# Complementarity in Reputation Building with Assortative Co-branding*

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**Abstract**

We present a leverage theory of reputation building with assortative co-branding. We show that under certain conditions, co-branding that links unknown firms in a new sector with established firms in a mature sector allows the unknown firms to signal a high product quality and establish their own reputation. We compare this situation with a benchmark in which both sectors are new and firms signal their quality only with prices. We investigate how this comparison is affected by the nature of the technology linking the two sectors and a cross-sector inference problem that consumers might face in identifying the true cause of product failure. We find that assortative co-branding facilitates the process in which a firm in the new sector signals its product quality only if the co-branding sectors produce complementary inputs and consumers face a cross-sector inference problem. We apply our insight to economics of superstars, multinational firms and co-authorship.

*Key Words*: Leverage, Co-branding, Complementarity in Reputation Building, Cross-Sector Inference Problem.

*JEL Codes*: D82, L15, M31

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# 1 Introduction

This paper considers an adverse selection model in which we analyze how the reputation of one firm can be transferred to another that sells a different product. More specifically, we consider a situation that there are two sectors in an economy. One sector is mature in that firms in that sector have already established their reputation and consumers are informed about the quality of their products. The other sector is new and hence firms in that sector have yet to establish their reputation. The question we ask in this context is whether there exists a mechanism to leverage reputation from the firms in the mature sector to the ones in the new sector.

In our model, the mechanism through which such leveraging of reputation takes place is *assortative co-branding* in which high quality firms are paired together. We show that there exist conditions under which being associated with a good brand in the mature sector allows a firm in the new sector to signal a high quality product and establish their own reputation. The case in which one sector is mature and one sector is new is called *the asymmetric case*. We compare this case with a benchmark of the *symmetric case* in which both sectors are new. In both cases, reputation building is modelled in terms of signaling. It is important to note that in the symmetric case, assortative co-branding itself does not convey any information about a pair since no firm in the pair has yet established its reputation. Thus, in the symmetric case, the only way to convey the quality of the pair is though pricing. We first investigate the condition under which there exists a separating equilibrium with prices being the only instrument of signaling in the symmetric case. In the asymmetric case in which one sector is mature, co-branding process itself can be a signal if it is assortative and hence we study the condition to have a separating equilibrium with co-branding and prices. We say that there exists complementarity in reputation building if the condition for the existence of a separating equilibrium is less stringent in the asymmetric case than in the symmetric case. We investigate how this comparison is affected by the nature of the technology linking the two sectors (i.e. whether components produced by both sectors are complementary or not) and a cross-sector inference problem that consumers might face in identifying the true cause of product failure. To investigate this issue, we consider a setting in which a final product requires two complementary components that are produced by separate entities. This setting provides a natural environment in which co-branding and an inference problem concerning the performance of each component arise. In particular, we consider two kinds of information structures. In the *component performance observability (CPO) case*, consumers can identify which

component is at fault when the final product fails to perform. In contrast, in the *product performance observability (PPO) case*, consumers can only observe the performance at the product level. In the PPO case, when the final product fails to perform, consumers have a cross-sector inference problem and cannot identify which component is responsible for the failure at the product level.

We find that complementarity in reputation building with assortative co-branding exists if and only if the co-branding sectors produce complementary inputs and consumers can only observe product performance and thus face a cross-sector inference problem. Thus, complementarity in technological relationship between the two sectors induces complementarity in reputation building if only the product performance is observable. We apply our insight to the role of movie stars in the motion picture industries, co-branding of global brands by multinational firms and local brands in brand transitions, and co-authorship between established scholars and young scholars.

Examples of co-branding are abound. First, all team productions such as movie, artistic performance, scientific article etc. in which team members' names are publicly observable can be regarded as what we call "natural co-branding". Second, co-branding is an extensively used marketing strategy that is especially popular in introducing new consumer products.[1] It is typically defined as "pairing of two or more branded products (constituent brands) to form a separate and unique product (composite brand)."[2] The marketing literature points out that co-branding plays a key role in changing consumers' perception that the two constituent products will be regarded as being similar quality. Thus, linking with a well-known brand through co-branding can be a particularly effective strategy for an unknown brand in increasing assimilation in the eyes of consumers (Abratt ad Motlana, 2002). Rao, Qu, and Ruekert (1999), for instance, suggest that when an unknown brand cannot effectively signal its high quality, co-branding with a well-known brand of high quality can be an effective way to enter a new market. The marketing literature, however, does not model the mechanism through which the reputation of one brand can be leveraged to build reputation of another firm's brand. They rely either on case studies or experimental tests of hypothesis they posit without any rigorous micro-foundation. Our paper complements the marketing literature by providing a theoretical model that analyzes the mechanism through which the leverage of reputation takes place.

Our paper is closely related to Tadelis (1999) and Cabral (2000). Tadelis studies name

[1]Simonin and Ruth (1998) report that co-branding activities broadly defined have increased with an annual growth rate of 40% in the US.

[2]Washburn, Till and Priluck (2000), p. 591.

trade in an adverse selection framework with overlapping generations of firms.[3] Cabral applies Tadelis' framework to the issue of brand extension.[4,5] We extend Tadelis' framework to multiple sectors and study how firms with established names (or brands) in one sector may affect the ability of firms in the other sector to build their own reputation. In particular, we find that complementarity in reputation building is related to complementarity in underlying technology and consumers' cross-sector inference problems concerning the cause of failure. Our focus thus is on name trading between contemporaneous firms in different sectors, instead of name trading between different generations of firms within the same sector as in Tadelis (1999). Another important distinction is that Tadelis and Cabral abstract from the possibility of signaling through price by assuming that prices are equal to the consumers' willingness to pay. In contrast, we allow for the possibility of signaling through price in the symmetric case and compare it to signaling with prices and co-branding in the asymmetric case.

The special case of single component products in our model is similar to Bagwell and Staiger (1989) who study export subsidy as a strategic trade policy to overcome information barrier to entry. Major differences are that we study matches among firms producing complementary inputs and name trades between different sectors. In addition, their model assumes that once a consumer is matched to a firm, the relationship is maintained for both periods while the relationship is short-lived in our model. This difference arises since we need to entertain the possibility that firms change their partners based on the newly revealed information about the product outcomes realized in the first period, which they do not need to consider.

Bar-Isaac (forthcoming) develops a theory of reputation in teams to explain organizational design of many professional service providers such as law firms, consulting firms, and medical practices where reputation both at the level of the firm and the individual are crucial. He shows how a partnership structure where a senior works with a junior in

[3]The first paper on name trades is Kreps (1990) in a moral hazard setting with repeated games.

[4]Choi (1998) studies brand extension in a model of infinite horizon in which a firm can launch a new product every period. An equilibrium in which the firm extends its brand only to products of high quality is sustained by the threat of a breakdown of trust in case a low quality product is sold under the same brand.

[5]Thal (2007) studies brand extension through licensing in an adverse selection framework similar to Cabral (2000). In her model, there are two firms: an incumbent who produces an old product under a brand name and an innovator who launches a new product. The incumbent decides whether to license its brand to the innovator. In contrast, in Cabral (2000), both the old and the new products are produced by the incumbent. Both papers, however, assume that prices are equal to consumers' willingness to pay whereas we allow that prices are used as a signaling device.

tandem provides proper incentives for both senior and junior to work hard: the junior's incentives arise from a concern for her own reputation whereas the senior's incentives arise from a concern for the reputation of her firm which she plans to sell out to the junior. His model employs a joint production technology that is similar to ours in the following sense: individual outputs of a senior and a junior cannot be observed and only combined output of a team of senior and junior is observable. This non-observability of individual contribution in joint production is crucial to his main results and is akin to our assumption that consumers cannot infer the true cause of failure when the final product fails to perform. The incentive problem in his paper is mainly of moral hazard while we are mainly concerned with adverse selection problem. As such, Bar-Isaac's and our papers complement each other.

Our paper is also related to the literature on the economics of superstars (Rosen 1981 and MacDonald 1988), and in particular to Kremer (1993) who studies how the return to talent is affected by technological complementarity and matching. Kremer, however, studies a situation in which agents's types (skill) are common knowledge while our focus is on complementarity in reputation building with the types of firms at least in one sector being imperfect information to consumers. We identify a premium to established names (or superstars) accruing from the ability to allow other complementary inputs to signal their quality through matching.

In Section 2, we present the basic model of adverse selection with complementary components. To highlight the importance of technological complementarity, in section 3 we study the case of single component product as a benchmark. We compare signaling with prices and signaling with name trades and prices. We find that conditions under which a signaling equilibrium exists are identical in both cases. This result suggests that there is no complementarity in reputation building in the absence of technological complementarity between name traders. What matters for signaling is each type's gain from building reputation. Since the gain does not depend on the mode of signaling, the condition to have a separating equilibrium is invariant to the mode of signaling.

Section 4 considers the case of complementary components and compares two information structures regarding firms' types. In the symmetric case in which both sectors are new, no firm's type is known to consumers. In contrast, in the asymmetric case in which one sector is mature, types of the firms in that sector are known to consumers whereas types of the firms in the new sector are unknown. We show that if consumers can observe the performance of each component (i.e. the CPO case), co-branding is irrelevant in relaxing the condition for the existence of separating equilibrium in which only pairs of

high types produce. However, if consumers can observe only the performance of the final product (i.e. the PPO case), co-branding can alleviate the adverse selection problem.

A rather obvious result is that regardless of the information structure on performance observability, the condition to have a separating equilibrium is *weakly* less stringent in the asymmetric case than in the symmetric case. The reason is that the deviation of a pair of low type firms simply cannot exist in the asymmetric case but such a deviation can exist in the symmetric case. The question is to know when the condition is *strictly* less stringent in the former than in the latter. In the CPO case, it turns out that the total gain from building reputation (or from masquerading as a pair of high types) for any pair of matched types producing complementary components is equal to the sum of each type's stand-alone gain from building reputation. Therefore, a separating equilibrium in which pairs of high type firms produce exists, both in the symmetric and asymmetric cases, as long as a high-type's stand-alone gain from building reputation is larger than a low type's one. This is similar to what happens in the case of single component product. In the PPO case, when a firm owning a good brand chooses a low type firm as its partner, it suffers from a negative feedback effect on its reputation. A low type firm is likely to produce a bad component and therefore the final product produced together with a low type is likely to fail. Since consumers are unable to identify which component caused the failure of the final product, a good brand is likely to acquire a failure track record at the end of period one. This negative feedback effect facilitates signaling through co-branding since a firm having a good brand has little incentive to choose a low type firm as its partner. In contrast, when signaling is done with prices in the symmetric case, a pair of low type firms can masquerade as a pair of high type firms by charging the same introductory price as the latter. Since a low type firm is less likely to produce a good component than a high type firm, the former suffers less from the negative feedback effect than the latter when each of them is matched with a low type firm: this is obvious when a low type firm produces a bad quality component for sure. Therefore, preventing a pair of low types firms from masquerading as a pair of high types firms can be more difficult than preventing a pair of high type and low type firms from masquerading. This explains why co-branding relaxes the condition to have a separating equilibrium. In Section 5, we apply our insight to different contexts.

# 2 The Model with Complementary Components

We consider a final product that requires two complementary components, $x$ and $y$, to provide useful services to consumers.[6] There is a continuum of firms producing component $x$ whose mass is normalized to 1. There is another mass 1 of firms producing component $y$. We consider a two period model in which firms live for two periods. Each firm can produce at most one unit of component in each period. Consumers are homogeneous in their willingness to pay for the final product and their mass is assumed to be more than 1 in each period, implying that the sellers are on the short side of the market.

The performance realization of each component produced is uncertain; it can be either a success (denoted by $S$) or a failure (denoted by $F$). In our model, the complementarity of the two components means that the weakest-link principle prevails in the determination of the final product's quality, that is, the quality of the final product is determined by the lowest quality component. More specifically, we assume that the final product is successful in rendering useful service if and only if both components are successful. In such a case, the value of service provided by the final product is normalized to 1. If at least one of the components fails, the final product fails to provide any service to consumers and the product is of zero value.

One implication of the complementary components is that when the final product fails, consumers may not be able to identify the true cause of failure due to the technological sophistication of the product, i.e., whether it is due to the failure of component $x$, $y$, or both. We consider two cases depending on the ability of consumers to identify the true cause of failure. We first analyze the *component performance observability (CPO) case* in which consumers can observe the success or failure of each component. We then investigate the *product performance observability (PPO) case* in which consumers observe only the success or failure of each final product. Consumers face a cross-sector inference problem in the PPO case (since they cannot identify the true cause of a product failure) while they don't in the CPO case. Firms, however, are assumed to observe the performance of each component and this information is shared among all firms. This assumption that firms have no cross-sector inference problem is innocuous and simply allows us to focus on the most efficient outcome through firms' rematch in the second period. This assumption reflects the fact that firms are better informed than consumers due to their experience in production and superiority in technical knowledge.[7]

[6] We can extend the model to the case of more than two complementary inputs ($n > 2$).

[7] Levin and Tadelis (2005) makes a similar assumption in their model of partnerships. More specifically, they consider industries where human capital plays an important role in determining product quality.

Firms differ in their productivity. Let $\theta_i^j$ represent the type of firm $i \in [0,1]$ that produces component $j$, where $j = x, y$. There are two types of firms, high ($H$) and low ($L$) in each sector, that is, $\theta_i^j \in \{H, L\}$. The proportion of high type firms in sector $j$ is given by $\nu_j \in (0,1)$. Our main results hold regardless of $\nu_x \leq \nu_y$ or $\nu_x > \nu_y$ although we believe that $\nu_x < \nu_y$ is more relevant since in this case good brands are relatively scarce and command a premium for their name value. For simplicity and to isolate the effect of the consumers' cross-sector inference problem, we assume symmetric production technology in that it does not depend on $j = x, y$. More precisely, if a firm is of high type (*resp.* of low type), in period one it can produce a successful component with probability $q_H$ (*resp.* $q_L$) where $1 > q_H > q_L \geq 0$. The performance realization of each component given each firm's type is independently and identically distributed. The per-period cost of producing a component is given by $c_H$ and $c_L$ for high and low type firms. Since if $c_H < c_L$ holds, separating equilibrium trivially exists in our model, we assume $c_H \geq c_L$. Each firm knows its own type while consumers only know the distribution of types. In addition, we assume that all firms know each other's type. Our focus is on imperfect information on the consumer side. This assumption is meant to reflect the fact that firms are able to discern the capability of other firms better than consumers by virtue of being in the same line of business.[8]

We assume that the first period performance of each final product ($S$ or $F$) becomes known to all firms and consumers. Furthermore, we assume that there is some technological learning for the firms from their first period market production. More specifically, each firm's probability of making a successful quality component in period two depends both on its type and on the performance of its component in period one. To reflect this path dependency, we denote a type $\theta$ firm's probability of producing a successful component in period two as $q_{\theta S}$ and $q_{\theta F}$, respectively, depending on whether its component was successful or not in the first period, where $\theta = H, L$. In particular, we assume that for high type firms the probability of producing a successful component is strictly higher when they produced a successful component in period one than otherwise, that is,

$$1 \geq q_{HS} > q_{HF} \geq 0 \text{ for } \theta = H, L.$$

This assumption together with $1 > q_H > 0$ creates some residual uncertainty about

They assume that clients are at a disadvantage relative to firms in assessing the ability of employees.

[8]Biglaiser (1993) makes a similar assumption in his model of middlemen as experts. A middleman is assumed to be able to ascertain the quality of the good through his investment in human capital such as education or apprenticeships whereas such investment is costly enough so that no buyer is would gain by making an investment.

firms' productivity in our adverse selection model. When $c_H > c_L$, $q_{HS} > q_{HF}$ plays the role of a single-crossing condition without which no separating equilibrium exists. $q_{HS} > q_{HF}$ would arise when a high type firm produces the same component for both periods and initially has to choose one among $m(> 2)$ different ideas to produce it. Then, if the idea chosen in period one is successful, the firm can use it for period two, implying $q_{HS} = 1 > q_{HF}$. Furthermore, even in the case of failure, the firm can discard the idea already experimented and choose one among $m - 1$ ideas in period two, implying $q_{HF} > q_{HN}$ where $N$ means that no production occurred in period one. We assume that a high type firm that does not produce in period one is penalized since it does not acquire experience useful for period two production (i.e. $q_{HN} < q_{HF}$ holds). The role of this assumption is to eliminate the possibility that a high type firm who produced in period one (and hence whose type is revealed to the consumers in a separating equilibrium) prefers as its period two partner a high type who did not produce in period one to a high type that has a failure record. Let $\delta$ denote the discount factor for the second period payoffs, which is common for all the players.

To demonstrate the role of co-branding as a mechanism to leverage an established firm's reputation, we compare two information structures about firms' types. First, as a benchmark, we consider *the symmetric case* where both sectors are new and types of firms in neither sector are known to consumers. This case captures a situation in which no firm has yet established its reputation at the beginning of the game. Second, we analyze *the asymmetric case* in which types of firms in one sector ($x$) are already known (hence, a high type firm in sector $x$ is a firm having a good brand) whereas types of firms in the complementary sector ($y$) are not. This case represents a situation in which firms producing component $x$ have already established their reputation while firms producing component $y$ have yet to establish their reputation. We compare signaling through prices in the symmetric case with signaling through co-branding and prices in the asymmetric case to derive conditions under which co-branding can play a role in transmitting information about the types of unknown firms.

Concerning the determination of prices in the market, we consider the following matching technology and bargaining game. In each period there is a random match between a unit of final product and a buyer. The sellers are on the short side. As a result, the sellers of the final product choose a price and make a take-or-leave-it offer. In particular, this provides the sellers with the possibility to signal their types.

In summary, we make the following assumptions.

A1: (Information Structure regarding firms' types) Firms know each other's types.

In the symmetric case, consumers do not know any firm's type in either sector. In the asymmetric case, they know the types of firms producing component $x$ while they do not know the types of firms producing component $y$.

A2: (Information structure regarding performance observability) In the CPO case (in the PPO case), if a final product fails, consumers know (do not know) whether it is due to the failure in component $x$ or $y$ (or due to both). However, firms observe the performance of each component.

A3: (Incomplete and Short-Term Contracts) Although the outcome of each final product realized in period one becomes known to every player, firms cannot use a contract contingent on the realized outcomes. Firms cannot use a long-term contract. This is true among themselves and with respect to consumers.

A3 is a standard assumption in the incomplete contract literature à la Hart-Grossman (1986) and Hart-Moore (1990), which is also adopted by Tadelis (1999), Cabral (2000) and Bar-Issac (forthcoming). The inability to write contingent contracts can be justified by the problems of verifiability in courts. If outcomes are contractible, firms can use a long-term contract that specifies period two actions contingent on period one outcomes. Because of the incompleteness of contracts, it is natural to consider short-term contracts.

A4: (i) $(q_H)^2 - 2c_H > 0 > \max\left\{(q_L)^2 - 2c_L, q_H q_L - c_H - c_L\right\}$

(ii) $q_{HF}q_{HF} - 2c_H > 0 > \max\left\{q_{LS}q_{LS} - 2c_L, q_{HS}q_{LS} - c_H - c_L\right\}$ where we assume $q_{LS} \geq \max\left\{q_{LF}, q_{LN}\right\}$.

A4 implies that it is socially desirable for a pair of high type firms to produce in each period while it is socially undesirable for any pair that involves a low type firm to produce in any period. This assumption allows us to focus on the separating equilibrium in which only pairs of high type firms produce.

In the symmetric case, the timing in each period is given as follows:

1. Firms producing each component search for partners in the complementary sector with each firm producing component $x$ being matched to a firm producing component $y$.

2. After matching, each pair of matched firms decide whether or not to produce each component (and hence a unit of final product). The production takes place only when both parties agree. Upon producing a final product, each firm chooses a name to attach to the final product.[9]

[9]Hence, a product has two names. This is natural in particular in the CPO case in which each

3. Each final product is randomly matched to a consumer. Each pair of firms choose a price for the final product and makes a take-it-or-leave-it offer to the consumer. Each consumer decides whether to accept or reject the offer. In case the consumer rejects it, no trade is realized in period one.

4. The performance of each component is realized and known to firms. In the CPO case (in the PPO case), the performance of each component (product) is known to consumers.

Therefore at the end of period one, each firm's name is associated with $S$ or $F$ and the match in period two is based not only on the information about each firm's type but also on each firm's past record (the record perceived by consumers and the true record known by firms). Furthermore, when choosing a name in period two, a firm can keep the name it used in period one or use a new name.

We assume that the match among firms takes place on voluntary basis in that the match will be materialized only when both parties wish to be matched with the other. We also assume that the matching process is instantaneous and does not entail any search costs.

In what follows, we first study a benchmark in which each component $j(= x, y)$ is a final product that can be consumed independently of each other. This benchmark allows us to isolate the effect of having technological complementarity, as is assumed in our model. In all our analysis, we focus on separating equilibria. When there are multiple equilibria, we apply the Cho-Kreps (1987) criterion to eliminate unreasonable equilibria.

# 3 Benchmark: Single component product

In this benchmark, we consider the case of the single component product. Since each component $j(= x, y)$ can be consumed as an independent final product, the CPO case is equivalent to the PPO case. We below consider two scenarios. In the symmetric case, there is no established name in the economy. In the asymmetric case, there are some established names in the economy and hence firms in sector $j$ can buy them to signal their types. Since each product can be analyzed separately, for notational simplicity, we will dispense with the superscript $j$ in this section if there is no ambiguity. Straightforward modification of A4 to the single component product and $c_H \geq c_L$ leads to:

A4': (i) $q_H > c_H \geq c_L > q_L$ (ii) $q_{HF} > c_H \geq c_L > q_{LS}$.

firm's name carries the performance record of the component it produced in period one when firms are rematched in period two.

## 3.1 Price as a Signal without Name Trades

Consider first the symmetric case in which there is no established name in the economy and therefore the only instrument to signal one's type is the price chosen in period one. From A4', in any separating equilibrium, only high type firms sell their products while low type firms do not. Let $p_H^1$ be the price that high type firms choose in period one in the separating equilibrium. In the separating equilibrium, when a firm charges $p^1$ in period one, consumers' period one posterior belief about the probability that the firm's type is high is $\nu_1(p^1) = 1$ for $p^1 = p_H^1$ and $\nu_1(p^1) = 0$ for all $p^1 \neq p_H^1$. In addition, let $p_S^2$ and $p_F^2$ respectively denote the price that high type firms with the first period track record of $S$ and $F$ charge in period two. Since firms have bargaining power with respect to consumers, we have:

$$p_S^{2*} = q_{HS}, \quad p_F^{2*} = q_{HF};$$

where we use $*$ to denote the symmetric case.

The sufficient and necessary conditions to have a separating equilibrium in which only high types sell in the market are:

$$(\mathrm{IC}_L) \quad p_H^1 - c_L + \delta \left[ q_L p_S^{2*} + (1 - q_L) p_F^{2*} - c_L \right] \leq 0; \tag{1}$$

$$(\mathrm{IR}_H) \quad p_H^1 - c_H + \delta \left[ q_H p_S^{2*} + (1 - q_H) p_F^{2*} - c_H \right] \geq 0. \tag{2}$$

Condition ($\mathrm{IC}_L$) is an incentive compatibility constraint such that a low type firm has no incentive to mimic a high type firm in both periods.[10] Condition ($\mathrm{IR}_H$) is an individual rationality constraint for high type firms. For ($\mathrm{IC}_L$) and ($\mathrm{IR}_H$) to be compatible, it is required that

$$c_H - \delta[q_H q_{HS} + (1 - q_H) q_{HF} - c_H] \leq c_L - \delta \left[ q_L q_{HS} + (1 - q_L) q_{HF} - c_L \right],$$

which is equivalent to the following condition $SP^{S11}$:

$$\begin{aligned} c_H - c_L &< (q_H - q_L)(q_{HS} - q_{HF}) \text{ and } \delta \geq \delta^{S*} \\ \text{where } \delta^{S*} &\equiv \frac{c_H - c_L}{(q_H - q_L)(q_{HS} - q_{HF}) - (c_H - c_L)}. \end{aligned}$$

[10] We also need to check another IC condition in which a low type firm sells in period 1 and exits the market in period 2. But from A4'(ii), this condition is satisfied if ($\mathrm{IC}_L$) holds.

[11] $SP$ means separating equilibrium and the superscript $S$ means the single component case.

This implies that when the condition $SP^S$ holds, any $p_H^1 \in [c_H - \delta[q_H q_{HS} + (1 - q_H)q_{HF} - c_H],\ c_L - \delta\left[q_L q_{HS} + (1 - q_L)q_{HF} - c_L\right]]$ can be supported as a separating equilibrium with the belief $\nu_1(p_H^1) = 1$ and $\nu_1(p^1) = 0$ for all $p^1 \neq p_H^1$. Among the continuum of equilibria, the only one surviving the Cho-Kreps (1987) refinement is $p_H^{1*} \equiv c_L - \delta\left[q_L q_{HS} + (1 - q_L)q_{HF} - c_L\right]$. To show this, suppose that in equilibrium high type firms are supposed to choose $p_H^1$ belonging to the interval but different from $p_H^{1*}$. Then, high type firms can deviate and choose a price of $(p_H^1 + \epsilon)$, where $\epsilon$ is an infinitesimally small positive number. Since this new price is equilibrium dominated for low type firms, consumers should believe that this price is chosen by a high type firm. Thus, high type firms can profitably deviate upsetting the putative equilibrium. Thus, the only reasonable separating equilibrium price in the first period is $p_H^{1*} \equiv c_L - \delta\left[q_L q_{HS} + (1 - q_L)q_{HF} - c_L\right]$.

Note that high type firms make a loss in period 1 since the first period equilibrium price $p_{1H}^{S*}$ is less than its cost of $c_H$ to signal their type (i.e., $p_H^{1*} < c_L \leq c_H$). Therefore, a separating equilibrium exists only if the second period profit for high type firms is sufficiently large to recoup its loss in the first period.

## 3.2 Name trades

In this subsection we consider the asymmetric case and investigate whether the possibility of name trades as in Tadelis (1999) helps in establishing reputation of firms of unknown type. Hence, a firm in sector $j$ can purchase a good name to signal its type. For instance, many perfumes use names of famous fashion designers as product names.[12] We assume that each brand name can be sold at most to one firm in sector $j$ and any firm can introduce a new name without any cost. Let $G$ represents the set of good brand names, $B$ the set of bad brand names, $N$ the set of new brand names. Let $g$, $b$, $n$ respectively represent an element of each set. We are interested in a separating equilibrium in which a high type firm in sector $j$ buys a good brand name to be attached to its product and charges price $p_H^1$ in period one. Therefore, the revised priors in period one are $\nu_1(g, p_H^1) = 1$, $\nu_1(g, p^1) = 0$ for $p^1 \neq p_H^1$ and $\nu_1(b, p^1) = \nu_1(n, p^1) = 0$ for any $p^1$ including $p_H^1$.

When only high type firms sell in the first period, the second period prices with the track record of success and failure are respectively given by $p_S^{2**} = q_{HS} (= p_S^{2*})$, $p_F^{2**} = q_{HF} (= p_F^{2*})$ as in section 3.1 where we use $**$ to denote the asymmetric case. This implies that a $\theta$ type firm's maximum willingness to pay for a good brand name, denoted by $b_\theta$,

[12] See chapter 8 in Castarède (2007).

is given by:

$$b_H(p_H^1) = p_H^1 - c_H + \delta\left[q_H p_S^{2**} + (1-q_H)p_F^{2**} - c_H\right]; \tag{3}$$

$$b_L(p_H^1) = p_H^1 - c_L + \delta\left[q_L p_S^{2**} + (1-q_L)p_F^{2**} - c_L\right]. \tag{4}$$

Let $\nu_g(>0)$ denote the mass of good names. If $\nu_g \leq \nu_j$, a separating equilibrium in which only high types buy good names and charge $p_H^1$ exists if and only if $b_H(p_H^1) \geq b_L(p_H^1)$ and $b_H(p_H^1) \geq 0$. The first condition does not depend on $p_H^1$ and is equivalent to the condition $SP^S$. The second condition determines a lower bound on $p_H^1$. Therefore, any $p_H^1 \in [c_H - \delta[q_H q_{HS} + (1-q_H)q_{HF} - c_H],\, q_H]$ can be supported as a separating equilibrium. If $\nu_g > \nu_j$, the separating equilibrium exists if and only if $b_H(p_H^1) \geq 0 \geq b_L(p_H^1)$. Compared to the case with $\nu_g \leq \nu_j$, an additional condition $0 \geq b_L(p_H^1)$ must be satisfied, which determines an upper bound on $p_H^1$. Therefore, any $p_H^1 \in [c_H - \delta[q_H q_{HS} + (1-q_H)q_{HF} - c_H],\, c_L - \delta\left[q_L q_{HS} + (1-q_L)q_{HF} - c_L\right]]$ can be supported as a separating equilibrium.[13]

Summarizing, we have:

**Proposition 1** *Consider the benchmark of single component product. Suppose that there is a mass $\nu_g$ (>0) of good names in the economy.*

*(i) Without name trade, the condition to have a separating equilibrium is given by*

$$\begin{aligned} c_H - c_L &< (q_H - q_L)(q_{HS} - q_{HF}) \text{ and } \delta \geq \delta^{S*} \\ \text{where } \delta^{S*} &\equiv \frac{c_H - c_L}{(q_H - q_L)(q_{HS} - q_{HF}) - (c_H - c_L)}. \end{aligned}$$

*(ii) The possibility to buy name does not affect the condition to have a separating equilibrium*

*(iii) Therefore, in the single component product case, there is no complementarity in reputation building.*

The intuition for the result of no complementarity in reputation building in Proposition 1(iii) can be given in terms of the gain from building reputation (i.e. from being perceived as a high type firm). What matters for the possibility of signaling is the difference between a high type's gain and a low type's one, which does not depend on the mode of signaling.

[13] When $\nu_g > \nu_j$, using Cho-Kreps refinement makes us predict $p_H^{1**} = c_L - \delta\left[q_L q_{HS} + (1-q_L)q_{HF} - c_L\right]$ as the unique equilibrium price. However, when $\nu_g \leq \nu_j$, the refinement does not allow us to pin down a unique equilibrium price. For instance, when $\nu_g < \nu_j$, competition among high type firms to buy good brand names leaves them zero profit for any $p_H^1 \in [c_H - \delta[q_H q_{HS} + (1-q_H)q_{HF} - c_H],\, q_H]$.

# 4 Complementary Case with Two Sectors

We now consider the case in which a final product is composed of two complementary components as described in the model. In this section, we investigate a mechanism in which the reputation of firms in one component sector can be leveraged to build reputation of other firms in the complementary sector. Although our results do not depend on whether $\nu_x \leq \nu_y$ or $\nu_x > \nu_y$, for expositional simplicity, we only consider $\nu_x < \nu_y$ in this section and relegate the analysis of $\nu_x \geq \nu_y$ to the appendix. We investigate a separating equilibrium in which the firm types are revealed to consumers through signaling. With our parametric assumption A4, this implies that only high types are able to sell in the first period and all remaining firms in the second period are high types. As usual, we analyze the conditions for the existence of a separating equilibrium by applying backward induction.

## 4.1 The Second Period

When $\nu_x < \nu_y$, in a separating equilibrium, $\nu_x$ mass of high type firms in each sector produce in period one and their types are revealed to consumers. Therefore, the second period analysis is the same regardless of whether we analyze the symmetric or the asymmetric case. In other words, regardless of the information structure about firms' types at the beginning of the game, both sectors are symmetric *ex post* in the beginning of the second period. Thus, we assume that in period two, revenue is equally divided within a pair of high type firms with the same past record (perceived by consumers). We below show that there exists an equilibrium with positive sorting in the second period. Note that in the second period, every pair of firms charges a price equal to a consumer's maximum willingness to pay for the final product given his information about the firms' types and their previous records.

Consider first the CPO case. Let $p_{SF}^{2C}$ denote the period two price charged by a pair composed of a high type in sector $x$ with a success record and a high type in sector $y$ with a failure record: the superscript $C$ means CPO. We define $p_{SS}^{2C}$, $p_{FS}^{2C}$ and $p_{FF}^{2C}$ in a similar way. Then, we have:

$$p_{SS}^{2C} = (q_{HS})^2 , p_{SF}^{2C} = p_{FS}^{2C} = q_{HS}q_{HF}, p_{FF}^{2C} = (q_{HF})^2 .$$

With the assumption of the equal division of revenue within an $SS$ pair (and within an $FF$ pair), the expected payoff of a firm with a success record (respectively, with a failure record) under positive sorting is $p_{SS}^{2C}/2 - c_H$ (respectively, $p_{FF}^{2C}/2 - c_H$). We now show that

positive sorting takes place as an equilibrium, that is, no firm with a success record has an incentive to be matched with a firm with a failure record. Suppose, to the contrary, that a firm in sector $x$ with a success record is matched with a firm in sector $y$ with a failure record. In this case, their joint revenue is $p_{SF}^{2C}$. For this deviation to be profitable, their joint revenue ($p_{SF}^{2C}$) should exceed that achievable under positive sorting, that is, $\left(p_{SS}^{2C} + p_{FF}^{2C}\right)/2$. However, we have

$$p_{SF}^{2C} = q_{HS}q_{HF} < \frac{p_{SS}^{2C} + p_{FF}^{2C}}{2} = \frac{\left(q_{HS}\right)^2 + \left(q_{HF}\right)^2}{2}.$$

This implies that such a deviation cannot be profitable for both parties.

Consider now the PPO case. Let $p_{SF}^{2P}$ denote the period two price charged by a pair composed of a high type in sector $x$ with a success record and a high type in sector $y$ with a failure record: the superscript $P$ means PPO. We define $p_{SS}^{2P}$, $p_{FS}^{2P}$ and $p_{FF}^{2P}$ in a similar way. Obviously, we have $p_{SS}^{2P} = \left(q_{HS}\right)^2 \left(= p_{SS}^{2C}\right)$. We focus on the most efficient equilibrium in which among the firms with a failure record (i.e. among those whose final products failed in the first period), the successful producers of component $x$ are matched with the successful producers of component $y$ and similarly for the producers of failed components. In such an equilibrium, we have

$$p_{FF}^{2P} = \frac{q_H}{1+q_H}\left(q_{HS}\right)^2 + \frac{1}{1+q_H}\left(q_{HF}\right)^2.$$

The equal division of revenue within an $SS$ pair (and within an $FF$ pair) implies that the expected payoff of a firm with a success record (respectively, with a failure record) is $p_{SS}^{2P}/2 - c_H$ (respectively, $p_{FF}^{2P}/2 - c_H$). Once again, we show that positive sorting arises as an equilibrium. As in the CPO case, let us consider a deviation in which a firm in sector $x$ with success record is matched with a firm in sector $y$ with failure record. If consumers expect that the firm with a failure record is randomly chosen for matching with a success record, their maximum willingness to pay for a final product produced by an $SF$ pair is given by:

$$p_{SF}^{2P}\left(= p_{FS}^{2P}\right) = \frac{q_H}{1+q_H}\left(q_{HS}\right)^2 + \frac{1}{1+q_H}q_{HS}q_{HF}.$$

For the deviation to be profitable, their joint revenue ($p_{SF}^{2P}$) should exceed that achievable under positive sorting, that is, $\left(p_{SS}^{2P} + p_{FF}^{2P}\right)/2$. However, we have

$$\begin{aligned} p_{SF}^{2P} &= \frac{q_H}{1+q_H}\left(q_{HS}\right)^2 + \frac{1}{1+q_H}q_{HS}q_{HF} \\ &< \frac{q_H}{1+q_H}\left(q_{HS}\right)^2 + \frac{1}{1+q_H}\frac{\left(q_{HS}\right)^2 + \left(q_{HF}\right)^2}{2} \end{aligned}$$

$$= \frac{1}{2}\left(q_{HS}\right)^2 + \frac{1}{2}\left[\frac{q_H}{1+q_H}\left(q_{HS}\right)^2 + \frac{1}{1+q_H}\left(q_{HF}\right)^2\right] = \frac{p_{SS}^{2P} + p_{FF}^{2P}}{2}.$$

Therefore, such a deviation is not profitable and positive sorting with the same record is an equilibrium.

Finally, we check a deviation in which a high-type firm which produced in period one is matched with a firm which did not produce in period one. Let $\nu_2(n)$ denote consumers' posterior belief in period two when they see a firm with a new brand name. When $\nu_2(n) = 0$, consumers have pessimistic belief since they believe that the firm's type is low. A4(ii) implies that such a deviation is not profitable when consumers have pessimistic belief and this result holds both for the CPO case and the PPO case.

## 4.2 The First Period

### 4.2.1 General result

We first derive a general result. To present a result that holds both for the CPO case and the PPO case, we use superscript $h = C, P$.

Consider the symmetric case in which both sectors are new and the only instrument of signaling is price. Suppose that a pair of high types charges $p_{HH}^1$ in the first period in a separating equilibrium. Given information structure $h$ and price $p_{HH}^1$, let $V_{HH}^h(p_{HH}^1)$ denote the present discounted values of the total payoffs that a pair of high types realize over two periods in a separating equilibrium. Then, we have

$$V_{HH}^C(p_{HH}^1) \equiv p_{HH}^1 - 2c_H + 2\delta\left\{q_H\left[p_{SS}^{2C}/2\right] + (1-q_H)\left[p_{FF}^{2C}/2\right] - c_H\right\}; \tag{5}$$

$$V_{HH}^P(p_{HH}^1) \equiv p_{HH}^1 - 2c_H + \delta\left\{(q_H)^2\left[p_{SS}^{2P}\right] + (1-(q_H)^2)\left[p_{FF}^{2P}\right] - 2c_H\right\}. \tag{6}$$

Note that in the PPO case, the firms will be able to charge the price $p_{SS}^{2P}$ in period two only if both components were successful in period one.

Similarly, let $V_{HL}^h(p_{HH}^1)$ denote the present value of total joint payoffs that an $HL$ pair (i.e., a pair composed of a high type in sector $x$ and a low type in sector $y$) realize over two periods when they masquerade as an $HH$ pair and hence charge $p_{HH}^1$ in period one. $V_{LH}^h(p_{HH}^1)$ and $V_{LL}^h(p_{HH}^1)$ are similarly defined and by symmetry, we have $V_{HL}^h(p_{HH}^1) = V_{LH}^h(p_{HH}^1)$. More specifically,

$$V_{HL}^C(p_{HH}^1) \equiv p_{HH}^1 - c_H - c_L + \delta\left\{q_H\left[p_{SS}^{2C}/2\right] + (1-q_H)\left[p_{FF}^{2C}/2\right] - c_H\right. \tag{7}$$

$$+q_L\left[p_{SS}^{2C}/2\right]+(1-q_L)\left[p_{FF}^{2C}/2\right]-c_L\Big\};$$

$$V_{LL}^{C}(p_{HH}^{1})\equiv p_{HH}^{1}-2c_L+2\delta\left\{q_L\left[p_{SS}^{2C}/2\right]+(1-q_L)\left[p_{FF}^{2C}/2\right]-c_L\right\}; \tag{8}$$

$$V_{HL}^{P}(p_{HH}^{1})\equiv p_{HH}^{1}-c_H-c_L+\delta\left\{q_Hq_Lp_{SS}^{2P}+(1-q_Hq_L)p_{FF}^{2P}-c_H-c_L\right\}; \tag{9}$$

$$V_{LL}^{P}(p_{HH}^{1})\equiv p_{HH}^{1}-2c_L+\delta\left\{(q_L)^2\,p_{SS}^{2P}+(1-(q_L)^2)\left[p_{FF}^{2P}\right]-2c_L\right\}. \tag{10}$$

Let us define $\Delta_{HH,HL}^{h}\equiv V_{HH}^{h}(p_{HH}^{1})-V_{HL}^{h}(p_{HH}^{1})$, $\Delta_{HL,LL}^{h}\equiv V_{HL}^{h}(p_{HH}^{1})-V_{LL}^{h}(p_{HH}^{1})$, and $\Delta_{HH,LL}^{h}\equiv V_{HH}^{h}(p_{HH}^{1})-V_{LL}^{h}(p_{HH}^{1})$. Hence, $\Delta_{HH,LL}^{h}=\Delta_{HH,HL}^{h}+\Delta_{HL,LL}^{h}$. It is important to note that none of $\Delta_{HH,HL}^{h}$, $\Delta_{HL,LL}^{h}$and $\Delta_{HH,LL}^{h}$ does depend on $p_{HH}^{1}$.

We now derive the sufficient and necessary conditions for a separating equilibrium. Consider first the symmetric case in which both sectors are new. Necessary conditions that $p_{HH}^{1}$ has to satisfy to constitute a separating equilibrium in which only $HH$ pairs produce in the first period are:

$$(\mathrm{IC}_{LL}^{h*})\quad V_{LL}^{h}(p_{HH}^{1})\leq 0; \tag{11}$$

$$(\mathrm{IR}_{HH}^{h*})\quad V_{HH}^{h}(p_{HH}^{1})\geq 0. \tag{12}$$

Furthermore, when $\nu_x<\nu_y$, there remain some high type firms in sector $y$ which cannot be matched to high types in sector $x$. These firms can be matched to low types in sector $x$. Therefore, we have another necessary condition:

$$(\mathrm{IC}_{LH}^{h*})\quad V_{LH}^{h}(p_{HH}^{1})\leq 0. \tag{13}$$

Note that from symmetry, $(\mathrm{IC}_{LH}^{h*})$ is equivalent to

$$(\mathrm{IC}_{HL}^{h*})\quad V_{HL}^{h}(p_{HH}^{1})\leq 0. \tag{14}$$

We now show that $(\mathrm{IC}_{LL}^{h*})$, $(\mathrm{IR}_{HH}^{h*})$ and $(\mathrm{IC}_{HL}^{h*})$ are sufficient as well. To show this, we note first that any deviation involves the participation of at least one low type firm. However, from $(\mathrm{IC}_{LL}^{h*})$ and $(\mathrm{IC}_{HL}^{h*})$, it is clear that any pair including at least one low type has no incentive to masquerade as an $HH$ pair. We have:

**Lemma 1** *Consider the symmetric case in which both sectors are new. For given information structure regarding performance observability $h=C$ or $P$, a separating equilibrium with price exists if and only if $\Delta_{HH,HL}^{h}\geq 0$ and $\Delta_{HH,LL}^{h}\geq 0$.*

**Proof.** ($\text{IC}^{h*}_{LL}$), ($\text{IR}^{h*}_{HH}$) and ($\text{IC}^{h*}_{HL}$) imply $\Delta^h_{HH,HL} \geq 0$ and $\Delta^h_{HH,LL} \geq 0$. We now prove the reverse part, that is, if the conditions $\Delta^h_{HH,HL} \geq 0$ and $\Delta^h_{HH,LL} \geq 0$ hold, there exists a $p^1_{HH}$ satisfying ($\text{IC}^{h*}_{LL}$), ($\text{IR}^{h*}_{HH}$) and ($\text{IC}^{h*}_{HL}$). To prove this, notice that each of $V^h_{HH}(p^1_{HH})$, $V^h_{LL}(p^1_{HH})$ and $V^h_{HL}(p^1_{HH})$ linearly increases in $p^1_{HH}$. Let $\overline{p}^{1h*}_{HH}$ and $\underline{p}^{1h*}_{HH}$ be uniquely defined by $max\left\{V^h_{HL}(\overline{p}^{1h*}_{HH}), V^h_{LL}(\overline{p}^{1h*}_{HH})\right\} = 0$ and $V^h_{HH}(\underline{p}^{1h*}_{HH}) = 0$, respectively. If $\Delta^h_{HH,HL} \geq 0$ and $\Delta^h_{HH,LL} \geq 0$ , then it is clear that $\overline{p}^{1h*}_{HH} \geq \underline{p}^{1h*}_{HH}$. Then, any $p^{1h}_{HH}$ $\in [\underline{p}^{1h*}_{HH}, \overline{p}^{1h*}_{HH}]$ satisfies ($\text{IC}^{h*}_{LL}$), ($\text{IR}^{h*}_{HH}$) and ($\text{IC}^{h*}_{HL}$). Therefore, we can have a continuum of separating equilibria that can be supported with appropriate beliefs if $\overline{p}^{1h*}_{HH} > \underline{p}^{1h*}_{HH}$. However, the only equilibrium that survives the Cho-Kreps refinement criterion is the equilibrium in which $HH$ pairs charge $p^{1h*}_{HH} = \overline{p}^{1h*}_{HH}$. ■

Consider now the asymmetric case in which sector $x$ is mature and sector $y$ is new. We investigate the possibility that the existence of established firms in sector $x$ enables new firms in sector $y$ to signal their quality with co-branding and price. In a separating equilibrium with co-branding and price, no firm can sell its product without being associated with a good brand in sector $x$. Hence, neither ($\text{IC}^{h**}_{LH}$) nor ($\text{IC}^{h**}_{LL}$).exists. One obvious necessary condition that an introductory price $p^1_{HH}$ should satisfy to constitute a separating equilibrium with co-branding is:

$$(\text{IR}^{h**}_{HH}) \quad V^h_{HH}(p^1_{HH}) \geq 0. \tag{15}$$

When $\nu_x < \nu_y$, good brand names in sector $x$ are scarce with respect to the number of high type firms in sector $y$. Therefore, good brand firms in sector $x$ will have all bargaining power with respect to the high type firms in sector $y$, implying that firms in sector $y$ will have zero payoff. The unique possible deviation for a high-type firm in sector $x$ is to be matched with a low type firm in sector $y$. This deviation is not profitable if the following incentive constraint is satisfied.

$$(\text{IC}^{h**}_{HL}) \quad V^h_{HL}(p^1_{HH}) \leq V^h_{HH}(p^1_{HH}). \tag{16}$$

The previous argument shows that ($\text{IR}^{h**}_{HH}$) and ($\text{IC}^{h**}_{HL}$) are sufficient and necessary conditions to have a separating equilibrium with co-branding. We have the following lemma:

**Lemma 2** *Consider the asymmetric case in which sector $x$ is mature and sector $y$ is new. For given information structure regarding performance observability $h = C$ or $P$, a separating equilibrium with co-branding and price exists if and only if $\Delta^h_{HH,HL} \geq 0$.*

**Proof.** ($\text{IR}^{h**}_{HH}$) and ($\text{IC}^{h**}_{HL}$) imply $\Delta^h_{HH,HL} \geq 0$ since ($\text{IC}^{**}_{HL}$) is equivalent to $\Delta^h_{HH,HL} \geq 0$.

The reverse part is also trivial: if $\Delta^h_{HH,HL} \geq 0$, we have a $p^1_{HH}$ satisfying $(\text{IR}^{h**}_{HH})$ and $(\text{IC}^{h**}_{HL})$: for instance, it is enough to choose $p^{1h}_{HH}$ satisfying $V^h_{HL}(p^{1h}_{HH}) = 0$. ■

From both lemmas, we have the following Proposition.

**Proposition 2** *For given information structure regarding performance observability $h = C$ or $P$, if a separating equilibrium with price exists in the symmetric case, then a separating equilibrium with co-branding and price exists in the asymmetric case.*

The proposition essentially says that the condition to have a separating equilibrium is *weakly* less stringent in the asymmetric case than in the symmetric case. The intuition is simply that in the asymmetric case $(\text{IC}^{h**}_{LL})$ does not exist while it exists in the symmetric case. We now investigate when the condition for a separating equilibrium is *strictly* less stringent in the asymmetric case. We demonstrate that in the CPO case, the incentive compatibility constraint $(\text{IC}^{C*}_{LL})$ is not binding under signaling with prices. As a result, co-branding has no bite in the absence of the consumers' cross-sector inference problem. However, in the PPO case, the incentive compatibility constraint $(\text{IC}^{P*}_{LL})$ can be binding. In such a case, co-branding strictly relaxes the condition for the existence of a separating equilibrium and has the potential to be a more effective instrument of signaling.

### 4.2.2 Component performance observability (CPO) case

Consider the CPO case in which consumers can identify the culprit of the failure when the final product fails to perform. In such a case, the following lemma states that the additional constraint under signaling with prices, $\Delta^C_{HH,LL} \geq 0$, is redundant.

**Lemma 3** *In the component performance observability (CPO) case, $\Delta^C_{HH,HL} = \Delta^C_{HL,LL}$.*

**Proof.** From the definition of $V^C_{HH}(p^1_{HH})$, $V^C_{HL}(p^1_{HH})$, $V^C_{LL}(p^1_{HH})$ in section 4.2.1, we have

$$\Delta^C_{HH,HL} = \Delta^C_{HL,LL} = -(1+\delta)(c_H - c_L) + \delta\frac{(q_H - q_L)}{2}\left[(q_H)^2 - (q_L)^2\right].$$

■

Note that $\Delta^C_{HH,LL} = \Delta^C_{HH,HL} + \Delta^C_{HL,LL}$. Lemma 3 thus implies that $\Delta^C_{HH,LL} = 2\Delta^C_{HH,HL}$. Therefore, sign $(\Delta^C_{HH,LL}) = \text{sign}(\Delta^C_{HH,HL})$, that is, $\Delta^C_{HH,HL} \geq 0$ if and only if $\Delta^C_{HH,LL} \geq 0$. As a consequence, we have the following proposition that states the equivalence between signaling with prices and signaling with co-branding and price in the CPO case.

**Proposition 3** *In the component performance observability (CPO) case, technological complementarity does not imply complementarity in reputation building (i.e. the condition to have a separating equilibrium is the same both for the symmetric case and the asymmetric case). The condition ($\Delta^C_{HH,LL}(= 2\Delta^C_{HH,HL}) \geq 0$) can be stated equivalently in terms of primitive parameters as follows:*

$$\begin{aligned} A^{C*} &\equiv \frac{[q_H - q_L]}{2}\left[(q_{HS})^2 - (q_{HF})^2\right] > (c_H - c_L) \\ \textit{and } \delta &\geq \delta^{C*} \equiv \frac{c_H - c_L}{A^{C*} - (c_H - c_L)}. \end{aligned}$$

In order to provide an intuition of the result of the above proposition, define the stand-alone gain from building reputation as follows:[14]

$$\pi^{SA}_\theta(p^1_{HH}; p^{2C}_{SS}, p^{2C}_{FF}) = \frac{p^1_{HH}}{2} - c_\theta + \delta\left[q_\theta(p^{2C}_{SS}/2) + (1-q_\theta)(p^{2C}_{FF}/2) - c_\theta\right] \text{ for } \theta = H, L. \tag{17}$$

$\pi^{SA}_\theta$ represents a $\theta$-type firm's (expected) profit over two periods when consumers believe that it is a high type firm. Then, a $\theta\theta'$ pair's total payoff from masquerading as an $HH$ pair is given as follows:

$$V^C_{\theta\theta'}(p^1_{HH}) = \pi^{SA}_\theta(p^1_{HH}) + \pi^{SA}_{\theta'}(p^1_{HH}) \text{ for } \theta, \theta' \in \{H, L\}^2. \tag{18}$$

Equation (18) tells that in the CPO case, the type of a firm's partner has no impact on the firm's gain from building reputation. Therefore, we have:

$$V^C_{HH} - V^C_{LL} = 2(V^C_{HH} - V^C_{HL}) = 2(\pi^{SA}_H - \pi^{SA}_L). \tag{19}$$

Hence, co-branding is irrelevant for the existence of separating equilibrium. The separating equilibrium exists as long as a high type's stand-alone gain from building reputation is higher than that of a low type, which is similar to what happens in the single component product case.

**Remark 1**: The rematch in period two is a necessary condition to get the result of no complementarity in reputation building in the CPO case in proposition 3. For instance, if we consider a once-and-for-all match, the negative feedback effect that a low type partner inflicts on a high type firm's reputation, identified in section 4.2.3, kicks in even in the CPO case since a success record is more appreciated when the partner has

[14] In the definition, we assume equal sharing of the period one revenue for expositional facility; this assumption is not needed for the arguments we make since none of $\Delta^h_{HH,HL}$, $\Delta^h_{HL,LL}$and $\Delta^h_{HH,LL}$ does depend on $p^1_{HH}$.

also a success record than when it has a failure record. Therefore, co-branding strictly facilitates signalling even in the CPO case, which makes the complementary case with two sectors different from the single component product case.

### 4.2.3 Product performance observability (PPO) case

Consider now the PPO case in which consumers face a cross-sector inference problem in that they cannot identify the true cause of failure when a final product fails. In such a case, we demonstrate that co-branding relaxes the condition for the existence of a separating equilibrium. In other words, in the asymmetric case in which one sector is mature and firms in that sector have established reputation, co-branding may allow firms in the new sector signal the quality that they could not signal with prices when both sectors are new. The following lemma shows that ($\text{IC}_{LL}^{P*}$) the extra incentive constraint in the symmetric case is more stringent than ($\text{IC}_{HL}^{P**}$) the incentive constraint in the asymmetric case and thus can be a binding constraint.

**Lemma 4** *In the product performance observability (PPO) case,* $\Delta_{HH,HL}^{P} - \Delta_{HL,LL}^{P} = \delta\frac{(q_H-q_L)^2}{1+q_H}\left[(q_H)^2-(q_L)^2\right] > 0$.

**Proof.** From the definition of $V_{HH}^{P}(p_{HH}^{1})$, $V_{HL}^{P}(p_{HH}^{1})$, $V_{LL}^{P}(p_{HH}^{1})$ in section 4.2.1, we have

$$\begin{aligned}\Delta_{HH,HL}^{P} - \Delta_{HL,LL}^{P} &= [V_{HH}^{P}(p_{HH}^{1}) - V_{HL}^{P}(p_{HH}^{1})] - [V_{HL}^{P}(p_{HH}^{1}) - V_{LL}^{P}(p_{HH}^{1})] \\ &= \delta\left(q_H - q_L\right)^2\left[p_{SS}^{2P} - p_{FF}^{2P}\right] = \delta\frac{\left(q_H-q_L\right)^2}{1+q_H}\left[(q_{HS})^2-(q_{HF})^2\right] > 0.\end{aligned}$$

■

The lemma implies that it is possible to have $\Delta_{HH,HL}^{P} \geq 0$ and $\Delta_{HH,LL}^{P} < 0$ at the same time. To see this, notice that $\Delta_{HH,LL}^{P} = \Delta_{HH,HL}^{P} + \Delta_{HL,LL}^{P}$. Therefore, the condition for $\Delta_{HH,LL}^{P} < 0$ can be written as $\Delta_{HH,LL}^{P} = \Delta_{HH,HL}^{P} + \Delta_{HL,LL}^{P} = \{\Delta_{HL,LL}^{P} + \delta\frac{(q_H-q_L)^2}{1+q_H}\left[(q_H)^2-(q_L)^2\right]\} + \Delta_{HL,LL}^{P} < 0$. As a result, we can have $\Delta_{HH,HL}^{P} \geq 0$ and $\Delta_{HH,LL}^{P} < 0$ be satisfied simultaneously when the following condition holds:

$$-\delta\frac{\left(q_H-q_L\right)^2}{1+q_H}\left[(q_{HS})^2-(q_{HF})^2\right] \leq \Delta_{HL,LL}^{P} < -\delta\frac{\left(q_H-q_L\right)^2}{2\left(1+q_H\right)}\left[(q_{HS})^2-(q_{HF})^2\right].$$

By using the explicit expression for $\Delta_{HL,LL}^{P} = -(1+\delta)(c_H-c_L)+\delta\frac{q_L[q_H-q_L]}{(1+q_H)}\left[(q_{HS})^2-(q_{HF})^2\right]$), we can rewrite the condition above in terms of primitive parameters:

$$\frac{\left(q_H+q_L\right)\left(q_H-q_L\right)}{2(1+q_H)}\left[(q_{HS})^2-(q_{HF})^2\right] < \frac{(1+\delta)}{\delta}(c_H-c_L) \leq \frac{q_H[q_H-q_L]}{1+q_H}\left[(q_{HS})^2-(q_{HF})^2\right].$$

We have:

**Proposition 4** *In the product performance observability (PPO) case, technological complementarity implies complementarity in reputation building (i.e. the condition to have a separating equilibrium is strictly less stringent in the asymmetric case than in the symmetric case). More precisely, if the following inequality holds*

$$\frac{(q_H+q_L)(q_H-q_L)}{2(1+q_H)}\left[(q_{HS})^2-(q_{HF})^2\right]<\frac{(1+\delta)}{\delta}(c_H-c_L)\leq\frac{q_H[q_H-q_L]}{1+q_H}\left[(q_{HS})^2-(q_{HF})^2\right]$$

*the separating equilibrium exists under signaling with co-branding and price in the asymmetric case but does not exist under signaling with price in the symmetric case.*

In order to provide an intuition of the result of the above proposition, consider the simple case in which $1>q_H>q_L=0$. In this particular case, we have

$$\begin{aligned}V_{HL}^P(p_{HH}^1) &= p_{HH}^1-c_H-c_L+\delta\left\{p_{FF}^{2P}-c_H-c_L\right\}\\ V_{LL}^P(p_{HH}^1) &= p_{HH}^1-c_L-c_L+\delta\left\{p_{FF}^{2P}-c_L-c_L\right\}.\end{aligned}$$

Therefore, when $c_H>c_L$, we have $V_{LL}^P(p_{HH}^1)>V_{HL}^P(p_{HH}^1)$. As a consequence, satisfying $\Delta_{HH,LL}^P=V_{HH}^P-V_{LL}^P\geq 0$ is more difficult than satisfying $\Delta_{HH,HL}^P=V_{HH}^P-V_{HL}^P\geq 0$. This implies that signaling through prices is more difficult than signaling through co-branding and prices. Intuitively, in the asymmetric case, a good brand firm in the mature sector has little incentive to choose as its partner a low type firm in the new sector because of the *negative feed-back effect* on its reputation; when $q_L=0$, the final product produced together with a low type will fail for sure and therefore the good brand's reputation will be sullied from a failure record. By contrast, in the symmetric case, both an $LL$ pair and an $HL$ pair can masquerade as an $HH$ pair by charging price $p_{HH}^1$. When $q_L=0$, a firm in an $LL$ pair does not suffer from any negative feedback on its reputation while the total production cost of an $LL$ pair is lower than that of an $HL$ pair. Therefore, preventing an $LL$ pair from masquerading as an $HH$ pair is more difficult than preventing an $HL$ pair from masquerading as an $HH$ pair, which implies that co-branding relaxes the condition to have a separating equilibrium.

**Remark 2**: The condition for the existence of a separating equilibrium in the asymmetric case with PPO is given by:

$$\begin{aligned}A^{P**} &\equiv \frac{q_H[q_H-q_L]}{1+q_H}\left[(q_{HS})^2-(q_{HF})^2\right]>(c_H-c_L)\\ \text{and } \delta &\geq \delta^{P**}\equiv\frac{c_H-c_L}{A^{P**}-(c_H-c_L)}.\end{aligned}$$

This condition is more stringent than the existence condition in the CPO case as long as $q_H < 1$. Therefore, given an initial information structure on firms' types (i.e. the symmetric case or the asymmetric case), the change from the CPO case to the PPO case makes signaling more difficult. Hence, Proposition 4 does not imply that the change from the CPO case to the PPO case is desirable for signalling purpose.

# 5 Applications

**1. Movie Stars and Blockbusters in the Motion Picture Industry**

We apply our model to the role movie stars play in the motion picture industry. Motion pictures are an experience good whose quality can be only ascertained with consumption. The final product also requires many complementary inputs, most important of them are actors, directors, and scripts among other things. For an executive of a studio who has a project, it is essential to convince (i.e., signal) the quality of the project to outsider financiers and final consumers to get the necessary funding for the project and generate considerable grosses.

In addition, in the motion picture industry, it is a reasonable assumption that movie stars and top directors are much more recognizable to financiers and final consumers than other key complementary inputs such as scripts. Thus, the involvement of established movie stars in a movie project may be associated with the high quality of other key inputs, and thus overall quality of the movie. Ravid (1999), for instance, describes the institutional details of the motion picture industry in which star participation can signal superior information, that is, a star will commit to a project because he or she knows that it is of high quality. According to Ravid (1999), we can view that the commitment of move stars in an early stage of the project "signals the quality of the project to the studio or to outside financiers (p. 465)."

The empirical literature also corroborates the signaling hypothesis, suggesting that the participation of a movie superstar acts as a signal of the quality of the project. For instance, Albert (1998) finds that stars serve as a signal or "markers" of film types and the value of stars is partly due to their ability to signal the quality of films in a consistent and predictable way.[15] In addition, Chisholm (2004) finds a strong support for the hypothesis that the presence of stars plays a valuable role for studio executives *ex ante* even though an established star does not ensure a successful film *ex post*, and hence her data are

[15] According to Albert (1998), a superstar movie actor, "either through his talent, ability to choose, or ability to acquire investment, is a marker of many successful films (p. 255)."

consistent with the signaling hypothesis.

**2. Multinationals and Outsourcing**

Co-branding can be also used for a strategy to penetrate foreign markets by multinationals. Abratt and Motlana's (2002) case studies of Danone, a French multinational of yogurt manufacturer, and McCain Foods, a Canadian frozen food multinational, document how these firms penetrated into the South African market by co-branding with "well-known" local brands, Clover SA and Irvin and Johnson, respectively. These case studies show how co-branding can be used in brand transitions.[16] Our model may also shed some light on the rising activities in outsourcing we observe recently.[17] Nike, for instance, does most manufacturing activities through contractual arrangements with oversee factories whereas the design and marketing of its products is conducted in the United States. What distinguishes this kind of outsourcing activities from the purchase of any other foreign products by American consumers is the use of outsourcing firms' brand names. Perhaps foreign producers may have a hard time in penetrating the American market due to low recognition of their own brand names; American consumers may be reluctant to purchase foreign products with unknown brand names due to quality concerns. In contrast, they are willing to buy Nike products with a high price even though they are well aware that the actual manufacturing has been carried out by foreign manufacturers. They trust that Nike knows the capability of foreign manufacturers better than they do and the fact that Nike chose a particular foreign manufacturer certifies the capability of the foreign producer.

**3. Co-authorship with Established Scholars**

The same intuition can be applied to co-authorship decisions in the academic market. The currency in the academic market is recognition. Even though there is no monetary price attached to academic work, we can interpret the amount of attention readers pay to any particular work as its market price. When there are co-authors for a particular paper, the quality of the paper depends on the quality of contributions by all authors. When the quality of the paper is low, it may be difficult for outside readers to ascertain who is responsible for the quality. It is a reasonable assumption that potential collaborators

[16]The example of Lenovo can be interpreted in a similar vein. When IBM sold its PC division to China-based Lenovo Group and take a minority stake, the deal allowed Lenovo to use IBM's logo for five years. After the sale, Lenovo marketed its PC with the brand name of "Lenovo IBM Thinkpad." The rationale behind the deal for Lenovo was believed to be that the co-branding arrangement helps Lenovo expand in the global market where it is relatively unknown.

[17]See Feenstra (1998).

know each other's ability compared to outsiders. It may be difficult for unknown young scholars to convince other academics to read their papers. In such a case, one effective way to signal their ability and build reputation may be to work with established scholars. Established scholars will be selective in choosing their co-authors as long as there is residual uncertainty about senior scholar's ability as assumed in our model. As a result, co-authorship with established scholars can be used to signal young scholar's ability when it would be difficult to do on their own with single authorship. The following remark from a Nobel laureate illustrates well our point:

"... it clearly did my student... no harm at all to have me as a second author of the paper. It called people's attention to the paper who might otherwise not [have] read it at all... Nor as a matter of fact, did it do me any harm, even if I was heavily responsible for it, to have him as co-author." (Zuckerman, 1967, p.396)

# 6 Concluding Remarks

In this paper, we have developed a theory of reputation leverage in which the reputation of an established firm is used to help an unknown firm establish its own reputation through co-branding. The marketing literature points out an increasing popularity of co-branding in the introduction of new consumer products in recent years and posits that the practice plays a key role in changing consumers' perception that the two constituent products will be regarded as being similar quality. Our model provides a micro-foundation of the mechanism through which such a linkage in consumers' perception takes place and identifies conditions under which co-branding can be a signaling device of product quality in addition to pricing. We have considered a setting in which a final product requires two complementary components. This provides a natural environment in which non-observability of component performance arises. One consequence of the technological complementarity assumption is that every firm needs to find a partner and needs co-branding. However, what is important is the non-observability of individual performance. If we consider two independent products without any technological complementarity, then co-branding would be an endogenous choice. As long as the non-observability of individual component performance arises with co-branding, our main results will be robust with independent products.

We believe that a similar leverage mechanism can be applied to a wide range of contexts that entail alliances of multiple parties, as we discussed in the previous section. Our theory also suggests that in the formation of strategic alliances reputation can constitute

a key complementary asset that an established agent can bring to the table. We intend to extend our framework to explain various types of partnerships and hope that our theory shed light on the role of reputation in the formation of such alliances.

In particular, our perspective is useful to understand the role that brands of multinational firms play in the era of globalization. As the examples of our previous section illustrate, we can distinguish two types of leverage when reputation leverage occurs through co-branding between a global brand and a local one. Either the co-branding leverages the latter's reputation to the former (if the local brand enjoys high recognition and the global brand is relatively unknown in the local context) and thereby helps the former to penetrate local markets or it leverages the latter's reputation to the former. Through the second type of leverage, globalization can bring important benefit to a developing economy which is poor in reputation capital. However, a multinational would be reluctant to leverage its reputation to a local firm if the latter has an ambition to be a global player; in this case, it might prefer outsourcing to co-branding. It would be interesting to study global brands' incentives to choose among different strategies in penetrating local markets.

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# 7 Appendix: when $\nu_x \geq \nu_y$

## 7.1 The second period

Since the proof of the case $\nu_x < \nu_y$ also applies to the case $\nu_x = \nu_y$, we only need to consider $\nu_x > \nu_y$. In the symmetric case in which no firm has initially established reputation, the previous analysis of $\nu_x < \nu_y$ carries over to $\nu_x > \nu_y$. However, in the asymmetric case in which types of firms in sector $x$ are known to consumers from the beginning, a difference with respect to $\nu_x \leq \nu_y$ arises because a high-type firm in sector $y$ which produced in period one can deviate and be matched with a good brand (i.e. a high-type) firm in sector $x$ which did not produce in period one. In general, this will strengthen the (period two) bargaining position of high-type firms in sector $y$ with respect to high-type firms in sector $x$ that produced in period one that the former will have more than a half of the revenue. But it will not affect the positive sorting result as long as $q_{HN} < q_{HF}$ holds. Since in section 4.2 we focus on the case of equal sharing of revenue among high type firms with the same past record (perceived by consumers) in period two for simplicity, we here provide a sufficient condition under which the deviation in question is not profitable under the equal sharing: $2q_{HN} < q_{HF}$. When $2q_{HN} < q_{HF}$ holds, no high type firm in sector $y$ which produced in period one has an incentive to be matched with a high type firm in sector $x$ which did not produce in period one both for the CPO case and the PPO case.

## 7.2 The first period

Note first that the results in section 4.2.2 and 4.2.3 do not depend on whether $\nu_x < \nu_y$ or not. Therefore, it is enough to prove lemma 1 and lemma 2 for the case $\nu_x \geq \nu_y$.

**Proof of lemma 1 for $\nu_x \geq \nu_y$**

Consider first $\nu_x > \nu_y$. This case is completely symmetric to the case $\nu_x < \nu_y$. This symmetry and $V^h_{HL}(p^1_{HH}) = V^h_{LH}(p^1_{HH})$ imply that the case $\nu_x > \nu_y$ is identical to the case $\nu_x < \nu_y$.

Consider now $\nu_x = \nu_y$. Without loss of generality, we can consider equal revenue sharing between high-type firms in period one. In this case, $(\text{IC}^{h*}_{LL})$ and $(\text{IR}^{h*}_{HH})$ remain unchanged but $(\text{IC}^{h*}_{LH})$ is written as follows:

$$(\text{IC}^{h*}_{LH})' \quad V^h_{LH}(p^1_{HH}) \leq V^h_{HH}(p^1_{HH})/2.$$

From the complete symmetry, $(\text{IC}^{h*}_{LH})'$ is equivalent to $(\text{IC}^{h*}_{HL})'$. It is easy to see that $(\text{IC}^{h*}_{LL})$, $(\text{IC}^{h*}_{HL})'$ and $(\text{IR}^{h*}_{HH})$ are sufficient and necessary conditions to have a separating equilibrium. Obviously, $(\text{IC}^{h*}_{LL})$, $(\text{IC}^{h*}_{HL})'$ and $(\text{IR}^{h*}_{HH})$ imply $\Delta^h_{HH,HL} \geq 0$ and $\Delta^h_{HH,LL} \geq 0$. Furthermore, from the proof of lemma 1, we know that $\Delta^h_{HH,HL} \geq 0$ and $\Delta^h_{HH,LL} \geq 0$ implies that there exists a price $p^1_{HH}$ that satisfy $(\text{IC}^{h*}_{LL})$, $(\text{IC}^{h*}_{HL})$ and $(\text{IR}^{h*}_{HH})$. Since $(\text{IC}^{h*}_{HL})$ implies $(\text{IC}^{h*}_{HL})'$, the same price satisfies $(\text{IC}^{h*}_{LL})$, $(\text{IC}^{h*}_{HL})'$ and $(\text{IR}^{h*}_{HH})$.

**Proof of lemma 2 for $\nu_x \geq \nu_y$**

Consider first $\nu_x > \nu_y$. In this case, $(\text{IR}^{h**}_{HH})$ remains unchanged but $(\text{IC}^{h**}_{HL})$ is written as follows:

$$(\text{IC}^{h**}_{HL})' \quad V^h_{HL}(p^1_{HH}) \leq 0.$$

It is because there are some high type firms in sector $x$ which are not matched with high type firms in sector $y$ and they can be matched with low type firms in sector $y$. It is easy to see that (i) $(\text{IR}^{h**}_{HH})$ and $(\text{IC}^{h**}_{HL})'$ are sufficient and necessary conditions to have a separating equilibrium and (ii) $(\text{IR}^{h**}_{HH})$ and $(\text{IC}^{h**}_{HL})'$ are equivalent to $\Delta^h_{HH,HL} \geq 0$.

Consider now $\nu_x = \nu_y$. In this case, $(\text{IR}^{h**}_{HH})$ remains unchanged but $(\text{IC}^{h**}_{HL})$ is written as follows:

$$(\text{IC}^{h**}_{HL})'' \quad V^h_{HL}(p^1_{HH}) \leq \alpha_x V^h_{HH}(p^1_{HH}),$$

where $\alpha_x \in [0,1]$ is the fraction of $V^h_{HH}(p^1_{HH})$ that goes to a high type firm in sector $x$. It is easy to see that (i) $(\text{IR}^{h**}_{HH})$ and $(\text{IC}^{h**}_{HL})''$ are sufficient and necessary conditions to have a separating equilibrium and (ii) $(\text{IR}^{h**}_{HH})$ and $(\text{IC}^{h**}_{HL})''$ are equivalent to $\Delta^h_{HH,HL} \geq 0$.