Interconnection among Academic Journal Platforms: Multilateral versus Bilateral Interconnection*

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Abstract

Electronic academic journal platforms provide new services of text and/or data mining and linking, indispensable for efficient allocation of attention among over-abundant sources of scientific information. Fully realizing the benefit of these services requires interconnection among the platforms. Motivated by CrossRef, a multilateral citation linking backbone, this paper performs a comparison between a multilateral interconnection regime and a bilateral one and finds that publishers are fully interconnected in the former while they may partially break connectivity in the latter for exclusion or differentiation motives. Surprisingly, if partial interconnection arises for differentiation motive, exclusion of small publisher(s) occurs more often under the multilateral regime than under the bilateral one. In addition, we show that our main result is robust in the case of Internet Backbone interconnection. Finally, when publishers can interconnect both in a multilateral way and in a bilateral way, a conflict between a private incentive and a social incentive may arise when large publishers prefer excluding small publishers by opting for a bilateral interconnection. In this case, a light-handed regulation imposing no discrimination among rivals would foster full interconnection.

Key Words: Multilateral Interconnection, Bilateral Interconnection, Academic Journals, Internet, Platforms

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"A wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it" (Simon 1971, p. 40-41).

1 Introduction

Electronic publishing has been bringing fundamental changes in the market for academic journals.[1] In particular, digitalization of text, data and image is transforming the system of academic communication as an interactive one based on the new techniques of text, data and image mining and linking.[2] These techniques are extremely useful for the effective dissemination of scientific knowledge as the volume of scientific information grows exponentially. For instance, in biology where large amount of data are accumulating on genes, proteins etc., it is all but impossible for a single researcher to keep pace with new information about just a handful of genes even though he or she has access to information about 30000 genes. The techniques help researchers to make efficient allocation of their attention among the overabundant information sources by allowing them to extract meaning from digitized text and data and to search for the relevant information. The techniques are useful not only in data-rich hard science but also have many applications in social sciences and humanities.[3]

Fully realizing the benefit from techniques of text and data mining and linking requires interconnection (i.e. interoperability) among different platforms of scholarly publications such that seamless cross-platform search and navigation can be made. In this paper, we study publishers' incentive to interconnect their journal platforms.

Actually, a large number of publishers provide links through CrossRef, a backbone offering a collaborative reference linking service that allows users to click on a citation and be taken directly to the target content. It currently has over 1,462 participating publishers and societies[4] and more than 16 million content items are registered in the

[1] For instances, it has allowed publishers to practice bundling and price discrimination based on usage. Furthermore, open access journals which provide free on-line access have been introduced as an alternative to the traditional reader (or library)-pays model of journal pricing.

[2] For instance, in biology, there is a software which can recognize a two-dimensional image of a molecule and search for all the articles studying the same molecule.

[3] For instance, "digitized corpus can be analyzed in ways scholars whose work is confined to printed volumes are not able to explore."(European Commission, 2007, p.15)

[4] The Board of Directors currently comprises representatives from AAAS (Science), AIP, ACM, APA, Blackwell Publishers, Elsevier Science, IEEE, Wolters Kluwer, Nature, Sage, Springer, Taylor & Francis, Thieme, University of California Press, University of Chicago Press and Wiley.

CrossRef system.[5] In particular, it allows publishers to avoid bilateral linking agreements since a single agreement with CrossRef serves as a linking agreement with all participating publishers.

Motivated by the practice of CrossRef, this paper performs a comparison between a multilateral interconnection regime à la CrossRef and a bilateral interconnection regime in terms of incentives to interconnect. We also investigate how interconnection affects profits and social welfare. As the main result, we find that all active publishers interconnect under the multilateral regime even when they are asymmetric while there can be partial interconnection under the bilateral regime even among symmetric publishers. The result suggests that a clause of no discrimination among rival companies facilitates full interconnection among all firms.

Interconnection (or compatibility) choice among firms in network industry has been a subject of intensive investigation.[6] The seminal papers on economics of network externalities (Farrell and Saloner 1985, 1986, Katz and Shapiro, 1985) study the compatibility issue. For instance, Katz and Shapiro (1985) show that a dominant firm may choose to remain incompatible with a rival because it will suffer a substantial decline in market share if it becomes compatible.[7] More recently, the literature on two-way access pricing among telecommunication networks, initiated by Armstrong (1998) and Laffont-Rey-Tirole (1998a,b), studies how access prices affect retail competition and interconnection through telecommunications networks' choice of retail tariffs. In addition, Crémer-Rey-Tirole (henceforth, CRT, 2000) and Laffont-Marcus-Rey-Tirole (2003) study interconnection among Internet Backbone providers (IBP) in a peering regime or in a regime of access pricing respectively.[8] A general finding in the literature on interconnection without access pricing is that when networks are asymmetric, big networks might have an incentive to make the networks incompatible (or break connectivity) since complete compatibility (or interconnection) means that big and small networks become equal (Katz-Shapiro, 1985, CRT, 2000). Since we study interconnection without access pricing, we focus on asymmetric publishers although we analyze symmetric publishers as well. We contribute to the literature by studying a multilateral interconnection and comparing it with a bilateral one: the existing literature on interconnection typically considers two firms and hence does not make distinction between the two modes of interconnection.[9]

[5]See its annual report 2004-05 at http://www.crossref.org/07annual/index.html

[6]See for instance Church and Gandal (2005) for a recent survey.

[7]Farrell and Saloner (1985, 1986) examine a dynamic issue of how network externalities affect the adoption of a new technology and identify inefficiency in terms of both "excess momentum" and "excess inertia".

[8]See Economides (2006) for a survey on the economics of the Internet Backbone market.

[9]To our knowledge, Matutes-Padilla (1994) and CRT (2000) are the only papers that consider more

Regarding interconnection among electronic academic journal platforms, U.K. Competition Commission (2001) mentions big publishers' incentive not to provide links to other publishers' platforms in its report about the merger between Reed Elsevier (RE) and Harcourt:[10]

"... we had received some expressions of concerns from others in the industry that RE might try to undermine its competitors by denying them links with ScienceDirect, ...(p.22)".

The recent report on the market for academic journals commissioned by European Commission, Dewatripont et al. (2006), devotes a section to the issue of interoperability and recommends to foster interoperability by supporting research and development and by promoting wide implementation of linking technologies. To the best of our knowledge, no paper has studied interconnection among academic journal platforms in a formal model.

Interestingly, academic journal platforms differ from other typical platforms in network industries (such as mobile phone, Internet access, ATM cards) in their multi-homing nature. More precisely, a user must get access to both journal platforms in order to enjoy the benefit from seamless navigation across them while in the case of mobile phone networks, for instance, it is enough to subscribe to one of them to benefit from interconnection. There are other industries such as railroads (for instance, Eurail), ski resorts etc. that share the features of the academic journal industry in that facilitating navigation across platforms owned by different firms generates significant value to consumers. Furthermore, the anti-competitive issue related to refusal to provide links to rivals in academic journals is similar to the one in the well-known case, Aspen Skiing Company v. Aspen Highlands Skiing Company, 472 U.S. 585 (1985), in which the former owning three among all four ski resorts in Aspen refused to market all-Aspen ticket in order to weaken the competitive position of the latter owning only one resort.

than two firms. However, none of them makes a distinction between a multilateral and a bilateral interconnections. Matutes-Padilla study compatibility choice among three ATM networks but use a very strong equilibrium concept, the Perfect Coalition-Proof Nash Equilibria. As a consequence, they find that complete compatibility never arises even though networks are symmetric, which is in contrast to the result of full interconnection among symmetric networks obtained by CRT (2000). In Matutes-Padilla, starting from complete compatibility, any coalition of two networks has an incentive to deviate by breaking the connectivity with the third but maintaining the connectivity between themselves. Their equilibrium concept may be appropriate to an asymmetric situation in which two incumbents face an entrant but does not allow to capture the insight from LRT that interconnection, compared to no interconnection, allows to improve a network's competitive position when networks are symmetric.

[10] At the time of the merger, RE's ScienceDirect was the most developed website and offered access to around 1,150 journals and Harcourt's IDEAL offered access to 320 journals.

We study games of interconnection and pricing among heterogenous publishers. To model price competition among publishers, we build on our previous work, Jeon and Menicucci (2006): assuming price discrimination based on usage,[11] we consider a situation in which (for-profit) publishers owning different portfolio of journals compete to sell them to a library[12] which faces a budget constraint. Since we know from Jeon and Menicucci (2006) that each publisher has an incentive to bundle its journals, we assume, without loss of generality, that all publishers practice bundling. In this model, the number of publishers which succeed in making sales with positive profits increases with the size of the budget.

Our analysis focuses on how interconnection affects which bundles of journals are sold and at what prices. In our model, there are three publishers (the large, the middle and the small one) and each publisher competes for relative standing as in the Hotelling or circular city model since the industry profit is equal to the budget of the library, which is given. We assume that the value created by interconnection of two bundles of journals exhibits economies of scale and increases with the stand-alone value of each bundle.[13] This implies that full interconnection among all publishers improves the relative standing of the large publisher and worsens that of the small one.

When the budget is large enough that all publishers remain active regardless of interconnection profiles, we find that each publisher chooses to interconnect with all rival publishers both in the multilateral and in the bilateral interconnection regime. This is because interconnection strictly improves one's relative standing compared to no interconnection. However, if interconnection profiles affect the small publisher's ability to remain active in the market, larger publishers may be tempted to break connectivity to exclude the small one.

We first consider the case in which publishers can interconnect only through a multilateral regime. This case corresponds to the case in which the government imposes a no-discrimination clause in terms of interconnection. In this case, we find that exclusion motive does not affect incentives to interconnect and all active publishers are fully interconnected. Note first that since full interconnection among all publishers weakens the small publisher's relative standing, the latter is excluded more often than in the absence of interconnection. If publisher 1 (the large one), for instance, does not interconnect in order

[11] For instance, Derk Haank (2001), the CEO of Elsevier Science, says: "What we are basically doing is to say that you pay depending on how useful the publication is for you—estimated by how often you use it." See also Bolman (2002) and Key Perspectives (2002) with regard to price discrimination.

[12] In section 4 we consider one library and, in section 6, we extend it to any number of heterogeneous libraries.

[13] This assumption is standard in the literature: the value of interconnection increases with the size of the interconnected networks. Furthermore, in section 2, we give a microfoundation to this assumption.

to exclude publisher 3 (the small one), both publishers 2 and 3 respond by interconnecting. This weakens 1's relative standing while improving the rivals' standing.

Under the bilateral interconnection, interconnection between two publishers occurs only if each of them chooses to interconnect with the other. We find that partial interconnections may arise for two different motives: *exclusion motive and differentiation motive*. First, when the big and the middle ones are similar, in order to exclude the small one, the large one (or the middle one) may break connectivity with the small one while maintaining connectivity with the middle one (or the large one). In this case, the exclusion of the small one occurs more often under the bilateral interconnection regime than under the multilateral one. Second, when the big one is much larger than the middle one and the middle one is similar to the small one, surprisingly, the big one may prefer maintaining connectivity only with the small one (and hence allowing it to be active in the market) to maintaining connectivity only with the middle one (and hence excluding the small one). The first strategy allows the big one to improve its relative standing with respect to the middle one (i.e. to further differentiate itself from the middle one) so that its profit is larger than its duopoly profit obtained with the second strategy. In this case, the exclusion of the small one occurs less often under the bilateral interconnection regime than under the multilateral one since the small one is the only one interconnected with both other publishers.

We find qualitatively the same result in the case of symmetric publishers. All publishers are active and fully interconnected under the multilateral interconnection regime and the same outcome is obtained under the bilateral interconnection regime as long as the budget is larger than a threshold. However, if the budget is smaller than the threshold, surprisingly, exclusion of one publisher occurs due to partial interconnection.

Therefore, the multilateral interconnection regime provides stronger incentives to interconnect than the bilateral one. From social welfare point of view, the multilateral regime is (at least weakly) better than the bilateral one if publishers are symmetric. However, if they are asymmetric, the multilateral regime is preferred to the bilateral one if partial interconnection arises for exclusion motive while the ranking is reversed if partial interconnection arises for differentiation motive.

Finally, we endogenize the interconnection regime by analyzing a hybrid interconnection game in which publishers are free to interconnect in any of the two regimes. We find that the market outcome coincides with the social optimum except when the big publisher wants to exclude the small one by opting for the bilateral interconnection. When the conflict arises, our previous results suggest that a light-handed regulation forbidding discrimination among rivals would foster full interconnection and restore efficiency.

As a robustness check, we compare the multilateral interconnection regime with the bilateral one in the case of interconnection among IBPs in the setting of CRT (2000) and find that our main insight is robust: firms have stronger incentives to interconnect under the multilateral regime than under the bilateral one.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the pricing game given any interconnection profile. Section 4 studies the game without interconnection, the game of multilateral interconnection, the game of bilateral interconnection, compares profits and social welfare and also study the case of symmetric publishers. Section 5 considers a hybrid interconnection game and endogenizes the interconnection regime. Section 6 performs robustness checks: section 6.1 extends the multilateral interconnection game to any number of heterogenous libraries and section 6.2 compares the two modes of interconnection in the case of Internet interconnection. Section 7 provides our conclusion and discusses policy implications. All the proofs except that of proposition 7 are gathered in Appendix.

2 Model: Moves, information, preferences

There are three publishers and a library. Publisher i is often simply denoted by i, for $i = 1, 2, 3$. We consider only profit-maximizing publishers[14] and assume that they practice bundling[15] and price discrimination based on usage and budget. B_i denotes i's bundle. Note that since publishers practice price discrimination based on usage and budget, considering only one library does not involve loss of generality at the pricing stage. We later on (in section 6) extend the analysis of the multilateral interconnection to any number of heterogenous libraries.[16]

In this section, we introduce the basic games that we analyze in most of the paper. The hybrid game in which publishers can interconnect both in a multilateral way and in a bilateral way is introduced (and analyzed) in section 5.

2.1 Games with and without interconnection

We study two games of interconnection which differ depending on whether the interconnection regime is multilateral or bilateral. The game of multilateral interconnection à la

[14]Not-for-profit publishers have no strategic reasons to refuse interconnections that enhance values to subscribers of their journals.

[15]We can mimic the arguments in the proof of Proposition 2(i) in Jeon and Menicucci (2006) to show that, for each publisher, bundling the journals weakly dominates the alternative of no bundling (i.e., independent sales).

[16]Considering only one library is necessary for tractability in the case of the bilateral interconnection since it is complicated even with one library.

CrossRef is denoted by Γ^m and the game of bilateral interconnection is denoted by Γ^b. We also consider as a benchmark the game without interconnection denoted by Γ^0.

We first describe Γ^m.

- (Sequential Interconnection) In stage $i = 1, 2, 3$, publisher i decides (i) whether to be active or not and, if active, (ii) whether or not to interconnect through CrossRef.

When i is active, we let $x_i = 1$ if he[17] has joined CrossRef and $x_i = 0$ otherwise. The actions of publisher i are observed by the two other publishers and by the library. We use $A \subseteq \{1, 2, 3\}$ to denote the set of active publishers.

- (Pricing) In stage four, each active publisher i simultaneously chooses a price $P_i > 0$ for B_i.

- In stage five, the library chooses bundles to buy from the set $\{\text{B}_i : i \in A\}$.

In this game, by joining CrossRef a publisher makes *multiple* interconnections, i.e. with all the other publishers who are also members of CrossRef. Conversely, if publisher i does not join CrossRef then he is not interconnected with any other publisher. We use $\mathbf{y} \equiv (y_{12}, y_{13}, y_{23})$ to represent the interconnection profile by defining $y_{12} \equiv x_1 x_2$, $y_{13} \equiv x_1 x_3$ and $y_{23} \equiv x_2 x_3$.[18] Clearly, $y_{ij} \in \{0, 1\}$: $y_{ij} = 1$ means that the platforms of i and j are interconnected and $y_{ij} = 0$ means that there is no interconnection between them.

In Γ^b, publisher i can refuse to interconnect with publisher j while being interconnected with $k(\neq j)$. Formally, Γ^b is such that:

- In stage $i = 1, 2, 3$, publisher i (i) decides whether to be active or not and, if active, (ii) chooses $x_{ij} \in \{0, 1\}$ and $x_{ik} \in \{0, 1\}$ which denote his willingness to interconnect with publisher j and with publisher $k(\neq j)$, respectively.

The actions of i are publicly observed and A is the set of active publishers.

- Stages four and five are like in Γ^m.

In Γ^b, the variables in $\mathbf{y} \equiv (y_{12}, y_{13}, y_{23})$ are defined as follows: $y_{12} \equiv x_{12}x_{21}$, $y_{13} \equiv x_{13}x_{31}$ and $y_{23} \equiv x_{23}x_{32}$. In Γ^b, publishers i and j can interconnect among themselves (i.e., $x_{ij} = x_{ji} = 1$) and at the same time i (j) can make his own platform not interconnected

[17] We use "he" to for publishers.

[18] CRT (2000) uses the same technology of interconnection for the most part of their paper.

with k's one by choosing $x_{ik} = 0$ ($x_{jk} = 0$). In Γ^m, in contrast, in order for the platforms of i and j to be interconnected it is necessary that $x_i = x_j = 1$ and then k can interconnect to both platforms by choosing $x_k = 1$. Another interconnection configuration of interest that does not exist in Γ^m but can exist in Γ^b is that the platforms of 1 and 3, and of 2 and 3 are interconnected, but those of 1 and 2 are not. This occurs if $(x_{12}, x_{13}) = (0, 1)$, $(x_{21}, x_{23}) = (0, 1)$ and $(x_{31}, x_{32}) = (1, 1)$, so that $\mathbf{y} = (0, 1, 1)$.

The reason why we study the sequential game of interconnection is that the simultaneous game of bilateral interconnection has too many equilibria.[19] Considering the sequential game significantly reduces the number of equilibria such that we obtain a unique outcome. In contrast, in the case of multilateral interconnection, the strategic interactions among publishers is much simpler than in the case of bilateral interconnection and therefore we analyze both the sequential game and the simultaneous one (see sections 4.2 and 6.1) and show that our main result holds for both games.

In order to isolate the effects of interconnection, we first analyze (in section 4.1) as a benchmark the game Γ^0 in which interconnection is infeasible. This means that, in Γ^0, in stage $i = 1, 2, 3$, publisher i only decides whether to be active or not and $\mathbf{y} = (0, 0, 0)$; stages four and five are like in Γ^m. We use Γ to denote an unspecified game in $\{\Gamma^m, \Gamma^b, \Gamma^0\}$ and for any Γ we let γ represent the subgame which starts in stage four, in which active publishers choose prices for their bundles and then the library makes her purchases. Clearly, γ depends on the active publishers and on the interconnection profile $\mathbf{y}$, but we do not emphasize this fact in the notation.

2.2 The agents' preferences

In order to determine the library's purchases in stage five it is necessary to specify the library's preferences. A publisher's platform provides several services, which can be regrouped into two categories: basic service (reading, printing and downloading articles) and advanced service (text and data mining and linking). Let b_i represent the utility that the library obtains from basic service after buying B_i and assume that $b_1 > b_2 > b_3 > 0$. The library's utility from basic service when it purchases $\mathrm{B}_i\&\mathrm{B}_j$ or $\mathrm{B}_1\&\mathrm{B}_2\&\mathrm{B}_3$ is $b_i + b_j$ or $b_1 + b_2 + b_3$, respectively.

The library's utility from advanced service depends both on the bundles it buys and on the interconnection profile among them. In case the library subscribes only to B_i, its utility from advanced service is $I(b_i)$, where $I(0) = 0$ and $I(.)$ is assumed to be strictly

[19]For instance, when M is larger than $\bar{M}(1, 0, 0)$ but smaller than $U - I_{12}$ (where $\bar{M}(1, 0, 0)$ is defined in section 3 and U and I_{12} are defined in section 2.2), all three are active and any interconnection profile among them can be an equilibrium.

increasing and strictly convex (for instance, $I(b) = \alpha b^2$ with $\alpha > 0$); thus, the marginal surplus from advanced service increases as b_i becomes larger. If the library purchases $B_i\&B_j$, its utility from advanced service is $I(b_i + b_j)$ if $y_{ij} = 1$,[20] is $I(b_i) + I(b_j)$ if $y_{ij} = 0$. Notice that $I(b_i + b_j) > I(b_i) + I(b_j)$ since $I(.)$ is strictly convex; hence, interconnection between two platforms creates a positive value for the library.[21] Finally, if the library buys $B_1\&B_2\&B_3$, its utility from advanced service is determined as follows:[22][23]

$$\begin{array}{c}\text{library's utility from advanced}\\ \text{service if it buys } B_1\&B_2\&B_3\end{array} = \begin{cases} I(b_1) + I(b_2) + I(b_3) & \text{if } y_{12} = y_{13} = y_{23} = 0 \\ I(b_i + b_j) + I(b_k) & \text{if } y_{ij} = 1 \text{ and } y_{ik} = y_{jk} = 0 \\ I(b_i + b_j) + I(b_i + b_k) - I(b_i) & \text{if } y_{ij} = y_{ik} = 1 \text{ and } y_{jk} = 0 \\ I(b_1 + b_2 + b_3) & \text{if } y_{12} = y_{13} = y_{23} = 1 \end{cases}$$

In order to simplify notation, we introduce $U_i \equiv b_i + I(b_i)$, $\mathbf{U} \equiv (U_1, U_2, U_3)$, $U \equiv U_1 + U_2 + U_3$, $I_{ij} \equiv I(b_i + b_j) - I(b_i) - I(b_j) > 0$ and $\mathbf{I} \equiv (I_{12}, I_{13}, I_{23})$. While U_i represents the "stand-alone" utility that the library obtains from B_i when $y_{ij} = y_{ik} = 0$, I_{ij} is the increase in surplus for the library, with respect to $U_i + U_j$, from interconnection between i and j. The assumption $b_1 > b_2 > b_3$ implies $U_1 > U_2 > U_3$ and $I_{12} > I_{13} > I_{23}$.

The library's total utility from buying one or more bundles is given by its utility from basic service plus the utility from advanced service, minus the money spent. We assume that the library has a fixed budget $M > 0$ which can be used only to buy journals.[24] Therefore, the publishers compete for the library's budget and we assume that they have complete information about $(M, \mathbf{U}, \mathbf{I})$.

If we assume $M \geq U$, then we find that in Γ^0 (the benchmark without interconnection) there is no competition among publishers since publisher $i = 1, 2, 3$ can extract the full surplus from the library by charging a price equal to U_i. While this fact suggests to restrict attention to $M < U$, we actually assume $M \leq U - I_{12}$, which is stronger than

[20] We may also add a parameter representing the quality of interconnection, but it would not affect our results while adding notational complexity.

[21] The value added from interconnection would be zero if $I(.)$ were linear (i.e., if $I(b) = \alpha b$) and would be negative if $I(.)$ were strictly concave. Therefore, we view convexity of $I(.)$ as a reasonable assumption.

[22] By the strictly convexity of $I(.)$, this utility increases as the number of interconnected platforms increases.

[23] For instance, when $I(b) = \alpha b^2$ with $\alpha > 0$, we have: $I(b_1) + I(b_2) + I(b_3) = \alpha\left[(b_1)^2 + (b_2)^2 + (b_3)^2\right]$, $I(b_i + b_j) + I(b_k) = I(b_1) + I(b_2) + I(b_3) + 2\alpha b_i b_j$, $I(b_i + b_j) + I(b_i + b_k) - I(b_i) = I(b_1) + I(b_2) + I(b_3) + 2\alpha b_i b_j + 2\alpha b_i b_k$, $I(b_1 + b_2 + b_3) = I(b_1) + I(b_2) + I(b_3) + 2\alpha b_i b_j + 2\alpha b_i b_k + 2\alpha b_j b_k$.

[24] In Jeon and Menicucci (2006) we allow the library to use the budget to buy journals and books and assume that the library obtains utility $v(m)$ from spending $m \leq M$ to purchase books, with v increasing and concave. Hence, we can see our current setting as one in which $v(m) = m$. Since interconnection complicates the analysis, we make this simplification to obtain closed form formulas for equilibrium prices.

$M < U$, in order to simplify the exposition.[25] We show later on that this implies that the equilibrium prices always add up to M, which means that the library always exhausts its budget on buying journals. When we study Γ^m or Γ^b, we adopt the following simplifying assumption:

A1: $I(.)$ is not too convex such that the following inequalities hold: $U_1 \geq U_2 + I_{23}$, $U_2 \geq U_3 + I_{13}$, $U_3 \geq I_{12}$.

$U_1 \geq U_2 + I_{23}$ and $U_2 \geq U_3 + I_{13}$ imply that interconnection does not alter the ranking of profits among the publishers with respect to the benchmark of no interconnection. This is a simplifying assumption that allows us to focus on publisher 3's decision to be active or not when we study how interconnection affects the set of active publishers. $U_3 \geq I_{12}$ implies that the value from interconnection is small relative to the value from the original bundle of journals. This simplifies the statements of our results but does not affect the results themselves.[26] For robustness check, we also consider the symmetric cases (i.e. $U_1 = U_2 = U_3$) in section 4.5.

Since we would like to study publishers' incentives to interconnect and how interconnection affects which publishers succeed in selling their bundles and at what prices, we assume that the fixed cost of producing the first electronic copy of each journal has already been incurred and that the marginal cost of distributing each electronic journal through Internet is zero. Furthermore, we assume that interconnection is costless in order to focus on strategic interconnection incentives related to journal pricing.[27] Therefore, the profit of an active publisher i is equal to P_i if the library purchases B_i; i's profit is 0 otherwise.

In the next sections we use the concept of subgame perfect Nash equilibrium (SPNE) to determine the publishers' behavior in Γ. To find SPNE, from backward induction, we need to find the Nash equilibria (NE henceforth) of γ under various possible scenarios regarding decisions to be active and to interconnect. However, when all three publishers are active, in some cases γ has infinitely many NE in which the prices of the bundles the library buys depend on the prices of bundles the library does not buy.[28] In order to eliminate this indeterminacy, we adopt the following tie-breaking rule as in Jeon and

[25] Our results below hold also when $U - I_{12} < M < U$, but then we need to deal with a multiplicity of equilibria (off the equilibrium path), which requires to consider a large number of cases depending on the parameter values.

[26] For instance, if $I_{12} > 2U_2 + U_3$, then $M - I_{12}$ is smaller than $U_1 - U_2$ and only 1 is active. This case is not interesting since interconnection plays no role.

[27] We remark, however, that our results would qualitatively hold if we introduce a small interconnection fee $c > 0$.

[28] For instance, suppose that $U_1 = 10$, $U_2 = 2$, $U_3 = 1$ and $M = 9$. Then, for any $\beta \in [0, \frac{2}{3}]$, there exists a SPNE of Γ^0 in which $P_1 = 9 - \beta$, $P_2 = P_3 = \beta$ and the library buys B_1&B_2.

Menicucci (2006).

T1: In each game $\Gamma \in \{\Gamma^m, \Gamma^b, \Gamma^0\}$, any publisher i prefers being non-active to being active but unable to make a strictly positive profit.

T1 can be justified if a publisher would incur a very small but positive cost of contracting the library. Therefore, in a SPNE, publisher i is active if and only if in stage five the library purchases B_i at some price $P_i > 0$.

In what follows, we let P_i^* denote the equilibrium price for B_i and, likewise, $(A^*, x_i^*, x_{ij}^*, y_{ij}^*)$ represents (A, x_i, x_{ij}, y_{ij}) in equilibrium. For each $\Gamma \in \{\Gamma^m, \Gamma^b, \Gamma^0\}$, under A1, an intuitive result holds, which can be stated as follows: in any SPNE, publisher 1 is active; if publisher 3 is active, then publisher 2 is active as well. In other words, the following lemma applies.

Lemma 1 *Under A1, in any SPNE of* $\Gamma \in \{\Gamma^m, \Gamma^b, \Gamma^0\}$, *either* $A^* = \{1\}$, *or* $A^* = \{1, 2\}$, *or* $A^* = \{1, 2, 3\}$.

3 The pricing game

In this section, we describe the NE of γ as a function of A and $\mathbf{y}$, which is useful to determine the SPNE of $\Gamma \in \{\Gamma^m, \Gamma^b, \Gamma^0\}$. The next lemma considers the cases of $A = \{1, 2, 3\}$ and $A = \{1, 2\}$, provides the condition on M under which all active publishers realize positive profits and characterizes the equilibrium prices.

Lemma 2 *(pricing game) (i) Under A1, for* $A = \{1, 2, 3\}$ *and for given* $\mathbf{y}$, *there exists a NE of* γ *in which the library buys* $B_1\&B_2\&B_3$ *and all publishers realize strictly positive profits if and only if* $M > \bar{M}(\mathbf{y}) \equiv U - 3U_3 + 2I_{12}y_{12} - I_{13}y_{13} - I_{23}y_{23}$. *Furthermore, for any* M *between* $\bar{M}(\mathbf{y})$ *and* $U - I_{12}$, *the NE is unique and prices are given by:*

$$\begin{aligned}
P_1^* &= \bar{P}_1(\mathbf{y}) \equiv U_1 + \frac{1}{3}(M - U + \bar{I} - 3I_{23}y_{23}), \\
P_2^* &= \bar{P}_2(\mathbf{y}) \equiv U_2 + \frac{1}{3}(M - U + \bar{I} - 3I_{13}y_{13}), \\
P_3^* &= \bar{P}_3(\mathbf{y}) \equiv U_3 + \frac{1}{3}(M - U + \bar{I} - 3I_{12}y_{12});
\end{aligned}$$

where $\bar{I} \equiv I_{12}y_{12} + I_{13}y_{13} + I_{23}y_{23}$. *Therefore,* $\bar{P}_1(\mathbf{y}) + \bar{P}_2(\mathbf{y}) + \bar{P}_3(\mathbf{y}) = M$.
(ii) For $A = \{1, 2\}$ *and for any* y_{12}, *there exists a NE of* γ *in which the library buys* $B_1\&B_2$ *and both publishers realize strictly positive profits if and only if* $M > U_1 - U_2$.

Furthermore, for any M between $U_1 - U_2$ and $U_1 + U_2$, the NE is unique and prices are given by

$$P_1^* = \hat{P}_1 \equiv \frac{1}{2}(M + U_1 - U_2), \qquad P_2^* = \hat{P}_2 \equiv \frac{1}{2}(M + U_2 - U_1)$$

Therefore, $\hat{P}_1 + \hat{P}_2 = M$.

Lemma 2 has some results that help us to understand the number of active publishers in $\Gamma \in \{\Gamma^m, \Gamma^b, \Gamma^0\}$. First, independently of $\mathbf{y}$, if $M \leq U_1 - U_2$ then there is no NE of γ in which both publishers 1 and 2 (or all the three publishers) make a strictly positive profit. This implies that, when $M \leq U_1 - U_2$, only 1 will be active in SPNE and $P_1^* = M$. The reason is that if publisher 2 (for instance) is active as well, he makes no profit because the library's payoff from buying only B_1 is $U_1 - M$, which is higher than the payoff $U_2 - P_2$ obtained by purchasing B_2, for any $P_2 > 0$. When $M > U_1 - U_2$, this argument does not apply because $U_2 - P_2 > U_1 - M$ for P_2 close to 0; thus A^* will include at least $\{1, 2\}$.

Second, when $M > U_1 - U_2$, whether 3 also will be active or not depends on whether M is larger or smaller than $\bar{M}(\mathbf{y})$. Thus, while the interconnection profile $\mathbf{y}$ does not affect the set of parameter values for which only 1 is active, it determines the values of M for which all publishers are active: it is easy to see that $\bar{M}(\mathbf{y})$ is increasing (decreasing) with respect to y_{12} (with respect to y_{13} and y_{23}). We now briefly explain how $\bar{M}(\mathbf{y})$ is determined. Clearly, 3 is active if and only if he is able to sell B_3 at a positive price when $A = \{1, 2, 3\}$. Hence, we need to know how the equilibrium prices are determined. (P_1^*, P_2^*, P_3^*) should be such that the library does not purchase B_i anymore if publisher i increases P_i above P_i^*. It turns out that this condition is satisfied for each i if and only if the library is indifferent between buying B_i&B_j and buying B_i&B_k, for any i, j, k; this is equivalent to the following equalities

$$U_1 + U_2 + I_{12}y_{12} - P_1^* - P_2^* = U_1 + U_3 + I_{13}y_{13} - P_1^* - P_3^*; \tag{1}$$

$$U_1 + U_2 + I_{12}y_{12} - P_1^* - P_2^* = U_2 + U_3 + I_{23}y_{23} - P_2^* - P_3^*. \tag{2}$$

Furthermore, the assumption $M < U - I_{12}$ implies $P_1^* + P_2^* + P_3^* = M$ and using this equality[29] and (1)-(2) we find $(P_1^*, P_2^*, P_3^*) = (\bar{P}_1(\mathbf{y}), \bar{P}_2(\mathbf{y}), \bar{P}_3(\mathbf{y}))$ as in Lemma 2(i). From A1, we have $\bar{P}_1(\mathbf{y}) > \bar{P}_2(\mathbf{y}) > \bar{P}_3(\mathbf{y})$ for any interconnection profile $\mathbf{y}$ and $\bar{P}_3(\mathbf{y}) > 0$ if and only if $M > \bar{M}(\mathbf{y})$. $\bar{P}_i(\mathbf{y})$ is increasing in y_{ij} and y_{ik} and decreasing in y_{jk}. The reason is that library's payoff from buying B_i&B_j (for instance) is higher when $y_{ij} = 1$ than when $y_{ij} = 0$, and (1)-(2) imply that P_i needs to increase in order to make the library remain indifferent between B_i&B_j and B_j&B_k.

[29] If $P_1^* + P_2^* + P_3^* < M$, then $P_i^* < U_i$ holds for at least one i. As a consequence, publisher i can slightly increase P_i above P_i^* and still succeed in selling B_i.

In the intermediate case of $U_1 - U_2 < M \leq \bar{M}(\mathbf{y})$, the active publishers are 1 and 2 and prices are determined by a principle similar to the one explained above: given (P_1^*, P_2^*), publisher i $(i = 1, 2)$ has no incentive to increase P_i above P_i^* if and only if the library is indifferent between purchasing only B_1 and purchasing only B_2:

$$U_1 - P_1^* = U_2 - P_2^* \tag{3}$$

This condition and $P_1^* + P_2^* = M$ yield $(\hat{P}_1, \hat{P}_2)$. As (3) shows, the interconnection profile has no impact on the equilibrium prices when only two publishers are active.

4 The Interconnection games

In this section we analyze Γ^0, Γ^m and Γ^b.

4.1 Benchmark: The game without interconnection Γ^0

As a benchmark, we first consider the game Γ^0 in which there is no interconnection: in stage $i = 1, 2, 3$, each publisher i only chooses whether to be active or not and thus $\mathbf{y} = (0, 0, 0)$. By using Lemmas 1-2, it is relatively easy to analyze Γ^0. The next proposition characterizes the unique SPNE of Γ^0.

Proposition 1 *(without interconnection) In Γ^0 there exists a unique SPNE and the equilibrium active publishers and prices are given by:*
(i) if $M \leq U_1 - U_2$, then $A^* = \{1\}$ and $P_1^* = M$;
(ii) if $U_1 - U_2 < M \leq \bar{M}(0, 0, 0)$, then $A^* = \{1, 2\}$ and $P_i^* = \hat{P}_i$ for $i = 1, 2$;
(iii) if $\bar{M}(0, 0, 0) < M \leq U - I_{12}$, then $A^* = \{1, 2, 3\}$ and $P_i^* = \bar{P}_i(0, 0, 0)$ for $i = 1, 2, 3$.

When there is no interconnection, Proposition 1 establishes that the library buys B_1&B_2&B_3 if and only if M is larger than $\bar{M}(0, 0, 0) = U - 3U_3$; otherwise some publisher is excluded from the market. We have already explained that when $M \leq U_1 - U_2$, only 1 is active and $P_1^* = M$ because if 2 (or 3) is active as well, the library prefers to buy B_1 at price M rather than B_2 (or B_3) at any positive price. When $M > U_1 - U_2$, instead, $\{1, 2\} \subseteq A^*$ and also 3 is active if and only if a NE of γ exists in which the library buys B_1&B_2&B_3. By lemma 2(i), such a NE exists if and only if $M > \bar{M}(0, 0, 0)$, and it is actually the unique NE of γ; thus 3 is active if $M > \bar{M}(0, 0, 0)$.

4.2 The multilateral interconnection game Γ^m

Now we examine the multilateral interconnection game Γ^m in which publishers can interconnect only through CrossRef. We start by describing each publisher's incentive to interconnect in equilibrium. Since this incentive depends on the value of M, it is useful to notice that the following inequalities hold

$$\begin{aligned} U_1 - U_2 &< \bar{M}(0,1,0) < \bar{M}(0,0,1) < \bar{M}(0,0,0) \\ &< \bar{M}(1,1,1) < \bar{M}(1,0,0) < U - I_{12} \end{aligned}$$

The inequalities follow from the fact that $\bar{M}(\mathbf{y})$ is increasing in y_{12} and decreasing in (y_{13}, y_{23}).

Lemma 3 below considers only values of M between $\bar{M}(0,1,0)$ and $U - I_{12}$; we already know what happens if $M \leq U_1 - U_2$ and, if $U_1 - U_2 < M \leq \bar{M}(0,1,0)$, 3 will certainly not be active since $M \leq \bar{M}(\mathbf{y})$ for any $\mathbf{y}$.

Lemma 3 *(incentives to interconnect in Γ^m) Under A1, in any SPNE of Γ^m we find that (i) 3 chooses $x_3 = 1$ whenever he is active (i.e. when $M > \bar{M}(y_{12}, x_1, x_2)$) and $\max\{x_1, x_2\} = 1$.*
(ii) (a) When $x_1 = 0$: 2 is indifferent between $x_2 = 0$ and $x_2 = 1$ for M between $\bar{M}(0,1,0)$ and $\bar{M}(0,0,1)$, and 2 strictly prefers $x_2 = 1$ for M between $\bar{M}(0,0,1)$ and $U - I_{12}$;
(b) When $x_1 = 1$: 2 strictly prefers $x_2 = 1$ for any M between $\bar{M}(0,1,0)$ and $U - I_{12}$.
(iii) 1 is indifferent between $x_1 = 1$ and $x_1 = 0$ for $\bar{M}(0,1,0) < M < \bar{M}(0,0,1)$ and strictly prefers $x_1 = 1$ for $\bar{M}(0,0,1) < M \leq U - I_{12}$.

This lemma establishes that in any SPNE of Γ^m, for each active publisher and for any M, interconnection is a best response and sometimes the unique best response. In particular, for $M > \bar{M}(0,0,1)$, publisher 1 strictly prefers interconnection and $x_1 = 1$ induces 2 and 3 (if active) to strictly prefer interconnection. In order to provide the intuition about this result, we remind that in our model each publisher competes for relative standing since the industry profit is equal to M and is constant.

Consider first the case in which $M > \bar{M}(1,0,0)$, which means that the budget is large enough that even the interconnection profile the least favorable for 3 allows him to sell B_3 at a strictly positive price. According to lemma 3, all publishers interconnect in this case and the reason is that when at least one rival publisher is interconnected, $x_i = 1$ increases the price $\bar{P}_i(\mathbf{y})$ of B_i because it increases the value of B_i&B_j and/or of B_i&B_k (see (1)-(2) and the discussion at the end of section 3). In other words, compared to $x_i = 0$, $x_i = 1$ strictly improves the relative standing of B_i and thereby increases i's payoff. This explains why 3 will play $x_3 = 1$ if $\max\{x_1, x_2\} = 1$; 1 and 2 anticipate this choice and both choose to interconnect in stages one and two. This intuition holds *regardless of* whether the mode of interconnection is multilateral or bilateral.

Consider now $M < \bar{M}(1,0,0)$. Then, whether 3 is active or not depends on the interconnection profile. Hence, exclusion motive may modify publisher 1's (or 2's) incentive to interconnect. Obviously, 3's interconnection increases his chance to be active because it lowers $\bar{M}$, and this is why 3 always chooses $x_3 = 1$ if he is active. If 1 is interconnected (i.e. $x_1 = 1$), 2 always prefers $x_2 = 1$ because 2's relative standing is the weakest when only 1 and 3 are interconnected and 3 is active. Finally, if $x_1 = 0$, then 2 interconnects in order to induce 3 to be active whenever it is possible (i.e. when $M > \bar{M}(0,0,1)$) since this improves 2's relative standing while worsening 1's. Anticipating 2's response, 1 strictly prefers interconnection for $M > \bar{M}(0,0,1)$. Therefore, exclusion motive does not modify each publisher's incentive to interconnect.

From Lemma 3, we can find the equilibrium interconnection decisions and the resulting set of active publishers and prices.

Proposition 2 *(multilateral sequential interconnection) Under A1, in any SPNE of* Γ^m *the active publishers and prices are*
(i) if $M \leq U_1 - U_2$*, then* $A^* = \{1\}$ *and* $P_1^* = M$*;*
(ii) if $U_1 - U_2 < M \leq \bar{M}(1,1,1)$*, then* $A^* = \{1,2\}$*,* $x_i^* = 1$ *and* $P_i^* = \hat{P}_i$ *for* $i = 1,2$*;*[30] *Therefore, the exclusion of publisher 3 is more likely in* Γ^m *than in* Γ^0*.*
(iii) if $\bar{M}(1,1,1) < M \leq U - I_{12}$*, then* $A^* = \{1,2,3\}$*,* $x_i^* = 1$ *and* $P_i^* = \bar{P}_i(1,1,1)$ *for* $i = 1,2,3$*.*

When $M \leq U_1 - U_2$ (even though interconnection is possible) the argument presented in the previous section on the pricing game establishes that only 1 is active and $P_1^* = M$. Second, for $M > U_1 - U_2$ we find that all active publishers join CrossRef and therefore 3 is active if and only if $M > \bar{M}(1,1,1)$. Since $\bar{M}(1,1,1) > \bar{M}(0,0,0)$, this implies that CrossRef's existence makes the exclusion of the smallest publisher, publisher 3, more likely than in the absence of interconnection. This fact benefits both 1 and 2 since $\hat{P}_i$, i's profit when $A^* = \{1,2\}$, is larger than $\bar{P}_i(0,0,0)$, i's profit in Γ^0 when all three are active, for any $M > \bar{M}(0,0,0)$ and for $i = 1,2$. Publisher 3 is excluded more often in Γ^m than in Γ^0 since full interconnection increases the value of B_1&B_2 more than the values of B_1&B_3 and B_2&B_3 from $I_{12} > I_{13} > I_{23}$ and thereby worsens 3's relative position.

[30] For the sake of completeness we remark that if $U_1 - U_2 < M \leq \bar{M}(0,0,1)$ (respectively, if $U_1 - U_2 < M \leq \bar{M}(0,1,0)$), there also exists SPNE with $A = \{1,2\}$, $x_1 = 0$, $x_2 = 1$ (respectively, $x_1 = 1$, $x_2 = 0$ i). However, in all of these equilibria, 3 is not active and 1 and 2 charge the duopoly prices $(\hat{P}_1, \hat{P}_2)$; the outcome in terms of the active publishers and their profits is the same as in the Proposition.

Finally, when $M > \bar{M}(1,1,1)$ all publishers are active in Γ^m as in Γ^0, but $I_{12} > I_{13} > I_{23}$ imply that the largest publisher's profit is larger in Γ^m than in Γ^0 while the smallest publisher's profit is smaller in Γ^m than in Γ^0: $\bar{P}_1(1,1,1) > \bar{P}_1(0,0,0)$ and $\bar{P}_3(1,1,1) < \bar{P}_3(0,0,0)$. Publisher 2's profit is larger in Γ^m than in Γ^0 (i.e. $\bar{P}_2(1,1,1) > \bar{P}_2(0,0,0)$) if and only if $I_{12} + I_{23} > 2I_{13}$. We get as a corollary:

Corollary 1 *Under A1, in Γ^m, (i) If $M \leq \bar{M}(0,0,0)$, multilateral interconnection does not affect firms' profits.*

(ii) if $\bar{M}(0,0,0) < M \leq \bar{M}(1,1,1)$, multilateral interconnection increases 1's and 2's profits but reduces 3's profit.

(iii) if $\bar{M}(1,1,1) < M \leq U - I_{12}$, multilateral interconnection increases 1's profit, reduces 3's profit, and increases 2's profit if and only if $I_{12} + I_{23} > 2I_{13}$.

We conclude this section by examining the case in which all publishers simultaneously choose whether to be active or not, and (if active) whether to join CrossRef or not; in short, we merge stages one, two and three into a single stage and denote with Γ^{ms} the resulting game. In this setting, it is clear that if only one active publisher interconnects, interconnection has no effect and the outcome, in terms of active publishers and prices, is identical to the one without interconnection. Therefore, there always exists a SPNE of Γ^{ms} in which no publisher interconnects since each publisher expects that no rival will interconnect; the outcome of this SPNE is identical to that of the SPNE of Γ^0 (proposition 1). As a consequence, in proposition 3 we neglect this trivial equilibrium and find the same outcome that is described by Proposition 2.

Proposition 3 *(multilateral simultaneous interconnection) Under A1, in any SPNE of Γ^{ms} in which at least one active publisher interconnects, the active publishers and prices are determined as follows:*
(i) if $M \leq U_1 - U_2$, then $A^* = \{1\}$ and $P_1^* = M$;
(ii) if $U_1 - U_2 < M \leq \bar{M}(1,1,1)$, then $A^* = \{1,2\}$, $x_i^* = 1$ and $P_i^* = \hat{P}_i$ for $i = 1,2$;[31]
(iii) if $\bar{M}(1,1,1) < M < U - I_{12}$, then $A^* = \{1,2,3\}$, $x_i^* = 1$ and $P_i^* = \bar{P}_i(1,1,1)$ for $i = 1,2,3$.

[31] If $U_1 - U_2 < M \leq \bar{M}(0,0,1)$ (i.e., M is small enough that 3 is out if $\mathbf{y} = (0,0,1)$), there also exists an SPNE with $A = \{1,2\}$, $x_1 = 0$, $x_2 = 1$ and prices $(\hat{P}_1, \hat{P}_2)$. Similarly, If $U_1 - U_2 < M \leq \bar{M}(0,1,0)$ (i.e., M is small enough that 3 is out if $\mathbf{y} = (0,1,0)$), there also exists an SPNE with $A = \{1,2\}$ $x_1 = 1$, $x_2 = 0$, and prices $(\hat{P}_1, \hat{P}_2)$.

4.3 The bilateral interconnection game Γ^b

We now consider Γ^b, the game with bilateral interconnections. It turns out that in Γ^b the relative values of I_{12}, I_{13}, I_{23} affect the very nature of the SPNE, which makes many cases arise. Therefore, for expositional simplicity, instead of considering all possible parameter values, we restrict them to focus on two most interesting cases.[32] In the first one, the SPNE of Γ^b is such that 3 is excluded more often than in the SPNE of Γ^m while in the second case, 3 is excluded less often than in the SPNE of $\Gamma \in \{\Gamma^0, \Gamma^m\}$. However, we find that in both cases, there is less interconnection in Γ^b than in Γ^m, as long as the number of active publishers is the same.

In order to analyze Γ^b, it is useful to notice the following. First, in this game any interconnection profile $\mathbf{y} \in \{0,1\}^3$ is feasible, unlike in Γ^m.[33] Second, since $\bar{M}(\mathbf{y})$ is increasing in y_{12} and decreasing in (y_{13}, y_{23}), the following inequalities hold

$$\begin{aligned} U_1 - U_2 &< \bar{M}(0,1,1) < \bar{M}(0,1,0) < \bar{M}(0,0,1) < \bar{M}(0,0,0) \\ &< \bar{M}(1,1,1) < \bar{M}(1,1,0) < \bar{M}(1,0,1) < \bar{M}(1,0,0) < U - I_{12} \end{aligned}$$

From the arguments made in sections 4.1-4.2, the following results are straightforward; (a) for $M \leq U_1 - U_2$, only 1 is active and $P_1^* = M$; (b) when $U_1 - U_2 < M \leq \bar{M}(0,1,1)$ (i.e. even the interconnection profile the most favorable for 3 does not allow 3 to sell B_3), only 1 and 2 are active and $(P_1^*, P_2^*) = (\hat{P}_1, \hat{P}_2)$; (c) when $\bar{M}(1,0,0) < M \leq U - I_{12}$ (even the interconnection profile the least favorable for 3 allows it to sell its bundle), all publishers are active, fully interconnected and $P_i^* = \bar{P}_i(1,1,1)$ for $i = 1,2,3$; (d) publisher 3 always chooses to interconnect with both publishers.

However, it is quite long to describe, for all possible values of M and $\mathbf{I}$, 2's best response for each (x_{12}, x_{13}) and 1's choice. Therefore, in the following lemma, we describe only some interconnection choices that are relevant to understand the equilibrium choices.

Lemma 4 *(incentives to interconnect for 2 and 3 in Γ^b)*

(i) 3 chooses $x_{3j} = 1$ whenever it is active and $x_{j3} = 1$ for $j = 1, 2$.

(ii) For $\bar{M}(0,1,1) < M \leq \bar{M}(0,1,0)$: If $(x_{12}, x_{13}) = (0,1)$, 2 excludes 3 by choosing $x_{23} = 0$. Otherwise, 3 is excluded regardless of 2's interconnection decision.

(iii) For $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,0)$: If $(x_{12}, x_{13}) = (1,1)$, 2 chooses $(x_{21}, x_{23}) = (1,0)$ and excludes 3. If $(x_{12}, x_{13}) = (0,1)$, 2 and 3 choose $x_{23} = 1$ and $(x_{31}, x_{32}) = (1,1)$ and 3 is active.

[32] We can study the other cases since the analysis of the bilateral interconnection game basically follows from the analysis of the pricing game (presented in lemma 2) and lemma 2 is proved for general parameter values. We do not present these cases since it will take too much space without adding interesting insight.

[33] In Γ^m, the profiles $\mathbf{y} = (1,1,0)$, $\mathbf{y} = (1,0,1)$ and $\mathbf{y} = (0,1,1)$ cannot arise because, for instance, $y_{12} = y_{13} = 1$ requires $x_1 = x_2 = x_3 = 1$ and thus $y_{23} = 1$.

(iv) For $\bar{M}(1,1,0) < M \leq \bar{M}(1,0,1)$

a. $2I_{12} \geq I_{13}+2I_{23}$*: If* $(x_{12},x_{13}) = (1,1)$*, 2 and 3 choose* $(x_{21},x_{23}) = (x_{31},x_{32}) = (1,1)$*. If* $(x_{12},x_{13}) = (1,0)$*, 2 chooses* $x_{21} = 1$ *and excludes 3.*

b. $3I_{23} \geq 2I_{12}$*: If* $(x_{12},x_{13}) = (1,1)$*, 2 and 3 choose* $(x_{21},x_{23}) = (x_{31},x_{32}) = (1,1)$*. If* $(x_{12},x_{13}) = (1,0)$*, 2 chooses* $(x_{21},x_{23}) = (1,1)$ *and let 3 be active.*

(v) For $\bar{M}(1,0,1) < M \leq \bar{M}(1,0,0)$*: If* $(x_{12},x_{13}) = (1,1)$*, 2 and 3 choose* $(x_{21},x_{23}) = (x_{31},x_{32}) = (1,1)$*. If* $(x_{12},x_{13}) = (1,0)$*, 2 chooses* $(x_{21},x_{23}) = (1,1)$ *and let 3 be active.*

First, when $\bar{M}(0,1,1) < M \leq \bar{M}(0,1,0)$, 3 can be active only if 3 is interconnected with both 1 and 2 while 1 and 2 are not interconnected. Hence, any publisher j (with j=1,2) can unilaterally exclude 3 by choosing $x_{j3} = 0$ and the exclusion occurs since 2's duopoly profit is larger than $\bar{P}_2(0,1,1)$.

Second, when $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,0)$, as long as 1 and 2 are interconnected, 2 can exclude 3 by choosing $x_{23} = 0$. Hence, if 1 chooses $(x_{12},x_{13}) = (1,1)$, 2 excludes 3 since its duopoly profit is larger than the profit it obtains by letting 3 be active. In contrast, if 1 chooses $(x_{12},x_{13}) = (0,1)$ and hence breaks connectivity with 2, 2 cannot exclude 3 even if it chooses $x_{23} = 0$. Therefore, in this case, 2 chooses $x_{23} = 1$ and 3 is active and fully interconnected with 1 and 2. Finally, whether 1 will let 3 be active or not depends on whether $\hat{P}_1 < \bar{P}_1(0,1,1)$ or not. We find that $\hat{P}_1 \geq \bar{P}_1(0,1,1)$ for *all* M *between* $\bar{M}(0,1,0)$ *and* $\bar{M}(1,1,0)$ (resp. $\hat{P}_1 < \bar{P}_1(0,1,1)$) if and only if $4I_{23} \geq 3I_{13}$ (resp. $3I_{13} > 2I_{12} + 4I_{23}$).

Third, consider $\bar{M}(1,1,0) < M \leq \bar{M}(1,0,1)$. In this case, if 1 chooses $(x_{12},x_{13}) = (1,1)$, 2 cannot exclude 3 (since $M > \bar{M}(1,1,0)$) and hence all publishers are active and fully interconnected. If 1 wants to exclude 3, it should choose $(x_{12},x_{13}) = (1,0)$. Then 2 can exclude 3 by choosing $x_{21} = 1$ or can let 3 be active by choosing $x_{21} = 0$: in the latter case, 2 will choose $x_{23} = 1$. Obviously, whether 2 will exclude 3 or not depends on whether $\hat{P}_2 > \bar{P}_2(0,1,1)$ or not. The inequality $\hat{P}_2 \geq \bar{P}_2(0,1,1)$ holds *for all* M *between* $\bar{M}(1,1,0)$ *and* $\bar{M}(1,0,1)$ (resp. $\hat{P}_2 \leq \bar{P}_2(0,1,1)$) if and only if $2I_{12} \geq I_{13} + 2I_{23}$ (resp. $3I_{23} \geq 2I_{12}$). Finally, when $\hat{P}_2 \geq \bar{P}_2(0,1,1)$, 1 indeed chooses $(x_{12},x_{13}) = (1,0)$ to let 2 exclude 3 if $\hat{P}_1 \geq \bar{P}_1(1,1,1)$; $\hat{P}_1 \geq \bar{P}_1(1,1,1)$ holds *for all* M *between* $\bar{M}(1,1,0)$ *and* $\bar{M}(1,0,1)$ if and only if $4I_{23} \geq 3I_{13}$.

Last, consider $\bar{M}(1,0,1) < M \leq \bar{M}(1,0,0)$. In this case, in order to exclude 3, 1 and 2 should cooperate by choosing $x_{13} = x_{23} = 0$. Obviously, excluding 3 maximizes joint profits of 1 and 2. However, there is a coordination failure. If 1 chooses $(x_{12},x_{13}) = (1,0)$, then 2 chooses $(x_{21},x_{23}) = (1,1)$ and let 3 be active since $\hat{P}_2 < \bar{P}_2(1,0,1)$. Therefore, 1 chooses $(x_{12},x_{13}) = (1,1)$ and all three are active and fully interconnected.

The next proposition describes the SPNEs of Γ^b: the results are less neat when

$\bar{M}(0,1,0) < M \leq \bar{M}(1,0,1)$, since then A^* is determined by (I_{12}, I_{13}, I_{23}) (recall $I_{12} > I_{13} > I_{23}$) as described by Proposition 4(iii)-(iv).

Proposition 4 *(bilateral sequential interconnection) Under A1, in any SPNE of* Γ^b *the active publishers and prices are*
(i) if $M \leq U_1 - U_2$*, then* $A^* = \{1\}$ *and* $P_1^* = M$*;*
(ii) if $U_1 - U_2 < M \leq \bar{M}(0,1,0)$*, then* $A^* = \{1,2\}$*,* $(x_{12}^*, x_{13}^*) = (x_{21}^*, x_{23}^*) = (1,0)$ *and* $(P_1^*, P_2^*) = (\hat{P}_1, \hat{P}_2)$*;*
(iii) if $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,0)$*, then*
a. When $4I_{23} \geq 3I_{13}$*,* $A^* = \{1,2\}$*,* $(x_{12}^*, x_{13}^*) = (1,1)$*,* $(x_{21}^*, x_{23}^*) = (1,0)$ *and* $(P_1^*, P_2^*) = (\hat{P}_1, \hat{P}_2)$*.*
b. When $3I_{13} > 2I_{12} + 4I_{23}$*,* $A^* = \{1,2,3\}$*,* $(x_{12}^*, x_{13}^*) = (x_{21}^*, x_{23}^*) = (0,1)$*,* $(x_{31}^*, x_{32}^*) = (1,1)$ *and* $P_i^* = \bar{P}_i(0,1,1)$ *for* $i = 1,2,3$*.*
(iv) if $\bar{M}(1,1,0) < M \leq \bar{M}(1,0,1)$*, then*
a. When $4I_{23} \geq 3I_{13}$ *and* $2I_{12} \geq I_{13} + 2I_{23}$*,* $A^* = \{1,2\}$*,* $(x_{12}^*, x_{13}^*) = (x_{21}^*, x_{23}^*) = (1,0)$ *and* $(P_1^*, P_2^*) = (\hat{P}_1, \hat{P}_2)$*.*
b. When $3I_{23} \geq 2I_{12}$*,* $A^* = \{1,2,3\}$*,* $(x_{12}^*, x_{13}^*) = (x_{21}^*, x_{23}^*) = (x_{31}^*, x_{32}^*) = (1,1)$ *and* $P_i^* = \bar{P}_i(1,1,1)$ *for* $i = 1,2,3$*.*
(v) if $\bar{M}(1,0,1) < M < U - I_{12}$*, then* $A^* = \{1,2,3\}$*,* $(x_{12}^*, x_{13}^*) = (x_{21}^*, x_{23}^*) = (x_{31}^*, x_{32}^*) = (1,1)$ *and* $P_i^* = \bar{P}_i(1,1,1)$ *for* $i = 1,2,3$*.*

Proposition 4 shows that the exclusion of 3 can occur more often or less often under the bilateral interconnection than under the multilateral one. We remind that under the multilateral interconnection, the exclusion occurs if $M \leq \bar{M}(1,1,1)$ where $\bar{M}(1,1,1) \in (\bar{M}(0,1,0), \bar{M}(1,1,0))$.

First, if either 1 or 2 decides to break connectivity with 3 for *exclusion motive*, exclusion occurs more often under the bilateral interconnection than under the multilateral one. For instance, according to Proposition 4 (iii)a, this happens if $4I_{23} \geq 3I_{13}$. Since $I_{13} > I_{23}$ always holds, $4I_{23} \geq 3I_{13}$ implies that 1 is not much bigger than 2. For $M \in (\bar{M}(0,1,0), \bar{M}(1,1,0))$, if asymmetry between 1 and 2 is small, 1 and 2 can maintain duopoly: more precisely, by choosing full interconnection (i.e. $(x_{12}^*, x_{13}^*) = (1,1)$), 1 incentivizes 2 to exclude 3 through partial interconnection (i.e. $(x_{21}^*, x_{23}^*) = (1,0)$). Proposition 4 (iv)a shows that for $M \in (\bar{M}(1,1,0), \bar{M}(1,0,1))$, the bilateral interconnection regime can induce exclusion of 3 while exclusion never occurs under the multilateral interconnection: in this case, 1 actively excludes 3 through partial interconnection (i.e. $(x_{12}^*, x_{13}^*) = (1,0)$).

Second, if publisher 1 breaks connectivity with 2 but maintains connectivity with 3 *for differentiation motive*, exclusion occurs less often under the bilateral interconnection

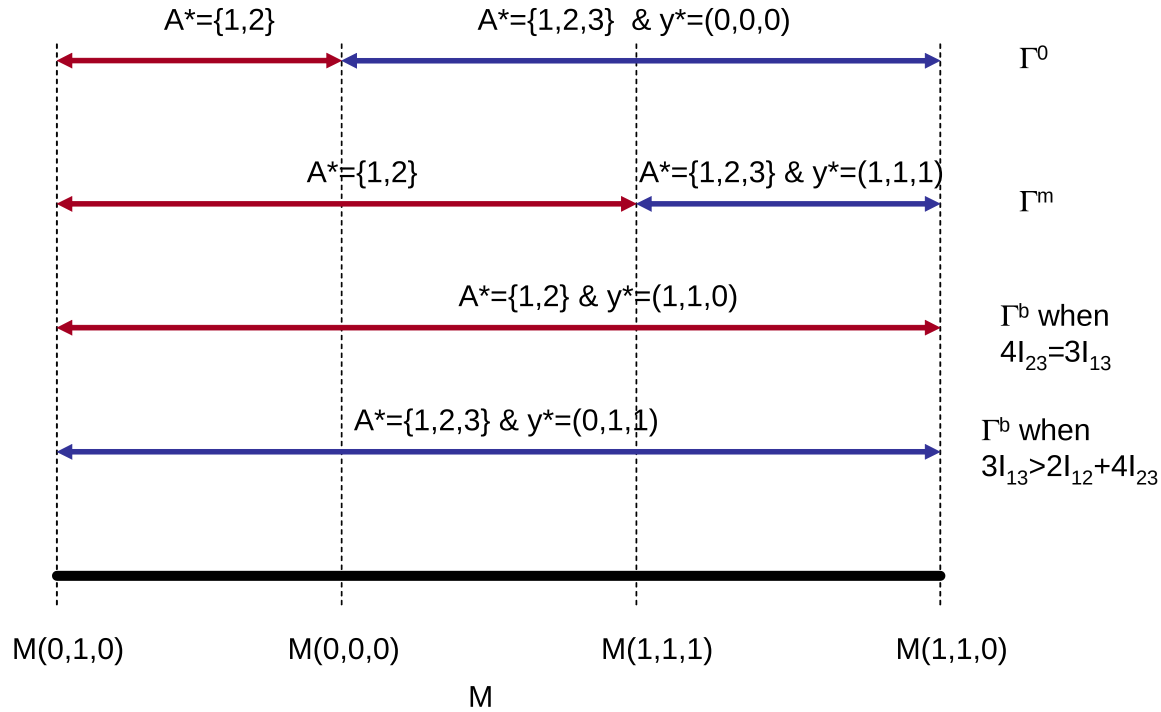

Figure 1: Comparison among $\Gamma^0, \Gamma^m, \Gamma^b$ for $M \in \left(\bar{M}(0,1,0), \bar{M}(1,1,0)\right]$.

than under the multilateral one. More precisely, according to proposition 4 (iii)b, this happens for $M \in (\bar{M}(0,1,0), \bar{M}(1,1,0))$, if $3I_{13} > 2I_{12} + 4I_{23}$. $3I_{13} > 2I_{12} + 4I_{23}$ means that 2 and 3 are similar (so that I_{13} is only slightly smaller than I_{12}) but 1 is much larger than 2 (so that I_{13} is much larger than I_{23}). In this case, 1 has an incentive to break connectivity with 2 and let 3 be active by interconnecting with 3: then, 2 responds by interconnecting with 3 and therefore 3 becomes fully interconnected with 1 and 2. Since I_{13} is much larger than I_{23} because of the large asymmetry between 1 and 2, this strategy improves 1's relative standing such that its profit $\bar{P}_1(0,1,1)$ is larger than its duopoly profit $\hat{P}_1$. In short, when 1 is much larger than 2, it pays for 1 to differentiate itself from 2 by breaking connectivity with 2 but maintaining it with 3.

Finally, the above arguments show that publishers have less incentive to interconnect under the bilateral interconnection than under the multilateral one: the bilateral interconnection induces publishers to employ partial interconnection for exclusion motive or for differentiation motive.

Figure 1 summarizes our findings under each $\Gamma \in \left\{\Gamma^0, \Gamma^m, \Gamma^b\right\}$ for $M \in \left(\bar{M}(0,1,0), \bar{M}(1,1,0)\right]$.

4.4 Social welfare

In our setting the sum of the publishers' profits is always equal to M; hence, social welfare coincides with the library's payoff. It follows from A1 that social welfare (and the library's payoff) increases with the number of bundles the library is able to buy regardless of $\mathbf{y}$; social welfare also increases as more platforms are interconnected for a given set of subscribed bundles. We below give detailed comparisons among Γ^0, Γ^m, Γ^b partly because we use it in section 5.

Corollary 2 *(social welfare) Under A1, the ranking among* Γ^0, Γ^m, Γ^b *in terms of social welfare is given by*
(i) For $U_1 - U_2 < M \leq \bar{M}(0,1,0)$ *and* $\bar{M}(1,0,1) < M < U - I_{12}$, $\Gamma^m \approx \Gamma^b \succ \Gamma^0$
(ii) For $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,0)$,
a. When $4I_{23} \geq 3I_{13}$
For $M \in \left(\bar{M}(0,1,0), \bar{M}(0,0,0)\right]$, $\Gamma^m \approx \Gamma^b \succ \Gamma^0$. *For* $M \in \left(\bar{M}(0,0,0), \bar{M}(1,1,1)\right]$, $\Gamma^0 \succ \Gamma^m \approx \Gamma^b$. *For* $M \in \left(\bar{M}(1,1,1), \bar{M}(1,1,0)\right]$, $\Gamma^m \succ \Gamma^0 \succ \Gamma^b$.
b. When $3I_{13} \geq 2I_{12} + 4I_{23}$
For $M \in \left(\bar{M}(0,1,0), \bar{M}(0,0,0)\right]$, $\Gamma^b \succ \Gamma^m \succ \Gamma^0$. *For* $M \in \left(\bar{M}(0,0,0), \bar{M}(1,1,1)\right]$, $\Gamma^b \succ \Gamma^0 \succ \Gamma^m$ *For* $M \in \left(\bar{M}(1,1,1), \bar{M}(1,1,0)\right]$, $\Gamma^m \succ \Gamma^b \succ \Gamma^0$.
(iii) For $\bar{M}(1,1,0) < M \leq \bar{M}(1,0,1)$,
a. When $4I_{23} \geq 3I_{13}$ *and* $2I_{12} \geq I_{13} + 2I_{23}$, $\Gamma^m \succ \Gamma^0 \succ \Gamma^b$.
b. When $3I_{13} \geq 2I_{12}$, $\Gamma^m \approx \Gamma^b \succ \Gamma^0$.

Obviously, interconnection increases the library's payoff as long as it does not modify the set of active publishers and this explains why Γ^m is strictly better than Γ^0 except the case $\bar{M}(0,0,0) < M \leq \bar{M}(1,1,1)$. In this case, interconnection in Γ^m reduces the library's consumption of bundles from B_1&B_2&B_3 to B_1&B_2 (even though its outlay is still M): The library receives the benefit I_{12}, but $I_{12} < U_3$ from A1 and thus social welfare is smaller in Γ^m than in Γ^0. Finally, Γ^m is at least weakly better than Γ^b except the case $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,1)$ and $3I_{13} \geq 2I_{12} + 4I_{23}$. In this case, the library buys all bundles in Γ^b while in Γ^m we have $A^* = \{1,2\}$.

4.5 Symmetric publishers

We have assumed until now that A1 holds, which means that publishers are sufficiently asymmetric. In the opposite case of symmetric publishers we obtain the following result:

Proposition 5 *(symmetric publishers) Suppose that publishers are symmetric in the sense that* $b_1 = b_2 = b_3 = b_s$ *(and thus* $U_1 = U_2 = U_3 = U_s > I_{12} = I_{13} = I_{23} = I_s$*).*

(i) For each $\Gamma \in \{\Gamma^0, \Gamma^m\}$ *there exists a unique SPNE and it is such that* $A^* = \{1, 2, 3\}$, $P_i^* = M/3$ *for* $i = 1, 2, 3$, *for any* $M \in (0, U - I_s)$. *Furthermore, if* $\Gamma = \Gamma^m$, *then* $x_1^* = x_2^* = x_3^* = 1$
(ii) When $\Gamma = \Gamma^b$

a. For $M \leq I_s$, *there are two equilibrium outcomes: either* $A^* = \{1, 2\}$ *and* $(P_1^*, P_2^*) = (M/2, M/2)$ *or* $A^* = \{1, 3\}$ *and* $(P_1^*, P_3^*) = (M/2, M/2)$.

b. For $M \in [I_s, U - I_s)$, *there is a unique SPNE and it is such that* $A^* = \{1, 2, 3\}$, $P_i^* = M/3$, $x_{ij}^* = 1$ *for* $i, j = 1, 2, 3$ *with* $i \neq j$.
(iii) Social welfare (and the library's payoff) in Γ^m *is always as least as high as in* Γ^b, *and strictly larger than in* Γ^0.

Proposition 5 shows that symmetric publishers have incentives to fully interconnect under the multilateral interconnection regime regardless of the size of the budget and under the bilateral interconnection regime for $M > I_s$. What is rather surprising is that under the bilateral regime, when $M \leq I_s$, even symmetric publishers have incentives to partially break connectivity to exclude a publisher: in all equilibria either 2 or 3 is excluded. For instance, the following is an equilibrium: 1 is active and chooses $x_{12}^* = x_{13}^* = 1$ and 2 is active and chooses $x_{21}^* = 1$ and $x_{23}^* = 0$. In this situation, 3 decides not to be active since he cannot realize any positive profit: in particular, if 3 becomes active and chooses $x_{31}^* = 1$, a sort of Bertrand competition arises between 2 and 3 at the pricing stage such that $(P_1, P_2, P_3) = (M, 0, 0)$ is the unique equilibrium.

5 Endogenizing interconnection regime

In the previous sections, we analyzed the outcome of a given (either multilateral or bilateral) regime of interconnection. In this section, we study which regime between the two emerges endogenously when publishers are free to interconnect in either regime. In the absence of regulation imposing a clause of no discrimination among rivals in terms of interconnection, the mere existence of a multilateral regime such as CrossRef does not eliminate the possibility for publishers to engage in bilateral interconnection.

To address the above question, we examine the following hybrid game of sequential interconnection Γ^h:[34] at stage $i = 1, 2, 3$, firm i decides whether to interconnect or not through CrossRef – by selecting $x_i \in \{0, 1\}$ – and also chooses $x_{ik} \in \{0, 1\}$, $x_{ij} \in \{0, 1\}$, which indicate his willingness to interconnect bilaterally with firm k and firm j. Firms i and j are interconnected if and only if both of them join CrossRef ($x_i = x_j = 1$) or they

[34]The superscript "h" means "hybrid".

have reached a bilateral agreement ($x_{ij} = x_{ik} = 1$). In this game, therefore, publisher i may for instance not join CrossRef, but leave the door open to bilateral interconnection with j. A few interesting features of this game are presented in the next lemma.

Lemma 5 *In the hybrid interconnection game* Γ^h
(i) 1 can obtain the same outcome as the one in Γ^b *by playing* $x_1 = 0$*;*
(ii) If $x_1 = 1$*, 3 plays* $x_3 = 1$*. If* $x_2 = 0$*, then 3 plays* $x_{32} = 1$ *if* $x_{23} = 1$*;*
(iii) If 1 plays $x_1 = 1$ *and* $x_{12} = 0$*, then the equilibrium outcome of* Γ^m *arises;*
(iv) If 1 plays $x_1 = 1$ *and* $x_{12} = 1$*, then the outcome is the same as the one in* Γ^b *where 1 plays* $(x_{12}, x_{13}) = (1, 1)$*.*

The result in Lemma 5(i) is intuitive: if $x_1 = 0$, then at most 2 and 3 can be interconnected through CrossRef (this occurs if $x_2 = x_3 = 1$). But the bilateral interconnection between 2 and 3 can be equivalently achieved with $x_{23} = x_{32} = 1$ and firm 2 (3) has an incentive to choose $x_{23} = 1$ ($x_{32} = 1$) as long as it has an incentive to choose $x_2 = 1$ ($x_3 = 1$). If $x_1 = 1$, 3 plays $x_3 = 1$ by the same logic of lemma 3(i)[35] and hence the value of x_{13} is irrelevant. There remain hence only the following two alternatives for firm 1 to consider: $x_1 = 1$, $x_{12} = 0$, and $x_1 = 1$, $x_{12} = 1$. We prove that $x_1 = 1$, $x_{12} = 0$ is equivalent to $x_1 = 1$ in Γ^m, and therefore the equilibrium outcome of Γ^m arises; $x_1 = 1$, $x_{12} = 1$ is instead equivalent to $(x_{12}, x_{13}) = (1, 1)$ in Γ^b.

Because of Lemma 5, we can view Γ^h as a game which differs from Γ^b in that 1 has an additional strategy, which allows him to implement the same outcome as in Γ^m. The outcome in Γ^m is duopoly for $M \leq \bar{M}(1,1,1)$ and triopoly with full interconnection for $M > \bar{M}(1,1,1)$. Actually, Lemma 4 shows that 1 can achieve this outcome in Γ^b for $M \leq \bar{M}(1,1,1)$ and for $M > \bar{M}(1,1,0)$; but for M between $\bar{M}(1,1,1)$ and $\bar{M}(1,1,0)$, in Γ^b, 1 is unable to implement the triopoly outcome with full interconnection.[36] The next proposition summarizes the outcome of Γ^h and the next corollary compares 1's profit in Γ^m with its profit in Γ^b to predict the outcome of Γ^h.[37]

[35] Firm 3 (if it is active) always wants to maximize his own interconnections.

[36] The reason is that full interconnection in Γ^b requires $(x_{12}, x_{13}) = (1, 1)$, but then (from Lemma 4) 2 reacts with $(x_{21}, x_{23}) = (1, 0)$ in order to exclude 3. In Γ^h, instead, $(x_1 = 1, x_{12} = 0)$ forces 2 to play $x_2 = 1$ in order to avoid being interconnected only with 3.

[37] Alternatively, we can study the game in which each firm must choose between joining CrossRef and interconnecting bilaterally with other firms (i.e. joining CrossRef is incompatible with bilateral interconnections). We find exactly the same outcome we have described in this section as it turns out that (i) if 1 does not join CrossRef, then none of 2 and 3 does either, and thus 1 can always obtain the same outcome as in Γ^b; (ii) by playing $x_1 = 1$, 1 induces 2 to play $x_2 = 1$ and this induces the same outcome as in Γ^m.

Proposition 6 *(hybrid interconnection) In the hybrid interconnection game* Γ^h*,*

(i) 1 can achieve the maximum between its profit in game Γ^m *and its profit in game* Γ^b*.*

(ii) The outcome of Γ^h *coincides with that of* Γ^m *if 1's profit is larger in* Γ^m *than in* Γ^b*; otherwise, it coincides with that of* Γ^b*.*

Corollary 3 *(1's profit) The ranking between* Γ^m *and* Γ^b *in terms of 1's profit is given by*

(i) For $U_1 - U_2 < M \leq \bar{M}(0,1,0)$ *and* $\bar{M}(1,0,1) < M < U - I_{12}$*,* $\Gamma^m \approx \Gamma^b$

(ii) For $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,0)$*,*

a. When $4I_{23} \geq 3I_{13}$

For $M \in \left(\bar{M}(0,1,0), \bar{M}(1,1,1)\right]$*,* $\Gamma^m \approx \Gamma^b$*. For* $M \in \left(\bar{M}(1,1,1), \tilde{M}\right]$*,* $\Gamma^m \succ \Gamma^b$ *where* $\tilde{M} \equiv U - 3U_3 + 2(I_{12} + I_{13} - 2I_{23})$*. For* $M \in \left(\tilde{M}, \bar{M}(1,1,0)\right]$*,* $\Gamma^b \succ \Gamma^m$*.*

b. When $3I_{13} \geq 2I_{12} + 4I_{23}$

For $M \in \left(\bar{M}(0,1,0), \bar{M}(1,1,1)\right]$*,* $\Gamma^b \succ \Gamma^m$*. For* $M \in \left(\bar{M}(1,1,1), \bar{M}(1,1,0)\right]$*,* $\Gamma^m \succ \Gamma^b$*.*

(iii) For $\bar{M}(1,1,0) < M \leq \bar{M}(1,0,1)$*,*

a. When $4I_{23} \geq 3I_{13}$ *and* $2I_{12} \geq I_{13} + 2I_{23}$*,* $\Gamma^b \succ \Gamma^m$

b. When $3I_{13} \geq 2I_{12}$*,* $\Gamma^m \approx \Gamma^b$*.*

Proposition 6 and Corollary 3 suggest that the market outcome can sometimes coincide with that of the multilateral interconnection. Given that full interconnection improves relative standing of big publishers because of economies of scale in the value generated by interconnection, big publishers may have an incentive to endorse a multilateral interconnection regime and this can explain the success of CrossRef.

Finally, comparing corollary 2 (social welfare) and corollary 3 (1's profit) leads to:

Corollary 4 *In the hybrid interconnection game* Γ^h*, the conflict between the market outcome and social welfare arises only when* $4I_{23} \geq 3I_{13}$ *in the following case: either* $M \in \left(\tilde{M}, \bar{M}(1,1,0)\right]$ *or* $M \in \left(\bar{M}(1,1,0), \bar{M}(1,0,1)\right]$ *and* $2I_{12} \geq I_{13} + 2I_{23}$*. In this case, the outcome of* Γ^h *is equivalent to that of* Γ^b *and involves exclusion of 3 while in* Γ^m *all publishers are active and therefore social welfare is higher in* Γ^m *than in* Γ^b*.*

Note first that 1's duopoly profit increases faster than 1's triopoly profit as M increases. This suggests that 1's incentive to exclude 3 increase with M and therefore the conflict between 1's profit and social welfare is more likely as M increases. However, if M is large enough (i.e. larger than $\bar{M}(1,0,1)$), all publishers are active both in Γ^m and Γ^b. Therefore, the conflict can arise only for an intermediate level of M i.e. $M \in \left(\tilde{M}, \bar{M}(1,0,1)\right]$. Then, the outcome of the hybrid game can involve exclusion of 3 since either 1 and/or 2 break(s) connectivity with 3. In this case, our results suggest that a light-handed regulation banning discrimination among rivals in terms of interconnection would foster full interconnection and induce all publishers to be active.

6 Robustness

To show that our main result is robust, in this section we first show that, under the multilateral interconnection regime, all publishers interconnect for any number of heterogenous libraries and regardless of whether we consider a sequential or a simultaneous game. Second, we perform comparison between the multilateral interconnection regime and the bilateral one in the Internet interconnection model of CRT (2000).

6.1 Multilateral interconnection and heterogenous libraries

There are $m(\geq 1)$ number of heterogenous libraries: libraries differ in terms of the budget and the value that they obtain from the bundle of publisher i. Let $U_i^h(>0)$ represent the stand-alone utility that library h obtains from B_i; we assume that all libraries have the same ranking of the bundles: $U_1^h > U_2^h > U_3^h > 0$ for $h = 1, ..., m$. Let $I_{ij}^h(>0)$ represent the value that library h obtains from interconnection between the platforms of publishers i and j; thus $I_{12}^h > I_{13}^h > I_{23}^h$ for $h = 1, ..., m$. Let $\mathbf{U}^h \equiv (U_1^h, U_2^h, U_3^h)$ and $\mathbf{I}^h \equiv (I_{12}^h, I_{13}^h, I_{23}^h)$. Finally, $M^h(>0)$ represents library h's budget; we assume $M^h \leq U^h - I_{12}^h$ for $h = 1, ..., m$ and also that A1 holds for each library – we call A1' the joint assumption for all libraries:

A1': $U_3^h \geq I_{12}^h$, $U_1^h \geq U_2^h + I_{23}^h$, $U_2^h \geq U_3^h + I_{13}^h$ for $h = 1, ..., m$.

Each publisher practices price discrimination based on usage and budget: let P_i^h represent the price of the bundle that publisher i charges to library h. We consider the following sequential game of multilateral interconnection and pricing.

- (Sequential Interconnection) In stage $i = 1, 2, 3$, publisher i (i) decides whether to be active or not; (ii) if he is active, i decides whether or not to interconnect through CrossRef.
- (Pricing) In stage four, all active publishers simultaneously decide whether to be active or not in the market for library h, for $h = 1, ..., m$. In stage five, each publisher i, if active in market h, simultaneously chooses a price $P_i^h > 0$, for $h = 1, ..., m$.

- In stage six, each library h chooses bundles to buy.

Note that publishers' interconnection decisions affect all libraries in the same way. As before, we adopt the tie-breaking rule that if publisher i expects to make zero profit in the market for library h, then he prefers not to be active in that market in stage four: if publisher i expects to realize a total profit equal to zero, he prefers not to be active in stage $i = 1, 2, 3$. Then we get the following corollary from the analysis of section 4.2:

Corollary 5 *Suppose there are $m(> 1)$ number of libraries that are heterogenous in terms of budgets and preferences, but such that the ranking of bundles is the same for each library, and A1' is satisfied. Then, in the sequential game of multilateral interconnection we have:*

(i) If $\bar{M}^h(0,0,1) < M^h < U^h - I^h_{12}$ for at least one h, then a unique SPNE exists and it is such that (a) all publishers which are active in at least one market interconnect; (b) for $h = 1, ..., m$, the set of active publishers and prices in the market for library h are determined as specified by Proposition 2, as a function of $\mathbf{U}^h, \mathbf{I}^h, M^h$.

(ii) If $M^h \leq \bar{M}^h(0,0,1)$ for $h = 1, ..., m$, then there exists different SPNE but in all of them we have that, for $h = 1, ..., m$, $A^{*h} = \{1,2\}$ and $(P_1^{*h}, P_2^{*h}) = (\hat{P}_1, \hat{P}_2)$ if $M^h > U_1^h - U_2^h$, while $A^{*h} = \{1\}$ and $P_1^{*h} = M$ if $M^h \leq U_1^h - U_2^h$.

This corollary says that in the setting of multilateral interconnection it is not restrictive to limit the analysis to the case of a single library, as the game with many heterogenous libraries replicates the outcomes of the various single-library games. The reason for this result is that in the game with multiple libraries, the markets for different libraries are linked one to another only by the firms' interconnection decisions. However, we know from section 4 that if $M^h > \bar{M}^h(0,0,1)$, then all active firms have a strict incentive to interconnect if they consider only the market for library h. But then they will definitely interconnect because choosing $x_i = 1$ never hurts firm i in any other market (see Lemma 3).

We now consider the simultaneous game of multilateral interconnection in which stages one, two, and three are merged into a single stage, like in the game that is considered at the end of subsection 4.2. Thus, all publishers decide at the same time whether to be active or not and, if active, whether to interconnect or not.[38] Consistently with the findings of subsection 4.2, we obtain the same result we have derived in Corollary 5.

Corollary 6 *Suppose there are $m(> 1)$ number of libraries that are heterogenous in terms of budgets and preferences, but such that the ranking of bundles is the same for each library, and A1' is satisfied. Then, in any SPNE of the simultaneous game of multilateral*

[38] As in Section 4, we neglect the "trivial" SPNE in which no publisher interconnects since each publisher expects that no rival will interconnect.

interconnection in which at least one publisher interconnects, the set of active publishers and prices are uniquely determined as specified by Proposition 2, as a function of $\mathbf{U}^h, \mathbf{I}^h, M^h$.

6.2 Application to Internet Interconnection

In order to check the robustness of our main result that firms have stronger incentives to interconnect under a multilateral interconnection regime than under a bilateral one, we perform comparison between the two regimes in the Internet interconnection model of CRT (2004). They consider a Cournot setting as in Katz-Shapiro (1985) and study interconnection with peering among Internet Backbone Companies (IBP) in a bilateral interconnection regime. An important difference between their model and our model is that in their model, a customer needs to subscribe only to one of the interconnected firms in order to get benefit from the interconnection while in our model, the library needs to subscribe to each interconnected journal platform in order to get benefit from the interconnection. Another difference is that in our model the industry profit is constant (equal to the library's budget) while in their model, an increase in IBPs' network sizes in terms of installed base expands the market (i.e. increases the number of new customers) and hence increases the industry profit.

More precisely, in section VI, CRT consider an interconnection game in which each IBP i simultaneously chooses a binary interconnection decision with respect to each rival and the cost of interconnection is zero as in our model. They study the game when there are three IBPs: a big IBP having a half of the total installed base and two IBPs sharing the remaining installed base equally. They find in proposition 6 that under certain parameter conditions, the large IBP interconnects with only one of the two small ones in order to prevent the small one without interconnection from attracting any new customer. This is similar to what happens in our proposition 4 in which publisher 1 interconnects only with 2 or 3 for exclusion or differentiation motive. However, since the small firms are symmetric in their framework, they are not able to distinguish the two motives.

We now study what happens in their Internet interconnection model if we consider the multilateral interconnection regime.

Proposition 7 *In Crémer-Rey-Tirole (2000), consider the following distribution of installed base among three IBPs* $(\beta_1, (\beta - \beta_1)/2, (\beta - \beta_1)/2)$ *where* β *is the total installed base. Under the multilateral interconnection regime, there exists a* $\beta_1^*(> \beta/2)$ *such that all IBPs fully interconnect for any* $\beta_1 \leq \beta_1^*$.

Proof. First, it is easy to see that the small IBPs will always interconnect since this improves their competitive positions: in fact, CRT find that they will maintain connectivity even under the bilateral interconnection regime (see their proposition 5). Suppose now that IBP 1 interconnects. Then, all IBPs are fully interconnected and hence they become completely symmetric (each IBP's network size in terms of installed base becomes β). Suppose now that IBP1 does not interconnect. In this case, after the interconnection game, each firm's network size in terms of installed base is given by $(\beta_1, \beta - \beta_1, \beta - \beta_1)$: in particular when $\beta_1 = \beta/2$, all firms have the same network size equal to $\beta/2$. Therefore, when $\beta_1 = \beta/2$, because of the market expansion effect, IBP1 strictly prefers interconnection to no interconnection. By continuity, there must be a $\beta_1^*(> \beta/2)$ such that all IBPs fully interconnect for any $\beta_1 \leq \beta_1^*$. ■

Proposition 7 shows that as long as the dominant IBP's share in the installed base is smaller than or close to a half, all IBPs fully interconnect under the multilateral interconnection regime. Therefore, our main result that firms have stronger incentives to interconnect under the multilateral interconnection regime than under the bilateral one is robust.

7 Conclusion and policy implications

Interconnection among platforms (or compatibility among networks) is an important issue that has been studied intensively. We contribute to the literature by comparing a multilateral interconnection regime with a bilateral one. We find that publishers have an incentive to fully interconnect under the multilateral interconnection regime while partial interconnection may arise under the bilateral interconnection regime for exclusion or for differentiation motives. We also find that the main result is robust in the case of interconnection among Internet Backbone Companies.

A general lesson from our paper is that allowing firms to have finer instruments to discriminate interconnection results in less interconnection than banning the discrimination. In fact, Jeon-Laffont-Tirole (2004) find a similar lesson in a different context: allowing telecommunication networks to discriminate between on-net calls and off-net calls can induce networks to break connectivity while without such discrimination, networks remain interconnected. In addition, both Jeon-Laffont-Tirole and our paper show that even symmetric firms may break connectivity when discrimination is allowed.

We show that when publishers can freely interconnect both in a multilateral way and in a bilateral way, large publishers may voluntarily endorse a multilateral interconnection regime since full interconnection among all publishers strengthens large publishers' relative standing. This might explain large commercial publishers support for CrossRef.[39] However, we also find that there can be a conflict between the market outcome and social welfare since large publishers can have an incentive to opt for bilateral interconnection in order to exclude small publishers. Then, our results suggest that a light-handed regulatory intervention banning discrimination among rivals will foster full interconnection. For instance, in the case of interconnection among internet backbone companies, the clause of no discrimination among rivals supplemented with a merger control forbidding the creation of a dominant firm would induce full interconnection, which is considered one of the four components of Internet Neutrality by Weitzer (2006)[40]

Finally, there is a potential governance issue related to the pricing of interconnection by CrossRef, an issue that we did not address. More precisely, if large commercial publishers control CrossRef, they may try to exclude small publishers by charging high membership fees: Actually, World Summit On the Information Society (WSIS) expressed such a concern.[41]

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[40] Actually, Weitzer (2006) argues that "non-discriminatory peering of backbone networks" is one of the four components of Internet Neutrality. Our clause of no discrimination among rivals is much weaker than his non-discriminatory peering, which is equivalent to mandatory full interconnection.

[41] WSIS argues that CrossRef charges fees with discount for large volume users and thereby treat small publishers on an unequal basis. See http://www.wsis-si.org/DOI/index.html. Concerning the fees charged by CrossRef, see http://www.crossref.org/02publishers/20pub_fees.htm

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8 Appendix

Before starting proofs, it is useful to introduce a new piece of notation. For any non-empty subset Z of $\{1,2,3\}$, let $u(Z)$ denote the library's payoff when she buys the bundles $\{\mathrm{B}_i : i \in Z\}$. For instance, $u(1) = U_1 - P_1$ and $u(13) = U_1 + U_3 + I_{13}y_{13} - P_1 - P_3$.

8.1 Proof of Lemma 1

Suppose by contradiction that there exists a SPNE of Γ in which publisher 1 is not active. This requires that if 1 is active (and chooses $x_1 = 1$ if we are considering Γ^m or $(x_{12}, x_{13}) = (1,0)$ if we are considering Γ^b), then the active publishers – 1, 2 and 3, for instance[42] – play a NE of γ in which 1 makes no profit because the library does not buy B_1; consider for instance the case in which the library buys B_2&B_3. The equilibrium prices (P_1^*, P_2^*, P_3^*) must be such that $P_2^* + P_3^* = M$, otherwise 1 can make a profit.[43] However, if 1 chooses $P_1 < P_3^*$, then from A1 we have $u(12) = U_1 + U_2 + I_{12}y_{12} - P_1 - P_2^* > u(23) = U_2 + U_3 + I_{23}y_{23} - P_2^* - P_3^*$. We also have $u(23) \geq \max\{u(2), u(3)\}$ since the library was supposed to buy B_2&B_3. Therefore, the library will buy B_1 (and maybe also B_2 or B_3). This means that if 1 is active, in any NE of γ he makes a profit; thus 1 is active in any SPNE of Γ.

[42] The case in which $A = \{1,2\}$ or $A = \{1,3\}$ is even simpler to deal with.

[43] Let $P_1(>0)$ be smaller than $M - P_2^* - P_3^*$ and smaller than U_1. Then $u(123) \geq u(23) + U_1 - P_1 > u(23)$, so that the library will buy B_1&B_2&B_3 rather than only B_2&B_3.

We now prove that A cannot be equal to $\{1,3\}$ in a SPNE. Suppose by contradiction that a SPNE with $A = \{1,3\}$ exists. Then, in γ with only 1 and 3 active, these publishers play a NE in which the library buys B_1&B_3, at some prices (P_1^*, P_3^*). This implies that $M > U_1 - U_3$, because if $M \leq U_1 - U_3$ then 1 can induce the library to buy B_1 at the price $P_1 = M$, given that the library's payoff is $U_1 - M$ and this is higher than the payoff $U_3 - P_3$ obtained by purchasing B_3, for any $P_3 > 0$. Therefore, $M > U_1 - U_3$ is necessary if 3 is to make a profit.

Now suppose that 2 is active (and chooses $x_2 = 1$, respectively $(x_{21}, x_{23}) = (1,0)$ if we are considering Γ^m or Γ^b). Then the SPNE needs to specify a NE of γ in which the library does not buy B_2; we prove below that no such NE exists. If the NE of γ is such that the library buys B_1&B_3 and not B_2,[44] then the equilibrium prices must be such that $P_1^* + P_3^* = M$. However, if publisher 2 chooses $P_2 < P_3^*$, then from A1 we have $u(12) = U_1 + U_2 + I_{12}y_{12} - P_1^* - P_2 > u(13) = U_1 + U_3 + I_{13}y_{13} - P_1^* - P_3^*$. We also have $u(13) \geq \max\{u(1), u(3)\}$ since the library was supposed to buy B_1&B_3. Therefore, the library will buy B_2 (and maybe also B_1 or B_3), which means that there exists no SPNE with $A = \{1,3\}$.

8.2 Proof of Lemma 2

(i) Let (P_1^*, P_2^*, P_3^*) be the equilibrium prices; then we show that $P_1^* + P_2^* + P_3^* = M$. If $P_1^* + P_2^* + P_3^* < M$, then $M < U - I_{12}$ implies that $P_i^* < U_i$ for at least one i and then a profitable deviation for i exists. Indeed, the library would be still willing to buy B_1&B_2&B_3 even if P_i were larger than P_i^*, provided that $P_i < U_i$ and $P_i + P_j^* + P_j^* \leq M$. In order to determine the other conditions which must be satisfied by (P_1^*, P_2^*, P_3^*), we notice that if 1 increases P_1 above P_1^*, the library cannot buy anymore the three bundles. Thus, the deviation of 1 is profitable if and only if the library buys B_1, or B_1&B_2, or B_1&B_3. In other words, 1's deviation does not occur if and only if (4) below is satisfied at prices (P_1^*, P_2^*, P_3^*); conditions (5)-(6) are derived likewise.

$$\max\{0, u(2), u(3), u(23)\} \geq \max\{u(1), u(12), u(13)\} \quad (4)$$
$$\max\{0, u(1), u(3), u(13)\} \geq \max\{u(2), u(12), u(23)\} \quad (5)$$
$$\max\{0, u(1), u(2), u(12)\} \geq \max\{u(3), u(13), u(23)\} \quad (6)$$

In what follows, we prove the following claim.

Claim: (4)-(6) are equivalent to $u(12) = u(13) = u(23)$.

[44] If the NE is such that the library buys only B_1, then the same arguments given when proving that $M > U_1 - U_3$ show that the library will not buy only B_1 if $M > U_1 - U_2$, and this inequality is implied by $M > U_1 - U_3$.

Note that $u(12) = u(13) = u(23)$ reduce to (1)-(2). Therefore, proving the claim is enough to prove Lemma 2(i). We prove the claim by showing that in each L.H.S. or R.H.S. of (4)-(6), the maximum is obtained when the library buys two bundles.

We first observe that if the left hand side of (4) is zero, then $u(12) \leq 0$, $u(13) \leq 0$, $u(23) \leq 0$ or, equivalently, $P_1 + P_2 \geq U_1 + U_2 + I_{12}y_{12}$, $P_1 + P_3 \geq U_1 + U_3 + I_{13}y_{13}$, $P_2 + P_3 \geq U_2 + U_3 + I_{23}y_{23}$. These three inequalities imply $M \geq U$, which contradicts $M < U - I_{12}$. Therefore, it is necessary that $\max\{0, u(2), u(3), u(23)\} > 0$ and we distinguish three cases, depending on the value of $\max\{0, u(2), u(3), u(23)\}$.

The case in which $u(2) \geq u(3)$ and $u(2) > \max\{0, u(23)\}$

From $u(2) > u(23)$ follows that $P_3 > U_3 + I_{23}y_{23}$ and $u(3) < 0$. From (5) we see that either $u(1) > 0$ and $u(1) \geq u(13)$, or $u(13) > \max\{0, u(1)\}$.

Suppose first that $u(1) > 0$ and $u(1) \geq u(13)$. Then $u(2) \geq u(1) \geq u(2)$ by (4) and (5). Thus $u(1) = u(2) > 0$ and $u(12) = u(1) + u(2) + I_{12}y_{12} > u(1) = u(2)$, violating (4): contradiction.

Suppose now that $u(13) > \max\{0, u(1)\}$. Since $P_3 > U_3$, the inequality $u(13) > u(1)$ requires $y_{13} = 1$ and $P_3 < U_3 + I_{13}$. We find that (4)-(6) boil down to $u(2) = u(13) \geq u(12)$, thus prices satisfy

$$U_2 - P_2 = U_1 + U_3 + I_{13} - P_1 - P_3, \qquad P_1 + P_2 + P_3 = M$$

This yields $P_1 + P_3 = \frac{1}{2}(M + U + I_{13} - 2U_2)$, $P_2 = \frac{1}{2}(M + 2U_2 - U - I_{13})$ and $u(13) = u(2) = \frac{1}{2}(U + I_{13} - M)$. Furthermore, $u(12) = U_1 + U_2 + I_{12}y_{12} - (M - P_3)$ and thus $u(2) \geq u(12)$ reduces to $M \geq U - 2U_3 + 2I_{12}y_{12} - I_{13} + 2P_3$. This right hand side is larger than $U - I_{12}$ since $P_3 > U_3 + I_{23}y_{23}$; then we get $M > U - I_{12}$, a contradiction.

The case in which $u(3) > \max\{0, u(2), u(23)\}$

From $u(3) > u(23)$ follows that $P_2 > U_2 + I_{23}y_{23}$ and $u(2) < 0$. From (6) we infer that either $u(1) > 0$ and $u(1) \geq u(12)$, or $u(12) > \max\{0, u(1)\}$.

Suppose first that $u(1) > 0$ and $u(1) \geq u(12)$. Then $u(3) \geq u(1) \geq u(3)$ by (4) and (6). Thus $u(1) = u(3) > 0$ and $u(13) > u(1) = u(3)$, which violates (4).

Suppose now that $u(12) > \max\{0, u(1)\}$. Since $P_2 > U_2$, the inequality $u(12) > u(1)$ requires $y_{12} = 1$ and $P_2 < U_2 + I_{12}$. Then (4)-(6) boil down to $u(3) = u(12) \geq u(13)$ and thus prices satisfy

$$U_3 - P_3 = U_1 + U_2 + I_{12} - P_1 - P_2, \qquad P_1 + P_2 + P_3 = M$$

This yields $P_3 = \frac{1}{2}(M + 2U_3 - U - I_{12})$, $P_1 + P_2 = \frac{1}{2}(M + U + I_{12} - 2U_3)$ and thus $u(3) = u(12) = \frac{1}{2}(U + I_{12} - M)$. Furthermore, $u(13) = U_1 + U_3 + I_{13}y_{13} - (M - P_2)$

and then $u(3) \geq u(13)$ reduces to $M \geq U - 2U_2 + 2I_{13}y_{13} - I_{12} + 2P_2 > U - I_{12}$ since $P_2 > U_2 + I_{23}y_{23}$: contradiction.

The case in which $u(23) > 0$ **and** $u(23) \geq \max\{u(2), u(3)\}$

Now (4)-(6) reduce to [notice that (7) is used to obtain (8)-(9)]:

$$u(23) \geq \max\{u(1), u(12), u(13)\} \tag{7}$$

$$\max\{u(1), u(3), u(13)\} \geq u(23) \tag{8}$$

$$\max\{u(1), u(2), u(12)\} \geq u(23) \tag{9}$$

We want to prove that (7)-(9) are equivalent to $u(12) = u(13) = u(23)$, thus we exclude all other cases. With reference to (8), suppose that $u(1) \geq u(23)$. Then using (7) we find $u(23) = u(1) \geq \max\{u(12), u(13)\}$ and prices satisfy

$$P_3 \geq U_3 + I_{13}y_{13} \qquad U_1 - P_1 = U_2 + U_3 + I_{23}y_{23} - P_2 - P_3, \qquad P_1 + P_2 + P_3 = M$$

The two equalities yield $P_1 = \frac{1}{2}(M + 2U_1 - U - I_{23}y_{23})$, $P_2 + P_3 = \frac{1}{2}(M + U + I_{23}y_{23} - 2U_1)$ and $u(1) = u(23) = \frac{1}{2}(U + I_{23}y_{23} - M)$. Furthermore, $u(12) = U_1 + U_2 + I_{12}y_{12} - (M - P_3)$ and $u(1) \geq u(12)$ reduces to $M \geq U - 2U_3 + 2I_{12}y_{12} - I_{23}y_{23} + 2P_3 \geq U - I_{12}$ since $P_3 \geq U_3 + I_{13}y_{23}$: contradiction. This allows to delete $u(1)$ from (7)-(9).
Now, still with reference to (8), suppose that $u(3) = u(23)$. Then $P_2 = U_2 + I_{23}y_{23}$ and $u(2) \leq 0$, $u(3) = u(23) = u(12) \geq u(13)$. We can argue like in the case in which $u(3) > \max\{0, u(2), u(23)\}$ (see above) to derive that $M \geq U - I_{12}$ from $u(3) = u(12) \geq u(13)$: contradiction. This implies that (7)-(8) reduce to $u(13) = u(23) \geq u(12)$.
Finally, with reference to (9), suppose that $u(2) = u(23)$. Then $P_3 = U_3 + I_{23}y_{23}$ and $u(2) = u(13) \geq u(12)$; thus we can argue like in the case in which $u(2) \geq u(3)$ and $u(2) > \max\{0, u(23)\}$ (see above) to derive that $M \geq U - I_{12}$ from $u(2) = u(13) \geq u(12)$: contradiction.

Thus, we have found that (7)-(9) are equivalent to $u(12) = u(13) = u(23)$ which reduce to (1)-(2); using $P_1^* + P_2^* + P_3^* = M$ we obtain $P_1^* = \bar{P}_1(\mathbf{y})$, $P_2^* = \bar{P}_2(\mathbf{y})$, $P_3^* = \bar{P}_3(\mathbf{y})$. Notice that $\bar{P}_1(\mathbf{y}) > \bar{P}_2(\mathbf{y}) > \bar{P}_3(\mathbf{y})$ for any $\mathbf{y}$ (given A1), and $\bar{P}_3(\mathbf{y}) > 0$ if and only if $M > \bar{M}(\mathbf{y})$. Furthermore, the conditions $u(23) \geq \max\{u(2), u(3)\}$ and $u(13) \geq u(1)$ are equivalent, respectively, to $U + 2I_{12}y_{12} - I_{13}y_{13} + 2I_{23}y_{23} \geq M$, $U - I_{12}y_{12} + 2I_{13}y_{13} + 2I_{23}y_{23} \geq M$ and $U + 2I_{12}y_{12} + 2I_{13}y_{13} - I_{23}y_{23} \geq M$ and are satisfied for any M between $\bar{M}(\mathbf{y})$ and $U - I_{12}$.

(ii) Given any $M \leq U_1 + U_2$, the equilibrium prices satisfy $P_1^* + P_2^* = M$, otherwise $P_i^* < U_i$ for at least one i and then i can profitably increase P_i slightly above P_i^* (we are arguing here as in the beginning of the proof of Lemma 2(i)). Given P_2^*, if 1 chooses

$P_1 > P_1^*$ then the library cannot afford to buy B_1&B_2. It will buy B_1 if and only if $U_1 - P_1 > U_2 - P_2^*$, and this condition is violated for any $P_1 > P_1^*$ if and only if $U_1 - P_1^* \leq U_2 - P_2^*$. Likewise, 2 has no incentive to deviate if and only if $U_1 - P_1^* \geq U_2 - P_2^*$. Then we find $P_1^* = \hat{P}_1$, $P_2^* = \hat{P}_1$ and $\hat{P}_1 > \hat{P}_2$; furthermore, $\hat{P}_2 > 0$ if and only if $M > U_1 - U_2$.

8.3 Proof of Proposition 1

Recall that, in Γ^0, $y_{12} = y_{13} = y_{23} = 0$.
(i) When $M \leq U_1 - U_2$ and $A = \{1,2\}$ or $A = \{1,2,3\}$, Lemma 2 states that there exists no NE of γ such that the library buys B_1&B_2 or B_1&B_2&B_3. Thus, by Lemma 1, the only SPNE of Γ^0 is such that $A^* = \{1\}$ and $P_1^* = M$.
(ii) When $U_1 - U_2 < M \leq \bar{M}(0,0,0)$ and $A = \{1,2,3\}$, Lemma 2(i) states that there exists no NE of γ such that the library buys B_1&B_2&B_3. Thus, by Lemma 1, $A^* = \{1\}$ or $A^* = \{1,2\}$. However, A^* cannot be $\{1\}$ because if also 2 is active, then γ has a unique NE and it is such that the library buys B_1&B_2 (see Lemma 2(ii)); 2's profit is then $\hat{P}_2 > 0$. Hence 2 will be active, i.e. $A^* = \{1,2\}$, and $(P_1^*, P_2^*) = (\hat{P}_1, \hat{P}_2)$ by Lemma 2(ii).
(iii) When $\bar{M}(0,0,0) < M \leq U - I_{12}$ and $A = \{1,2,3\}$. Lemma 2(i) states that each publisher succeeds in selling his bundle. Hence, $A^* = \{1,2,3\}$ and $P_i^* = \bar{P}_i(0,0,0)$.

8.4 Proof of Lemma 3

(i) The same proof given for Proposition 1(i) applies.
(ii)-(iii) If $M > U_1 - U_2$, we can argue as in the proof of Proposition 1(ii) to rule out that $A^* = \{1\}$. Then we apply backward induction to find out when $A^* = \{1,2\}$ occurs and when $A^* = \{1,2,3\}$ occurs.

Stage three Given (x_1, x_2), publisher 3 stays out if $M \leq \bar{M}(y_{12}, x_1, x_2)$ because then in γ there is no NE in which he makes a positive profit, regardless of x_3. Conversely, if $M > \bar{M}(y_{12}, x_1, x_2)$, then 3 is active and joins CrossRef because by doing so he makes a profit equal to $\frac{1}{3}[M - \bar{M}(y_{12}, x_1, x_2)] > 0$ – actually, if $x_1 = x_2 = 0$ and $M > \bar{M}(0,0,0)$, then 3 is indifferent between $x_3 = 0$ and $x_3 = 1$.

Stage two Publisher 2 chooses x_2 as a function of x_1, by taking into account the future behavior of 3 as described above.
The case of $x_1 = 0$. When $M < \bar{M}(001)$, 2 is indifferent between $x_2 = 0$ and $x_2 = 1$ because in any case 3 will stay out and 2's profit will be $\hat{P}_2$. When instead $\bar{M}(0,0,1) < M \leq \bar{M}(0,0,0)$, 3 is active if and only if $x_2 = 1$ and then 2 chooses $x_2 = 1$ because this will yield him $\bar{P}_2(0,0,1)$, which is larger than his profit $\hat{P}_2$ if $x_2 = 0$. Finally, when

$M > \bar{M}(0,0,0)$, 3 is active for any x_2 and thus 2 will interconnect because $\bar{P}_2(0,0,1) > \bar{P}_2(0,0,0)$.

The case of $x_1 = 1$. When $M < \bar{M}(0,1,0)$, 2 is indifferent between $x_2 = 0$ and $x_2 = 1$ because 3 will not be active in any case. If $M > \bar{M}(1,1,1)$, then 3 is active for any x_2 and therefore 2 interconnects. When $\bar{M}(0,1,0) < M \leq \bar{M}(1,1,1)$, 3 is active if and only if $x_2 = 0$; then 2 chooses $x_2 = 1$ to exclude 3 because $\hat{P}_2 > \bar{P}_2(0,1,0)$.

Stage one Publisher 1 is indifferent between $x_1 = 0$ and $x_1 = 1$ when $M \leq \bar{M}(0,1,0)$ because in any case 3 will not be active. When $\bar{M}(0,1,0) < M < \bar{M}(0,0,1)$, 3 is active if and only if $x_1 = 1$ and $x_2 = 0$, but we know that 2 will instead choose $x_2 = 1$. When $M > \bar{M}(0,0,1)$, 3 is active (and interconnected) if and only if $M > \bar{M}(x_1,x_1,1)$. Thus, for M between $\bar{M}(0,0,1)$ and $\bar{M}(1,1,1)$, 1 can exclude 3 by playing $x_1 = 1$ and obtain $\hat{P}_1$, while his profit with $x_1 = 0$ is $\bar{P}_1(0,0,1)$. Since $\hat{P}_1 > \bar{P}_1(0,0,1)$ holds, 1 will interconnect for these values of M. Furthermore, 1 will join CrossRef also for $M > \bar{M}(1,1,1)$ because $\bar{P}_1(\mathbf{y})$ is increasing in (y_{12}, y_{13}).

8.5 Proof of Proposition 2

The proof is straightforward from lemma 3.

8.6 Proof of Proposition 3

(i) The same proof given for proposition 1(i) applies.
(iii) We show that no SPNE of Γ^{ms} is such that $A^* = \{1,2,3\}$ and $(x_i^*, x_j^*, x_k^*) = (1,0,0)$ or $(x_i^*, x_j^*, x_k^*) = (1,1,0)$. In other terms, in any SPNE with $A^* = \{1,2,3\}$ and $x_1^* + x_2^* + x_3^* \geq 1$ we find $x_1^* = x_2^* = x_3^* = 1$. By lemma 2(i), this requires that $M > \bar{M}(1,1,1)$ and prices are $\bar{P}_1(1,1,1), \bar{P}_2(1,1,1), \bar{P}_3(1,1,1)$.
No SPNE of Γ^{ms} is such that $\mathbf{A}^* = \{\mathbf{1},\mathbf{2},\mathbf{3}\}$ and $(\mathbf{x}_i^*, \mathbf{x}_j^*, \mathbf{x}_k^*) = (\mathbf{1},\mathbf{0},\mathbf{0})$. When $(x_i, x_j, x_k) = (1,0,0)$, we prove that publisher j and/or publisher k has an incentive to interconnect. With $(x_i, x_j, x_k) = (1,0,0)$, the equilibrium prices of γ are $\bar{P}_1(0,0,0)$, $\bar{P}_2(0,0,0)$, $\bar{P}_3(0,0,0)$ by lemma 2(i) and it is necessary that $M > \bar{M}(0,0,0)$ in order for $\bar{P}_3(0,0,0)$ to be positive. In the case that $i = 3$, then it is profitable for 1 to set $x_1 = 1$ because $M > \bar{M}(0,0,0)$ implies $M > \bar{M}(0,1,0)$, thus the equilibrium prices will be $\bar{P}_1(0,1,0)$, $\bar{P}_2(0,1,0)$, $\bar{P}_3(0,1,0)$ and $\bar{P}_1(0,1,0) > \bar{P}_1(0,0,0)$. Likewise, if $i = 2$ then it is profitable for 3 to set $x_3 = 3$ because the equilibrium prices will be $\bar{P}_1(0,0,1)$, $\bar{P}_2(0,0,1)$, $\bar{P}_3(0,0,1)$ and $\bar{P}_3(0,0,1) > \bar{P}_3(0,0,0)$. The same argument applies if $i = 1$.

No SPNE of Γ^{ms} is such that $\mathbf{A}^*= \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$ and $(\mathbf{x}_i^*, \mathbf{x}_j^*, \mathbf{x}_k^*) = (\mathbf{1}, \mathbf{1}, \mathbf{0})$. When $(x_i, x_j, x_k) = (1, 1, 0)$, we prove that publisher k has an incentive to interconnect; notice that this induces the full interconnection profile $(1, 1, 1)$. We start by observing that when $(x_i, x_j, x_k) = (1, 1, 0)$, it is necessary that $M > \bar{M}(\mathbf{y})$ in order for $\bar{P}_3(\mathbf{y})$ to be positive. In the case that $k = 3$, profits are $\bar{P}_1(1, 0, 0), \bar{P}_2(1, 0, 0), \bar{P}_3(1, 0, 0)$ and $\bar{M}(\mathbf{y}) > \bar{M}(1, 1, 1)$; thus profits become $\bar{P}_1(1, 1, 1), \bar{P}_2(1, 1, 1), \bar{P}_3(1, 1, 1)$ if $x_3 = 1$ and $\bar{P}_3(1, 1, 1) > \bar{P}_3(1, 0, 0)$. In the case of $k = 1$, we have that $\mathbf{y} = (0, 0, 1)$ and thus $M > \bar{M}(\mathbf{y})$ does not imply $M > \bar{M}(1, 1, 1)$. If M is larger than $\bar{M}(1, 1, 1)$, then the same argument above applies: 1 gains from playing $x_1 = 1$ because $\bar{P}_1(1, 1, 1) > \bar{P}_1(0, 0, 1)$. If instead $M < \bar{M}(1, 1, 1)$, then 3 does not make any profit if publisher 1 plays $x_1 = 1$. In particular, we find a NE in the pricing game in which $P_1 = \frac{1}{2}(M + U_1 + I_{13} - U_2 - I_{23})$, $P_2 = \frac{1}{2}(M + U_2 + I_{23} - U_1 - I_{13})$ and P_3 is close to 0, and the library buys B_1&B_2. Since $\frac{1}{2}(M + U_1 + I_{13} - U_2 - I_{23}) > \bar{P}_1(0, 0, 1)$ for any $M > \bar{M}(0, 0, 1)$, we conclude that it pays for 1 to interconnect. The case of $k = 2$ is similar, and thus is omitted.
(ii) By the virtue of the proof of part (iii), we know that when $U_1 - U_2 < M \leq \bar{M}(1, 1, 1)$, there is no SPNE of Γ^{ms} with $A^* = \{1, 2, 3\}$ and $x_1^* + x_2^* + x_3^* \geq 1$. Thus, $A^* = \{1, 2\}$ and it is necessary that 3 has no incentive to be active. If $x_1 = x_2 = 1$, then this conditions is satisfied for any M between $U_1 - U_2$ and $\bar{M}(1, 1, 1)$.[45]

8.7 Proof of Lemma 4

(i) The same proof given for Proposition 1(i) applies.
(ii) and (v) The proofs of these results have been provided just after the statement of Lemma 4.
(iii) **Stage three** Given (x_{12}, x_{13}) and (x_{21}, x_{23}), publisher 3 is not active if $M \leq \bar{M}(y_{12}, x_{13}, x_{23})$ because in γ there is no NE in which he makes a positive profit, regardless of (x_{31}, x_{32}). Conversely, 3 is active when $M > \bar{M}(y_{12}, x_{13}, x_{23})$ by playing $x_{31} = 1$ if $x_{13} = 1$ and $x_{32} = 1$ if $x_{23} = 1$.

Stage two Publisher 2 chooses (x_{21}, x_{23}) as a function of (x_{12}, x_{13}), by taking into account the future behavior of 3 as described above.
In the case of $(x_{12}, x_{13}) = (1, 1)$, publisher 2 can exclude 3 for any $M \in (\bar{M}(0, 1, 0), \bar{M}(1, 1, 0)]$ by playing $(x_{21}, x_{23}) = (1, 0)$. If instead 2 plays $(x_{21}, x_{23}) \neq (1, 0)$, 3 is active for $M > \bar{M}(x_{21}, 1, x_{23})$ and then 2's profit is $\bar{P}_2(x_{21}, 1, x_{23})$; however, for these values of M we have $\widehat{P}_2 > \bar{P}_2(x_{21}, 1, x_{23})$ and thus 2 prefers $(x_{21}, x_{23}) = (1, 0)$.

[45] Actually, for M close to $U_1 - U_2$ the condition is satisfied also if $x_1 x_2 = 0$, but for $M > \bar{M}(0, 0, 0)$ it is necessary that $x_1 = x_2 = 1$.

In the case of $(x_{12}, x_{13}) = (0, 1)$ it is clear that the value of x_{21} does not matter. Furthermore, $\bar{M}(0, 1, x_{23}) \leq \bar{M}(0, 1, 0)$ and thus 3 will be active for any x_{23}; 2's best action is then $(x_{21}, x_{23}) = (0, 1)$.
In the case of $(x_{12}, x_{13}) = (1, 0)$ or $(x_{12}, x_{13}) = (0, 0)$, we do not actually need to find the optimal (x_{21}, x_{23}) for 2: (i) if (x_{21}, x_{23}) excludes 3, then 1 can achieve the same outcome by playing $(x_{12}, x_{13}) = (1, 1)$. If (x_{21}, x_{23}) is such that 3 is active, then y_{12} is necessarily equal to 0 and thus 1's profit is $\bar{P}_1(0, 0, 1)$, smaller than the profit $\bar{P}_1(0, 1, 1)$ he earns by playing $(x_{12}, x_{13}) = (0, 1)$.

8.8 Proof of Proposition 4

(i) (ii), (v) are obvious from Lemma 4 and the main texts in section 4.
(iii) The arguments in the proof of lemma 4 imply that 1 needs to compare $(x_{12}, x_{13}) = (1, 1)$ – which makes 3 inactive – with $(x_{12}, x_{13}) = (0, 1)$ – which makes 3 active and yields 1 the profit $\bar{P}_1(0, 1, 1)$. Therefore we verify that for any $M \in (\bar{M}(0, 1, 0), \bar{M}(1, 1, 0)]$, the inequality $\widehat{P}_1 > \bar{P}_1(0, 1, 1)$ holds if $4I_{23} \geq 3I_{13}$ while $\widehat{P}_1 < \bar{P}_1(0, 1, 1)$ if $3I_{13} \geq 2I_{12} + 4I_{23}$. This is a straightforward result, as $\widehat{P}_1 > \bar{P}_1(0, 1, 1)$ is equivalent to $M > U - 3U_3 + 2I_{13} - 4I_{23}$ and $\bar{M}(0, 1, 0) \geq U - 3U_3 + 2I_{13} - 4I_{23}$ when $4I_{23} \geq 3I_{13}$; on the other hand, $\bar{M}(1, 1, 0) \leq U - 3U_3 + 2I_{13} - 4I_{23}$ if $3I_{13} \geq 2I_{12} + 4I_{23}$.
(iv) When $3I_{23} \geq 2I_{12}$: Publisher 3 is not active if and only if $y_{12} = 1$ and $y_{13} = 0$. This requires that 1 plays $(x_{12}, x_{13}) = (1, 0)$, and then 2 can exclude 3 by playing $(x_{21}, x_{23}) = (1, 0)$ (earning $\widehat{P}_2$) otherwise his best option is to induce 3 to be active by playing $(x_{21}, x_{23}) = (0, 1)$, which gives him $\bar{P}_2(0, 0, 1)$. Since $\widehat{P}_2 \leq \bar{P}_2(0, 0, 1)$ is equivalent to $M \leq U - 3U_3 + 2I_{23}$, we conclude that $\widehat{P}_2 \leq \bar{P}_2(0, 0, 1)$ for any M between $\bar{M}(1, 1, 0)$ and $\bar{M}(1, 0, 1)$, given that $3I_{23} \geq 2I_{12}$. Therefore, it is optimal for 1 to play $x_{12}, x_{13}) = (1, 1)$. Then, publisher 2 cannot exclude 3 by playing $(x_{21}, x_{23}) = (1, 0)$ as $M > \bar{M}(1, 1, 0)$.and hence chooses $(x_{21}, x_{23}) = (1, 1)$, implying that all publishers are active and fully interconnected.
When $2I_{12} \geq I_{13} + 2I_{23}$ and that $4I_{23} \geq 3I_{13}$: In this case, if 1 plays $(x_{12}, x_{13}) = (1, 0)$, 2 finds it optimal to exclude 3 by playing $(x_{21}, x_{23}) = (1, 0)$ since $\widehat{P}_2 > \bar{P}_2(0, 0, 1)$. If 1 plays $(x_{12}, x_{13}) = (1, 1)$, as we have already seen, 2 is unable to exclude 3 and therefore all three are active and fully interconnected. Finally, 1 prefers to exclude 3 since $\bar{P}_1(1, 1, 1) < \widehat{P}_1$ is equivalent to $M > U - 3U_3 + 2(I_{12} + I_{13} - 2I_{23})$ and $\bar{M}(1, 1, 0) \geq U - 3U_3 + 2(I_{12} + I_{13} - 2I_{23})$ holds from $4I_{23} \geq 3I_{13}$.

8.9 Proof of Proposition 5

(i) and (iii) The proofs of (i) and (iii) are straightforward and are omitted.

(ii) We first introduce some notation: $\tilde{P} = \frac{1}{3}(M+I_s)$, $\bar{P} = \frac{1}{3}(M+2I_s)$, $\hat{P} = \frac{1}{3}(M-2I_s)$, $P^* = \frac{1}{3}(M - I_s)$. Note first that:

- The case of $\mathbf{y} = (1, 0, 0)$. Then we have a NE in which all three make a profit if and only if $M > 2I_s$, and profits are $P_1 = P_2 = \tilde{P}$, $P_3 = \hat{P}$. If instead $M \leq 2I_s$, then profits are $P_1 = P_2 = M/2$, and zero for 3.

- The case of $y = (1, 1, 0)$. Then we have a NE in which all three make a profit if and only if $M > I_s$, and profits are $P_1 = \bar{P}$, $P_2 = P_3 = P^*$. If instead $M \leq I_s$, then profits are $P_1 = M$, and zero for 2 and 3.

Stage 3

1. Say $\mathbf{x}_1 = (., 0)$, $\mathbf{x}_2 = (., 0)$, $y_{12} = 0$. Then $\mathbf{x}_3$ is irrelevant and profits are $M/3, M/3, M/3$ for $M \in (0, U - I_s)$. If instead $\mathbf{x}_1 = \mathbf{x}_2 = (1, 1)$, then $\mathbf{x}_3 = (1, 1)$ yields $M/3$ to publisher 3; $\mathbf{x}_3 = (0, 0)$ yields 0 or $\hat{P}$; $\mathbf{x}_3 = (1, 0)$ [like $\mathbf{x}_3 = (0, 1)$] yields 0 or P^*. Thus, 3 plays $\mathbf{x}_3 = (1, 1)$ and profits are $M/3, M/3, M/3$ for $M \in (0, U - I_s)$.

2. Say $\mathbf{x}_1 = \mathbf{x}_2 = (1, 0)$. Then $\mathbf{x}_3$ is irrelevant, 3 stays out for $M \in (0, 2I_s]$ and profits are $M/2, M/2, 0$. 3 is active for $M \in (2I_s, U - I_s)$ and profits are $\tilde{P}, \tilde{P}, \hat{P}$.

3. Say $\mathbf{x}_1 = (., 1)$, $\mathbf{x}_2 = (., 0)$, $y_{12} = 0$. Then 3 is active for any M and plays $\mathbf{x}_3 = (1, .)$, so that profits are $M/2, 0, M/2$ for $M \in (0, 2I_s]$, are $\tilde{P}, \hat{P}, \tilde{P}$ for $M \in (2I_s, U - I_s)$.

4. Say $\mathbf{x}_1 = (1, 1)$, $\mathbf{x}_2 = (1, 0)$. Then 3 stays out for $M \leq I_s$, plays $\mathbf{x}_3 = (1, .)$ for $M > I_s$ and profits are $M/2, M/2, 0$ for $M \in (0, I_s]$, are $\bar{P}, P^*, P^*$ for $M \in (I_s, U - I_s)$.

5. Say $\mathbf{x}_1 = (., 0)$, $\mathbf{x}_2 = (., 1)$, $y_{12} = 0$. Then 3 is active for any M and plays $\mathbf{x}_3 = (., 1)$ and profits are $0, M/2, M/2$ for $M \in (0, 2I_s]$, are $\hat{P}, \tilde{P}, \tilde{P}$ for $M \in (2I_s, U - I_s)$.

6. Say $\mathbf{x}_1 = (1, 0)$, $\mathbf{x}_2 = (1, 1)$. Then 3 stays out for $M \leq I_s$, plays $\mathbf{x}_3 = (., 1)$ for $M > I_s$ and profits are $M/2, M/2, 0$ for $M \in (0, I_s]$, are $P^*, \bar{P}, P^*$ for $M \in (I_s, U - I_s)$.

7. Say $\mathbf{x}_1 = (., 1)$, $\mathbf{x}_2 = (., 1)$, $y_{12} = 0$. Then 3 is active for any M, plays $\mathbf{x}_3 = (1, 1)$ and profits are $0, 0, M$ for $M \in (0, I_s]$, are $P^*, P^*, \bar{P}$ for $M \in (I_s, U - I_s)$.

The bottom line is that 3 is active if and only if $M > \bar{M}(y_{12}, x_{13}, x_{23})$, and plays $x_{31} = x_{13}$, $x_{32} = x_{23}$.

Stage 2

1. Say $\mathbf{x}_1 = (0, 0)$. Then

   - $\mathbf{x}_2 = (., 1)$ induces line 5 above.
   - $\mathbf{x}_2 = (., 0)$ induces line 1 above.

   Therefore, $\mathbf{x}_2 = (., 1)$ is best.

2. Say $\mathbf{x}_1 = (0, 1)$. Then

   - $\mathbf{x}_2 = (., 1)$ induces line 7 above..
   - $\mathbf{x}_2 = (., 0)$ induces line 3 above..

   Therefore, 2 stays out if $M \leq I_s$, while $\mathbf{x}_2 = (., 1)$ is best if $M > I_s$.

3. Say $\mathbf{x}_1 = (1, 1)$. Then

   - $\mathbf{x}_2 = (0, 0)$ induces line 3 above.;
   - $\mathbf{x}_2 = (0, 1)$ induces line 7 above.;
   - $\mathbf{x}_2 = (1, 0)$ induces 3 to stay out (with profits $M/2, M/2, 0$) for $M \leq I_s$, then line 4 above.;
   - $\mathbf{x}_2 = (1, 1)$ induces line 1 above..

   Therefore, $\mathbf{x}_2 = (1, 0)$ if $M \leq I_s$, $\mathbf{x}_2 = (1, 1)$ if $M > I_s$.

4. Say $\mathbf{x}_1 = (1, 0)$. Then

   - $\mathbf{x}_2 = (0, 0)$ induces line 1 above;
   - $\mathbf{x}_2 = (0, 1)$ gives 1 profit $\frac{M}{2}$ until $2I_s$, then line 5 above;
   - $\mathbf{x}_2 = (1, 0)$ gives 1 profit $\frac{M}{2}$ until $2I_s$, then line 2 above;
   - $\mathbf{x}_2 = (1, 1)$ induces 3 to stay out (with profits $M/2, M/2, 0$) for $M \leq I_s$, then line 6 above;

   Therefore, if $M \leq I_s$, 2 is indifferent between $(0, 1)$, or $(1, 0)$, or $(1, 1)$. If instead $I_s < M$, the best $\mathbf{x}_2$ is $(1, 1)$.

Stage 1

1. If $M \leq I_s$, 1 can get $M/2$ with $\mathbf{x}_1 = (0, 1)$ (then $A = \{1, 3\}$) or $\mathbf{x}_1 = (1, 1)$ (then $A = \{1, 2\}$), or $\mathbf{x}_1 = (1, 0)$ if 2 plays $(1, 0)$ or $(1, 1)$ (then $A = \{1, 2\}$).

2. If $I_s < M \leq 2I_s$, 1 cannot get more than $\frac{1}{3}M$, and getting $\frac{1}{3}M$ requires $\mathbf{x}_1 = (1, 1)$.

3. If $2I_s < M$, 1 cannot get more than $\frac{1}{3}M$, and getting $\frac{1}{3}M$ requires $\mathbf{x}_1 = (1, 1)$.

8.10 Proof of Lemma 5

The proof of (i)-(ii) is given in the text immediately after the statement of the lemma.
(iii) We start by noticing that if $x_2 = 1$, then 3 can maximize his interconnections with $x_3 = 1$, which induces $\mathbf{y} = (1,1,1)$. Hence, $x_2 = 1$ implies that duopoly occurs if $M \le \bar{M}(1,1,1)$, while 3 will be active (with full interconnection) if $M > \bar{M}(1,1,1)$.
If instead $x_2 = 0$, then 3 will choose $x_{32} = 1$ if and only if $x_{23} = 1$, and given $x_1 = 1$ and $x_{12} = 0$, we simply need to evaluate the alternatives of $x_{23} = 0$ and $x_{23} = 1$ for firm 2. In the first case, 2 earns $\hat{P}_2$ for $M \le \bar{M}(0,1,0)$ and $\bar{P}_2(0,1,0)$ for $M > \bar{M}(0,1,0)$; in the second case he earns $\hat{P}_2$ for $M \le \bar{M}(0,1,1)$ and $\bar{P}_2(0,1,1)$ for $M > \bar{M}(0,1,1)$. Since $\hat{P}_2 > \bar{P}_2(0,1,1) > \bar{P}_2(0,1,0)$ for any $M > \bar{M}(0,1,1)$, we conclude that 2 prefers $x_2 = 1$, and thus the same outcome as in Γ^m results.[46]
(iv) When $x_1 = 1$ and $x_{12} = 1$, the incentives of 2 are exactly the same as the incentives he has in Γ^b when $(x_{12}, x_{13}) = (1,1)$. This yields the statement of the lemma. From Lemma 4 we know that in such a case 2 plays $(x_{21}, x_{23}) = (1,0)$ (in order to exclude 3) for $M \le \bar{M}(1,1,0)$, but plays $(x_{21}, x_{23}) = (1,1)$ since he cannot exclude 3, when $M > \bar{M}(1,1,0)$.

8.11 Proof of Proposition 6

Propositoin 6 is a direct consequence of lemma 5 and the proof is omitted.

[46] $\widehat{P}_2 > \overline{P}_2(0,1,1)$ if and only if $M > U - 3U_3 + 2I_{23} - 4I_{13}$, and $U - 3U_3 + 2I_{23} - 4I_{13}$ is smaller than $M(0,1,1)$. Thus $\widehat{P}_2 > \overline{P}_2(0,1,1)$ holds for any $M > \overline{M}(0,1,1)$.