Problem 1 Consider a set \( A \subseteq \mathbb{R}^n \). Show that the closure of the set obtained by forming all strictly positive convex combinations of any finite subset of \( A \) is the convex hull of \( A \).

Problem 2 Let \( X \) be an arbitrary nonempty set and consider \( d(p, q) = I_{p \neq q} \) defined for \( p, q \in X \). Show that \( d \) is a metric. Determine which sets are open, closed, bounded, and compact.

Problem 3 Let \( C \subseteq \mathbb{R}^n \) be a convex set and let \( \delta > 0 \). Show that if \( x_1, \ldots, x_k \in \mathbb{R}^n \) are such that \( D(C, x_i) < \delta \) for all \( i = 1, \ldots, k \) then any convex combination \( y \) of \( x_1, \ldots, x_k \) satisfies \( D(C, y) < \delta \).

Problem 4 Let \( X, Y \) be metric spaces and let \( f : X \to Y \) be uniformly continuous. Show that if \( \{x_n\} \) is a Cauchy sequence in \( X \) then \( \{f(x_n)\} \) is Cauchy in \( Y \). Does the statement remain true if \( f \) is continuous on \( X \) but not necessarily uniformly?

Problem 5 Show that any closed and convex set \( C \) in \( \mathbb{R}^n \) equals the intersection of all closed halfspaces containing \( C \).

Problem 6 Let \( X \) be a metric space and let \( f : X \to X \) be a function such that for all \( x, y \in X \),

\[ d(f(x), f(y)) < d(x, y). \]

Give an example when \( f \) does not have any fixed point. Show that if \( X \) is compact, \( f \) has exactly one fixed point. \textit{Hint:} The function \( d(x, f(x)) \) achieves its minimum on \( X \).

Problem 7 Let \( f \) be a real-valued function defined on an open set \( E \subseteq \mathbb{R}^n \), and assume that the partial derivatives \( D_1 f, \ldots, D_n f \) are bounded in \( E \). Prove that \( f \) is continuous in \( E \).

Problem 8 Prove that there is no continuous one-to-one and onto function \( f \) mapping the interval \([0, 1]\) onto the square \([0, 1] \times [0, 1] \).