When Is the Optimal Lending Contract in Microfinance State Non-Contingent?*

Doh-Shin Jeon† and Domenico Menicucci‡

May 29, 2008

Abstract

Whether a microfinance institution should use a state-contingent repayment or not is very important since a state-contingent loan can provide insurance for borrowers. However, the Grameen bank uses state non-contingent repayments, which is puzzling since it forces poor borrowers to make their payments even under hard circumstances in which their projects failed. This paper provides an explanation to this puzzle. More precisely, we characterize the optimal lending and supervisory contracts when a staff member (a supervisor) can embezzle borrowers' repayments by misrepresenting realized returns. After identifying the main trade-off between insurance gain and audit cost in a single borrower setting, we study how the mode of liability (joint versus individual liability) affects the trade-off when a staff member supervisors multiple (= two) borrowers. We find that joint liability dominates individual liability by providing more insurance or by saving audit costs and that a state-contingent loan is more likely to be optimal under joint liability than under individual liability.

JEL Code: O16, D82, G20

Key words: Microfinance, Repayment, Contract, Embezzlement, Insurance, Joint Liability.

*We thank seminar participants at Universidad Carlos III, Universitat Pompeu Fabra, Université de Toulouse, ASSET 05 (Crete) and FEMES 06 (Beijing). We also thank Martin Besfamille, Bruno Biais, Vicente Cuñat, Xavier Freixas, Patrick Rey, Jean-Charles Rochet and Joel Shapiro for helpful comments. The first author gratefully acknowledges financial support from the Spanish Ministry of Science and Technology under BEC2003-00412 and Ramon y Cajal grant.

†Universitat Pompeu Fabra, IESE (SP-SP), Toulouse School of Economics. doh-shin.jeon@upf.edu

‡Università degli Studi di Firenze, Italy. domenico.menicucci@dmd.unifi.it

1 Introduction

The remarkable success of microfinance programs in making loans to (and recovering them from) poor people has received world-wide attention and generated a global microfinance movement which has been growing rapidly: according to the report from the Microcredit Summit Campaign, by the end of 2002, 67.6 million clients were served worldwide by over 2,500 microfinance institutions (Armendáriz de Aghion and Morduch, 2005, p.3). The original ideas of microfinance are due to Muhammad Yunus, the founder of the Grameen bank and Nobel peace prize laureate in 2006, who started making small loans to groups of poor people in rural area in Bangladesh in the 1970s. Today the Grameen bank is a large financial organization: it disbursed \$ 755.58 million during the 12 months from March 2007 to February 2008 to about 7.34 millions borrowers with a loan recovery rate of approximately 98.15 percent.[1]

There exists a large economic literature on microfinance and most of it focuses on how group lending, in particular joint liability, affects adverse selection (Ghatak, 1999, 2000, Van Tassel, 1999, Armendáriz de Aghion and Gollier 2000, Laffont and N'Guessan 2000, Laffont, 2003), moral hazard in terms of loan repayment (Besley and Coate, 1995, Armendáriz de Aghion 1999, Sadoulet 2000, Rai and Sjöström, 2004), and moral hazard before return realization in terms of work incentives (Stiglitz 1990, Varian 1990, Conning 1999, Che 2002, Laffont and Rey, 2003).[2] Despite the variety of the issues that these papers examine, most of them, with a few exceptions mentioned later on, consider only borrowers' incentives and do not study the incentive issues of the bank staff managing the loans. Furthermore, all papers studying the optimal lending contracts find that state-contingent repayments are optimal where a state refers to a realized return of a borrower's project. This finding is in stark contrast with the practice of the Grameen bank, which specifies a repayment schedule that does not depend on the realization of the state of nature.[3] The Grameen bank's practice is very puzzling since it means that poor borrowers of the Grameen bank make their payments even under hard circumstances in which their projects failed. We try to explain this puzzle from the perspective of the agency problem of the bank staff. Note that the standard perspective focusing on borrowers' moral hazard

[1] http://www.grameen-info.org/bank/GBGlance.htm.

[2] See Ghatak and Guinnane (1999) and Morduch (1999) for surveys. The book written by Armendáriz de Aghion and Morduch (2005) also reviews some recent papers that focus on issues different from joint liability such as dynamic incentives, competition, the use of collateral, etc.

[3] Yunus (1998, p.110) describes the repayment mechanism of the Grameen bank as: (i) one year loan (ii) equal weekly installments (iii) repayment starts one week after the loan etc.

in terms of repayment does not provide a satisfactory explanation to the puzzle since a poor borrower has little incentive to repay the loan under a state non-contingent lending contract if his project fails: this would favor the adoption of a state contingent loan.

Whether a microfinance institution should use a state-contingent contract or not is an extremely important issue since a state-contingent loan can provide insurance to borrowers by linking repayments to the success or failure of their projects.[4] However, the spectacular growth of microfinance institutions raises the issue of staff's quality and their misbehavior.[5] In particular, when a loan is state-contingent, a staff member managing the loan has some discretion; since the bank does not know the realized state and depends on the staff member's report for this information, he[6] might manipulate his report either unilaterally (to embezzle some of the repayment) or under collusion with borrower(s). In this paper, we study the optimal lending contract and the optimal supervisory contract when a staff member (called a supervisor) can embezzle repayments by misrepresenting the realized state; although we focus on the embezzlement, our model can be used to deal with collusion with minor modifications and our insight is applicable to collusion issue as well (see section 5).[7] We first study the least expensive mix of incentive pay and auditing that would induce the supervisor to behave well for a given lending contract. Building on the optimal supervisory contract, we maximize the borrowers' payoff and identify the condition determining when the optimal repayment is state-contingent or not and analyze how the mode of liability (joint versus individual liability) affects the condition.

Given that embezzlement and corruption are very frequent in most organizations in underdeveloped countries[8], incentive schemes in microfinance institutions should be designed to reduce the scope for such misconduct of staff. For instance, according to an employee of the Grameen bank,

[4]For instance, the Bank for Agriculture and Agricultural Cooperatives (BAAC) in Thailand uses state-contingent repayments in that a nonrepayment can be rescheduled if it is due to *force majeur*, but is penalized otherwise. In the former case, depending on the nature of the adverse shock, borrowers may get partial relief of principal (Townsend and Yaron, 2001, p.36).

[5]Bazoberry (2001, p.13) describes six unauthorized activities that staff members of some microfinance organizations in Bolivia engaged in, such as creation of "ghost" loans to hide the fact that goals are not met, utilization of inactive saving accounts to pay for outstanding debts, etc. Bond and Rai (2002) give several examples of supervisor frauds around the world.

[6]We use she for the lender, and he for a staff member (i.e., a supervisor) and for a borrower.

[7]The reason why we focus on embezzlement instead of collusion is that borrowers' illiteracy together with the fact that staff form a bottleneck in the communication channel between the lender and borrowers enlarge the scope for staff's unilateral abuse of discretion.

[8]For instance, Angolan officials are accused of embezzling 10 percent of the country's GDP during the last five years. (Fantaye, 2004, p.173).

“Every other organization in Bangladesh is full of corruption. Here, you can be an honest person and it’s possible to remain so.” (Bornstein 1997, p. 167)

The above quotation illustrates well the focus of our paper. The first sentence suggests that Yunus must have been particularly concerned about the staff’s agency problem when he designed the architecture of his bank. The second sentence suggests that Yunus successfully designed the architecture to contain the agency problem.

In particular, in the case of microfinancing programs in poor countries, most borrowers are illiterate[9] and means of transportation are primitive; therefore, they get informed about the loan conditions exclusively through the bank staff member who visits their villages to collect repayments. In other words, a staff member forms a bottleneck in the communication channel between his borrowers and the lender. This creates significant scope for the staff’s misconduct and for embezzlement, as is documented by Bornstein (1996)[10] and Mknelly and Kevane (2002). For instance, according to Mknelly and Kevane (2002), embezzlement occurs because illiterate borrowers cannot maintain their account books.[11]

We consider a simple model of hierarchy: there are a lender, a supervisor and borrower(s). The lender maximizes the borrower(s)’ payoff subject to her own break-even constraint. The supervisor must check the success or failure of the project undertaken by each borrower and collect the repayments. We assume that the supervisor can discover the return realization by visiting the borrowers’ village and can enforce repayments.[12] In other words, we assume that a borrower is able to repay the loan even when the project fails: although this is a departure from the standard assumption in the microfinance literature, we stress that it is consistent with the 98% repayment rate in the Grameen bank. However, we suppose that a borrower’s marginal cost of paying back one unit of money is higher when his project fails than when it succeeds. Therefore, if the supervisor were honest, the lender would provide full insurance to the borrower by recovering all financing cost only through a repayment upon success.

[9]According to Yunus (1998, p. 24), ”We have worked with ... women who cannot read and write.....”

[10]Bornstein writes about embezzlement in the early period of the Grameen bank (pp. 169-174).

[11]For this reason, staff often take the responsibility of maintaining account books. However, when the authors investigated a staff member’s accounting book, they found that ”... the account book had no entry for the group fund. There was no carryover from one cycle to the next, and no entries in the log of deposits and withdrawals”(p. 2028).

[12]The enforcement can be done through (i) pecuniary punishment such as denial of future loan (chapter 5.2 of Armendáriz de Aghion and Morduch, 2005) and seizure of income or assets (Besley and Coate, 1995) and (ii) non-pecuniary punishment of being ‘hassled’ by the bank (Besley and Coate, 1995).

The lender can design either a state non-contingent lending contract in which a borrower makes the same payment regardless of the realization of his project return, or a state-contingent contract in which the repayment depends on the realization of the return. If the lender uses the latter, the supervisor has some discretion in that when a project succeeded, he can report that the project failed and embezzle the difference between the payment upon success and the payment upon failure. The lender can use incentive pay and/or audit to induce the supervisor to behave well, but the supervisor is protected by limited liability: in case auditing reveals that embezzlement occurred, the supervisor returns the stolen money and is not paid any wage, but is not liable for a further fine.[13] The lender's objective is to design optimally both the contract for the supervisor and the contract for the borrower.

We first consider the simple case of a single borrower (in section 3) in order to identify the main trade-off between the optimal state-contingent lending contract and the optimal state non-contingent one. We start by characterizing the optimal supervisory contract for any given lending contract, and find that there is a threshold discretion level such that it is optimal to use incentive pay only (respectively, audit only) when the amount of discretion is smaller (respectively, larger) than the threshold. Optimization with respect to the amount of discretion reveals that zero discretion (respectively, maximum discretion) is optimal when the amount of discretion is smaller (respectively, larger) than the threshold. Therefore, the optimal lending contract involves either zero or the maximum discretion. In the latter case, the borrower makes a repayment only when his project succeeds, but the lender should conduct an audit when the supervisor reports that the project failed. Hence, the optimal contract is state non-contingent when the audit cost is larger than the borrower's gain from full insurance.

In section 4, we consider the case in which a staff member supervises multiple (=two) borrowers and each borrower undertakes a project. After characterizing the optimal supervisory and lending contracts under given mode of liability (joint liability or individual liability), we study how the mode of liability affects the main trade-off. Although the analysis is involved even for the case of two borrowers, it turns out that the optimal contracts are quite intuitive. Note first that in the benchmark of an honest supervisor, the first-best outcome with full insurance can be achieved regardless of the liability mode. We find that the optimal contracts under individual liability are similar to those in the single borrower case. In the case of joint liability, the contracts provide some insurance

[13] Note that with unlimited liability, the agency problem can be solved at almost zero cost by imposing a fine large enough when embezzlement is discovered.

by requiring that only the successful borrower pays when only one borrower's project succeeds; furthermore, the aggregate repayment is constant when at least one borrower's project succeeds.[14]

We find that joint liability is strictly preferred to individual liability for two reasons. When the optimal contract is state non-contingent, joint liability provides partial insurance while, when it is state-contingent, joint liability reduces the audit cost since audit occurs only when both projects fail. Therefore, joint liability can be regarded as an optimal response to the embezzlement problem. However, we show that the outcome of the optimal non-contingent contract under joint liability can be achieved also under individual liability, if the borrowers can sign a side-contract for mutual insurance before return realization.[15] Finally, we find that a state-contingent contract is more likely to be optimal under joint liability than under individual liability.

Agency problems in hierarchies have been a major theme of research in economics. For instance, Williamson (1967) and Calvo and Wellisz (1978) study when and how loss of control in a hierarchy limits the size of firm. Our paper is more closely related to the literature on collusion between a supervisor and an agent in mechanism design theory (Tirole 1986, Laffont and Tirole 1991, Kofman and Lawarrée 1993 and Faure-Grimaud, Laffont and Martimort 2003). This literature derives the optimal collusion-proof contract when the supervisor can manipulate the information he reports to the principal about the agent's type in exchange for a bribe. We do not consider collusion but focus on the supervisor's incentive to unilaterally manipulate his report to embezzle payments.

Although our paper is related to the literature on costly state verification (Townsend 1979, Diamond 1984, Gale and Hellwig 1985) since the lender can conduct an audit to verify the realized return, to our knowledge it is the first paper that considers a hierarchy and analyzes the supervisor's incentive to embezzle borrowers' repayments. Another difference is that while the previous literature assumes that a borrower cannot repay the loan in some states of nature and derives the optimality of a state-contingent contract, we assume that the borrower is able to repay the loan even when the project fails and focus

[14] Sadoulet (2002) considers a multiperiod setting in which a borrower obtains a loan in period t only if he has not defaulted on the previous loans. The terms of the loans are exogenously fixed in his setting. He studies providing repayment insurance for borrowers on the basis of their credit records.

[15] Laffont and Rey (2003) find that joint liability is preferred to individual liability when borrowers can sign a side-contract before return realization. In contrast, Rai and Sjöström (2004) find that the benefit of joint liability over individual liability disappears if side-contacting occurs before return realization, but it remains if side-contracting occurs after return realization. None of these papers consider a staff member's incentive.

on the trade-off between a state-contingent repayment and a state non-contingent one.

In the microfinance literature, only a few papers (Conning 1999 and Aubert, Janvry and Sadoulet 2005) consider hierarchy.[16] For instance, Conning (1999) studies both a borrower's incentive to divert funds and a staff member's incentive to monitor the former's misbehavior.[17] However, he does not study the staff's incentive to embezzle repayments since he assumes that the realized return is common knowledge, as many papers on microfinance do.

Section 2 presents the model for the case of a single borrower and section 3 analyzes this case. Section 4 analyzes the case of two borrowers. Section 5 discusses our results and section 6 concludes. All the proofs are gathered in Appendix.

2 Basic model

We consider a hierarchy composed of a lender, a supervisor and a borrower. The borrower borrows one unit of money from the lender and invests it in a project. The lender is risk neutral and designs the contracts to maximize the borrower's payoff under her own break-even constraint. To break even, she needs to recover the opportunity cost of the loan ρ (> 1) plus the wage bill paid to the supervisor and the cost of audit.

Let Y denote the revenue generated by the borrower's investment: with probability $p \in (0, 1)$, the project succeeds and $Y = Y_S > 0$; with probability $1 - p$, it fails and $Y = Y_F = 0$. A lending contract $\{r_S, r_F\}$ specifies a repayment contingent on the state: the borrower should pay r_i when $Y = Y_i$, for $i = S, F$. Without loss of generality, we require r_i to be non-negative.[18]

The lender does not ask for any collateral. In the case of failure, the borrower is still able to generate the amount of cash $r_F > 0$,[19] but in order to do so he must reduce his consumption (or sell his assets, or borrow money from local money lenders charging

[16] See also chapter 10 of Armendáriz de Aghion and Morduch (2005).

[17] Aubert, Janvry and Sadoulet (2005) study the conflict between a not-for-profit microfinance organization and its staff member in terms of the incentive to reach poor borrowers instead of less poor. However, they consider only state-contingent lending contracts.

[18] We obtain the same results even though r_i can be negative, but allowing for this case makes the proofs longer without adding economic insights. In short, even though the constraint $r_F \geq 0$ binds in our analysis, one cannot strictly improve the borrower's payoff by allowing $r_F < 0$, as we explain in remark 1 in section 3.

[19] As we pointed out in the introduction, this contrasts with a standard assumption microfinance literature, but the high repayment rate of Grameen bank supports our assumption.

usurious interest rates[20]). This is costly in the following sense: generating r units of money in state F costs $\psi(r)$ to the borrower, with $\psi(0) = 0$, $\psi'(r) > 1$ and $\psi''(r) \geq 0$ for any $r > 0$; hence, $\psi(r) > r$ for $r > 0$. Let $U(Y_i, r_i)$ denote the borrower's utility in state i. We assume:

$$U(Y_i, r_i) = Y_i - r_i \quad \text{if } Y_i - r_i \geq 0;$$
$$U(Y_i, r_i) = -\psi(r_i - Y_i) \quad \text{if } Y_i - r_i < 0.$$

Thus, given a state $i = S, F$, the borrower's marginal utility of one unit of money is 1 if $Y_i - r_i \geq 0$, but it is larger than 1 if $Y_i - r_i < 0$. This decreasing marginal utility of money makes the borrower risk averse. We say that the borrower is *fully insured* if $r_F = 0$ and $r_S \leq Y_S$; then, he makes a payment to the lender only in state S and the marginal utility from additional money is equal to one regardless of the realized state. When $r_S \leq Y_S$, his expected payoff upon accepting the contract is given by:

$$p(Y_S - r_S) - (1-p)\psi(r_F). \tag{1}$$

The supervisor has the task to observe and report the state (whether $i = S$ or $i = F$) to the bank, and to collect the borrower's repayment r_S or r_F. We assume that the supervisor observes the state by visiting the borrower and can enforce the repayment. Since recollecting the repayment anyway requires him to visit the borrower, we assume that his cost of visiting the borrower is zero for simplicity. A supervisory contract specifies a wage for the supervisor contingent on the state he reports: the wage is w_S if he reports $i = S$ and w_F if he reports $i = F$. Furthermore, we suppose that the supervisor's wage cannot be lower than a certain minimum wage $\underline{w} \geq 0$ (i.e., $\min\{w_S, w_F\} \geq \underline{w}$), which is the supervisor's reservation utility: hence, we can neglect his participation constraint.[21]

We focus on the moral hazard of the supervisor, who can misrepresent the state to the lender. For instance, if $\Delta \equiv r_S - r_F > 0$, by reporting $i = F$ when the true i is S, the supervisor can embezzle Δ. The lender can audit the actual payment made by the borrower at a cost of $k(> 0)$. When cheating is discovered, the lender can recover Δ and refuse to pay any wage to the supervisor. However, we assume that the lender cannot

[20] For instance, Bornstein (1997, p.149) documents the story of a woman who purchased a cow with the money borrowed from the Grameen bank. Since the cow stopped lactating midway through the year, she had to cut down on her family's eating to pay back the weekly installments. Jain and Mansuri (2003) provide evidence of microfinance members borrowing money from local lenders.

[21] It is common to assume that the minimum wage is defined with respect to the reservation utility: Laffont and Martimort (2002, section 4.8.1), Conning (1999). In Conning, $\underline{w} = 0$. In the opposite case without the minimum wage constraint, the principal can achieve the first-best by making the supervisor's wage depend on his report.

impose any further fine on him since the supervisor is protected by limited liability. If the lender can levy a large fine, it is easy to show that she can eliminate embezzlement at almost zero cost: for instance, by conducting an audit with a small probability $\varepsilon(>0)$ and imposing a large fine if embezzlement is discovered, she can force honest behavior at the cost of εk. Hence, we focus on the case of zero fine. A supervisory contract is represented by $\{q_i, w_i\}$ with $i = S, F$, where q_i represents the probability of audit when the supervisor reports i. The supervisor is assumed to be risk-neutral.

A *grand-contract* $\{r_i, w_i, q_i\}_{i=S,F}$ is composed of a lending contract $\{r_S, r_F\}$ and a supervisory contract $\{w_S, w_F, q_S, q_F\}$. We assume that the lender makes a take-it-or-leave-it offer to both the supervisor and the borrower. In the benchmark of an honest supervisor, $\{r_S = (\rho + \underline{w})/p, r_F = 0, q_F = q_S = 0, w_S = w_F = \underline{w}\}$ is the optimal grand-contract provided that $pY_S \geq \rho + \underline{w}$. In particular, it provides the borrower with full insurance since no repayment is required when the project fails (i.e. $r_F = 0$).

For expositional facility, we define two kinds of lending contracts, depending on whether or not it is state-contingent, and two kinds of supervisory contracts, with a carrot and with a stick.

Definition: A lending contract is said to be state non-contingent if $\Delta = 0$. A lending contract is said to be state-contingent (or equivalently, discretionary) if $\Delta \neq 0$. Given a total cost C of financing the project, a contract with $r_S = C/p$ and $r_F = 0$ is called a contract with maximum discretion.

Definition: A supervisory contract with $(w_S - w_F)(r_S - r_F) > 0$ and $q_F = q_S = 0$ is called a supervisory contract with a carrot. A supervisory contract with $w_S = w_F$ and $q_F > 0$ or $q_S > 0$ is called a supervisory contract with a stick.

In a state non-contingent lending contract, the supervisor has no discretion since the borrower's payment does not depend on the state of nature. By contrast, the supervisor has some discretion when $\Delta \neq 0$; the amount of discretion is given by $|\Delta|$. We show later on (in lemma 1) that we need to consider only lending contracts with $\Delta \geq 0$. Given a total cost C of financing the project, a contract with $r_S = C/p$ and $r_F = 0$ gives the maximum discretion to the supervisor while providing full insurance for the borrower. When $\Delta > 0$, there are two different ways to induce the supervisor not to embezzle Δ: either w_S is sufficiently larger than w_F, or the probability of audit is sufficiently large. In the former case the lender uses a carrot (i.e. the incentive pay), while in the latter case she uses a stick in that if the embezzlement is detected, the supervisor loses his wage and the stolen repayment is recovered by the lender.

In section 3, in which we consider the case of one borrower, we first study the optimal supervisory contract given $\{r_S, r_F\}$ and then derive the optimal lending contract by comparing the optimal state non-contingent contract with the optimal state-contingent contract. Since we are mainly interested in this comparison, we make the following assumption A1, in which $\Delta^{\max}$ denotes the largest solution to the equation

$$\rho + \underline{w} + (1-p)\frac{\Delta}{\Delta + \underline{w}}k = p\Delta \tag{2}$$

The value of $\Delta^{\max}$ is important since the analysis in section 3 shows that the cost of the supervisor's moral hazard is given by $(1-p)\min\{\psi(\rho+\underline{w}) - \rho - \underline{w}, k\frac{\Delta^{\max}}{\Delta^{\max}+\underline{w}}\}$. Because of this reason we make the following assumption.

A1: Y_S is large enough such that the net present value (NPV) of the project (i.e., the borrower's expected utility) is positive at the equilibrium: $pY_S > \rho+\underline{w}+(1-p)\min\{\psi(\rho+\underline{w}) - \rho - \underline{w}, k\frac{\Delta^{\max}}{\Delta^{\max}+\underline{w}}\}$.

For instance, when the supervisor is honest, the project's NPV is positive if and only if $pY_S - \rho - \underline{w} > 0$. A1 means that pY_S is sufficiently larger than $\rho + \underline{w}$ such that the NPV is positive even in the presence of the cost generated by the supervisor's moral hazard. This allows us to neglect the borrower's participation constraint.

3 Single borrower case

In this section, we derive the optimal grand-contract in the case of a single borrower. From the revelation principle, there is no loss of generality in restricting our attention to direct revelation mechanisms[22] that induce the supervisor to report the true state to the lender. Therefore, a grand-contract should satisfy the following incentive constraints to induce the supervisor to report truthfully the state of the world:

$$\begin{array}{lccl} (IC_{SF}) & w_S & \geq & (1-q_F)(r_S - r_F + w_F); \\ (IC_{FS}) & w_F & \geq & (1-q_S)(r_F - r_S + w_S). \end{array}$$

In order to help understand the right hand sides in (IC_{SF}) and (IC_{FS}), we remind that if embezzlement is discovered then the supervisor receives zero as he loses both the embezzled money and his wage.

[22] According to Laffont and Martimort (2002, section 3.6), the revelation principle holds when the principal can commit to audit mechanisms as in our case.

A grand-contract must also satisfy the lender's break-even constraint, given by:

$$(BE) \quad pr_S + (1-p)r_F \geq \rho + pw_S + (1-p)w_F + [pq_S + (1-p)q_F]\,k, \tag{3}$$

where the right hand side represents the total financing cost that the lender needs to recover. The lender's optimization problem, denoted by (L^S),[23] is defined as follows:

$$\max_{\{r_i, w_i, q_i\}_{i=S,F}} \quad p(Y_S - r_S) - (1-p)\psi(r_F)$$

subject to

$$(BE), \quad (IC_{SF}), \quad (IC_{FS}), \quad r_i \geq 0, \quad w_i \geq \underline{w} \quad \text{for } i = S, F. \tag{4}$$

The following useful lemma (i) proves an important property of the borrower's utility function which is repeatedly used in our paper and (ii) shows that we can focus on contracts such that $r_S \geq r_F$.

Lemma 1 *When there is a single borrower,*
(i) Suppose that the lending contracts $\{r_S, r_F\}$ *and* $\{r'_S, r'_F\}$ *have the same expected payment and* $r_F > r'_F \geq 0$. *Then the borrower prefers* $\{r'_S, r'_F\}$ *to* $\{r_S, r_F\}$.
(ii) The optimal lending contract is such that $\Delta \geq 0$.

The intuition for lemma 1(i) is that although the expected payment is the same, the borrower pays less in state F with $\{r'_S, r'_F\}$ than with $\{r_S, r_F\}$; thus, with the former contract he bears a smaller cost of reducing consumption when his project fails. Lemma 1(ii) relies on lemma 1(i) to show that we can restrict our attention to lending contracts with $\Delta \geq 0$. Precisely, it proves that if $r_S < r_F$, then increasing the payment in state S and decreasing the payment in state F without modifying the expected payment relaxes the incentive constraints and increases the borrower's expected utility.

When $\Delta \geq 0$ holds, the supervisor has no incentive to misrepresent the state from F to S in the absence of any incentive pay or audit. Accordingly, we consider the relaxed problem in which (IC_{FS}) is neglected, but later on we prove that (IC_{FS}) is satisfied in the solution of the relaxed problem. We observe that in the relaxed problem it is optimal to set $q_S = 0$ in order to minimize the financing cost. For notational simplicity, in the rest of this section we let $q \equiv q_F$.

We now solve (L^S) in two steps. First, given a lending contract $\{r_S, r_F\}$, we find the optimal supervisory contract $\{w_S, w_F, q\}$ that minimizes the cost of financing the project,

[23] The large L means the "lender" and the superscript S means a "single" borrower.

$\rho + pw_S + (1-p)w_F + (1-p)qk$, subject to (IC_{SF}), $w_S \geq \underline{w}$ and $w_F \geq \underline{w}$; it is important to notice that $\{r_S, r_F\}$ affects (IC_{SF}) only through $\Delta(\geq 0)$. Second, we maximize the borrower's expected utility with respect to (r_S, r_F) subject to (BE).[24]

3.1 The optimal supervisory contract given Δ

In this subsection we find the optimal supervisory contract given a lending contract (i.e. given $\Delta \geq 0$) by solving the following problem, denoted by (S^S):[25]

$$\min_{w_S, w_F, q} \quad \rho + pw_S + (1-p)w_F + (1-p)qk$$

subject to

$$(IC_{SF}), \quad w_S \geq \underline{w} \quad \text{and} \quad w_F \geq \underline{w}.$$

A first step is given by a simple lemma:

Lemma 2 *When there is a single borrower, the solution to (S^S) for a given $\Delta \geq 0$ is such that*
(i) $w_F = \underline{w}$;
(ii) if $\Delta = 0$, then $w_S = \underline{w}$ and $q = 0$;
(iii) if $\Delta > 0$, then (IC_{SF}) binds and $q \leq \overline{q}(\Delta) \equiv \Delta/(\Delta + \underline{w})$;
(iv) (IC_{FS}) is satisfied.

In what follows we consider the case of $\Delta > 0$. Since (IC_{SF}) binds, we find $w_S = (1-q)(\Delta + \underline{w})$. The objective function after replacing w_S with $(1-q)(\Delta + \underline{w})$ in (S^S) is

$$\rho + p(1-q)(\Delta + \underline{w}) + (1-p)\underline{w} + (1-p)qk$$

We need to minimize this function with respect to q, for $q \in [0, \overline{q}(\Delta)]$. Since the function is linear in q, the optimum is easily found as follows. We first let $\overline{\Delta} \equiv (1-p)k/p - \underline{w}$, and assume that $\overline{\Delta} > 0$.[26] Then, $q = 0$ is optimal if $\Delta \leq \overline{\Delta}$ while $q = \overline{q}(\Delta)$ is optimal if $\Delta > \overline{\Delta}$. The following proposition summarizes these results by characterizing the optimal supervisory contract as a function of $\Delta \geq 0$ and computes the associated financing cost.

[24] This methodology is similar to the one followed in a standard model of moral hazard in which (i) given an effort level to implement, one finds the incentive scheme which minimizes the cost of implementing that effort and (ii) one finds the optimal effort.

[25] The large S means a "supervisory contract".

[26] This assumption is made only in order to simplify the exposition. Our results can easily be extended to cover the case in which $\overline{\Delta} \leq 0$, but without adding significant insights.

Proposition 1 *When there is a single borrower, given a lending contract with* $\Delta \geq 0$*, the optimal supervisory contract is characterized as follows. There exists* $\overline{\Delta} \equiv (1-p)k/p - \underline{w}$ *such that*
(i) (Carrot regime) For $0 \leq \Delta \leq \overline{\Delta}$*,* $q = 0$ *and* $w_S = \underline{w} + \Delta$*,* $w_F = \underline{w}$*; the financing cost is* $\rho + p\Delta + \underline{w}$*;*
(ii) (Stick regime) For $\Delta \geq \overline{\Delta}$*,* $q = \overline{q}(\Delta)$ *and* $w_S = w_F = \underline{w}$*; the financing cost is* $\rho + \underline{w} + (1-p)\overline{q}(\Delta)k$*.*

Proposition 1 says that incentive pay should be offered to the supervisor for small discretion ($\Delta \leq \overline{\Delta}$), while auditing should be used to induce truthful reporting for large discretion ($\Delta \geq \overline{\Delta}$). More formally, we note first that because the total financing cost in the objective function and (IC_{SF}) are linear with respect to q, the optimal q is either zero or $\overline{q}(\Delta)$, where $\overline{q}(\Delta)$ is the minimal q satisfying (IC_{SF}) at the minimum wage $w_S = \underline{w}$. With $q = 0$, the lender must give a carrot ($w_S - w_F$) equal to the amount of discretion (Δ) to satisfy (IC_{SF}); when $q = \overline{q}(\Delta)$, instead, the contract induces truthtelling by using only stick and then it is feasible to set $w_S = \underline{w}$. In order to see which method between the two performs better, we compare the extra financing cost generated by the supervisor's moral hazard with respect to the cost in the absence of moral hazard. Without moral hazard, the financing cost is simply $\rho + \underline{w}$. The extra cost under the carrot contract is $p\Delta$, while the extra cost under the stick contract is $(1-p)\overline{q}(\Delta)k = (1-p)\Delta k/(\Delta + \underline{w})$. The two are equivalent when $\Delta = \overline{\Delta}$, but the former increases faster than the latter as $\Delta \geq \overline{\Delta}$ increases, and this yields the result. For ease of exposition, we say that a supervisory contract belongs to the *stick regime* when $\Delta \geq \overline{\Delta}$, and belongs to the *carrot regime* when $0 \leq \Delta \leq \overline{\Delta}$ (obviously, if $\Delta = 0$ then the lender needs neither carrot nor stick).

3.2 The optimal lending contract

We now find the optimal lending contract by maximizing the borrower's payoff subject to the lender's break-even constraint (BE). Given a lending contract $\{r_S, r_F\}$, proposition 1 identifies the minimum financing cost associated with $\{r_S, r_F\}$, and therefore the right hand side of (BE). Furthermore, since any increase in r_S or r_F reduces the borrower's payoff, (BE) must bind in the optimum. We first find the optimal lending contract conditional on the carrot regime or on the stick regime, and then derive the overall optimal contract.

Consider first the carrot regime, which occurs if $0 \leq \Delta \leq \overline{\Delta}$. From the binding ($BE$), we obtain

$$r_F = \rho + \underline{w}.$$

Since r_F is equal to $\rho + \underline{w}$ regardless of the value of Δ, the objective of (L^S) is given by

$$p(Y_S - \rho - \underline{w} - \Delta) - (1-p)\psi(\rho + \underline{w}). \tag{5}$$

It is clear that $\Delta = 0$ is optimal. The reason is that $\Delta > 0$ increases the financing cost (with respect to $\Delta = 0$) without affecting r_F. Therefore, we have $r_S = r_F = \rho + \underline{w}$.

Consider now the stick regime, which occurs if $\Delta \geq \overline{\Delta}$. Now the binding (BE) is given by

$$r_F = \rho + \underline{w} + (1-p)\overline{q}(\Delta)k - p\Delta (\equiv r_F^{stick}(\Delta)). \tag{6}$$

Since r_F cannot be negative, a value of Δ satisfying $\Delta \geq \overline{\Delta}$ is feasible if and only if $r_F^{stick}(\Delta) \geq 0$. It is possible to verify that $r_F^{stick}(\Delta^{\max}) = 0$, where $\Delta^{\max}$ is defined through (2), and that $r_F^{stick}(\Delta) \geq 0$ if and only if $\Delta \in [\overline{\Delta}, \Delta^{\max}]$.[27] Hence, the lender chooses Δ in this interval to maximize the following objective of (L^S):

$$p\left[Y_S - r_F^{stick}(\Delta) - \Delta\right] - (1-p)\psi[r_F^{stick}(\Delta)]. \tag{7}$$

Let Δ^{*S} denote the maximizer of (7). In order to have an idea of what determines Δ^{*S}, notice from (6) that an increase in Δ has two opposing effects on r_F^{stick}. On the one hand, r_F^{stick} is reduced because of the term $-p\Delta$ which shifts the borrower's payment from state F to state S without changing his expected payment; this increases his payoff by providing insurance (see lemma 1). On the other hand, increasing Δ raises the auditing cost because, for a given $q < 1$, it gives the supervisor a higher incentive to embezzle and a larger q is needed to satisfy (IC_{SF}) with $w_S = \underline{w}$. This increases r_F^{stick} through the term $(1-p)\overline{q}(\Delta)k$ and reduces the borrower's payoff. This trade-off makes it difficult to find the exact value of Δ^{*S}, as it is clear from the first order derivative of (7), which is $-p[\frac{dr_F^{stick}(\Delta)}{d\Delta} + 1] - (1-p)\psi'[r_F^{stick}(\Delta)]\frac{dr_F^{stick}(\Delta)}{d\Delta}$.

The next proposition characterizes the optimal grand contract. After summarizing the general findings in (i), it gives more detailed results by considering specific cases in (ii) and (iii). Since, without further assumptions, it is impossible to derive Δ^{*S} and compare the optimal state non-contingent contract with the optimal state-contingent contract, we consider the specific cases in which ψ is linear or $\underline{w} = 0$.[28]

[27] From $\frac{dr_F^{stick}}{d\Delta} = (1-p)\frac{w}{(\Delta+\underline{w})^2}k - p$ we see that r_F^{stick} is decreasing in Δ for any $\Delta \geq \overline{\Delta}$ if $\overline{\Delta} \geq 0$; r_F^{stick} is first increasing and then decreasing in $\Delta \in [0, \Delta^{\max}]$ if $\overline{\Delta} < 0$.

[28] We abuse a little bit of terminology by using the term "the optimal state contingent contract" in the

Proposition 2 *When there is a single borrower, under A1,*
(i) The optimal grand contract is the best one between the two following contracts:
a. a state non-contingent lending contract $\{r_S = r_F = \rho + \underline{w}\}$ *with* $\{w_S = w_F = \underline{w}, q = 0\}$
b. a lending contract $\{r_S = \Delta^{*S} + r_F, r_F = r_F^{stick}(\Delta^{*S})\}$ *with* $\{w_S = w_F = \underline{w}, q = \overline{q}(\Delta^{*S})\}$,
which is state-contingent if $\Delta^{*S} > 0$.
(ii) When $\psi(r) = \theta r$ *with* $\theta > 1$, *we find that* Δ^{*S} *is either equal to* $\overline{\Delta}$ *or to* $\Delta^{\max}$. *The state non-contingent lending contract is optimal if and only if*

$$k\overline{q}(\Delta^{\max}) \geq \psi(\rho + \underline{w}) - \rho - \underline{w}. \tag{8}$$

Otherwise, the state-contingent contract in (i)b with $\Delta^{*S} = \Delta^{\max}$ *(and* $r_F = 0$*) is optimal.*
(iii) When $\underline{w} = 0$, *we find* $\Delta^{*S} = \Delta^{\max} = [\rho + (1-p)k]/p$ *and* $\overline{q}(\Delta) = 1$. *The state non-contingent contract is optimal if and only if* $k \geq \psi(\rho) - \rho$, *otherwise the state-contingent contract in (i)b with* $r_F = 0$ *and* $q = 1$ *is optimal.*

When ψ is linear, it is quite intuitive that the solution is bang bang: the optimal lending contract is either a state non-contingent contract ($\Delta = 0$) or a contract with maximum discretion ($\Delta = \Delta^{\max}$) and stick. In the former case, the lender needs neither carrot nor stick and therefore the financing costs are equal to $\rho+\underline{w}$, the lowest feasible level. However, $r_F = \rho + \underline{w} > 0$ implies that the borrower is badly insured. The contract with maximum discretion provides instead full insurance for the borrowers because it specifies $r_F = 0$. However, the possibility of embezzlement requires the lender to conduct an audit with probability $\overline{q}(\Delta^{\max}) \in (0, 1)$ whenever the supervisor reports that the project failed; this increases the borrower's expected payment to $\rho + \underline{w} + (1-p)k\overline{q}(\Delta^{\max})$. Therefore, the optimal lending contract is state non-contingent if and only if the expected audit cost $(1-p)k\overline{q}(\Delta^{\max})$ is larger than the expected gain from full insurance $(1-p)[\psi(\rho+\underline{w})-\rho-\underline{w}]$, as is stated by (8).[29]

Another setting in which a clear-cut result can be obtained is the one with $\underline{w} = 0$; then we have $\overline{q}(\Delta) = 1$ for any $\Delta \geq \overline{\Delta}$. In this case, since an increase of Δ in $[\overline{\Delta}, \Delta^{\max}]$ does not increase the audit cost but reduces r_F^{stick} (and increases r_S) without modifying the borrower's expected payment, it is clear that $\Delta^{*S} = \Delta^{\max}$ by lemma 1(i). Then, as we explained above, the trade-off between insurance and audit cost determines the optimum between the state non-contingent contract and the contract with maximum discretion.

following sense. Although the contract is the optimal one among all the state contingent contracts if it is better than the optimal state non-contingent contract, it may not be optimal among state contingent contracts if it is worse than the optimal state non-contingent contract.

[29] Actually, proposition 2(ii) holds as long as (7) is convex in Δ, which occurs even though ψ is not linear but ψ'' is close to 0.

Remark 1 *The characterization of the optimal contract in proposition 2 applies even if r_F can be negative. First, the optimal state non-contingent contract is obviously not affected. Second, concerning the optimal state-contingent contract, setting $r_F < 0$ does not improve insurance provision with respect to $r_F = 0$ but weakly increases the audit cost $k\overline{q}(\Delta)$ since Δ must be larger than $\Delta^{\max}$.*

4 Two-borrower case

Suppose now that there are two borrowers living in the same village. Each borrower finances a project with one unit of money borrowed from the lender and a supervisor monitors both of them. For simplicity, we assume that the revenue of each project is identically and independently distributed, and let $p \in (0, 1)$ denote the probability of success of a project. As in section 3, the supervisor observes costlessly whether each project succeeds or fails. We distinguish two cases depending on whether there is joint liability or individual liability between the two borrowers.

Under individual liability, the lender signs a lending contract with each borrower which takes the same form $\{r_S, r_F\}$ as before, which means that each borrower's repayment depends only on the success or failure of his own project. However, the supervisory contract is different since the supervisor now monitors two borrowers. Let w_n represent the supervisor's wage and q_n the probability of conducting an audit when he reports that n number of projects succeeded, with $n \in \{0, 1, 2\}$. Therefore, a supervisory contract is given by $\{w_n, q_n\}_{n=0,1,2}$ and the minimum wage constraint requires $w_n \geq \underline{w}$ for each n. Since both borrowers live in the same village, the cost of audit does not depend on whether the lender audits the payment of one borrower or those of both borrowers[30] and is equal to $k > 0$.

Introducing joint liability affects the form of the lending contract since a borrower's repayment may depend on the outcome of the project of the other borrower (but notice that the form of the supervisory contract is not affected). Formally, under joint liability the lender signs a lending contract with the group of the two borrowers which takes the form $\{r_{SS}, r_{SF}, r_{FS}, r_{FF}\}$ where, for instance, r_{SF} is the payment that a borrower whose project succeeded has to make when the other borrower's project failed. Without loss of

[30] Since the auditor can easily find out the actual amount paid by a borrower as long as he visits him, the main cost of audit is the cost of visiting the village and the marginal cost of visiting one more borrower in the same village is negligeable. Even if we assume that the cost of auditing two borrowers is $2k$, our main result would qualitatively hold.

generality, we impose $r_{ij} \geq 0$ for $i, j = S, F$.[31]

As usual, a *group lending contract* is defined as a lending contract with joint liability. We have two observations:

Observation 1: The set of lending contracts under individual liability is a strict subset of the set of group lending contracts since the former is equal to the set of group lending contracts that satisfy $r_{SS} = r_{SF} = r_S$ and $r_{FS} = r_{FF} = r_F$.

Observation 2: If the supervisor is honest, there is no need of joint liability to achieve the first-best outcome: it suffices to choose $pr_S = \rho + \frac{w}{2}$ and $r_F = 0$ under individual liability.

We now define a state non-contingent and a state-contingent group lending contract:

Definition: A group lending contract is state non-contingent if $2r_{SS} = r_{SF} + r_{FS} = 2r_{FF}$; otherwise, it is state contingent (or equivalently, discretionary).

We introduce $\Delta_1 \equiv r_{SF} + r_{FS} - 2r_{FF}$ and $\Delta_2 \equiv 2r_{SS} - r_{SF} - r_{FS}$. For the time being, we conduct our analysis by focusing on the case with $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$. At the end of section 4.2, in lemma 8, we show that the optimal contract satisfies $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$.

In this section we make the following assumption A1', where $\Delta_1^{\max}$ denotes the largest solution of

$$\rho + \frac{1}{2}\underline{w} + \frac{(1-p)^2 k}{2} \frac{\Delta_1}{\Delta_1 + \underline{w}} = \frac{p(2-p)}{2}\Delta_1 \tag{9}$$

A1': We assume (i) $pY_S > \rho + \frac{1}{2}\underline{w} + (1-p)\min\left\{\psi(\rho + \frac{1}{2}\underline{w}) - \rho - \frac{1}{2}\underline{w}, \frac{1+p}{2}k\right\}$ and (ii) $Y_S \geq \Delta_1^{\max}$.

Both A1'(i) and A1'(ii) require that Y_S is sufficiently large. Precisely, A1'(i) is similar to A1 and implies that the NPV of the project is positive at the equilibrium. A1'(ii) simplifies our analysis of the optimal lending contract under joint liability in that $r_{FS} = 0$ becomes optimal; without A1'(ii), the characterization of the optimal lending contract under joint liability is technically more demanding without generating new insights.

[31] As in the case of the single borrower, we obtain the same results even though r_{ij} can be negative, but considering this possibility makes the proofs much longer. Even though the constraint $r_{FF} \geq 0$ binds in our analysis, one cannot strictly improve the borrowers' payoff by allowing for $r_{FF} < 0$ for reasons similar to those explained in remark 1 in section 3.

Consider the case of joint liability. By the revelation principle, there is no loss of generality in restricting our attention to direct and truthful revelation mechanisms. Therefore, the following incentive constraints should be satisfied:

$$(\mathrm{IC}_{n\widehat{n}}) \quad w_n \geq (1-q_{\widehat{n}})\left[R(n)-R(\widehat{n})+w_{\widehat{n}}\right] \quad \text{for} \quad (n,\widehat{n}) \in \{0,1,2\}^2, \tag{10}$$

where $R(2) \equiv 2r_{SS}$, $R(1) \equiv r_{SF}+r_{FS}$, $R(0) \equiv 2r_{FF}$. The lender's break-even constraint is now

$$\begin{aligned} &(BE) \quad 2p^2 r_{SS} + 2p(1-p)\left(r_{SF}+r_{FS}\right) + 2(1-p)^2 r_{FF} \\ \geq \ & 2\rho + p^2(w_2+kq_2) + 2p(1-p)(w_1+kq_1) + (1-p)^2(w_0+kq_0). \end{aligned}$$

The lender's program under joint liability, denoted by $\left(L^J\right)$, is defined as follows:

$$\max p^2\left(2Y_S - 2r_{SS}\right) + 2p(1-p)\left(Y_S - r_{SF} - \psi(r_{FS})\right) - (1-p)^2 2\psi(r_{FF})$$
$$\text{with respect to } r_{SS}, r_{SF}, r_{FS}, r_{FF}, \{w_n, q_n\}_{n=0,1,2}$$

$$\text{subject to}$$
$$(BE), \quad (10), \quad r_{ij} \ \geq \ 0 \text{ for } i,j=S,F, \quad w_n \geq \underline{w} \text{ for } n=0,1,2.$$

Note that in the objective function in (L^J) we assume $r_{SF} \leq Y_S$ and $r_{SS} \leq Y_S$, which in the proof of proposition 4 we verify to be satisfied under A1'.

The program with individual liability, denoted by $\left(L^I\right)$, is defined as $\left(L^J\right)$ except that r_S replaces r_{SS} and r_{SF} and r_F replaces r_{FF} and r_{FS}; therefore, we have $\Delta_1 = \Delta_2$. Notice that under individual liability the proof of lemma 1(ii) applies, and thus we consider only lending contracts with $r_S - r_F \geq 0$.

Our next lemma presents one important effect of joint liability:

Lemma 3 *When there are two borrowers, under joint liability, it is optimal to choose* $r_{FS} = 0$ *and* $r_{SF} = R(1)$ *if* Y_S *is sufficiently large.*

Lemma 3 says that when only one borrower is successful, it is optimal that the unsuccessful borrower pays nothing ($r_{FS} = 0$) and hence $r_{SF} = R(1)$. The reason is that letting the unsuccessful borrower pay a positive amount requires him to reduce his consumption, which is more costly for borrowers than asking the successful borrower to pay for both. Therefore, joint liability provides borrowers with insurance when only one project is successful. Clearly, this approach is viable only if the successful borrower has enough money to pay $R(1)$; we show later on that $R(1) \leq Y_S$ holds in the optimal contract under A1'.

In $\left(L^J\right)$, with $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$, it is easy to see that the supervisor has no incentive to report a state $\widehat{n}$ larger than the true state n in the absence of any incentive pay or audit. Therefore, we consider a relaxed problem $\left(Lr^J\right)$ in which the upward incentive constraints $(\text{IC}_{02}), (\text{IC}_{01}), (\text{IC}_{12})$ are neglected and $(w_0, w_1, w_2, q_0, q_1, q_2)$ need to satisfy only (11)-(12):

$$\begin{cases} (\text{IC}_{21}) \quad w_2 \geq (1-q_1)(\Delta_2 + w_1); \\ (\text{IC}_{20}) \quad w_2 \geq (1-q_0)(\Delta_2 + \Delta_1 + w_0); \\ (\text{IC}_{10}) \quad w_1 \geq (1-q_0)(\Delta_1 + w_0); \end{cases} \tag{11}$$

$$(LL) \quad w_n \geq \underline{w} \quad \text{for} \quad n = 0, 1, 2. \tag{12}$$

The next lemma establishes some straightforward properties of (Lr^J) and, more importantly, shows that the solution to $\left(Lr^J\right)$ satisfies (IC_{02}), (IC_{01}), (IC_{12}) even though we do not know the solution yet. Therefore, the solution to $\left(Lr^J\right)$ is the optimal grand contract and for this reason, with some abuse of notation, we use $\left(L^J\right)$ to denote the relaxed problem.

Lemma 4 *The solution to the relaxed problem*
(i) is such that $q_2 = 0$ *and* $w_0 = \underline{w}$*;*
(ii) satisfies (IC_{02})*,* (IC_{01})*,* (IC_{12}) *and therefore it is just the optimal grand contract.*

As in the previous section, we perform our analysis in two steps: we first find the optimal supervisory contract given a lending contract, and then find the optimal lending contract.

4.1 The optimal supervisory contract

In this subsection, we find the optimal supervisory contract, given a lending contract $\{r_{SS}, r_{SF}, r_{FS}, r_{FF}\}$ satisfying $(\Delta_1, \Delta_2) \geq (0, 0)$, by minimizing the right hand side of (BE) with respect to (w_2, w_1, q_1, q_0) subject to (11)-(12). Let $\left(S^J\right)$ denote this program:

$$\min_{w_2, w_1, q_1, q_0} 2\rho + p^2 w_2 + 2p(1-p)(w_1 + kq_1) + (1-p)^2(\underline{w} + kq_0) \tag{13}$$

$$\text{subject to } (11) - (12)$$

We use $C^J(\Delta_1, \Delta_2)$ to represent the value of the objective function in (13) at the optimal supervisory contract.

Solving (S^J) is not straightforward because the incentive constraints are not linear in (w_2, w_1, q_1, q_0), although they are linear in each single variable. This leads to minimizing a function which is neither concave nor convex, over a non-convex feasible set. We first prove that there exist three regimes for the binding incentive constraints in the solution to (S^J):

Lemma 5 *When there are two borrowers, under joint liability, the optimal supervisory contract given $(\Delta_1, \Delta_2) \geq (0,0)$ is such that the binding incentive constraints are either (IC_{21}) and (IC_{20}), or (IC_{10}) and (IC_{21}), or all the three constraints in (11)*

The next lemma shows that by considering the three regimes, five different supervisory contracts may be optimal for different parameter values. The contracts are introduced below and are denoted by α, β, γ, δ, η; $C_h^J(\Delta_1, \Delta_2)$ represents the value of the objective function in (13) under contract $h = \alpha, \beta, \gamma, \delta, \eta$.

- α : $(q_0, q_1) = (0,0)$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w} + \Delta_1, \underline{w} + \Delta_1 + \Delta_2)$, $C_\alpha^J(\Delta_1, \Delta_2) = 2\rho + \underline{w} + p(2-p)\Delta_1 + p^2\Delta_2$;

- β : $(q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + \underline{w}}, 0)$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w}, \underline{w} + \Delta_2)$, $C_\beta^J(\Delta_1, \Delta_2) = 2\rho + \underline{w} + (1-p)^2 k \frac{\Delta_1}{\Delta_1 + \underline{w}} + p^2\Delta_2$;

- γ : $(q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + \underline{w}}, \frac{\Delta_2\Delta_1}{(\Delta_2 + \underline{w})(\Delta_1 + \underline{w})})$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w}, \underline{w} + \underline{w}\frac{\Delta_2}{\Delta_1 + \underline{w}})$, $C_\gamma^J(\Delta_1, \Delta_2) = 2\rho + \underline{w} + p^2\underline{w}\frac{\Delta_2}{\Delta_1 + \underline{w}} + 2p(1-p)k\frac{\Delta_2\Delta_1}{(\Delta_2 + \underline{w})(\Delta_1 + \underline{w})} + (1-p)^2 k \frac{\Delta_1}{\Delta_1 + \underline{w}}$;

- δ :[32] $q_0 = q_0^* \equiv 1 + \frac{\Delta_2}{\Delta_1 + \underline{w}} - \frac{1}{\Delta_1 + \underline{w}}\sqrt{\frac{2p(1-p)k\Delta_2(\Delta_2 + \Delta_1 + \underline{w})}{p(p\Delta_2 + (2-p)(\Delta_1 + \underline{w})) - (1-p)^2 k}}$, $q_1 = f(q_0^*)$ with $f(q_0) \equiv \frac{q_0\Delta_2}{\Delta_2 + (1-q_0)(\Delta_1 + \underline{w})}$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w} + (1-q_0^*)(\Delta_1 + \underline{w}), \underline{w} + (1-q_0^*)(\Delta_1 + \Delta_2 + \underline{w}))$, $C_\delta^J(\Delta_1, \Delta_2) = 2\rho + (1-q_0^*)[p^2\Delta_2 + (2-p)p(\Delta_1 + \underline{w})] + 2p(1-p)kf(q_0^*) + (1-p)^2(kq_0^* + \underline{w})$.

- η : $(q_0, q_1) = (\frac{\Delta_2 + \Delta_1}{\Delta_2 + \Delta_1 + \underline{w}}, \frac{\Delta_2}{\Delta_2 + \underline{w}})$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w}, \underline{w})$, $C_\eta^J(\Delta_1, \Delta_2) = 2\rho + \underline{w} + 2p(1-p)k\frac{\Delta_2}{\Delta_2 + \underline{w}} + (1-p)^2 k \frac{\Delta_2 + \Delta_1}{\Delta_2 + \Delta_1 + \underline{w}}$.

In order to give an interpretation of each different supervisory contract, we consider for the moment the simple case of $\underline{w} = 0$; then, contract η is equivalent to γ. Contract α specifies no audit but intensively uses carrots since $w_1 - w_0 = \Delta_1$ and $w_2 - w_1 = \Delta_2$. Contract γ, by contrast, gives no incentive pay but intensively uses sticks since the lender conducts an audit with probability one whenever the supervisor reports that at least one project failed. Contract β uses either carrot or stick depending on the supervisor's

[32] This contract is defined if and only if $2p(1-p)\frac{\Delta_2}{\Delta_2 + \Delta_1 + \underline{w}}k < p(p\Delta_2 + (2-p)(\Delta_1 + \underline{w})) - (1-p)^2 k < 2p(1-p)\frac{\Delta_2(\Delta_2 + \Delta_1 + \underline{w})}{(\Delta_2 + \underline{w})^2}k$; these conditions are equivalent to $q_0^* \in (0, \frac{\Delta_1}{\Delta_1 + \underline{w}})$.

report in the following sense. When the supervisor reports $n = 0$, the lender conducts an audit with probability one ($q_0 = 1$) while the supervisor receives a carrot $w_2 - w_1 = \Delta_2$ when he reports $n = 2$. Finally, contract δ mixes carrots and sticks not over different states of nature as β, but within the same state of nature; since $\Delta_1 > w_1 - w_0 > 0$ and $\Delta_2 > w_2 - w_1 > 0$, there is a positive probability of audit both if the supervisor reports $n = 1$ and if he reports $n = 0$.

Lemma 6 *When there are two borrowers, under joint liability,*
(i) Suppose that $\Delta_1 > 0$ and $\Delta_2 > 0$.

a. If only (IC_{21}) and (IC_{20}) bind in the optimum of (S^J), then η is the optimal supervisory contract.

b. If only (IC_{10}) and (IC_{21}) bind in the optimum of (S^J), then β is the optimal supervisory contract.

c. If all the three constraints in (11) bind in the optimum of (S^J), then the optimal supervisory contract belongs to $\{\alpha, \delta, \gamma\}$.
(ii) If $\Delta_1 = 0$ and/or $\Delta_2 = 0$, then the optimal supervisory contract belongs to $\{\alpha, \gamma, \eta\}$.

A consequence of lemma 6 is the following proposition, which characterizes the optimal supervisory contract for any given lending contract with $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$. The proposition includes the characterization of the optimal supervisory contract under individual liability as a special case with $\Delta_1 = \Delta_2 \geq 0$.

Proposition 3 *When there are two borrowers, under joint liability, the optimal supervisory contract is the lowest cost contract among $\alpha, \beta, \gamma, \delta, \eta$. Hence, $C^J(\Delta_1, \Delta_2) \equiv \min\left\{C^J_\alpha(\Delta_1, \Delta_2), C^J_\beta(\Delta_1, \Delta_2), C^J_\gamma(\Delta_1, \Delta_2), C^J_\delta(\Delta_1, \Delta_2), C^J_\eta(\Delta_1, \Delta_2)\right\}$.*

4.2 The optimal lending contract under joint liability

In this subsection we find the optimal lending contract under joint liability by solving $\left(L^J\right)$. When $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$, the next lemma says that we can restrict our attention to the set of lending contracts satisfying $\Delta_2 = 0$.

Lemma 7 *If Y_S is sufficiently large, then the best contract within the set of lending contracts satisfying $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ is such that $\Delta_2 = 0$. Thus, the optimal supervisory contract is either α or γ, with α such that $(q_0, q_1) = (0, 0)$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w} + \Delta_1, \underline{w} + \Delta_1)$, and γ such that $(q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + \underline{w}}, 0)$, $(w_0, w_1, w_2) = (\underline{w}, \underline{w}, \underline{w})$.*

The proof of the lemma basically shows that when $\Delta_1 \geq 0$ and $\Delta_2 > 0$, reducing Δ_2 and increasing Δ_1 while keeping the borrowers' expected payment constant does not hurt borrowers but can relax the incentive constraints in (11).[33] The reason is that $\Delta_2 > 0$ only generates more opportunities for embezzlement for the supervisor, without providing any insurance gains to the borrowers, given Lemma 3. Since $\Delta_2 = 0$ implies that (i) δ is not defined (see footnote 32); (ii) β and η are equivalent to γ, it follows that the optimal supervisory contract belongs to $\{\alpha, \gamma\}$. In particular, $\Delta_2 = 0$ implies that $w_1 = w_2$ and that no auditing occurs in states SF and FS. Thus, the lender has to choose only between a discretionary contract with $2r_{SS} = r_{FS} + r_{FS} > 2r_{FF}$ (and incentive pay or audit), and a state non-contingent contract with $2r_{SS} = r_{FS} + r_{FS} = 2r_{FF}$ and no audit nor incentive pay. His problem is therefore similar to the one studied in section 3.2

In order to solve $\left(L^J\right)$, we first determine $C^J(\Delta_1, 0)$ by comparing $C^J_\alpha(\Delta_1, 0)$ with $C^J_\gamma(\Delta_1, 0)$. For this purpose, we introduce $\bar{\Delta}_1 \equiv \frac{(1-p)^2}{p(2-p)}k - \underline{w}$ and assume, only to simplify the exposition, that $\bar{\Delta}_1 > 0$.[34] Then, it turns out that for $\Delta_1 \in [0, \bar{\Delta}_1]$, contract α is optimal and therefore we are in the carrot regime. By contrast, if $\Delta_1 > \bar{\Delta}_1$, contract γ is optimal and we are in the stick regime.

We now derive the optimal lending contract while assuming that Y_S is large enough. By using $r_{SF} = 2r_{SS} = \Delta_1 + 2r_{FF}$ ($r_{FS} = 0$ by lemma 3 and $\Delta_2 = 0$ by lemma 7) we can write the lender's program as follows:

$$\max_{\Delta_1 \geq 0, r_{FF} \geq 0} 2pY_S - p(2-p)(2r_{FF} + \Delta_1) - (1-p)^2 2\psi(r_{FF}) \tag{14}$$

subject to the break-even constraint, which binds in the optimum in the case of two borrowers as well.

$$(BE) \quad 2r_{FF} + p(2-p)\Delta_1 = C^J(\Delta_1, 0). \tag{15}$$

Consider first the carrot regime (i.e., $\Delta_1 \in [0, \bar{\Delta}_1]$). Then, (15) is equivalent to $2r_{FF} = 2\rho + \underline{w}$. Since increasing Δ_1 ends up only increasing the financing cost without improving insurance provision, it is optimal to choose $\Delta_1 = 0$ in the carrot regime as in the case of a single borrower.

Consider now the stick regime (i.e., $\Delta_1 \geq \bar{\Delta}_1$). Then (15) is equivalent to

$$r_{FF} = \rho + \frac{\underline{w}}{2} + \frac{(1-p)^2 k}{2}\bar{q}(\Delta_1) - \frac{p(2-p)}{2}\Delta_1 (\equiv r^{stick}_{FF}(\Delta_1)). \tag{16}$$

[33] As in lemma 3, this argument assumes that Y_S is larger than $R(1)$. We verify in the proof of proposition 4 that this condition is satisfied.

[34] Note that $\bar{\Delta}_1$ is similar to $\bar{\Delta}$ defined in section 3.1 in that k is multiplied by the probability of the state in which audit may occur in contract γ, given $\Delta_2 = 0$, and is divided by the probability of the states in which there is no audit.

It is simple to see that $r_{FF}^{stick}(\Delta_1^{\max}) = 0$ ($\Delta_1^{\max}$ has been introduced just before assumption A1'), and arguing as we have done for the case of a single borrower we find that $r_{FF}^{stick}(\Delta_1) \geq 0$ if and only if $\Delta_1 \leq \Delta_1^{\max}$. Hence, the lender maximizes with respect to $\Delta_1 \in [\bar{\Delta}_1, \Delta_1^{\max}]$ the following objective:

$$2pY_S - p(2-p)[2r_{FF}^{stick}(\Delta_1) + \Delta_1] - (1-p)^2 2\psi[r_{FF}^{stick}(\Delta_1)]. \tag{17}$$

This maximization problem is similar to the one in section 3.2. Let $\Delta_1^{*\gamma}$ denote the maximizer of (17) in $[\bar{\Delta}_1, \Delta_1^{\max}]$. Proposition 4 gives the general result and some clear-cut results under specific assumptions on ψ or $\underline{w}$, as proposition 2 in section 3.2.

Proposition 4 *When there are two borrowers, under A1' and joint liability,*
(i) the optimal grand contract is the best between the two following contracts:

a. A state non-contingent lending contract $\{2r_{SS} = 2r_{FF} = r_{SF} = 2\rho + \underline{w}, r_{FS} = 0\}$ *with* $\{q_0 = q_1 = q_2 = 0,\ w_0 = w_1 = w_2 = \underline{w}\}$ *(the supervisory contract is* α*).*

b. A lending contract $\{2r_{SS} = r_{SF} = \Delta_1^{*\gamma} + 2r_{FF}, r_{FS} = 0, r_{FF} = r_{FF}^{stick}(\Delta_1^{*\gamma})\}$ *with* $\{q_0 = \bar{q}(\Delta_1^{*\gamma}),\ q_1 = q_2 = 0,\ w_0 = w_1 = w_2 = \underline{w}\}$ *(the supervisory contract is* γ*), which is state-contingent if* $\Delta_1^{*\gamma} > 0$*.*

(ii) When ψ *is linear, we find that* $\Delta_1^{*\gamma}$ *is either equal to* $\bar{\Delta}_1$ *or to* $\Delta_1^{\max}$*. The state non-contingent contract is optimal if and only if* $k\bar{q}(\Delta_1^{\max}) \geq 2[\psi(\rho + \frac{1}{2}\underline{w}) - \rho - \frac{1}{2}\underline{w}]$*. Otherwise, the state-contingent contract in (i)b with* $\Delta_1^{*\gamma} = \Delta_1^{\max}$ *(and* $r_{FF} = 0$*) is optimal.*
(iii) When $\underline{w} = 0$*, we obtain* $\Delta_1^{*\gamma} = \Delta_1^{\max} = \frac{2\rho + (1-p)^2 k}{p(2-p)}$ *and the state non-contingent contract is optimal if and only if* $k \geq 2(\psi(\rho) - \rho)$*. Otherwise, the state-contingent contract in (i)b with* $q_0 = 1$ *and* $r_{FF} = 0$ *is optimal.*

As in the case of a single borrower, we can determine the exact value of $\Delta_1^{*\gamma}$ under the assumption that ψ is linear or $\underline{w} = 0$. In these cases the optimal contract is either state non-contingent or is the contract with maximum discretion in terms of Δ_1 (while $\Delta_2 = 0$ from lemma 7). The former provides insurance only when at least one project succeeds, while the latter provides insurance for all states. However, the latter involves an audit cost; the audit occurs only when both projects fail (and with probability equal to one if $\underline{w} = 0$). Therefore, the state-contingent contract is optimal when the audit cost $(1-p)^2 k\bar{q}(\Delta_1^{*\gamma})$ is larger than the gain from full insurance for both borrowers, $(1-p)^2 2[\psi(\rho + \frac{1}{2}\underline{w}) - \rho - \frac{1}{2}\underline{w}]$, as proposition 4(ii)-(iii) states.

Although the statement of proposition 4 is about the optimal grand contract, the proof of proposition 4 considers only lending contracts such that $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$.

The proposition is correct, nevertheless, since the following lemma shows that there is no loss of generality in restricting attention to contracts with $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$.[35]

Lemma 8 *When there are two borrowers, under joint liability and A1', the optimal lending contract satisfies* $\Delta_1 \geq 0$ *and* $\Delta_2 \geq 0$.

4.3 Individual liability

In the case of individual liability, we have $r_{SS} = r_{SF} = r_S$ and $r_{FS} = r_{FF} = r_F$, which implies $\Delta_1 = \Delta_2 \equiv \Delta$. We remind that in the case of joint liability, given $r_{FF} \geq 0$ and $\Delta_1 \geq 0$, it is possible to set $r_{FS} = 0$ and $r_{SF} = R(1) = 2r_{FF} + \Delta_1$ (if $2r_{FF} + \Delta_1 \leq Y_S$) and thereby to provide insurance for an unsuccessful borrower when the other borrower is successful (lemma 3). Under individual liability, by contrast, $r_{FS} = 0$ implies $r_{FF} = 0$ and therefore, for instance, the optimal state non-contingent contract under joint liability described by Proposition 4(i)a is not feasible. Notice also that while it is always optimal to set $\Delta_2 = 0$ under joint liability (lemma 7), in the setting of individual liability no similar result holds because $\Delta_2 = 0$ implies $\Delta_1 = 0$.

The optimal supervisory contract under individual liability is a special case of the optimal supervisory one under joint liability characterized in Proposition 3, after replacing Δ_1 and Δ_2 with Δ. Therefore, for $h = \alpha, \beta, \gamma, \delta, \eta$, let $C_h^I(\Delta) \equiv C_h^J(\Delta, \Delta)$ denote the cost of the supervisory contract h, given Δ. The first part of the next lemma however shows that we can neglect contract δ (since $\Delta_1 = \Delta_2$) and hence we define $C^I(\Delta) \equiv \min\{C_\alpha^I(\Delta), C_\beta^I(\Delta), C_\gamma^I(\Delta), C_\eta^I(\Delta)\}$ as the minimum financing cost (derived from the optimal supervisory) contract given Δ. The second part of the lemma proves a useful property of C^I.

Lemma 9 *(i) When there are two borrowers, under individual liability, contract* δ *is never an optimal supervisory contract;*
(ii) C^I *is concave in* Δ.

Arguing as when dealing with (L^J), we can write the lender's program under individual liability, denoted by (L^I), as follows:

$$\begin{cases} \max_{\Delta \geq 0, r_F \geq 0} \quad 2pY_S - 2p(r_F + \Delta) - 2(1-p)\psi(r_F) \\ \quad \text{subject to} \quad (BE) \quad 2r_F + 2p\Delta = C^I(\Delta); \end{cases} \tag{18}$$

[35] The lemma is stated at this point because its proof is easier to read with the knowledge of the analysis for the case with $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$.

We can express (18) by using (BE) to obtain $r_F(\Delta) = \frac{C^I(\Delta)}{2} - p\Delta$, which we insert into the objective function. Furthermore, Lemma 9(ii) implies that there exists a (unique) value of Δ, denoted by $\Delta_I^{\max}$, such that $r_F(\Delta) \geq 0$ if and only if $\Delta \in [0, \Delta_I^{\max}]$. Thus, we need to solve the program $\max_{\Delta\in[0,\Delta_I^{\max}]} 2pY_S - 2p(\frac{C^I(\Delta)}{2} - p\Delta + \Delta) - 2(1-p)\psi(\frac{C^I(\Delta)}{2} - p\Delta)$. In order to tackle this program it would be useful to know, for given $\Delta \in [0, \Delta_I^{\max}]$, which contract in $\{\alpha, \beta, \gamma, \eta\}$ is the optimal supervisory contract. Since comparing the four contracts is cumbersome, we consider the special case in which ψ is linear. Under this assumption the objective function is convex and therefore the optimal contract is either state non-contingent (i.e. $\Delta = 0$) or has the maximum discretion, in the sense that $r_F = 0$ and $r_S = \Delta_I^{\max}$. Unfortunately, without further restrictions it is hard to find the optimal supervisory contract at $\Delta = \Delta_I^{\max}$, and hence to determine the value of $\Delta_I^{\max}$ which is necessary to compare the optimal state-contingent contract with the state non-contingent contract. The next proposition provides a general result for a linear ψ and a more detailed result for the specific case with $\underline{w} = 0$.

Proposition 5 *When there are two borrowers, under individual liability, suppose that A1' is satisfied and $\psi(r) = \theta r$ with $\theta > 1$. Then*
(i) The optimal grand contract is the best one between the two following contracts:
a. A state non-contingent lending contract $\{r_S = r_F = \rho + \frac{1}{2}\underline{w}\}$ with $\{q_0 = q_1 = q_2 = 0, w_0 = w_1 = w_2 = \underline{w}\}$,
b. A state-contingent contract $\{r_S = \Delta_I^{\max}, r_F = 0\}$.
The state non-contingent contract is optimal if and only if $C^I(\Delta_I^{\max}) - C^I(0) \geq (1 - p)\{2\psi[\frac{1}{2}C^I(0)] - C^I(0)\}$, where $C^I(0) = 2\rho + \underline{w}$.
(ii) When $\underline{w} = 0$ and $2\rho \geq (1-p)(3-p)k$, the optimal grand contract is the best one between the two following contracts:
a. A state non-contingent lending contract $\{r_S = r_F = \rho\}$ with $\{q_0 = q_1 = q_2 = 0, w_0 = w_1 = w_2 = 0\}$ (the supervisory contract is α).
b. A state-contingent contract $\{r_S = [2\rho + (1-p^2)k]/2p, r_F = 0\}$ with $\{q_0 = q_1 = 1,\ q_2 = 0\ w_0 = w_1 = w_2 = 0\}$ (the supervisory contract is γ with $\Delta = [2\rho + (1-p^2)k]/2p$).
The state-contingent contract is optimal if and only if $(1 + p)k \geq 2(\psi(\rho) - \rho)$.

The result in Proposition 5(i) is quite consistent with Propositions 2(ii) and 4(ii) and shows the trade-off between the insurance gain and the cost to discourage embezzlement. A more specific result can be obtained if, furthermore, we assume that $\underline{w} = 0$ because then it is easy to find the optimal supervisory contract given Δ, and so $\Delta_I^{\max}$. Indeed, in this case contract γ is equivalent to η and direct computations show that

$$C^I(\Delta) = \begin{cases} 2\rho + 2p\Delta & \text{if } \Delta \leq \frac{(1-p)^2}{p(2-p)}k \\ 2\rho + (1-p)^2k + p^2\Delta & \text{if } \frac{(1-p)^2}{p(2-p)}k \leq \Delta \leq \frac{2(1-p)}{p}k \\ 2\rho + (1-p^2)k & \text{if } \frac{2(1-p)}{p}k \leq \Delta \end{cases} \tag{19}$$

From (19) it follows that $\frac{(1-p)^2}{p(2-p)}k < \Delta_I^{\max} < \frac{2(1-p)}{p}k$ if $2\rho < (1-p)(3-p)k$, while $\Delta_I^{\max} \geq \frac{2(1-p)}{p}k$ if $2\rho \geq (1-p)(3-p)k$. We assume in Proposition 5(ii) that the audit cost is relatively small with respect to the opportunity cost of capital and thus $2\rho \geq (1-p)(3-p)k$ is satisfied[36] (Remark 2 in this section examines the case in which $2\rho < (1-p)(3-p)k$). Then $\Delta_I^{\max} = \frac{2\rho+(1-p^2)k}{2p} > \frac{2(1-p)}{p}k$ and $C^I(\Delta_I^{\max}) = C_\gamma^I(\Delta_I^{\max})$.

The optimal contract under individual liability characterized in Proposition 5(ii) is very similar to the one in the single-borrower case characterized in Proposition 2(iii). The only difference is that under the optimal state-contingent contract in Proposition 5(ii)b, when both projects fail, the lender needs to incur the audit cost only once instead of twice. This is why the condition for the optimal contract to be state non-contingent is more restrictive in proposition 5(ii) than in proposition 2(iii).

Remark 2 *When $\underline{w} = 0$ and $(1-p)(3-p)k > 2\rho$ holds, we find $\Delta_I^{\max} = \frac{2\rho+(1-p)^2k}{p(2-p)}$ and the optimal grand contract is either the state non-contingent one described in proposition 5(ii)a or $\{r_S = \frac{2\rho+(1-p)^2k}{p(2-p)}, r_F = 0\}$ with $\{q_0 = 1,\ q_1 = q_2 = 0\ \ w_0 = w_1 = 0, w_2 = \frac{2\rho+(1-p)^2k}{p(2-p)}\}$ (the supervisory contract is β with $\Delta = \frac{2\rho+(1-p)^2k}{p(2-p)}$). The state non-contingent contract is optimal if and only if $p\rho + (1-p)^2k \geq (1-p)(2-p)(\psi(\rho) - \rho)$.*

4.4 Comparison: joint liability versus individual liability

The following table summarizes the total NPV when there are two borrowers (and ψ is linear, $\underline{w} = 0$ and $2\rho \geq (1-p)(3-p)k$) under the optimal state non-contingent or state-contingent contract, depending on the liability regime.

Liability	The optimal state non-contingent contract	The optimal state-contingent contract
Joint	$V - (1-p)^2 2(\psi(\rho) - \rho)$	$V - (1-p)^2k$
Individual	$V - [(1-p)^2 + p(1-p)]\, 2(\psi(\rho) - \rho)$	$V - [(1-p)^2 + 2p(1-p)]\, k$

where $V \equiv 2pY_S - 2\rho$ represents the NPV when the supervisor is honest.

[36] For instance, if $p = 1/2$ the inequality is equivalent to $\rho \geq \frac{5}{8}k$.

Conditional on that the lender uses the optimal state non-contingent contract, or the optimal state-contingent contract, the change from individual liability to joint liability strictly increases the NPV, as suggested by Observation 1 at the beginning of Section 4. In the case of the optimal state non-contingent contract, joint liability provides a partial insurance in that when only one project succeeds, the borrower whose project failed does not pay anything (i.e. $r_{FS} = 0$) while he has to pay ρ under individual liability. The increase in the NPV from this partial insurance is equal to $2p(1-p)(\psi(\rho)-\rho)$. In the case of the optimal state-contingent contract, the introduction of joint liability does not affect any insurance provision since full insurance is provided regardless of the type of liability. However, it reduces the cost of audit: by making the borrowers' aggregate repayment when both projects succeed equal to the aggregate repayment when only one succeeds (i.e. $\Delta_2 = 0$), the lender needs to conduct audit only when both projects fail. By contrast, under individual liability, the lender should conduct audit whenever at least one project fails. The reduction in the audit cost is equal to $2p(1-p)k$.

We also find that joint liability makes a state-contingent contract more likely to be optimal. If a state-contingent contract is optimal under individual liability, i.e. if $(1+p)k < 2(\psi(\rho)-\rho)$ holds (see proposition 5(ii)), then a state-contingent contract is optimal also under joint liability, i.e. $k < 2(\psi(\rho)-\rho)$ holds (see proposition 4(ii)): but the converse does not hold. The reason is the following: if a state-contingent contract is optimal under individual liability, even though joint liability provides some insurance under the state non-contingent contract, this insurance gain is not large enough with respect to the reduction in audit cost to make a state non-contingent contract optimal under joint liability.

Summarizing, we have

Proposition 6 *When there are two borrowers, suppose A1', $\underline{w} = 0$, $(1-p)(3-p)k \leq 2\rho$ and $\psi(r) = \theta r$ with $\theta > 1$.*
(i) When the supervisor is honest, joint liability does not increase the borrowers's payoff with respect to individual liability.
(ii) When the supervisor can misbehave, joint liability strictly dominates individual liability since it either reduces the financing cost or provides borrowers with more insurance.
(iii) A state-contingent contract is more likely to be optimal under joint liability than under individual liability.

4.5 Mutual insurance

Up to now, we have not considered the possibility that two borrowers can sign a side-contract between themselves, while some related literature considers side-contracting.[37] In our model, the borrowers might have an interest to sign a side-contract to provide mutual insurance, especially under individual liability. More precisely, consider the timing in which, after accepting the lending contract and before the realization of the state of nature, the borrowers sign a binding side-contract which specifies a state-contingent side-payment between themselves. Since the lending contract and the agents are symmetric, a side-contract does not need to specify any side-payment when both projects succeed or both fail.[38] Hence, a side-contract only specifies a monetary transfer x that a borrower whose project succeeds makes to a borrower whose project fails such that the latter uses x to make his repayment to the lender. Note first that given a grand-contract, side-contracting has no impact on the supervisor's incentives since it does not affect the borrowers' aggregate payment schedule.

Consider first joint liability. Then, we can show that the optimal grand-contract without side-contracting that we characterized in proposition 4 is still the optimal contract even though the borrowers can sign a side-contract. Indeed, the only instrument of the borrowers' coalition is x, the transfer from a successful borrower to the unsuccessful borrower in state SF or FS, and this instrument is useless under the optimal contract because of Lemma 3 (which shows that $r_{FS} = 0$ in the optimal contract). In other words, the lender has already done the best the borrowers can hope for in terms of insurance for these states of nature.

Consider now individual liability. First, we note that the outcomes that the lender can achieve under individual liability are a subset of the outcomes achievable under joint liability regardless of whether or not the borrowers can sign a side-contract. Therefore, the lender cannot obtain under individual liability an outcome superior to the best she can achieve under joint liability. Second, we can show that if side-contracting is possible under individual liability, the lender can achieve the outcome of the optimal state non-contingent contract under joint liability (see proposition 4(i)a) by offering $G^{I*} = \left\{ r_S = r_F = \rho + \frac{w}{2}, w_n = \underline{w}, q_n = 0 \text{ for } n = 0, 1, 2 \right\}$. Under G^{I*}, it is optimal for the borrowers to sign a side-contract specifying $x^* = \rho + \frac{w}{2}$ because it maximizes the bor-

[37] See Laffont and N'Guessan (2000), Laffont (2003), Laffont and Rey (2003), and Rai and Sjöström (2004)

[38] Actually, if ψ is strictly convex then a side payment in state FF from a borrower to the other borrower reduces the borrowers' sum of payoffs.

rowers' ex ante expected payoffs. Therefore, G^{I*} implements the outcome of the optimal state non-contingent contract under joint liability. Last, when there is individual liability, side-contracting does not allow the lender to achieve the outcome of the optimal state-contingent contract under joint liability in proposition 4(i)b. The latter contract specifies a repayment schedule such that $2r_{SS} = r_{SF}$, $r_{FS} = r_{FF} = 0$. This kind of schedule cannot be obtained under individual liability because $r_{SS} = r_{SF}$ and $r_{FS} = r_{FF}$ must hold. As the table in subsection 4.4 reveals, the consequence is a higher audit cost under individual liability. Summarizing, we have:

Proposition 7 *When there are two borrowers and A1' is satisfied,*
(i) Under joint liability, the optimal grand-contract is the same regardless of whether or not the borrowers can sign a side-contract and the possibility of side-contracting has no impact on their payoffs.
(ii) When the borrowers can sign a side-contract,

a. the borrowers' payoffs cannot be higher under individual liability than under joint liability.

b. If a state non-contingent contract is optimal under joint liability, the maximal payoffs under joint liability can be achieved also under individual liability by a grand-contact which induces the borrowers to sign a side-contract for mutual insurance.

c. If a state-contingent contract is optimal under joint liability, the borrowers' payoffs are strictly higher under joint liability than under individual liability.

Proposition 7(ii)b implies that, conditional on that a state non-contingent contract is optimal, our model does not necessarily predict that we should observe the use of joint liability. Actually, the Grameen bank seems to recently have discontinued its previous practice of joint liability.[39]

5 Discussions

Consider the case of a single borrower who can choose between two investment projects of different size, a small one and a large one. The small project is the one described in section 3. In the case of the large one, the borrower invests $a(> 1)$ units of money and the investment generates the revenue aY_S with probability $p \in (0, 1)$, the revenue zero with probability $1 - p$. The cost of capital is $a\rho$ and, in order to use our result in

[39] See http://www.grameen-info.org/bank/GBGlance.htm. We asked them when and why they eliminated joint liability but got no response.

proposition 2(iii), we assume $\underline{w} = 0$. When the supervisor is honest, choosing the large project is optimal if and only if $pY_S - \rho$ is positive because there are constant returns to scale. However, when the supervisor can be dishonest, it can be optimal to choose the small project even though $pY_S - \rho > 0$. For instance, suppose $k \geq \psi(a\rho) - a\rho$; then, a state non-contingent contract is optimal regardless of the size of the project. In this case, choosing the small project instead of the large one is optimal if and only if $(a-1)p(Y_S - \rho) < (1-p)[\psi(a\rho) - \psi(\rho)]$ and this inequality can hold even though $pY_S > \rho$. Therefore, *staff's incentive problem creates a bias in project selection toward projects of smaller scale.*

Suppose now that the borrower can choose between two investment projects of different risk: each of them requires one unit of money to invest but has a different probability of success. A (relatively) safe project produces a return of Y_S with probability p and zero return with probability $1-p$. A risky project produces a return of $Y'_S > Y_S$ with probability p' $(< p)$ and zero return with probability $1-p'$. We assume $pY_S = p'Y'_S$. Therefore, when the supervisor is honest, the borrower is indifferent between the two projects. When the supervisor can misbehave and $\underline{w} = 0$, from proposition 2(iii), the borrower's expected payoff conditional on choosing the safer project is $pY_S - \rho - (1-p)\min\{\psi(\rho) - \rho, k\}$, which is strictly larger than the expected payoff conditional on choosing the riskier project: $p'Y'_S - \rho - (1-p')\min\{\psi(\rho) - \rho, k\}$. Therefore, *staff's incentive problem creates a bias in project selection toward safer projects.*

Studying collusion between a supervisor and a borrower will not require a model much different from our model of embezzlement. In fact, our model is equivalent to a model of collusion where the supervisor has full bargaining power with respect to the borrower. More precisely, when repayment upon success is larger than repayment upon failure, the supervisor has an incentive to collude with the borrower in case of success to manipulate the report into failure, and share the gain. In the extreme case in which the supervisor has full bargaining power, the incentive constraint to undo the report manipulation is exactly equal to (IC_{SF}) in section 3, the binding incentive constraint in our model.

In this paper, we considered a three-tier hierarchy for simplicity. However, when we add additional layers into the hierarchy, a state non-contingent lending contract is more likely to be optimal than a state-contingent contract. Actually, the hierarchy of the Grameen bank is composed of head office, zonal office, area office, branch office, center, group, member. As long as the total payment that each staff member at the bottom of the hierarchy should collect does not depend on the realized returns of his borrowers, no staff member at a higher level of hierarchy has any discretion. Therefore, when a state non-contingent contract is used, adding layers of hierarchy in our setting involves no extra cost except the minimum wages paid to the staff. By contrast, when a state-contingent lending contract is used, even though the staff at the bottom are induced to behave well through audit or incentive pays, the staff at a higher level have also discretion and the bank has to incur additional costs to discourage embezzlement.

Even though making each borrower's payment responsive to his individual shock can be very costly, making it responsive to a publicly observable shock affecting a whole region such as natural catastrophes can be managed in a centralized way with little agency cost. Actually, the Grameen bank provides Disaster funds to areas affected by natural catastrophes.

In our model, the only discretion that a staff member can have is in terms of making a report about the realized returns. However, in reality, there are other sources of discretion. In particular, a staff member can exercise his discretion when deciding whether or not a villager is eligible for a loan or how much loan a member can obtain, etc. This might induce staff's misconduct. Therefore, even though fixed repayment schedules are used, some monitoring of the staff's actions should be made. In fact, according to Yunus, his bank could maintain excellence "only if its monitoring system can reach out to all the remote and dark corners of the system and keep them clean". (Bornstein, 1997, p. 171)

6 Conclusion

Our paper studied the question of when it is optimal for a microfinance institution to use a state non-contingent repayment by focusing on the bank staff's incentive to embezzle borrowers' repayments. We found that if the optimal lending contract is state-contingent, it is optimal to induce the staff member to behave well by using only audit even though the bank can use a mix of audit and incentive pay. Therefore, a state non-contingent schedule is optimal if the cost of monitoring the staff's behavior is larger than the borrowers' gain from full insurance. When the optimal lending contract is state non-contingent, joint liability is preferred to individual liability because it provides borrowers with partial insurance. However, under individual liability, the borrowers themselves have an incentive to mutually provide such an insurance. When the optimal lending contract is state-contingent, we showed that joint liability is strictly preferred to individual liability because it allows to save audit costs. We also found that a state-contingent contract is more likely to be optimal under joint liability than under individual liability.

Actually, Yunus expresses a strong concern about the burden of respecting a state non-contingent repayment schedule for borrowers. Yunus (2002) points out in "Grameen Bank II" that a major challenge of the bank lies in reducing the tension that arises between a borrower and a staff member when adverse shocks make it difficult for the former to honor the contract. For this purpose, Grameen Bank II considers introducing more flexibility into the system, for instance, by allowing borrowers to reschedule repayments. Our paper suggests that the gain from providing insurance should be carefully weighed against the cost of controlling staff's discretion.

Appendix

Proof of Lemma 1

(i) If $pr'_S + (1-p)r'_F = pr_S + (1-p)r_F \equiv r^e$, then $p(Y_S - r'_S) - (1-p)\psi(r'_F) = pY_S - r^e + (1-p)(r'_F - \psi(r'_F))$. This function of r'_F is equal to (1) at $r'_F = r_F$ and is decreasing with respect to r'_F.

(ii) Suppose that a grand contract $G = \{r_S, r_F, w_S, w_F, q_S, q_F\}$ satisfies (4) and is such that $r_S < r_F$. We now find $G' = \{r'_S, r'_F, w'_S, w'_F, q'_S, q'_F\}$ which satisfies (4) and increases the borrower's payoff with respect to G. Precisely, let $r'_S = r'_F = pr_S + (1-p)r_F$ and $w'_S = w'_F = \underline{w}$, $q'_S = q'_F = 0$. Then, (a) G' satisfies (IC_{SF}) and (IC_{FS}); (b) the financing cost with G' is equal to $\rho + \underline{w}$, the minimum feasible cost: therefore, it cannot be larger than the cost under G; (c) the borrower's expected payment (the left hand side of (BE)) with G' is the same as with G. Hence, G' satisfies (4) and the borrower's expected utility is higher with G' by lemma 1(i) since $r_F > r'_F$. ■

Proof of Lemma 2

(i) Reducing w_F improves the objective function and relaxes (IC_{SF}). Therefore w_F is equal to $\underline{w}$, the smallest feasible value.

(ii) When $\Delta = 0$, the supervisor has no discretion and there is no need to use incentive pay or audit.

(iii) Suppose that $\Delta > 0$ and (IC_{SF}) is slack. Then it is optimal to set $w_S = \underline{w}$ and $q = 0$, but a contradiction arises because (IC_{SF}) is violated, given $\Delta > 0$. If $q > \Delta/(\Delta + \underline{w})$, then the right hand side of (IC_{SF}) is smaller than $\underline{w}$ and (IC_{SF}) is slack.

(iv) By lemma 2(ii)-(iii), (IC_{SF}) binds both if $\Delta = 0$ and if $\Delta > 0$. Then (IC_{FS}) reduces to $\underline{w} \geq -\Delta + (1-q)(\Delta + \underline{w})$, equivalent to $q\underline{w} \geq -q\Delta$. ■

Proof of Proposition 2

(i) We have already proved Proposition 2(i)a-b in the text. We only need to verify that borrower's payoff is positive in the optimal grand contract. Since A1 implies that the payoff is positive in the best contract between the state non-contingent contract and the state-contingent contract with $\Delta = \Delta^{\max}$, the payoff is positive also in the optimal grand contract.

(ii) Since $\psi(r) = \theta r$ for some $\theta > 1$, we find that r_F^{stick} is concave in Δ, (7) is convex in Δ and thus Δ^{*S} is either equal to $\overline{\Delta}$ or to $\Delta^{\max}$.[40] However, if $\Delta^{*S} = \overline{\Delta}$, then the state non-contingent lending contract in Proposition 2(i)a is optimal because we already know that $\Delta = 0$ is strictly better than $\Delta = \overline{\Delta}$. Therefore, the state non-contingent contract is optimal if and only if $p(Y_S - \rho - \underline{w}) - (1-p)\psi(\rho + \underline{w})$ [which is (5) evaluated at $\Delta = 0$] is larger than $p(Y_S - \Delta^{\max})$ [which is (7) evaluated at $\Delta = \Delta^{\max}$]. Hence, the state non-contingent contract is optimal if and only if $p\Delta^{\max} \geq \rho + \underline{w} + (1-p)[\psi(\rho + \underline{w}) - \rho - \underline{w}]$; by recalling that $\Delta^{\max}$ satisfies (2), we can write the condition as $\rho + \underline{w} + (1-p)\frac{\Delta}{\Delta + \underline{w}}k \geq \rho + \underline{w} + (1-p)[\psi(\rho + \underline{w}) - \rho - \underline{w}]$, which is equivalent to (8).

(iii) It is obvious that if $\underline{w} = 0$, then $\bar{q}(\Delta) = 1$ for any $\Delta > 0$ and thus $\Delta^{*S} = \Delta^{\max}$ because $f(\Delta) = p(1-p)\{\psi'[r_F^{stick}(\Delta)] - 1\} > 0$ for any $\Delta \in [\overline{\Delta}, \Delta^{\max}]$. The condition $k \geq \psi(\rho) - \rho$ comes from (8) when $\underline{w} = 0$. ■

Proof of Lemma 3

Let $R(1)$ be given, which means that $r_{SF} + r_{FS}$ is given. We prove that if Y_S is sufficiently large, then it is optimal to set $r_{FS} = 0$ and thus $r_{SF} = R(1)$. Since $R(1)$ is given, (r_{SF}, r_{FS}) does not affect (10) but affects the borrowers' payoff only through the term

$$2p(1-p)[Y_S - r_{SF} - \psi(r_{FS})] = 2p(1-p)[Y_S - R(1) + r_{FS} - \psi(r_{FS})] \qquad (20)$$

where the equality comes from $r_{SF} = R(1) - r_{FS}$. In writing (20), we implicitly assumed that $r_{SF} \leq Y_S$. As long as this condition holds, maximizing (20) with respect to r_{FS} tells us that any $r_{FS} > 0$ is dominated by $r_{FS} = 0$ since $\psi'(r) > 1$ for any $r > 0$. We prove in proposition 4 that the condition $r_{SF} \leq Y_S$ is satisfied in the optimal lending contract given that A1'(ii) holds.[41] ■

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[40] We obtain the corner solutions since the marginal gain in terms of insurance from increasing Δ is constant because of the linear $\psi(r)$, while the marginal cost in terms of audit cost decreases because of concave $\bar{q}(\Delta)$.

[41] Furthermore, if we allow for $r_{FS} < 0$ then (20) becomes $2p(1-p)[Y_S - R(1)]$ and does not depend on r_{FS}. Intuitively, in state FS the marginal utility of money for the unsuccessful borrower is 1 when $r_{FS} < 0$ and so the precise value of $r_{FS} (< 0)$ is irrelevant, given $R(1)$. However, since $r_{SF} = R(1) - r_{FS}$, a negative r_{FS} makes r_{SF} larger than $R(1)$ and it is more difficult to satisfy $r_{SF} \leq Y_S$.

Proof of Lemma 4

(i) In (Lr^J), (a) q_2 appears only in the right hand side of (BE) and $q_2 = 0$ is the value which minimizes this right hand side; (b) w_0 appears in the right hand side of (BE) and in (11), thus $\underline{w}$ – the smallest feasible value of w_0 – most relaxes these constraints.

(ii) We first show a general result, which is independent of the particular relaxed problem that is considered. Let $\Delta_{n\hat{n}} \equiv R(n) - R(\hat{n})$; then $\Delta_{\hat{n}n} \equiv -\Delta_{n\hat{n}}$ and $\text{IC}_{n\hat{n}}$ and $\text{IC}_{\hat{n}n}$ are expressed as $w_n \geq (1 - q_{\hat{n}})(\Delta_{n\hat{n}} + w_{\hat{n}})$ and $w_{\hat{n}} \geq (1 - q_n)(-\Delta_{n\hat{n}} + w_n)$, respectively. We show that if $\Delta_{n\hat{n}} \geq 0$ and $\text{IC}_{n\hat{n}}$ is satisfied, then $\text{IC}_{\hat{n}n}$ holds as well. This fact is obvious if $w_n = \underline{w}$. If instead $w_n = (1 - q_{\hat{n}})(\Delta_{n\hat{n}} + w_{\hat{n}})$, then $\text{IC}_{\hat{n}n}$ reduces to $w_{\hat{n}} \geq (1 - q_n)[-q_{\hat{n}}\Delta_{n\hat{n}} + (1 - q_{\hat{n}})w_{\hat{n}}]$ and is satisfied.
This result proves that IC_{01} is satisfied in the solution to (Lr^J). Regarding IC_{02} and IC_{12}, we need to distinguish four different cases depending on whether IC_{20} and/or IC_{21} bind. It is clear from the above result that IC_{02} and IC_{12} are satisfied if either both IC_{20} and IC_{21} bind or both are slack. If only IC_{20} binds, then we know that IC_{02} holds and IC_{12} reduces to $w_1 \geq -q_0\Delta_2 + (1 - q_0)(\Delta_1 + \underline{w})$, which is weaker than IC_{10}. If only IC_{21} binds, then IC_{12} is satisfied and IC_{02} reduces to $\underline{w} \geq -\Delta_1 - q_1\Delta_2 + (1 - q_1)w_1$; this inequality holds in both in the case of $w_1 = \underline{w}$ and in the case of $w_1 = (1 - q_0)(\Delta_1 + \underline{w})$.

Proof of Lemma 5

We first prove that (IC_{21}) always binds. If (IC_{21}) is slack, then $q_1 = 0$ (otherwise it is profitable and feasible to reduce q_1) and $w_2 > \Delta_2 + w_1$ needs to hold. There are two cases to consider: when (IC_{20}) is slack and when (IC_{20}) binds. If (IC_{20}) is slack, then $w_2 = \underline{w}$ and so $w_2 > \Delta_2 + w_1$ is violated. If (IC_{20}) binds, then $w_2 = (1 - q_0)\Delta_2 + (1 - q_0)(\Delta_1 + \underline{w})$ and again $w_2 > \Delta_2 + w_1$ fails to hold because $w_1 \geq (1 - q_0)(\Delta_1 + \underline{w})$.
Given that (IC_{21}) binds, now we prove that it never arises the case in which both (IC_{20}) and (IC_{10}) are slack. If this were the case, then $q_0 = 0$ and $w_1 = \underline{w}$, since, otherwise, it is profitable and feasible to reduce q_0 and w_1. The inequality $\underline{w} > \Delta_1 + \underline{w}$ would be equivalent to (IC_{10}) slack, but it fails to hold since $\Delta_1 \geq 0$.

Proof of Lemma 6

(i)a. From (IC_{21}) and (IC_{20}) binding and (IC_{10}) slack we obtain

$$w_1 = \underline{w}, \quad w_2 = (1 - q_0)(\Delta_2 + \Delta_1 + \underline{w}), \quad q_1 = 1 - \frac{(1 - q_0)(\Delta_2 + \Delta_1 + \underline{w})}{\Delta_2 + \underline{w}} \tag{21}$$

Hence, $w_2 \geq \underline{w}$ and $q_1 \geq 0$ are equivalent to $q_0 \in [\frac{\Delta_1}{\Delta_2+\Delta_1+\underline{w}}, \frac{\Delta_2+\Delta_1}{\Delta_2+\Delta_1+\underline{w}}]$ and (IC$_{10}$) slack reduces to $q_0 > \frac{\Delta_1}{\Delta_1+\underline{w}}$. By plugging (21) into the objective function we find a linear function of q_0 which is minimized either at $q_0 = \frac{\Delta_1}{\Delta_2+\Delta_1+\underline{w}}$ or at $q_0 = \frac{\Delta_2+\Delta_1}{\Delta_2+\Delta_1+\underline{w}}$. In the second case, which implies $q_1 = \frac{\Delta_2}{\Delta_2+\underline{w}}$ and $w_2 = \underline{w}$, (IC$_{10}$) is slack and contract η is obtained. In the first case, instead, (IC$_{10}$) is not slack.

(i)b. From (IC$_{10}$) and (IC$_{21}$) binding we find

$$w_1 = (1-q_0)(\Delta_1+\underline{w}) \quad \text{and} \quad w_2 = (1-q_1)(\Delta_2+(1-q_0)(\Delta_1+\underline{w})) \tag{22}$$

Hence, $w_1 \geq \underline{w}$ is equivalent to $q_0 \leq \frac{\Delta_1}{\Delta_1+\underline{w}}$ and $w_2 \geq \underline{w}$ is equivalent to

$$(1-q_1)(\Delta_2+(1-q_0)(\Delta_1+\underline{w})) \geq \underline{w} \tag{23}$$

From (22) it follows that (IC$_{20}$) reduces to $q_1 \leq f(q_0)$, where $f(q_0) \equiv q_0 \frac{\Delta_2}{\Delta_2+\Delta_1+\underline{w}-q_0(\Delta_1+\underline{w})}$ is an increasing and convex function such that $f(0) = 0$ and $f(\frac{\Delta_1}{\Delta_1+\underline{w}}) = \frac{\Delta_2\Delta_1}{(\Delta_2+\underline{w})(\Delta_1+\underline{w})}$. The feasible set F for (q_0, q_1) is therefore $F = \left\{(q_0, q_1) \in \mathbb{R}^2_+ : q_0 \leq \frac{\Delta_1}{\Delta_1+\underline{w}} \text{ and } q_1 \leq f(q_0)\right\}$ since (IC$_{20}$) is equivalent to $w_2 \geq (1-q_0)\Delta_2 + w_1$ and in any point of F (a) (IC$_{20}$) holds; (b) $w_2 \geq (1-q_0)\Delta_2 + w_1 \geq \underline{w}$, thus (23) is satisfied. From (22) and (13) we derive the following objective function to minimize with respect to $(q_0, q_1) \in F$:

$$\begin{aligned} Q(q_0, q_1) &= 2\rho + p^2(1-q_1)(\Delta_2+(1-q_0)(\Delta_1+\underline{w})) \\ &\quad +2p(1-p)((1-q_0)(\Delta_1+\underline{w})+kq_1) + (1-p)^2(\underline{w}+kq_0) \end{aligned}$$

We now prove the statement in lemma 6(i)b. First notice that no point in the interior of F minimizes Q since the Hessian matrix of Q is indefinite for any (q_0, q_1):

$$\frac{\partial^2 Q}{\partial q_0^2} = \frac{\partial^2 Q}{\partial q_1^2} = 0 \qquad \frac{\partial^2 Q}{\partial q_0 \partial q_1} = p^2(\Delta_1+\underline{w}) > 0$$

This violates the second order condition for a minimum, which requires the Hessian matrix to be positive semi-definite.

We can neglect any point $(\widetilde{q}_0, 0)$ on the boundary of F such that $0 < \widetilde{q}_0 < \frac{\Delta_1}{\Delta_1+\underline{w}}$ because Q is linear in q_0 and this implies $\min\{Q(0,0), Q(\frac{\Delta_1}{\Delta_1+\underline{w}}, 0)\} \leq Q(\widetilde{q}_0, 0)$. Likewise, we can neglect any point $(\frac{\Delta_1}{\Delta_1+\underline{w}}, \widetilde{q}_1)$ on the boundary of F such that $0 < \widetilde{q}_1 < f(\frac{\Delta_1}{\Delta_1+\underline{w}})$ because Q is linear in q_1. As a consequence, only $(\frac{\Delta_1}{\Delta_1+\underline{w}}, 0)$ may be a minimum point for Q in the subset of F in which (IC$_{20}$) is slack and at $(q_0, q_1) = (\frac{\Delta_1}{\Delta_1+\underline{w}}, 0)$ we find contract β.

(i)c. When all the three constraints bind we have $q_1 = f(q_0)$ and $Q(q_0, q_1)$ is equal to

$$\begin{aligned} &\bar{Q}(q_0) \equiv Q(q_0, f(q_0)) = 2\rho + p^2(1-f(q_0))(\Delta_2+(1-q_0)(\Delta_1+\underline{w})) \\ &\quad +2p(1-p)((1-q_0)(\Delta_1+\underline{w})+kf(q_0)) + (1-p)^2(\underline{w}+kq_0) \\ &= 2\rho + 2p(1-p)kf(q_0) + (1-p)^2kq_0 + p(1-q_0)(p\Delta_2+(2-p)(\Delta_1+\underline{w})) \end{aligned}$$

Since f is convex, also $\bar{Q}$ is so. Hence, $\bar{Q}$ is minimized at $q_0 = 0$ if $\bar{Q}'(0) \geq 0$ (contract α), at $q_0 = \frac{\Delta_1}{\Delta_1+\underline{w}}$ if $\bar{Q}'(\frac{\Delta_1}{\Delta_1+\underline{w}}) \leq 0$ (contract γ), at $q_0^* \in (0, \frac{\Delta_1}{\Delta_1+\underline{w}})$ if $\bar{Q}'(0) < 0 < \bar{Q}'(\frac{\Delta_1}{\Delta_1+\underline{w}})$ (contract δ).[42]

(ii) Suppose that $\Delta_2 > 0 = \Delta_1$. Then $w_1 = \underline{w}$ since (IC_{10}) is $w_1 \geq (1-q_0)\underline{w}$. Lemma 5 implies that ($IC_{21}$) binds and therefore $w_2 = (1-q_1)(\Delta_2+\underline{w})$. ($IC_{20}$) reduces to $q_0 \geq q_1$ and therefore we have $q_0 = q_1$ because $q_0 > q_1$ implies that both (IC_{10}) and (IC_{20}) are slack, violating lemma 5. Thus, $q_1 = q_0 \leq \frac{\Delta_2}{\Delta_2+\underline{w}}$ in order to satisfy $w_2 \geq \underline{w}$. The objective function is then $2\rho + p^2(1-q_0)(\Delta_2+\underline{w}) + 2p(1-p)(\underline{w}+kq_0) + (1-p)^2(\underline{w}+kq_0)$, which is linear with respect to $q_0 \in [0, \frac{\Delta_2}{\Delta_2+\underline{w}}]$. Thus, the optimal q_0 is either 0 or $\frac{\Delta_2}{\Delta_2+\underline{w}}$ and contract α is obtained for $q_0 = 0$, contract η for $q_0 = \frac{\Delta_2}{\Delta_2+\underline{w}}$.

Suppose that $\Delta_2 = \Delta_1 = 0$. Then all incentive constraints are satisfied with $w_2 = w_1 = \underline{w}$, $q_1 = q_0 = 0$, which minimize the financing cost (as if the supervisor were honest). Contract α is obtained.

Suppose that $\Delta_2 = 0 < \Delta_1$. Then, (IC_{21}) is redundant because (IC_{20}) and (IC_{10}) jointly imply that (IC_{21}) is satisfied; hence, $q_1 = 0$. Notice that (IC_{20}) and (IC_{10}) reduce to $w_1 \geq (1-q_0)(\Delta_1+\underline{w})$ and $w_2 \geq (1-q_0)(\Delta_1+\underline{w})$, respectively, and either both of them bind or both are slack. But by Lemma 5 both bind. We plug $w_2 = w_1 = (1-q_0)(\Delta_1+\underline{w})$ into (9), with $q_0 \in [0, \frac{\Delta_1}{\Delta_1+\underline{w}}]$. Since the objective function is linear in q_0, we find either α or γ (which is equivalent to β if $\Delta_2 = 0$). ■

Proof of Lemma 7

In this proof we suppose that Y_S is sufficiently large so that lemma 3 applies. Suppose that $G = (R(0), R(1), R(2), w_0, w_1, w_2, q_0, q_1, q_2)$ is such that $\Delta_2 > 0$. We find G' which is weakly better than G and satisfies $\Delta_2' < \Delta_2$. Precisely, $R'(1) = R(1) + \frac{p}{2-p}\varepsilon$, $R'(2) = R(2) - \frac{2(1-p)}{2-p}\varepsilon$, $R'(0) = R(0)$ for a small $\varepsilon > 0$ and $q_n' = q_n$ for $n = 0, 1, 2$. In G', the borrowers' payoff and expected payment are unchanged and $\Delta_2' = \Delta_2 - \varepsilon$, $\Delta_1' = \Delta_1 + \frac{p}{2-p}\varepsilon$ and $\Delta_2' + \Delta_1' = \Delta_2 + \Delta_1 - \frac{2(1-p)}{2-p}\varepsilon$. We now determine wages w_2', w_1', w_0' such that G' satisfies (BE), and thus it is feasible, and yields the expected wage bill that is weakly smaller than the one under G.[43] By lemma 6, we need to distinguish three different cases depending on whether the binding constraints in (11) are (i) IC_{20} and IC_{21}; (ii) IC_{10} and IC_{21}; (iii) IC_{21}, IC_{20} and IC_{10}. In case (i), both IC_{20} and IC_{21} are relaxed in G' with respect to G,

[42] Since $\bar{Q}'(x) = \frac{2p(1-p)k\Delta_2(\Delta_2+\Delta_1+\underline{w})}{(\Delta_2+(1-x)(\Delta_1+\underline{w}))^2} + (1-p)^2k - p(p\Delta_2 + (2-p)(\Delta_1+\underline{w}))$, the condition $\bar{Q}'(0) < 0 < \bar{Q}'(\frac{\Delta_1}{\Delta_1+\underline{w}})$ is equivalent to the condition mentioned in footnote 32.

[43] In some cases the expected wage bill is strictly smaller than the one under G. Then, it is possible to strictly increase the borrowers' payoff by reducing $R'(2)$, $R'(1)$ and $R'(0)$ by a same amount.

thus we can choose $w'_2 = w_2, w'_1 = w_1, w'_0 = w_0$. In case (ii) we set $w'_1 = w_1 + (1-q_0)\frac{p}{2-p}\varepsilon$ and $w'_2 = w_2 + (1-q_1)[-\varepsilon + (1-q_0)\frac{p}{2-p}\varepsilon]$, thus the change in the expected wage paid to the supervisor is

$$2p(1-p)(1-q_0)\frac{p}{2-p}\varepsilon + p^2(1-q_1)[-\varepsilon + (1-q_0)\frac{p}{2-p}\varepsilon] \tag{24}$$

In order to show that (24) is negative, notice that from IC_{21} binding and IC_{20} slack it follows that $(1-q_1)(\Delta_2+w_1) > (1-q_0)(\Delta_2+\Delta_1+\underline{w})$ and then $1-q_1 > 1-q_0$. Thus, (24) is smaller than $2p(1-p)(1-q_0)\frac{p}{2-p}\varepsilon + p^2(1-q_0)[-\varepsilon + (1-q_0)\frac{p}{2-p}\varepsilon] = -\frac{p^3(1-q_0)q_0}{2-p}\varepsilon \leq 0$. In case (iii), we pick $w'_1 = w_1 + (1-q_0)\frac{p}{2-p}\varepsilon$ and $w'_2 = w_2 - (1-q_0)\frac{2(1-p)}{2-p}\varepsilon$ because the right hand side of IC_{20} decreases by $(1-q_0)\frac{2(1-p)}{2-p}\varepsilon$ while the right hand side of IC_{21} decreases by $(1-q_1)[\varepsilon-(1-q_0)\frac{p}{2-p}\varepsilon]$ and $1-q_1 > 1-q_0$ implies $(1-q_0)\frac{2(1-p)}{2-p}\varepsilon \leq (1-q_1)[\varepsilon-(1-q_0)\frac{p}{2-p}\varepsilon]$. The change in the expected wage bill is $2p(1-p)(1-q_0)\frac{p}{2-p}\varepsilon - p^2(1-q_0)\frac{2(1-p)}{2-p}\varepsilon = 0$.

In this way we have proved that whenever a lending contract is such that $\Delta_2 > 0$, we can find another contract which is at least as good and has a smaller Δ_2. Ultimately, this allows to look for the optimal contract within the set of contracts which satisfy $\Delta_2 = 0$. When $\Delta_2 = 0$, lemma 6(ii) establishes that the optimal supervisory contract belongs to $\{\alpha, \gamma, \eta\}$. However, it is straightforward to see that $\gamma = \eta$ if $\Delta_2 = 0$ and so we restrict our attention to $\{\alpha, \gamma\}$. ■

Proof of Proposition 4

(i) The proof of Proposition 4(i)a-b appears in the text. In order to verify that the borrowers' payoff is positive in the optimal grand contract, notice that the payoff under the state non-contingent contract described in (i)a is $2pY_S - p(2-p)(2\rho+\underline{w}) - (1-p)^2 2\psi(\rho+\frac{1}{2}\underline{w}) = 2pY_S - (2\rho+\underline{w}) - (1-p)^2(2\psi(\rho+\frac{1}{2}\underline{w}) - 2\rho - \underline{w})$ and it is obvious that assumption A1'(i) implies that this payoff is positive. The payoff in the optimal state-contingent contract is $2pY_S - p(2-p)[2r_{FF}(\Delta_1^{*\gamma}) + \Delta_1^{*\gamma}] - (1-p)^2 2\psi(\Delta_1^{*\gamma})$, which is at least as large as $2pY_S - p(2-p)[2r_{FF}(\Delta_1^{\max}) + \Delta_1^{\max}] - (1-p)^2 2\psi(\Delta_1^{\max})$ by definition of $\Delta_1^{*\gamma}$. Furthermore, $2pY_S - p(2-p)[2r_{FF}(\Delta_1^{\max}) + \Delta_1^{\max}] - (1-p)^2 2\psi(\Delta_1^{\max}) = 2pY_S - p(2-p)\Delta_1^{\max} = 2pY_S - C_\gamma^J(\Delta_1^{\max}, 0)$ (by definition of $\Delta_1^{\max}$) and $C_\gamma^J(\Delta_1, 0) = 2\rho + \underline{w} + (1-p)^2 k\frac{\Delta_1}{\Delta_1+\underline{w}} < 2\rho + \underline{w} + (1-p)^2 k$; thus, A1'(i) implies $2pY_S - C_\gamma^J(\Delta_1^{\max}, 0) > 0$.[44]
Finally, the condition $R(1) \leq Y_S$ is satisfied by the optimal state non-contingent contract since then $R(1) = 2\rho + \underline{w}$ while A1'(ii) requires that $Y_S \geq \Delta_1^{\max}$ and (9) implies $\Delta_1^{\max} >$

[44] Notice that A1' is more restrictive than needed for the case of joint liability because we want the same assumption to cover the case of individual liability as well, in which the borrowers' payoff is smaller than the one under joint liability (this fact is explained in detail in subsection 4.4).

$2\rho + \underline{w}$. In the optimal state-contingent contract we have $R(1) = 2r_{FF}(\Delta_1^{*\gamma}) + \Delta_1^{*\gamma} = 2\rho+\underline{w}+(1-p)^2k\bar{q}(\Delta_1^{*\gamma})+(1-p)^2\Delta_1^{*\gamma}$ and the function $2\rho+\underline{w}+(1-p)^2k\bar{q}(\Delta_1)+(1-p)^2\Delta_1$ is monotone and increasing with respect to Δ_1. Thus it achieves its maximum at $\Delta_1 = \Delta_1^{\max}$, where it takes the value $2r_{FF}(\Delta_1^{\max}) + \Delta_1^{\max} = \Delta_1^{\max}$; this is not larger than Y_S because of A1'(ii).

(ii) The proof that $\Delta_1^{*\gamma} \in \{\overline{\Delta}_1, \Delta_1^{\max}\}$ is straightforward since it follows from the convexity of (17) over the interval $[\overline{\Delta}_1, \Delta_1^{\max}]$. As in the case of proposition 2(ii), we know that if $\Delta_1^{*\gamma} = \overline{\Delta}_1$ then the optimal grand contract is such that $\Delta_1 = 0$ and thus we just need to compare the payoff at $\Delta_1 = 0$ with the payoff at $\Delta_1 = \Delta_1^{\max}$. By using (9) we find that the former payoff is larger than the latter if and only if $k\bar{q}(\Delta_1) \geq 2[\psi(\rho+\frac{1}{2}\underline{w})-\rho-\frac{1}{2}\underline{w}]$.

(iii) The proof is very similar to the proof of Proposition 2(iii) and thus is omitted.

Proof of Lemma 8

We consider all the possible cases in which $\Delta_1 < 0$ and/or $\Delta_2 < 0$ and in any such case we prove that it is possible to (weakly) increase the borrowers' payoff by satisfying $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$. In particular, we need to consider four different regimes.

1. $R(0) \geq \max\{R(1), R(2)\}$ (regime B)

Let G^B denote the best grand contract within the set of grand contracts satisfying $R(0) \geq R(1)$ and $R(0) \geq R(2)$. We show that $G^B = G'$, with $R'(0) = R'(1) = R'(2) = 2\rho + \underline{w}$ and $w'_0 = w'_1 = w'_2 = \underline{w}$, $q'_0 = q'_1 = q'_2 = 0$; thus, G^B satisfies $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$. It is easy to see that G' is feasible because it satisfies (10) and (BE). The borrowers' payoff in G' is $2pY_S - p^2R'(2) - 2p(1-p)R'(1) - (1-p)^2\psi(\frac{R'(0)}{2})$ because A1'(ii) implies that $Y_S > R'(1) = 2\rho + \underline{w}$ and thus lemma 3 applies. The borrowers' payoff with a grand contract G such that $R(0) > \max\{R(1), R(2)\}$ or $R(0) = \max\{R(1), R(2)\} > \min\{R(1), R(2)\}$ is smaller than the one with G' because of the following argument. First, suppose for the moment that the expected payment is the same in G as in G'. Then G' is better than G because of lemma 1, given that $R'(0) = R'(1) = R'(2)$ implies $R(0) > R'(0)$. Second, the expected payment in G is actually larger than in G' because $R(0) = R(1) = R(2)$ fails to hold and thus some cost must be borne to discourage embezzlement. Third, while $Y_S \geq R'(1)$ holds, it is possible to have $Y_S < R(1)$ and in this case the borrowers face some cost from reducing consumption in states SF and FS and not only in state FF.

2. $R(2) \geq R(0) \geq R(1)$ (regime C)

We show that within the set of grand contracts satisfying $R(2) \geq R(0) \geq R(1)$, the best contract G^C is such that $R^C(0) = R^C(1)$ and therefore $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ are satisfied. Let $\Delta_{20} \equiv R(2) - R(0) \geq 0$ and $\Delta_{01} \equiv R(0) - R(1) \geq 0$. We study the following reduced program in which we consider only IC_{21}, IC_{20} and IC_{01} among the incentive constraints.

$$\begin{array}{lrcl} IC_{21} & w_2 & \geq & (1-q_1)(\Delta_{20}+\Delta_{01}+w_1) \\ IC_{20} & w_2 & \geq & (1-q_0)(\Delta_{20}+w_0) \\ IC_{01} & w_0 & \geq & (1-q_1)(\Delta_{01}+w_1) \end{array}$$

A result analogous to lemma 4 holds and establishes that the solution to the relaxed problem is G^C because it satisfies IC_{12}, IC_{02} and IC_{10}. We prove that G^C is such that $\Delta_{01}^C = 0$ by showing that starting from any contract G satisfying $\Delta_{01} > 0$, we can find G' which is better than G and satisfies $\Delta'_{01} = 0$. Precisely, let $w'_n = w_n$, $q'_n = q_n$ for $n = 0, 1, 2$ and $R'(1) = R(1) + [p^2 + (1-p)^2]\Delta_{01}$, $R'(2) = R(2) - 2p(1-p)\Delta_{01}$, $R'(0) = R(0) - 2p(1-p)\Delta_{01}$; then $\Delta'_{01} = 0 < \Delta_{01}$ and $\Delta'_{20} = \Delta_{20}$. As a consequence, the incentive constraints are weakly relaxed and the borrowers' expected payment is unchanged; thus, G' satisfies (BE). Furthermore, $R'(1) > R(1)$ and $R'(0) < R(0)$. By lemmas 1 and 3, this fact implies that the borrowers' payoff is higher in G' than in G as long as $Y_S \geq R'(1)$. This proves that G^C is such that $\Delta_{01}^C = 0$ as long as $R^C(1) \leq Y_S$. Now we show that $R^C(1) \leq Y_S$ holds by finding G^C under the assumption that Y_S is large enough and then verifying that this assumption holds under A1'(ii). When $\Delta_{01} = 0$ and $\Delta_{20} \geq 0$, it is optimal to set (i) $w_1 = w_0 = \underline{w}$ because the right hand side of IC_{01} is not larger than $\underline{w}$; (ii) $q_1 = q_0 = q$ because if $q_1 > q_0$ (for instance), then it is profitable to reduce q_1 slightly. Furthermore, IC_{21} is equivalent to IC_{20} and it binds since otherwise it is profitable to reduce q (notice that for values of q close to 0, the right hand side of IC_{21} is larger than $\underline{w}$); this implies $q \in [0, \frac{\Delta_{20}}{\Delta_{20}+\underline{w}}]$. From (BE) binding we find $R(1) + p^2\Delta_{20} = 2\rho + \underline{w} + p^2\Delta_{20} + q[k(1-p^2) - p^2(\Delta_{20}+\underline{w})]$, where the right hand side is the financing cost. It is clear that this cost is minimized with respect to q at $q = 0$ if $(1-p^2)k - p^2(\Delta_{20}+\underline{w}) \geq 0$, at $q = \frac{\Delta_{20}}{\Delta_{20}+\underline{w}}$ otherwise. In any case, we have $R(1) \leq 2\rho + \underline{w}$ and we know that $2\rho + \underline{w}$ is smaller than Y_S under A1'(ii).

3. $R(1) \geq R(2) \geq R(0)$ (regime D).

We show that within the set of grand contracts satisfying $R(1) \geq R(2) \geq R(0)$, the best contract G^D is such that $R^D(1) = R^D(2)$, and therefore it satisfies $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$. Let $\Delta_{12} \equiv R(1) - R(2) \geq 0$ and $\Delta_{20} \equiv R(2) - R(0) \geq 0$. We study a reduced program in which we consider only IC_{12}, IC_{10} and IC_{20} among the incentive constraints:

$$\begin{cases} IC_{12} \quad w_1 \geq (1-q_2)(\Delta_{12}+w_2) \\ IC_{10} \quad w_1 \geq (1-q_0)(\Delta_{12}+\Delta_{20}+w_0) \\ IC_{20} \quad w_2 \geq (1-q_0)(\Delta_{20}+w_0) \end{cases} \tag{25}$$

A result analogous to lemma 4 holds and establishes that the solution to this relaxed problem satisfies IC_{21}, IC_{01} and IC_{02}. Suppose that G is such that $\Delta_{12} > 0$. We find G' which is weakly better than G and satisfies $\Delta'_{12} < \Delta_{12}$. Let $R'(2) = R(2) + \frac{2(1-p)}{2-p}\varepsilon$, $R'(1) = R(1) - \frac{p}{2-p}\varepsilon$ and $R'(0) = R(0)$ with $\varepsilon > 0$ and small; $q'_i = q_i$ for $i = 0, 1, 2$. Hence, $\Delta'_{12} = \Delta_{12} - \varepsilon$, $\Delta'_{20} = \Delta_{20} + \frac{2(1-p)}{2-p}\varepsilon$ and $\Delta'_{12} + \Delta'_{20} = \Delta_{12} + \Delta_{20} - \frac{p}{2-p}\varepsilon$. In G', the borrowers' expected payment is the same as in G and their expected payoff is at least as large as in G, since $R'(1) < R(1)$. We now determine w'_i, $i = 0, 1, 2$, such that G' satisfies (25) and (BE). This makes G' feasible and proves that within regime D, we can restrict our attention to contracts such that $\Delta_{12} = 0$, without loss of generality.

A result similar to lemma 5 holds and implies that we must distinguish three different cases, depending on the binding constraints in (25): (i) IC_{12}, IC_{10}; (ii) IC_{12}, IC_{20}; (iii) IC_{12}, IC_{10}, IC_{20}. In case (i), both IC_{12} and IC_{10} are relaxed in G' with respect to G, thus we can pick $w'_2 = w_2, w'_1 = w_1, w'_0 = w_0$. In case (ii) we set $w'_2 = w_2 + (1 - q_0)\frac{2(1-p)}{2-p}\varepsilon$ and $w'_1 = w_1 + (1 - q_2)[-\varepsilon + (1 - q_0)\frac{2(1-p)}{2-p}\varepsilon]$, thus the change in the expected wage paid to the supervisor is

$$2p(1-p)(1-q_2)[-\varepsilon + (1-q_0)\frac{2(1-p)}{2-p}\varepsilon] + p^2(1-q_0)\frac{2(1-p)}{2-p}\varepsilon \qquad (26)$$

In order to show that (26) is negative, notice that from IC_{12} binding and IC_{10} slack it follows that $(1 - q_2)[\Delta_{12} + (1 - q_0)(\Delta_{20} + \underline{w})] > (1 - q_0)(\Delta_{12} + \Delta_{20} + \underline{w})$ and then $1 - q_2 > 1 - q_0$. Thus, (26) is smaller than $2p(1-p)(1-q_0)[-\varepsilon + (1-q_0)\frac{2(1-p)}{2-p}\varepsilon] + p^2(1-q_0)\frac{2(1-p)}{2-p}\varepsilon = -\frac{4p(1-p)^2(1-q_0)q_0}{2-p}\varepsilon \leq 0$. In case (iii), we choose $w'_2 = w_2 + (1 - q_0)\frac{2(1-p)}{2-p}\varepsilon$ and $w'_1 = w_1 - (1 - q_0)\frac{p}{2-p}\varepsilon$ because the right hand side of IC_{10} decreases by $(1 - q_0)\frac{p}{2-p}\varepsilon$ while the right hand side of IC_{12} varies by $(1 - q_2)[-\varepsilon + (1 - q_0)\frac{2(1-p)}{2-p}\varepsilon]$. Given that $1 - q_2 \geq 1 - q_0$, we see that $(1 - q_0)\frac{p}{2-p}\varepsilon \leq (1 - q_2)[\varepsilon - (1 - q_0)\frac{2(1-p)}{2-p}\varepsilon]$. In this case, the change in the expected wage bill is $-2p(1-p)(1-q_0)\frac{p}{2-p}\varepsilon + p^2(1-q_0)\frac{2(1-p)}{2-p}\varepsilon = 0$.

In this way we have proved that whenever the lending contract is such that $\Delta_{12} > 0$, there exists another contract which is at least as good and has a smaller Δ_{12}. Therefore, in regime D we can consider only the contracts such that $\Delta_{12} = 0$.

4. $R(1) \geq R(0) \geq R(2)$ (regime E)

We show that within the set of grand contracts such that $R(1) \geq R(0) \geq R(2)$ the best contract G^E is such that $R^E(0) = R^E(2)$; therefore, it belongs to regime D. Let $\Delta_{10} \equiv R(1) - R(0) \geq 0$ and $\Delta_{02} \equiv R(0) - R(2) \geq 0$. We study the reduced program in which we consider only IC_{10}, IC_{12} and IC_{02} among the incentive constraints.

$$\begin{array}{lcll}\mathrm{IC}_{10} & w_1 & \geq & (1-q_0)(\Delta_{10}+w_0) \\ \mathrm{IC}_{12} & w_1 & \geq & (1-q_2)(\Delta_{10}+\Delta_{02}+w_2) \\ \mathrm{IC}_{02} & w_0 & \geq & (1-q_2)(\Delta_{02}+w_2)\end{array}$$

A result analogous to lemma 4 shows that the solution to this relaxed problem is G^E because it satisfies IC_{01}, IC_{21} and IC_{20}. Suppose that G is such that $\Delta_{02} > 0$. We find G' which is better than G and satisfies $\Delta'_{02} = 0$. Precisely, let $w'_n = w_n$, $q'_n = q_n$ for $n = 0, 1, 2$ and $R'(2) = R(2) + (1 - p^2)\Delta_{02}$, $R'(1) = R(1) - p^2\Delta_{02}$, $R'(0) = R(0) - p^2\Delta_{02}$; then $\Delta'_{02} = 0 < \Delta_{02}$ and $\Delta'_{10} = \Delta_{10}$. As a consequence, the incentive constraints are weakly relaxed while the borrowers' expected payment is unchanged. Further, $R'(1) < R(1)$ and $R'(0) < R(0)$ and thus, by lemma 1, the borrowers' payoff is larger in G' than in G. This proves that G^E satisfies $\Delta_{02} = 0$. ■

Proof of Lemma 9

(i) In contract δ, q_0^* is such that $\bar{Q}'(q_0^*) = 0$ and this condition is equivalent to $\frac{\partial Q[q_0^*, f(q_0^*)]}{\partial q_0} + \frac{\partial Q[q_0^*, f(q_0^*)]}{\partial q_1} f'(q_0^*) = 0$ (see the proof of lemma 6(i)c). Since $f'(q_0^*) > 0$, it must be the case that $\frac{\partial Q[q_0^*, f(q_0^*)]}{\partial q_0} < 0$ and $\frac{\partial Q[q_0^*, f(q_0^*)]}{\partial q_1} > 0$, or $\frac{\partial Q[q_0^*, f(q_0^*)]}{\partial q_0} \geq 0$ and $\frac{\partial Q(q_0^*, f(q_0^*)]}{\partial q_1} \leq 0$. In the first case, δ is not an optimal supervisory contract because it is possible to reduce the cost by slightly increasing q_0 above q_0^* and/or by slightly decreasing q_1 below $q_1^* = f(q_0^*)$. The proof is completed by showing that the second case cannot arise. Indeed, still using the proof of lemma 6(i)c, we find

$$\begin{aligned}\frac{\partial Q}{\partial q_0} &= -p^2(1-q_1)(\Delta + \underline{w}) - 2p(1-p)(\Delta + \underline{w}) + (1-p)^2 k \\ \frac{\partial Q}{\partial q_1} &= -p^2(\Delta + (1-q_0)(\Delta + \underline{w})) + 2p(1-p)k\end{aligned}$$

Then $\frac{\partial Q}{\partial q_1} \leq 0$ is equivalent to $k \leq \frac{p[\Delta + (1-q_0)(\Delta + \underline{w})]}{2(1-p)}$ and this inequality implies $\frac{\partial Q}{\partial q_0} \leq -p^2(1-q_1)(\Delta + \underline{w}) - 2p(1-p)(\Delta + \underline{w}) + \frac{(1-p)}{2} p[\Delta + (1-q_0)(\Delta + \underline{w})]$. The right hand side of the last inequality is smaller than $-\frac{1}{2}p(1-p)(2\Delta + 3\underline{w}) < 0$ for any $(q_0, q_1) \in [0, 1]^2$.

(ii) It is easy to see that C^I_α, C^I_β and C^I_η are concave. If also C^I_γ is concave, then the concavity of $C^I(\Delta) = \min\{C^I_\alpha(\Delta), C^I_\beta(\Delta), C^I_\gamma(\Delta), C^I_\eta(\Delta)\}$ follows from Theorem 5.5 in Rockafellar (1997). After some manipulations, we find that

$$\frac{d^2 C^I_\gamma}{d\Delta^2} = \frac{2\underline{w}}{(\Delta + \underline{w})^4}\{[k(3p-1)(1-p) - p^2\underline{w}]\underline{w} - [p^2\underline{w} + k(3p+1)(1-p)]\Delta\}$$

If $p \leq \frac{1}{3}$ or $\underline{w} \geq k$, then $k(3p-1)(1-p) - p^2\underline{w} \leq 0$ and thus C_γ^I is concave. If instead $k(3p-1)(1-p) - p^2\underline{w} > 0$, then $C_\gamma'''(\Delta) > 0$ for Δ close to 0; C_γ^I is convex in $[0, \tilde{\Delta}]$ and concave in $[\tilde{\Delta}, +\infty)$, where $\tilde{\Delta}$ is such that $\underline{w}[k(3p-1)(1-p) - p^2\underline{w}] - [p^2\underline{w} + k(3p+1)(1-p)]\tilde{\Delta} = 0$. In this case we prove below that $C_{\beta\gamma}^I(\Delta) \equiv \min\{C_\beta^I(\Delta), C_\gamma^I(\Delta)\}$ is concave. Since $C^I(\Delta) = \min\{C_\alpha^I(\Delta), C_{\beta\gamma}^I(\Delta), C_\eta^I(\Delta)\}$, we can apply Theorem 5.5 in Rockafellar (1997) to show that C^I is concave.

We need to prove that for any $\Delta'' > \Delta' \geq 0$ and any $t \in (0,1)$, the following inequality holds:

$$C_{\beta\gamma}^I(\Delta_t) \geq tC_{\beta\gamma}^I(\Delta') + (1-t)C_{\beta\gamma}^I(\Delta'') \tag{27}$$

where $\Delta_t = t\Delta' + (1-t)\Delta''$. Since $\frac{dC_\beta^I(0)}{d\Delta} = \frac{dC_\gamma^I(0)}{d\Delta} = p^2 + (1-p)^2\frac{k}{\underline{w}}$, the concavity of C_β^I in $[0, +\infty)$ and the convexity of C_γ^I in $[0, \tilde{\Delta}]$ imply $C_\gamma^I(\Delta) > C_\beta^I(\Delta) = C_{\beta\gamma}^I(\Delta)$ for any $\Delta \in (0, \tilde{\Delta}]$.

Suppose that $\tilde{\Delta} \geq \Delta'' > \Delta' \geq 0$. Then (27) holds because $C_{\beta\gamma}^I(\Delta) = C_\beta^I(\Delta)$ for any $\Delta \in (0, \tilde{\Delta}]$. Now consider the case in which $\Delta'' > \Delta' \geq \tilde{\Delta}$. Since C_β^I and C_γ^I are both concave in $[\tilde{\Delta}, +\infty)$, (27) is satisfied by Theorem 5.5 in Rockafellar (1997).

As a last case, suppose that $0 \leq \Delta' < \tilde{\Delta} < \Delta''$; then $C_{\beta\gamma}^I(\Delta') = C_\beta^I(\Delta')$. If $C_{\beta\gamma}^I(\Delta_t) = C_\beta^I(\Delta_t)$, then (27) holds because $C_\beta^I(\Delta_t) \geq tC_\beta^I(\Delta') + (1-t)C_\beta^I(\Delta'') \geq tC_{\beta\gamma}^I(\Delta') + (1-t)C_{\beta\gamma}^I(\Delta'')$. If instead $C_{\beta\gamma}^I(\Delta_t) = C_\gamma^I(\Delta_t)$, then it must be the case that $\Delta_t > \tilde{\Delta}$ and there exists $\bar{t} \in (0,1)$ such that $\Delta_t = \bar{t}\tilde{\Delta} + (1-\bar{t})\Delta''$. Since C_γ^I is concave in $[\tilde{\Delta}, +\infty)$, we find

$$\begin{aligned} C_{\beta\gamma}^I(\Delta_t) = C_\gamma^I(\Delta_t) \geq C_\gamma^I[\bar{t}\tilde{\Delta} + (1-\bar{t})\Delta''] \\ \geq \bar{t}C_\gamma^I(\tilde{\Delta}) + (1-\bar{t})C_\gamma^I(\Delta'') > \bar{t}C_\beta^I(\tilde{\Delta}) + (1-\bar{t})C_\gamma^I(\Delta'') \end{aligned} \tag{28}$$

Now let $\hat{t}$ satisfy $\hat{t}\Delta' + (1-\hat{t})\Delta'' = \tilde{\Delta}$. Then the extreme right hand side of (28) is at least as large as

$$\begin{aligned} \bar{t}[\hat{t}C_\beta^I(\Delta') + (1-\hat{t})C_\beta^I(\Delta'')] + (1-\bar{t})C_\gamma^I(\Delta'') \geq \\ \bar{t}\hat{t}C_{\beta\gamma}^I(\Delta') + \bar{t}(1-\hat{t})C_{\beta\gamma}^I(\Delta'') + (1-\bar{t})C_{\beta\gamma}^I(\Delta'') = \bar{t}\hat{t}C_{\beta\gamma}^I(\Delta') + (1-\bar{t}\hat{t})C_{\beta\gamma}^I(\Delta'') \end{aligned} \tag{29}$$

By combining (28) and (29) we see that (27) holds because $\bar{t}\hat{t}\Delta' + (1-\bar{t}\hat{t})\Delta'' = \Delta_t$ and thus $\bar{t}\hat{t} = t$. ■

Proof of Proposition 5

(i) The arguments in the text show that the optimal Δ belongs to $\{0, \Delta_I^{\max}\}$. The payoff when $\Delta = 0$ is $2pY_S - C^I(0) - (1-p)[2\psi(\frac{C^I(0)}{2}) - C^I(0)]$. The payoff when $\Delta = \Delta_I^{\max}$ is $2pY_S - 2p\Delta_I^{\max} = 2pY_S - C^I(\Delta_I^{\max})$. Since $C^I(\Delta_I^{\max}) < 2\rho + \underline{w} + (1-p^2)k$, A1'(i) implies that the payoff in the best of the two contracts is positive. In particular, by comparing the payoffs in the two contracts we see that the state non-contingent contract is optimal if and only if $C^I(\Delta_I^{\max}) - C^I(0) \geq (1-p)\{2\psi[\frac{1}{2}C^I(0)] - C^I(0)\}$.

(ii) The proof of part (ii) is straightforward, hence it is omitted. ■

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