proportion of High Skill Agents Changes Over Time When the Noise Term is Drawn from a t-Distribution with (B) $s = 1$ and (C) $s = 0.1$ (Based on 10,000 Simulations, Each with 50 Agents)



6.4. Fixed Selection Threshold

The proportion of high skill agents only decreases over time when the performance required for survival increases over time. The assumption that selection operates on relative performance—the agents with the w percent lowest levels of performances fail and are replaced—ensures that the performance required for survival does increase over time. Suppose, alternatively, that all agents with a performance above a fixed cutoff survive. In such a model, the proportion of high skill agents monotonically increases over time (if there is no replacement) or increases over time until it reaches an equilibrium (if there is replacement) and never decreases. The implication is that the proportion of high skill agents can only decrease if survival depends on relative rather than absolute performance.

6.5. Size Changes

Our model focused on the elimination of poorly performing units and assumed that all agents were of equal size. In contrast, several evolutionary models in management have focused on growth. For example, Nelson and Winter (1982) explored the consequences of the assumption that profitable firms grow and unprofitable firms contract. Changes in traits among survivors may be relatively uninteresting in such a model. What matters are size-weighted traits: whether large firms are more likely to use "efficient" technologies.

Do our results hold also in a model where relative performance determines growth? To explore this, we simulated a simple version of a model in which relatively high performance leads to growth in size while relatively low performance leads to contraction in size. The model is specified as follows. There are $n = 50$ firms. The performance of firm i is $p_i = u_i + \varepsilon_i$. There are only two levels of u_i: high ($u_i = 1$) and low ($u_i = 0$), each equally likely. The level of u_i remains the same during the lifetime of a firm, as does the noise term ε_i. There is no selection in this model: We ignore selection to focus on size changes. All firms survive all periods and there are no entrants. Firms do change in size, however. At the start, each firm has size $s_{i,1} = 1$. Firm size changes, in each period, as follows. Let z_t be the market-share weighted performance in period t: $z_t = \sum_{i=1}^{n} m_{i,t} p_i$, where $m_{i,t} = s_{i,t}/\sum_{j=1}^{n} s_{j,t}$ is the market share of firm i at the start of period t. Every firm with a performance above z_t increases in size by 10%: if $p_i > z_t$, then $s_{i,t+1} = 1.1 s_{i,t}$. Every firm with a performance below z_t contracts in size by 10%: if $p_i < z_t$, then $s_{i,t+1} = 0.9 s_{i,t}$. Firms with a performance equal to z_t do not change in size: if $p_i = z_t$, then $s_{i,t+1} = s_{i,t}$.

Because all firms survive in this model, the proportion of high skill firms remains the same in all periods, on average 50%. The market-share weighted average of skill, i.e., $\sum_{i=1}^{n} m_{i,t} u_i$, changes over time, however. Figure 9 plots how the market-share weighted average of skill changes over time when (A) ε_i is drawn from a normal distribution with mean 0 and variance 1 or (B) ε_i is drawn from a t-distribution with mean 0 and one degree of freedom. When ε_i are drawn from a normal distribution, the market-share weighted average of u_i increases over time. The reason is that high performing firms will grow and increase their market share. A high market share thus indicates relatively high performance. Moreover, when ε_i are drawn from a normal distribution, high performance is an indicator that the firm is likely to have high skill ($u_i = 1$). Eventually, the highest performing firm among the 50 firms will reach a market share close to one. When ε_i are drawn from a normal distribution, the highest performing firm is very likely to have high skill.

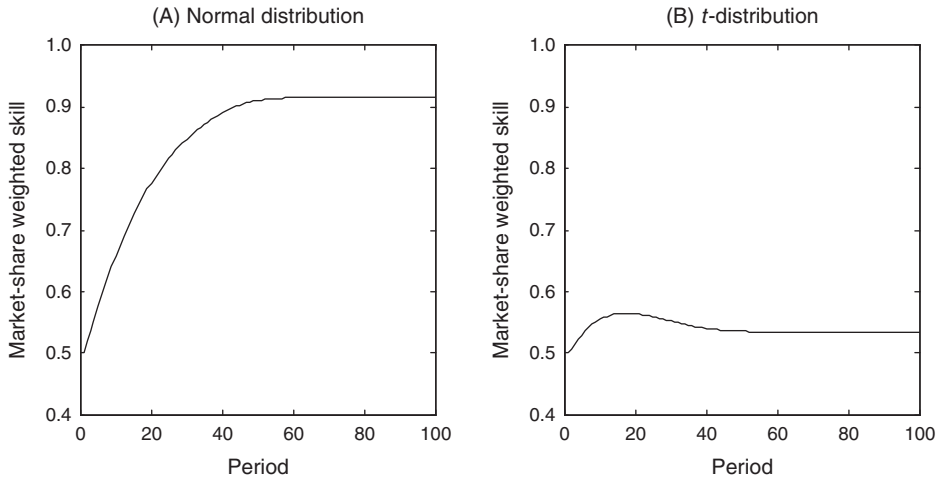
By contrast, when ε_i are drawn from a t-distribution, the market-share weighted average of skill initially increases but then decreases. The reason is that when ε_i are drawn from a t-distribution, higher performance is not necessarily an indication that the firm is more likely to have high skill. Firms with moderately high performance are more likely to have high skill than firms with average performance, but firms with very high levels of performance are no more likely than firms with average performance to have high skill. Firms with moderately high performance grow initially, but eventually only firms with very high levels of performance grow, while others contract. The firm with the highest level of performance eventually reaches a market share close to one. When ε_i are drawn from a t-distribution, a firm with such a high level of performance is not much more likely than an average performing firm to have high skill.

6.6. Imitation

Cultural selection can operate via imitation as well as replacement. An agent i may observe the performance and strategy of another agent j and switch strategy if agent j has superior performance and uses a strategy different from what i is using. A model with these features can also reproduce our basic result.

Specifically, suppose there are n agents. There is no selection or exit in this model: all agents survive all periods. The model focuses on how agents switch between two "strategies." Each agent can, in each period t, use one of two possible "strategies": $u_i = 1$ or $u_i = 0$. Thus, we treat the two "skill-levels" as two possible strategies that an agent can adopt. The performance of an agent that uses strategy $u_i = k$ in period t is $p_{i,t} = k + \varepsilon_i$. Here, ε_i is a noise term, drawn from a noise distribution f_ε. The noise term (and thus performance) remains the same until agent i changes strategy. Initially, at the start of period 1, 50% of all agents uses strategy $k = 1$. Strategy change occurs as follows. In each period t, each agent i selects at random an agent j

Figure 9. How the Market-Share Weighted Average of Skill Changes Over Time When Relatively High Performing Firms Grow and the Noise Term is Drawn from (A) a Normal Distribution or (B) a t-Distribution (Based on 5,000 Simulations, Each with 50 Firms)



and observes his or her performance and strategy. If agent j has higher performance than agent i ($p_{j,t} > p_{i,t}$), and j uses a strategy different from agent i (i.e., $u_j = 1$ while $u_i = 0$, or $u_j = 0$ while $u_i = 1$), then agent i switches strategy to the strategy that j uses. If agent i switches strategy, his or her new performance is $p_i = u_j + \varepsilon_i$, where u_j is the strategy that j used, and ε_i is redrawn, independently, from the noise distribution f_ε.

Simulations show that when f_ε is a normal distribution, then the proportion of agents that uses the strategy $k = 1$ increases over time (i.e., agents tend to switch to $k = 1$ over time). When f_ε is a t-distribution with one degree of freedom, however, the proportion of agents that uses the strategy $k = 1$ initially increases but eventually decreases. The initial increase occurs because agents that use $k = 1$ are initially more likely to have high performance. The eventual decrease occurs because after the initial periods of switching to $k = 1$, the agents that stick with $k = 0$ tend to have higher performance than the agents that stick with $k = 1$. The reason is that most agents having $k = 0$ initially that have low performance are likely to have switched strategy to $k = 1$. Eventually, the only agents that stick with $k = 0$ are those who were lucky, with a high value of ε_i.

7. Additional Robustness Checks

7.1. Auto-Regressive Performance

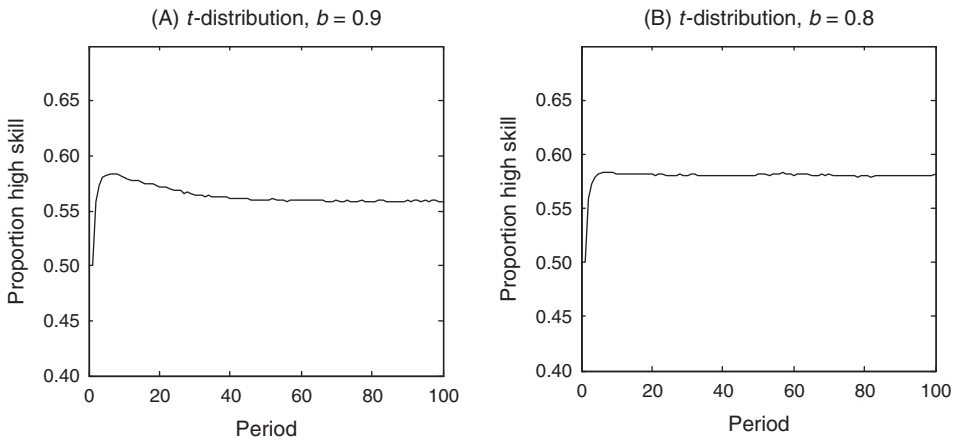
In Section 2, we showed that inefficient selection can happen when performance follows a random walk (Figure 3). The random walk specification assumes that a random draw during the first period of the lifetime of an agent $\varepsilon_{i,1}$ remains relevant during the lifetime of an agent. In some cases it may be more realistic to assume that the impact of $\varepsilon_{i,t}$ decays over time. This can be modeled by assuming that performance follows an autoregressive structure. Specifically, consider an agent that entered in period t. His or her performance in period t, $p_{i,t}$, equals his or her skill u_i plus a noise term, $\varepsilon_{i,t}$: $p_{i,t} = u_i + \varepsilon_{i,t}$. His or her performance in period $t+1$ equals $p_{i,t+1} = bp_{i,t} + \varepsilon_{i,t+1}$. Similarly, $p_{i,t+2} = bp_{i,t+1} + \varepsilon_{i,t+2}$, etc. When $b = 1$, this specification is identical to the random walk specification above. Lower values of b imply lower levels of dependence between performances in consecutive periods (i.e., lower autocorrelation).

Simulations show that the proportion of high skill agents can decline even when b is lower than 1. To illustrate this, suppose there are 50 agents. Each agent is equally likely to have high ($u_i = 1$) or low ($u_i = 0$) skill. In each period t the agents with the w percent lowest performance ($p_{i,t}$) are removed from the population. Each eliminated agent is replaced by a new agent. The probability that a new agent has high skill ($u_j = 1$) is 50%. Figure 10 plots how the proportion of high skill agents changes over time when (A) $\varepsilon_{i,t}$ are drawn from a t-distribution with one degree of freedom, $w = 0.35$, and $b = 0.9$; (B) $\varepsilon_{i,t}$ are drawn from a t-distribution with one degree of freedom, $w = 0.35$, and $b = 0.8$. As shown, the proportion of high skill agents eventually declines in the first case, when $b = 0.9$, while the proportion of high skill agents initially increases and then reaches a plateau when $b = 0.8$.

7.2. Many Levels of Skill

Our analysis assumed that there were two possible levels of skill. What if there are many levels of skill or possibly a continuous skill distribution? Simulations show that the basic result can hold even if there is a continuous skill distribution. Suppose, for example, that u_i is drawn from a normal distribution with mean 0 and variance 1 and remains the same during the lifetime of a firm. The performance of firm i in period t, $p_{i,t}$,

Figure 10. How the Proportion of High Skill Agents Changes Over Time When Performance Follows an Autoregressive Process, $p_{i,t+1} = bp_{i,t} + \varepsilon_{i,t+1}$, and (A) $\varepsilon_{i,t}$ are Drawn from a t-Distribution with One Degree of Freedom, $w = 0.35$, and $b = 0.9$ and (B) $\varepsilon_{i,t}$ are Drawn from a t-Distribution with One Degree of Freedom, $w = 0.35$, and $b = 0.8$



Note. Each graph is based on 10,000 simulations, each with 50 firms, where the w percent firms with the lowest performances are replaced in every period.

is equal to the sum of its capability, u_i, and a noise term, $\varepsilon_{i,t}$: $p_{i,t} = u_i + \varepsilon_i$. In each period t, the 10% lowest performing firms (based on $p_{i,t}$) are removed from the population. Each eliminated firm is replaced by a new firm with performance $p_{j,t} = u_j + \varepsilon_j$, where u_j is drawn from a normal distribution with mean 0 and variance 1 and ε_j is drawn from density f_ε. The noise term ε_j remains the same during the lifetime of a firm. Figure 11 shows how the average level of skill changes over time (based on 10,000 simulations each with $n = 50$ firms). When f_ε is a normal distribution with mean 0 and variance 1, the average level of skill increases over time. When f_ε is a t-distribution with one degree of freedom, the average level of skill initially increases but eventually decreases.

The case of a continuous skill distribution is more difficult to handle formally than a binary distribution. What can be demonstrated formally is that whenever the hazard function of the noise distribution is increasing (which is true, for example, for a normal distribution) then average skill increases over time. Formally, suppose that skill (i.e., u_i) is drawn from a distribution with density g_u, where g_u can be continuous or discrete. As before, we denote the density of the noise term by f_ε and suppose performance equals skill plus the noise term drawn in period 1: $p_{i,t} = u_i + \varepsilon_{i,1}$.

Theorem 5. *If the hazard function of the distribution of the noise term $h_\varepsilon(x)$ is an increasing function of x, then average skill in the population increases over time.*

Proof. See Appendix E. □

We have not been able to derive necessary or sufficient conditions for when average skill eventually declines. On the basis of simulations, however, we conjecture that average skill eventually decreases if the skill distribution g_u has an increasing hazard function (such as the normal) and the noise distribution f_ε has a decreasing or eventually decreasing hazard function (such as the t-distribution with one degree of freedom). By contrast, if the skill distribution g_u and the noise distribution f_ε both have decreasing hazard functions (for example, they are both t-distributions), then average skill increases over time. The general lesson seems to be that average skill can decrease over time when the noise terms are drawn from a distribution with a "longer" tail than the skill distribution (cf. Denrell and Liu 2012).

8. When Are the Results Relevant?

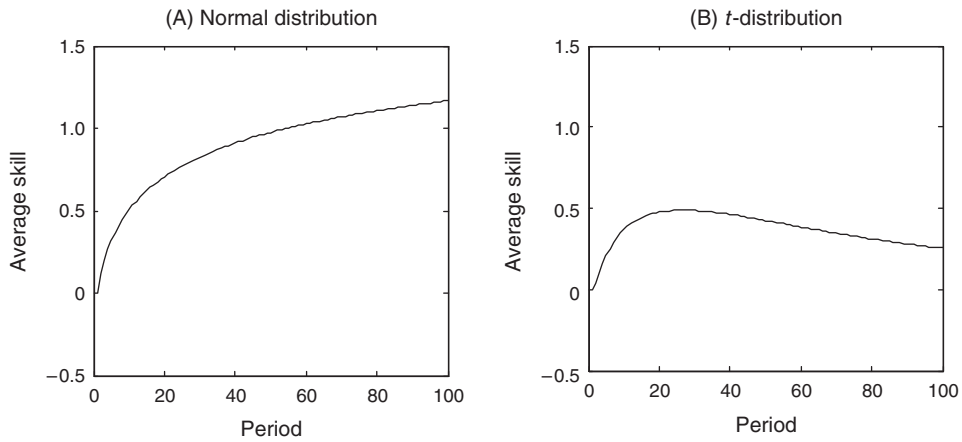
Our model shows that the proportion of high skill agents eventually declines as a result of selection when four conditions are satisfied:

(1) The hazard function of the noise distribution is always decreasing (Theorem 2(b)) or eventually decreasing (Theorem 2(c)). Theorem 3 also shows that the proportion of high skill agents will converge to 50% as $t \to \infty$ when the distribution of the noise term is "long-tailed," a sub-class of heavy-tailed distributions.

(2) The impact of skill on performance is limited. Theorem 2 assumes that there are only two levels of skill, but Figure 11 shows that the average skill also declines over time when skill is drawn from a normal distribution (which is not long-tailed) and the noise term is drawn from a t-distribution (which is long-tailed).

(3) Performance in a given period strongly depends on performance in earlier periods. Theorem 2 assumes that the level of performance remains the same during the lifetime of an agent, but Figure 3 shows that the proportion of high skill agents may also decline over

Figure 11. How the Average Level of Skill Changes Over Time When Skill is Drawn from a Normal Distribution with Mean 0 and Variance 1 and the Noise Term is Drawn from (A) a Normal Distribution with Mean 0 and Variance 1 (B) a t-Distribution with One Degree of Freedom



Note. Each graph is based on 10,000 simulations, each with 50 firms, where the 10% firms with the lowest performances are replaced in every period.

time when the noise term follows a random walk. Figure 10 shows that we can get the same result when performance follows an autoregressive process, $p_{i,t+1} = bp_{i,t} + \varepsilon_{i,t+1}$, as long as b is sufficiently close to one.

(4) The threshold level of performance required for survival increases over time, as a result of selection based on relative performance.

In what realistic settings do these technical conditions hold?

Consider first the conditions regarding the noise distribution. The concept of a "long-tailed" distribution is perhaps the easier one to understand intuitively. Informally, a distribution is long-tailed if the probability of an extremely high level of performance for an agent with high skill (the probability that $1 + \varepsilon_i > c$ when $c \to \infty$) is about the same as the probability of an extremely high level of performance for an agent with low skill (the probability that $\varepsilon_i > c$ when $c \to \infty$). Intuitively, if the error term is drawn from a long-tailed distribution, then the level of skill has almost no impact on the probability of an extreme event. It follows that an extreme event (a very high level of performance) is not informative about the level of skill of the agent.

Several well-known heavy-tailed distributions are long-tailed, including the Pareto distribution, the Log-normal distribution, and the Cauchy distribution (i.e., the t-distribution with one degree of freedom). Moreover, researchers have shown that these distributions fit several important economic outcomes and social indicators. The Pareto distribution and the log-normal fit the distribution of wealth (e.g., the wealth of the top 1% population follows a Pareto distribution whereas the wealth of the rest of the population follow a log-normal distribution, see Levy and Levy 2003). Stock market returns fit a t-distribution (Blattberg and Gonedes 1974). Our model applies in settings where the performance of an agent equals a skill component plus a random draw from these distributions. Our model does not apply in settings where performance is subject to a noise term drawn from a light-tailed distribution such as the normal distribution.

One setting where our results are relevant is firm size. The firm size distribution fits a log-normal distribution with an upper Pareto tail (Growiec et al. 2008). Moreover, the empirical evidence suggests that random variation, rather than systematic variation in growth rates, account for most of the variance in firm size (Geroski 2005, Coad 2007). This suggests that repeated selection based on firm size (which could occur if there are economies of scale, implying that size strongly impacts performance) can lead to a decline in average firm "capability" over time. Our results also have interesting implications for firm growth rates. Evidence suggests that firm growth rates follow a Laplace distribution (Bottazzi et al. 2001, Bottazzi and Secchi 2003)[3] with a low degree of autocorrelation (Coad 2007). The Laplace distribution is a heavy-tailed distribution, which is not "long-tailed." Thus, the result that average "capability" may decline as a result of repeated selection does not hold for this distribution. However, the hazard function for the Laplace distribution is eventually constant (see Table 1), and Theorem 2(d) implies that, for such a noise distribution, the proportion of high skill agents does not increase over time as a result of repeated selection, but reaches a plateau. This result may be relevant to explaining the puzzling weak association between productivity and growth (Bottazzi et al. 2001, 2010): Repeated selection based on growth will not lead to an increasing proportion of highly productive firms.

The second condition is realistic in settings where variation in skill is limited and unlikely to be responsible for extreme outcomes. Consider trading: it is possible that an individual without skill (e.g., without above average ability to make money in the stock market) might obtain a really high return from trading during one year. Systematic variation in trading ability may exist but explains only a small percentage of the variance in trading results. In other tasks, it is inconceivable that low skilled individuals will obtain extremely high outcomes. Consider the 100-meter dash. An unskilled individual who runs 100 meters in 15 seconds, on average, will not, by luck, be able to run under 10 seconds. Many economically relevant tasks are similar: low quality producers will not, by luck, be able to turn out high quality products. Nevertheless, economic outcomes such as profitability, which depend on demand in addition to technical skill, are subject to many uncontrollable factors. For example, demand for cultural products can be very difficult to forecast (Salganik et al. 2006). As a result, it is conceivable that a low-quality producer, who happened to produce what became a fashionable product, would become very profitable.

The third condition, regarding the dependence of current performance on past performance, is relevant in economic and social systems in which there are strong path-dependencies (Merton 1968, Lynn et al. 2009) such that high past performance makes future high performance more likely. For example, producers with a high market share may, as a result of network externalities or social influence, be more likely to obtain a high market share in the future. Similarly, individuals who have performed well in their job may, as a result of a self-fulfilling prophecy, be more likely to perform well in the future. Performance in consecutive periods will also be dependent when performance accumulates over time, as in a random walk (Levinthal 1991, Denrell 2004). The third condition is less likely to be satisfied in settings where technological or social changes imply that capabilities or resources developed in the past become less applicable or relevant in the future.

The fourth condition—an increasing threshold level of performance required for selection—seems realistic in many economic and social settings where there is repeated selection, and survival requires beating other agents. For example, students as well as managers get repeatedly selected for more advanced degrees or positions. Surviving to the next "stage" often requires beating the other available candidates. Survival in most markets requires offering a product or service that is competitive compared to the offerings of competing firms (Barnett and Hansen 1996). The fourth condition would not be satisfied if all agents that passed an absolute threshold survived.

When are these four conditions applicable to the selection of firms? An example consists in firms producing a product with success subject to strong network externalities. These firms may differ in their research and development capabilities, but their level of performance will strongly depend on whether their product became popular early on and generated a large installed base; an outcome which is related to product quality but also depends on many factors beyond the control of management. If product demand is subject to strong network externalities, performance will be subject to a rich-get-richer dynamic (Arthur 1989, Barabasi and Albert 1999) that can generate a heavy-tailed distribution of outcomes (Simon 1955, Barabasi and Albert 1999). Over time, firms with poor performance will exit the industry, and firms with high performance will grow. Eventually the industry will be dominated by one or a few firms with high market shares. Our model implies that the average research and development capabilities of survivors may decline systematically over time, after an initial increase (compare Figure 9).

When are the four conditions applicable to selection of individuals within organizations? Selection among individuals in academia provides a possible illustration. Academics are evaluated for jobs, tenure, and chairs based primarily on their research output such as high prestige publications, impact, and citations. Both the number of publications and citations follow heavy-tailed distributions such as the Pareto and the log-normal (Radicchi et al. 2008). Research on the Matthew Effect suggests that evaluation in academia is noisy, and good luck can have a persistent impact because good performance leads to increased attention and resources and improves the chances of high subsequent performance (Merton 1968). Performance is also persistent because both the number of publications and citations are added up throughout a career. Finally, advancement within schools and to universities of higher status often depends on performance relative to peers. Overall, these observations suggest that it is possible that average research skills decreases over time, as academics are subject to more and more selections.

These illustrations of the possibility that average "skill" declines over time are only relevant if evaluators care about such "skill" rather than performance. For example, hiring committees at universities may want to hire highly cited researchers because they believe citations indicate skill or because they believe having highly cited researchers attracts attention and enhances the reputation of the university. Our result would only be relevant in the first case. Similarly, an investor evaluating a firm may only care about its predicted future performance and not directly about its

capability. Even if evaluators only care about performance, however, evaluating skill may be important for predicting future performance. Consider an investor who wants to predict the future performance of a high-technology company that did achieve a dominant position in an industry that is now subject to technological disruption. Is this firm also likely to achieve dominance in the future? Evaluating this requires an assessment of the capabilities of the firm in developing and marketing new technologies. Was the success of the firm the result of such capabilities—which are likely to remain relevant in the future—or the result of good initial fortune—which is unlikely to be replicated? Similarly, evaluating whether a star individual will continue to perform well in a new job requires an assessment of whether his or her job performance is due to transferable skills or to idiosyncratic factors (e.g., favorable interpersonal relationships or affiliation with a high status firm) that do not apply to the new job (Groysberg 2010). Finally, one setting in which an evaluator may care about "skill" is the situation we described at the very beginning of the paper, i.e., an outside observer is interested in imitating a high performing firm. Consider a firm with a large installed base. An outside observer would ideally like to imitate all aspects of this firm, including its large installed base. But the large installed base is a consequence of skill and good initial fortune and an imitator may only be able to imitate the skill component.

A possibility that we have not considered is that "skill" evolves over time. Skills may increase as a result of being exposed to competition (Barnett 2008). If the skills of all firms increase at the same rate, our basic result still holds in the sense that an observer who examines firms that have been subject to moderate levels of selection is more likely to identify the most capable firm than an observer who observes firms subject to many periods of selection. It is possible, however, that the skills of high performing firms increase at a higher rate than the skills of poorly performing firms because high performing firms have the resources to invest in capability building. If so, our main result may not hold: the survivors of selection, with high performance, may end up with the highest level of skill. This does not necessarily imply that an outside observer should imitate the survivors of many periods. An outside observer may not be able to imitate the aspects of skill that are the result of investment. Only firms with high performance have the resources to make the necessary investments and develop these skills. As such, these aspects of skills cannot be immediately imitated but are the byproduct of good performance. The relevant aspects of skill for an outside observer might thus be the initial level of skill, and for this initial level of skill our main result still holds.

9. Implications for Selection Explanations

A selection explanation focuses on changes in the composition of a population instead of changes within the agents making up the population. According to a selection explanation, the reason why most agents in a population have a trait x rather than y is not that agents who had x have switched to y, but that agents with y have left the population through a selection process.

The results in this paper challenge the applicability of some seemingly intuitive selection explanations, but also suggest that a selection perspective can be extended to novel phenomena. The results challenge the intuition that the survivors of "more" selection are "better" than the survivors of "less" selection. It is not necessarily true that a trait x that increases expected performance will become more common in a population as selection continues for more periods (Theorem 2). Moreover, it is not necessarily true that the survivors of more "intense" selection, i.e., selection where a larger percentage of the population is removed (w is larger), have a higher proportion of the trait x that increases expected performance (Lemma 2 and Theorem 3).

At the same time, our results can also be said to extend the scope of selection explanations because our results show that a selection process can account for nonmonotonic effects; effects that may have seemed inconsistent with a selection explanation. For example, consider a researcher who observes that the average level of skill among survivors of selection initially increases but eventually decreases. If this researcher assumes that the survivors of more rounds of selection must have higher expected skill, then he or she would need to postulate some additional mechanism to account for the observed decline. Our results show that such additional mechanisms may not be necessary. Next, we comment briefly on the implications of these challenges and opportunities for selection explanations used in strategic management and organization theory.

9.1. Competition and Density Delay

Our results about more or less intense selection (Lemma 2 and Theorem 3) have implications for discussions about the effect of competition at founding. If only a fixed number of agents can survive, the threshold for survival increases when there is more competition in the sense that are more firms in the industry (Barnett et al. 2003). Are actors that survive such intense competition better? Barnett et al. (2003) suggest that they likely are, implicitly invoking an assumption that "tougher" selection leads to "better survivors." Our results show that this is not necessarily true. Survivors of tougher selection could in fact be worse. In fact, it is possible that very intense competition can lead to an uninformative selection process. Suppose that $p_i = u_i + \varepsilon_i$, where $u_i = 1$ with

probability 0.5 and $u_i = 0$ otherwise. Suppose, further, that higher competition increases the threshold performance, c, required for survival. Theorem 3 shows that if ε_i is drawn from a long-tailed distribution, then $\Pr(u_i = 1 \mid p_i > c) \to 0.5$ as $c \to \infty$. Formulated differently: very intense selection can be uninformative about skill. This observation also suggests a different interpretation of the finding that density at founding has a persistent negative effect on survival rates (Carroll and Hannan 1989). This finding has been attributed to a negative effect of competition on capabilities, but our results show it can be the result of selection.

9.2. Obsolescence

To explain the decline of organizational performance with organizational age for old organizations, researchers have postulated that age or experience has a detrimental effect or that the environment shifts in a way unfavorable to established firms (Hannan et al. 2007; Le Mens et al. 2015a, b). Our results suggest that continued selection can in fact lead to a type of obsolescence: the average skill of the survivors of selection may eventually go down implying that older firms could, as a result of selection rather than adaptation, have lower expected skill than younger firms.

More precisely, if the noise terms are dependent and drawn from a long-tailed distribution, average skill may initially increase, then eventually decrease. Thus, continued selection can explain a nonmonotonic association between age and organizational skill or "capability." Such a non-monotonic effect has previously been attributed to a combination of selection, generating the initial increase, and environmental drift, resulting in the eventual decline (Hannan et al. 2007). Alternatively, the eventual decrease has been attributed to the negative consequences for capability development of being buffered from competition (Barnett and Hansen 1996; Barnett 1997, 2008). Our results show that it is not necessary to postulate environmental drift or capability decline.

This selection-based explanation of obsolescence only predicts a decline in performance in activities that depend on skill and for which the "noise term" is redrawn. The empirical implication is that a decline in performance should only occur in "novel" activities; activities to which previous good fortune does not apply. In activities that continue to draw upon whatever good initial fortune a firm has had, performance should continue to increase. Interestingly, this is broadly consistent with empirical evidence that incumbent firms continue to do well in the absence of radical technological changes (Hill and Rothaermel 2003). For example, Sorensen and Stuart (2000), in their study of the effect of age on organizational innovation, note that the rate of patenting increases with age (suggesting that age has a positive effect). As firms age, however, the citations to their new patents by other firms eventually decrease. The usual interpretation of this decrease is that the knowledge of older firms gradually becomes less relevant for others due to environmental drift. An alternative selection explanation, suggested by our results, is that the patenting rate (which increases over time) reflects past good fortune (e.g., to survive, firms need successful innovations on which they subsequently can build; past success also attracts talented employees, etc.). By contrast, the extent to which new patents by the same firm are cited by others reflects innovative ability. Innovative ability ultimately declines because survival during many periods eventually becomes less diagnostic about innovative ability.

More research is needed to develop this idea further and to thoroughly compare it to the existing explanations. Prevailing explanations attribute the decline in average skill to changes within individual units. By contrast, our selection-based explanation attributes it to changes in the composition of the population: high skill organizations eventually become more likely to fail in any given period. This explanation is in the same spirit as the random-walk model of Levinthal (1991), although this model focused on the liabilities of newness (higher failure rate for young organizations) and adolescence (the failure rate increases and then decreases with age), but not on the liability of obsolescence.

9.3. Career Systems

Many professional organizations, such as consulting firms or university departments, have an "up or out" career system in which the lowest performers are forced to exit while the others are promoted to the next level. Our results suggest that the individuals promoted to the highest levels can be worse than the individuals who do not reach the highest levels. This occurs when initial good fortune has a long-lasting effect (constant noise terms) and noise can substantially impact the probability of extreme performance (long-tailed noise term distribution). In this scenario, the individuals promoted to the highest levels will continue to perform well (because performance depends on past good fortune as well as skill). If they move to a new environment (e.g., a new firm), their expected performance will be lower than those who did not reach the highest levels. Our results imply that this effect can occur due to selection and does not require any change at the individual level.

This analysis offers a novel interpretation of the finding that "stars" hired from another organization perform much worse for the new organization than for their previous employer (Groysberg et al. 2004). This finding has been attributed to laziness resulting from past success or resentment from employees in the

new organization (Groysberg et al. 2008). It has also been suggested that the decline may reflect context-dependent skills (Groysberg et al. 2008). The selection explanation is instead that the skills of the "stars," who have survived many rounds of selection, may not be exceptionally high. In fact, the expected skill of stars might be lower than the expected skill of those who have achieved an intermediate level of career success.

Acknowledgments

The authors thank Bill Barnett, Mike Hannan, Thorbjorn Knudsen, Dan Levinthal, Jim March, and Sidney Winter, for comments.

Appendix A. Proof of Theorem 1

We start by proving the following lemma that applies to the general model setup described in Section 4.1.

Lemma 1. *In the setting described in Section 4.1, the proportion of high skill agents, π_t, increases from period t to period $t+1$ whenever the probability that a low skill agent exits during period $t+1$ is higher than the probability that a high skill agent exits during period $t+1$.*

Proof. Let $\Pr_{H,t+1}$ be the probability that a high skill agent survives during period $t+1$. Similarly, let $\Pr_{L,t+1}$ be the probability that a low skill agent survives during period $t+1$. We can express the proportion of high skill agents at the end of period $t+1$ as a function of the proportion of high skill agents at the end of period t and the survival probabilities of high skill and low skill agents in period $t+1$. We have

$$\pi_{t+1} = \frac{\pi_t \Pr_{H,t+1}}{\pi_t \Pr_{H,t+1} + (1-\pi_t)\Pr_{L,t+1}} = \pi_t \frac{\Pr_{H,t+1}}{\pi_t \Pr_{H,t+1} + (1-\pi_t)\Pr_{L,t+1}}. \tag{A.1}$$

If $\Pr_{H,t+1} > \Pr_{L,t+1}$, then $\Pr_{H,t+1} > \pi_t \Pr_{H,t+1} + (1-\pi_t)\Pr_{L,t+1}$ and $\pi_{t+1} > \pi_t$. □

To prove Theorem 1 by application of this lemma, it is enough to show that in every period, the probability that a low skill agent exits is higher than the probability that a high skill agent exits.

Let $t \geq 1$. Let p_t^* be the maximum level of performance among the agents that failed during period t. The probability that a low skill agent exits in period t is $P(0+\varepsilon_{i,t} \leq p_t^*)$. The probability that a high skill agent exits in period t is $P(1+\varepsilon_{i,t} \leq p_t^*)$. Clearly, $P(0+\varepsilon_{i,t} \leq p_t^*) > P(1+\varepsilon_{i,t} \leq p_t^*)$. Therefore, the probability that a low skill agent exits is higher than the probability that a high skill agent exits in period t.

We have shown that in all periods, the exit probability of a high skill agent is higher than the exit probability of a low skill agent. Lemma 1 implies that π_t increases with t for all t. □

Appendix B. Proof of Theorem 2

Here we characterize the noise distributions for which π_t monotonically increases or monotonically decreases with t. We begin by analyzing how $P(u_i = 1 \mid p_i > c)$ varies with c.

Let $F_\varepsilon(x)$ denote the cumulative density function of the noise term and let $f_\varepsilon(x)$ denote the corresponding density function. We assume that the support of f_ε is of the form $(a, +\infty)$ with $a \in \{-\infty, \mathbb{R}\}$ (for all $x \in (a, +\infty)$, $f_\varepsilon(x) > 0$ and $f_\varepsilon(x) = 0$ elsewhere). Because $p_{i,t} = 0 + \varepsilon_{i,1} = \varepsilon_{i,1}$ for low skill actors, the probability density function of performance for low skill actors is $f_0(x) = f_\varepsilon(x)$, with support (a, ∞), and the cumulative density function is $F_0(x) = F_\varepsilon(x)$. For high skill actors, $p_{i,t} = 1 + \varepsilon_{i,1}$. Therefore, for high skill actors, the density of performance is $f_1(x) = f_\varepsilon(x-1)$, with support $(a+1, \infty)$, and the cumulative density function is $F_1(x) = F_\varepsilon(x-1)$. Let $S_0(x)$ be the survival function for actors with low skill, i.e., $S_0(x) = 1 - F_0(x)$, and let $S_1(x)$ be the survival function for actors with high skill, i.e., $S_1(x) = 1 - F_1(x-1)$. Finally, let $h_0(x) = f_0(x)/S_0(x)$ be the hazard function for low skill agents and $h_1(x) = f_1(x)/S_1(x)$ be the hazard function for high skill agents.

Lemma 2 (Skill and Intensity of Selection). *In the setting of Theorem 2 (noise terms drawn at time of entry from the density $f_\varepsilon(x)$ and no replacement), we have*

(i) *if for all $c > a+1$, $h_1(c) < h_0(c)$ then $P(u_i = 1 \mid p_i > c)$ is an increasing function of c.*

(ii) *if for all $c > a+1$, $h_1(c) > h_0(c)$, then $P(u_i = 1 \mid p_i > c)$ is a decreasing function of c.*

(iii) *if for all $c > a+1$, $h_1(c) = h_0(c)$, then $P(u_i = 1 \mid p_i > c)$ is a constant.*

Proof. Note that both $S_0(c)$ and $S_1(c)$ are decreasing in c (as are all survival functions). Using Bayes rule, the proportion of high skill agents is given by

$$P(u_i = 1 \mid p_i > c) = \frac{P(p_i > c \mid u_i = 1)\pi_0}{P(p_i > c \mid u_i = 1)\pi_0 + P(p_i > c \mid u_i = 0)(1-\pi_0)}. \tag{B.1}$$

Because $P(p_i > c \mid u_i = 1) = S_1(c)$, $P(p_i > c \mid u_i = 0) = S_0(c)$, and $\pi_0 = 0.5$ we get

$$P(u_i = 1 \mid p_i > c) = \frac{S_1(c)0.5}{S_1(c)0.5 + S_0(c)(1-0.5)} = \frac{1}{1 + S_0(c)/S_1(c)}. \tag{B.2}$$

From Equation (B.2), it is clear that $P(u_i = 1 \mid p_i > c)$ is an increasing function of c whenever $S_0(c)/S_1(c)$ is a decreasing function of c. Moreover,

$$\frac{dS_0(c)/S_1(c)}{dc} = \frac{-f_0(c)S_1(c) + f_1(c)S_0(c)}{S_1(c)^2},$$

which is negative when $f_1(c)S_0(c) < f_0(c)S_1(c)$, or, $f_1(c)/S_1(c) < f_0(c)/S_0(c)$, i.e., when $h_1(c) < h_0(c)$. Similarly, $P(u_i = 1 \mid p_i > c)$ is decreasing function of c when $h_1(c) > h_0(c)$ and $P(u_i = 1 \mid p_i > c)$ is a constant when $h_1(c) = h_0(c)$. Claims (i) to (iii) follow trivially. □

Lemma 2 shows that the shape of the hazard function determines whether $P(u_i = 1 \mid p_i > c)$ increases, decreases or is constant in c. If the hazard function of the noise term distribution is increasing for all values, implying that $h_1(c) < h_0(c)$ (because $h_1(c) = h_\varepsilon(c-1)$ and $h_0(c) = h_\varepsilon(c)$), then $P(u_i = 1 \mid p_i > c)$ is increasing in c. If the hazard function is decreasing, implying that $h_1(c) > h_0(c)$, $P(u_i = 1 \mid p_i > c)$ is decreasing in c. If the hazard function initially increases but eventually decreases with c then $P(u_i = 1 \mid p_i > c)$ may initially increase

and then decrease with c. To determine whether $P(u_i = 1 \mid p_i > c)$ increases with c, it is thus enough to know whether the hazard function is increasing or decreasing.

Whether the hazard function is increasing or decreasing can be determined from the shape of the density function as follows:

Lemma 3. *Let $g(x)$ be a differentiable density function such that $g(x) \to 0$ as $G(x) \to 1$. If $\ln g(x)$ is a concave (convex) function of x, then $h(x) = g(x)/(1 - G(x))$ is an increasing (decreasing) function of x.*

Proof. See Thomas (1971). □

Using this criterion it can be shown that the hazard function of the normal distribution is increasing (see Luce 1986, pp. 16–17). It follows that $P(u_i = 1 \mid p_i > c)$ is an increasing function of c when the noise term is drawn from a normal distribution.

Next, we prove that the threshold for survival increases over time.

Lemma 4. *Let p_t^* be the maximum level of performance among the agents that fail in period t. In the setting of Theorem 2, p_t^* increases with t and $\lim_{t\to\infty} p_t^* = \infty$.*

Proof. Because the noise distribution is continuous, the probability that two or more agents have the same level of performance is 0. It follows that all the survivors of selection in period t have a performance higher than p_t^*. Because some positive fraction of these survivors from period t are eliminated in period $t+1$, $p_{t+1}^* > p_t^*$. Hence, p_t^* increases with t.

Now we prove that p_t^* converges toward ∞. Suppose that $(p_t^*)_{t\geq 1}$ is bounded by $k < \infty$. Let γ denote the initial proportion of agents with performance at least as high as k: $\gamma = P(p_{i,1} \geq k)$. Note that $\gamma > 0$ because the support of f_ε is unbounded on the right. All agents with performance at least as high as k survive infinitely many periods because their performance is higher than all the performance thresholds (we showed that the sequence of performance thresholds is increasing and we assumed it was bounded). But at the same time, after t periods, the proportion of survivors is $(1-w)^t$. This converges toward 0 when t becomes large. This fact is incompatible with the fact that a proportion of at least $\gamma > 0$ agents survive all periods. Therefore, it cannot be the case that p_t^* is bounded. Thus, necessarily, $\lim_{t\to\infty} p_t^* = \infty$. □

By combining Lemmas 2 and 4 we can characterize the noise distributions for which π_t always increases with t and the noise distributions for which π_t eventually decreases with t. Note that in the setting of the theorem, $\pi_t = P(u_i = 1 \mid p_i > p_t^*)$.

(i) Suppose $t = 1$. The proportion of high skill agents who survive the first period, $P(1 + \varepsilon_i > p_1^*)$ is larger than the proportion of low skill agents who survive the first period, $P(\varepsilon_i > p_1^*)$. Hence, $\pi_1 > 0.5$.

(ii(a)) Suppose the noise distribution has a strictly increasing hazard function h_ε. Lemma 2(i) implies that $P(u_i = 1 \mid p_i > p_t^*)$ increases with p_t^*. Lemma 4 implies that p_t^* increases with t. By composition, this implies that $P(u_i = 1 \mid p_i > p_t^*) = \pi_t$ increases with t.

(ii(b)) Suppose h_ε is a decreasing hazard function. According to Lemma 4, p_t^* increases with t and converges toward ∞. Let t^a be the first period in which $p_t^* > a + 1$ (a is the lower bound of the support of f_ε). It follows from Lemma 2(ii) that when $t > t^a$, π_t decreases with t.

(ii(c)) Suppose $h_\varepsilon(c-1) < h_\varepsilon(c)$ for all $c < c^*$ and $h_\varepsilon(c-1) > h_\varepsilon(c)$ for all $c > c^*$. Let t^m be the first period in which $p_{t^m}^* > c^*$ (we can be sure that t^m exists because $\lim_{t\to\infty} p_t^* = \infty$, as implied by Lemma 4). Let t^a be the first period in which $p_t^* > a + 1$ (a is the lower bound of the support of f_ε). It follows from Lemma 2(ii) that when $t > \max(t^m, t^a)$, π_t decreases with t.

(ii(d)) Suppose $h_\varepsilon(c-1) = h_\varepsilon(c)$ for all $c > c^*$. Let t^m be the first period in which $p_{t^m}^* > c^*$ and t^a be the first period in which $p_t^* > a + 1$. It follows from Lemma 2(iii) that when $t > \max(t^m, t^a)$, π_t remains constant.

Appendix C. Proof of Theorem 3

(i) According to Equation (B.2), the proportion of high skill agents can be written in terms of the survival functions, and, in turn, in terms of the cumulative distribution function of the noise term:

$$\begin{aligned} &P(u_i = 1 \mid p_i > c) \\ &\quad = \frac{S_1(c)0.5}{S_1(c)0.5 + S_0(c)0.5} = \frac{1 - F_\varepsilon(c-1)}{1 - F_\varepsilon(c-1) + 1 - F_\varepsilon(c)} \\ &\quad = \frac{1}{1 + (1 - F_\varepsilon(c))/(1 - F_\varepsilon(c-1))}. \end{aligned}$$

Therefore, $\lim_{c\to\infty} P(u_i = 1 \mid p_i > c) = 0.5$ if and only if $\lim_{c\to\infty}(1 - F_\varepsilon(c))/(1 - F_\varepsilon(c-1)) = 1$. This later condition is exactly the necessary and sufficient condition for a distribution to be long-tailed.

(ii) This follows from (i) and Lemma 4. □

Appendix D. Proof of Theorem 4

To prove Theorem 4 note first that, after selection and replacement in period t,

$$\pi_t = (1 - w)P(u_i = 1 \mid p_i > p_t^*) + w0.5.$$

The first term represents the proportion of high skill agents among the agents that survived selection: their performance must all be above the threshold for survival in period t. The second term is the proportion of high skill agents among the new agents, which is 0.5. Because the change in π_t only depends on $P(u_i = 1 \mid p_i > p_t^*)$ (all other elements of the formula remain constant over time), the conclusion of Theorem 2 continues to hold as long the equivalent of Lemma 4, i.e., p_t^* is increasing without bound, continues to hold when there is replacement.

The proof that p_t^* is increasing follows almost the same reasoning as the proof of Lemma 4. All the survivors of selection in period t have a performance higher than p_t^*. Some positive fraction of these survivors from period t are eliminated in period $t+1$, because whenever p_t^* is finite, there is a positive probability that an entrant i in period $t+1$ will have a performance higher than a survivor j in period t, i.e., $P(p_{i,1} > p_{j,1} \mid p_{j,1} > p_t^*) > 0$. Hence, p_t^* increases with t.

Now we prove that p_t^* converges toward ∞. Suppose that $(p_t^*)_{t\geq 1}$ is bounded by $k < \infty$. Let γ denote the proportion of agents with a performance at least as high as k: $\gamma = P(p_{i,1} \geq k)$. Note that $\gamma > 0$ because the support of f_ε is unbounded on the right. All agents with performance at least as high as k survive infinitely many periods. Initially, the proportion of agents with performance of at least k is γ. The probability

that an entrant has performance of at least k is also γ. The fraction of entrants is w in each period. Over time, the proportion of agents with a performance of at least k becomes higher $(1-w)$. To see why, suppose that in period 1, $\gamma \le 1-w$. In period 2, we have w entrants that are replacing some of the agents with performance below k. Among the entrants, the proportion of agents with performance of at least k is equal to γ. So at the end of period 2, the proportion of agents with performance of at least k is equal to $\gamma + \gamma w$. If this proportion is higher than $1-w$, we have proved what we wanted. If this proportion lower than or equal to $1-w$, the same dynamics applies to period the following periods. If at the end of period $t-1$, the proportion of agents with performance of at least k is lower than $1-w$, then this proportion at the end of period t is $\gamma + (t-1)\gamma w$. This dynamic process necessarily implies that at some point, the proportion of agents with performance of at least k becomes higher than $1-w$. That means that the proportion of replaced agents becomes lower than $1-w$, which is incompatible with the definition of w. Therefore, it cannot be the case that p_t^* is bounded. Thus, necessarily, $\lim_{t\to\infty} p_t^* = \infty$.

Appendix E. Proof of Theorem 5

To prove Theorem 5 it is enough to prove that that the equivalent of Lemma 2(i) holds, for any skill distribution, whenever the hazard function h_ε is increasing.

Lemma 5. *In the setting of Theorem 5, if h_ε is increasing, then $E[u_i \mid p_i > c]$ is increasing in c.*

Proof. We denote by P the performance and by U the skill of a randomly chosen agent. Consider an agent i and its performance p_i. The survival function, conditional on skill, is defined as $S(c \mid U = u_i) = P(p_i > c \mid U = u_i) = 1 - F_\varepsilon(c - u_i)$.

Let c_1 and c_2 be performance thresholds such as $c_2 > c_1$. We want to show $E[u \mid p_i > c_2] > E[u \mid p_i > c_1]$. To do so, we will write $E[u \mid p_i > c_2]$ as the sum of $E[u \mid p_i > c_1]$ and a positive term.

Assuming a performance threshold of c_1, we can use Bayes' rule to write the skill distribution for the survivors:

$$g_1(u \mid p_i > c_1) = \frac{P(p_i > c_1 \mid U = u) g_u(u)}{\int_{-\infty}^{\infty} P(p_i > c_1 \mid U = v) g_u(v)\, dv}. \tag{E.1}$$

Because $P(p_i > c_1 \mid U = u) = S(c_1 \mid U = u)$ this can be written as

$$g_1(u \mid p_i > c_1) = \frac{S(c_1 \mid U = u) g_u(u)}{\int_{-\infty}^{\infty} S(c_1 \mid U = v) g_u(v)\, dv}. \tag{E.2}$$

Similarly, if the performance threshold is c_2, we can write

$$g_2(u \mid p_i > c_2) = \frac{S(c_2 \mid U = u) g_u(u)}{\int_{-\infty}^{\infty} S(c_2 \mid U = v) g_u(v)\, dv}, \tag{E.3}$$

$$= \frac{w(u) g_1(u \mid p_i > c_1)}{\int_{-\infty}^{\infty} w(v) g_1(v \mid p_i > c_1)\, dv}, \tag{E.4}$$

where $w(u) = S(c_2 \mid U = u)/S(c_1 \mid U = u)$. We get

$$E[u \mid p_i > c_2] = \frac{\int_{-\infty}^{\infty} u w(u) g_1(u \mid p_i > c_1)\, du}{\int_{-\infty}^{\infty} w(u) g_1(u \mid p_i > c_1)\, du} = \frac{E_1[w(u)u]}{E_1[w(u)]}, \tag{E.5}$$

where E_1 denotes that the expectation is with respect to the random variable with density $g_1(u \mid p_i > c_1)$. This can be expressed as

$$E[u \mid p_i > c_2] = \frac{E_1[w(u)]E_1[u] + \mathrm{Cov}_1[w(u), u]}{E_1[w(u)]}$$
$$= E[u \mid p_i > c_1] + \frac{\mathrm{Cov}_1[w(u), u]}{E_1[w(u)]}, \tag{E.6}$$

where Cov_1 denotes a covariance with respect to density $g_1(u \mid p_i > c_1)$. Then,

$$E[u \mid p_i > c_2]$$
$$= E[u \mid p_i > c_1] + \frac{\mathrm{Cov}_1[S(c_2 \mid U = u)/S(c_1 \mid U = u), u]}{E_1[S(c_2 \mid U = u)/S(c_1 \mid U = u)]}. \tag{E.7}$$

Whenever $S(c_2 \mid U = u)/S(c_1 \mid U = u)$ is a strictly increasing function of u, then the covariance term is positive (Ross 2000, p. 626), and thus $E[u \mid p_i > c_2]$ is larger than $E[u \mid p_i > c_1]$. Moreover, $S(c_2 \mid U = u)/S(c_1 \mid U = u)$ is a strictly increasing function of u iff

$$\frac{d}{du}\frac{S(c_2 \mid U = u)}{S(c_1 \mid U = u)} = \frac{f_\varepsilon(c_2)S(c_1 \mid U = u) - f_\varepsilon(c_1)S(c_2 \mid U = u)}{S(c_1 \mid U = u)^2} > 0, \tag{E.8}$$

This occurs when $f_\varepsilon(c_1)S(c_2 \mid U = u) < f_\varepsilon(c_2)S(c_1 \mid U = u)$, i.e., when $h_\varepsilon(c_1) = f_\varepsilon(c_1)/S(c_1 \mid U = u)$ is smaller than $h_\varepsilon(c_2) = f_\varepsilon(c_2)/S(c_2 \mid U = u)$. Since $c_1 < c_2$ and the hazard function h_ε is by assumption increasing, this condition is satisfied. Therefore, $\mathrm{Cov}_1[S(c_2 \mid U = u)/(S(c_1 \mid U = u)), u] > 0$. Equation (E.7) in turns implies that $E[u \mid p_i > c_2] > E[u \mid p_i > c_1]$. □

Endnotes

[1] Ancillary analyses and computer simulations show that most of our results still apply when the support of f_ε has an upper bound, but assuming that there is an upper bound leads to much more complicated proofs with many special cases that make it difficult to understand the mechanism at the core of our analysis.

[2] Formally, a distribution is heavy-tailed if the moment generating function is infinite (Foss et al. 2013).

[3] Fu et al. (2005), however, argue that the upper-tail is Pareto.

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