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SOME GENERIC PROPERTIES OF AGGREGATE EXCESS DEMAND AND AN APPLICATION

ANDREU MAS-COLELL AND WILHELM NEUEFEIND¹

The fact that preference maximizing consumers generate aggregate excess demand is utilized to prove (i) a statement on the values of the excess demand correspondence and (ii) that the economies having an excess demand function are dense in the set of all economies. This is applied to get a straightforward proof for the existence of an equilibrium distribution.

1. INTRODUCTION

WITHIN THE FRAMEWORK of the continuum of traders approach to economic equilibria (Aumann [2] and Hildenbrand [8]) we are concerned in this paper with properties of the mean excess demand correspondence of atomless pure exchange economies whose consumers have complete, continuous, monotone, but *not necessarily convex*, preferences. The underlying theme is that in this class of economies the preference maximization hypothesis does (at least in a generic sense) impose restrictions on aggregate demand which are both interesting and useful.

It is well known that the mean excess demand correspondence of an atomless economy (defined as in Hildenbrand [8]) is upper hemicontinuous and compact, convex-valued. As our first result we show that if we are given an atomless economy, then, except for a very small (i.e., meager and of measure zero) set of prices, the values the excess demand correspondence may take belong to a very special class of convex sets, i.e., the class of translates of ranges of atomless vector measures. Hence, not every upper hemicontinuous compact, convex-valued excess demand correspondence coincides, in an a priori given compact set of strictly positive prices, with the excess demand correspondence derived from some economy, even if a continuum of consumers with nonstrictly convex preferences is allowed (and even, we could remark, if a production sector is added). This is to be contrasted with the result of Debreu [4] (see also, Sonnenschein [17] and Mantel [11]) implying that, on a compact set of prices, every continuous excess demand *function* can be generated by a finite economy with consumers having continuous, monotone, strictly convex preferences.

Our second result is of a more positive nature. It asserts that, with respect to the standard topology for these matters (the same as used by Hildenbrand [8]) there is a dense (in fact, second category) set of economies giving rise to excess demand *functions* with a very "nice" boundary behavior. A similar result could be derived from some recent aggregation theorems (Sondermann [16], Araujo [1], and Mas-Colell [13]); our proofs, however, do not rely on differentiability techniques.

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Finally, we will show that the approximation result referred to in the last paragraph is a useful one. Combined with the recent approach to the modeling of competitive equilibrium for large economies proposed in Hart, Hildenbrand, and Kohlberg [7], it yields, straightforwardly, an existence of equilibrium theorem. So, we will have obtained it by neither requiring any “convexifying” result such as the Shapley–Folkman or Lyapunov theorems, nor making any use of the theory of integration of correspondences, i.e., there is something to be gained in equilibrium theory by keeping in mind that one is dealing with excess demand rather than arbitrary convex-valued correspondences.

Our whole analysis (and results) can be developed in a context of general consumption sets and locally nonsatiated preferences; this is done in Mas-Colell and Neufeind [15]. For the sake of clarity, however, we will here confine ourselves to the case of consumption sets equal to the nonnegative orthant of the commodity space and monotone preferences. No essential difficulties are involved in the extension to the general case.

The paper will be organized as follows. In Section 2 we introduce some notation and state some well-known facts. Section 3 contains a basic lemma on individual demand. In Section 4 we state, prove, and discuss the generic properties of aggregate excess demand for a fixed economy. Section 5 gives the results on generic properties, concerning aggregate excess demand, of economies. Section 6 contains the application of these results to the existence of equilibrium proof.

2. DEFINITIONS AND BASIC FACTS

An *agent* will be determined by specifying a couple (\succeq, e) of *characteristics*, where \succeq is a complete,² continuous, weakly monotone preorder on R^l_+ and $e \in R^l_+$ is an endowment vector such that³ $e \gg 0$. The space of agents is denoted by \mathcal{A} ; a is a generic element of \mathcal{A} and $(\succeq_a, e(a))$ designates the characteristics of agent a ; $>_a$ is the strict preference relation derived from \succeq_a in the usual manner. The symbol e will also denote the map $a \rightarrow e(a)$ from \mathcal{A} to R^l .

We endow \mathcal{A} with the same (separable) topology as used in Hildenbrand [8, p. 96], i.e., $a_n \rightarrow a$ if $\succeq_{a_n} \rightarrow \succeq$ in closed convergence and $e(a_n) \rightarrow e(a)$ for the Euclidean norm; \mathcal{A} becomes, then, a separable, metric space; it follows from Grodal [6, Propositions 1, 2(a) and (d), and 4(e)] that \mathcal{A} can be viewed as a G_δ subset of a complete, metric space (on the nature of which we shall not need to be more specific).

Denote $S := \{p \in R^l_+ | \sum_{i=1}^l p^i = 1\}$, $S^+ := \{p \in S | p^i > 0 \text{ for all } i\}$. For any $a \in \mathcal{A}$, $p \in R^l$, and $w \in R$ define the *budget set* $\beta(a, p, w) := \{x \in R^l_+ | px \leq w\}$ and the *demand set* $\varphi(a, p, w) := \{x \in \beta(a, p, w) | x \succeq_a y \text{ for all } y \in \beta(a, p, w)\}$. For any $a \in \mathcal{A}$ and $p \in R^l$ let $\varphi_p(a) := \varphi(a, p, pe(a))$; the point to set map from $\mathcal{A} \times S$ to R^l defined by $(a, p) \mapsto \varphi_p(a)$ has closed graph and is nonempty-valued and upper hemicon-

² For these and any other standard but here undefined terms, we refer to Hildenbrand [8]. A preorder \succeq is weakly monotone if $x \in R^l_+$ and $x \ll y$ implies $x < y$.

³ For convenience, we make this somewhat restrictive assumption on agent’s endowments. It would, of course, be possible to drop this assumption and to work with quasi-demand rather than with demand.

tinuous (u.h.c.) in the domain $\mathcal{A} \times S^+$ (Hildenbrand [8, Prop. 3, p. 103 and Th. 2, p. 99]).

Denote by $\mathcal{B}(\mathcal{A})$ the Borel σ -field of \mathcal{A} . Let \mathcal{E} be the set of probability measures μ on $(\mathcal{A}, \mathcal{B}(\mathcal{A}))$ such that $\int e \, d\mu < \infty$. We endow \mathcal{E} with the topology whose concept of convergence is the weak convergence of measures with the convergence of the mean endowments; since \mathcal{A} is a G_δ subset of a complete metric space, \mathcal{E} also enjoys this property (for a proof see Ichiishi [9, Th. 2.11, pp. 12 and 26]). Hence, \mathcal{E} is topologically complete (Kuratowski [10, Ch. 3, 29VI, p. 316]) and, in particular, a Baire space.

The basic entities that shall concern us in this paper are the elements of \mathcal{E} ; these stand not for economies but for classes of economies. To completely determine an economy (of the pure exchange variety) one would have to specify both the distribution μ of the characteristics of the traders, i.e., an element μ of \mathcal{E} , and the cardinality of the economy, i.e., the number N (possibly infinite) of traders; of course, pairs (μ, N) represent economies only if for any $B \in \mathcal{B}(\mathcal{A})$, $N\mu(B)$, if defined, is an integer (possibly infinite). This number is then interpreted as the total number of traders with characteristics in B . Observe that, with these definitions, any agent in an economy with $N = \infty$ will automatically be negligible. Hence, we could justifiably call any *economy* with $N < \infty$, *simple* and with $N = \infty$, *atomless*.⁴ At any rate, for the purposes of this paper, only the distribution of agents' characteristics matters and so we shall dispense with the formal definition of an economy.

3. A PRELIMINARY LEMMA

For any set $Q \subset R^l$ let F_Q be the flat generated by $\text{co } Q$ and L_Q be the subspace parallel to F_Q . We denote by λ_Q the Lebesgue measure in L_Q , which, obviously, can also be regarded as a measure in F_Q .

Let $X \subset R^l$ be a closed set and \succeq a complete, continuous, and locally nonsatiated preorder on X . For any $p \in R^l$ define $h(p) := \{x \in X \mid px \leq 1 \text{ and } y \in X, py \leq 1 \text{ implies } x \succeq y\}$. For any set $Q \subset R^l$ denote $C(Q) := \{p \in Q \mid \dim \text{co } h(p) > l - \dim \text{co } Q\}$.

LEMMA 1: *For any measurable set $Q \subset R^l$ one has $\lambda_Q(C(Q)) = 0$, i.e., the set $C(Q)$ has Lebesgue measure zero.*

REMARK 1: (i) The attentive reader will realize that the closedness of \succeq is only used for a measurability argument. He may verify that a "measurable" consumer (i.e., a consumer whose preferences are a measurable set) will do as well.

(ii) The conclusion of the lemma is related, but different, to a result in Mas-Colell [14].

⁴ The term atomless used here for economies is not to be confused with the same term used later on for measures.

PROOF: Let L^* be a countable dense subset of L_Q . For any $q \in L_Q$ define $H_q := \{p \in R^l \mid qx < 0 < qy \text{ for some } x, y \in h(p)\}$; $\hat{H}_q := H_q \cap Q$.

Suppose that $\dim \text{co } h(p) + \dim \text{co } Q > l$ for some $p \in R^l$. Letting $m := \dim \text{co } h(p)$ we can pick z_0, z_1, \dots, z_m from $h(p)$ such that $\{z_1 - z_0, \dots, z_m - z_0\}$ are linearly independent vectors. Let \bar{z}_j be the projection of z_j on L_Q . Since $m > l - \dim \text{co } Q = \dim L_Q^\perp$, we can assume $\bar{z}_0 \neq 0$. We will show that for some j , \bar{z}_0 and \bar{z}_j are linearly independent. Indeed, suppose not; then $z_j - \mu_j z_0 \in L_Q^\perp$ for all $1 \leq j \leq m$ and since $m > l - \dim \text{co } Q = \dim L_Q^\perp$ the vectors $z_j - \mu_j z_0, 1 \leq j \leq m$, are linearly dependent, i.e., $\sum_{j=1}^m \delta_j (z_j - \mu_j z_0) = 0$ with not all δ_j equal to zero. But then, remember $pz_0 = pz_j = 1, \sum_{j=1}^m \delta_j = \sum_{j=1}^m \delta_j \mu_j$ and this yields $\sum_{j=1}^m \delta_j (z_j - z_0) = 0$ which is impossible. Therefore, for some j , \bar{z}_0 and \bar{z}_j are linearly independent, which implies $qz_0 = q\bar{z}_0 < 0 < q\bar{z}_j = qz_j$ for some $q \in L_Q$. We can obviously pick q to be in L^* . We conclude $C(Q) \subset \bigcup_{q \in L^*} \hat{H}_q$.

We now show that for any $q \in L_Q, H_q$ is measurable. Indeed, $h: R^l \rightarrow R^l$ has a measurable graph (Hildenbrand [8, Prop. 3, p. 60]) and, letting $D := \{(x, y) \in R^{2l} \mid qx < 0 < qy\}$, H_q has the form $\{p \in R^l \mid h(p) \times h(p) \cap D \neq \emptyset\}$, hence it is measurable (Hildenbrand [8, Prop. 4, p. 61]). Since $\hat{H}_q = Q \cap H_q, \hat{H}_q$ is also measurable.

To prove the lemma we show that $\lambda_Q(\hat{H}_q) = 0$ for every $q \in L_Q$. Let $q \in L_Q$ and I_q be the subspace spanned by q . If for every $p \in Q, \# \hat{H}_q \cap (\{p\} + I_q) \leq 1$, then $\lambda_Q(\hat{H}_q) = 0$ according to Fubini's theorem (for a statement see Hildenbrand [8, p. 47]). Suppose that $p + tq, p + t'q \in \hat{H}_q$ with $t' > t$; then there are $x \in h(p + tq), y \in h(p + t'q)$ such that $qx < 0 < qy$; hence, $(p + t'q)x < 1$ and $(p + tq)y < 1$ which, because of the completeness and local nonsatiation of \succeq , yields, respectively, $y > x$ and $x > y$, a contradiction. Q.E.D.

4. THE MEAN EXCESS DEMAND CORRESPONDENCE OF AN ECONOMY

We shall assume familiarity in this section with the concept of the integral of a correspondence (see Hildenbrand [8, DII]).

Given $\mu \in \mathcal{E}$ and $p \in S^+$ the correspondence φ_p is integrably bounded on \mathcal{A} . We can define $\Phi(\mu, p) := \int \text{co } \varphi_p \, d\mu - \int e \, d\mu$; the convex set $\Phi(\mu, p)$ represents then the mean excess demand at prices p of any atomless economy with distribution of characteristics μ ; in general, the mean excess demand at prices p of any economy with characteristics distribution μ has $\Phi(\mu, p)$ as convex hull.

The purpose of this section is to show that, even for atomless economies, mean excess demand is subject to strong restrictions; the values of $\Phi(\mu, \cdot)$ are very special convex sets for most prices $p \in S^+$. Precisely, they are, up to translation, ranges of atomless vector measures. A necessary condition for a convex set to be in this class is for it to have a center of symmetry. But this is far from being sufficient; for $n > 2$ the set of translates of ranges of atomless n -measures is nowhere dense (for the topology of the Hausdorff distance) in the set of all compact, convex subsets of R^n having a center of symmetry (Bolker [3, Th. 5.4, p. 336]).

A subset of a complete topological space is *meager* (or of the first category) if it is contained in the union of a countable number of nowhere dense sets. A set which is not meager is of the *second category*. In Baire spaces the complement of a meager set is of the second category.

LEMMA 2: *Let $\mu \in \mathcal{E}$. If, for a fixed $p \in S^+$, $\text{co } \varphi_p(a)$ is a segment μ -a.e., then $\Phi(\mu, p)$ is, up to translation, the range of a vector measure.*

PROOF. Since $\varphi_p: \mathcal{A} \rightarrow R^l$ is u.h.c., $\text{co } \varphi_p$ has a closed graph. Applying in a straightforward manner Proposition 3 of Hildenbrand [8, p. 60], one gets two selections f and g of $\text{co } \varphi_p$ which are measurable with respect to the completion of $(\mathcal{A}, \mathcal{B}(\mathcal{A}), \mu)$ and have the property that $\text{co } \varphi_p(a) = \text{co } \{f(a), g(a)\}$ whenever $\text{co } \varphi_p(a)$ is a segment. Hence $\int \text{co } \varphi_p d\mu = \int \text{co } \{f(a), g(a)\} d\mu(a) = \text{co } \int \{f(a), g(a)\} d\mu(a) = \int g d\mu + \text{co } \{ \int_B (f - g) d\mu \mid B \in \mathcal{B}(\mathcal{A}) \}$; the last set is the range of an atomless measure (Bolker [3, Th. 1.6, p. 324]). Q.E.D.

THEOREM 1: *Let $\mu \in \mathcal{E}$. Then the set $C_\mu := \{p \in S^+ \mid \Phi(\mu, p) \text{ is not, up to translation, the range of an atomless vector measure}\}$ is meager in S and λ_S null (i.e., the Lebesgue measure of C_μ is zero).*

REMARK 2: (i) For $l=2$ the above theorem implies no restriction (every segment is the range of an atomless measure), but more is true: for this case there is *no* restriction; the methods of Sonnenschein [17] or Debreu [4] can be adapted to show that (for $l=2$) given any compact subset of S^+ , every convex- and compact-valued upper hemicontinuous correspondence coincides in this set with one generated by a two-trader pure exchange economy.

(ii) If μ has finite support, then the theorem implies that the values of $\Phi(\mu, \cdot)$ are polyhedra except in a meager set of prices, a result which was proved in McFadden, Mantel, Mas-Colell, and Richter [12]. The theorem, however, is much stronger. In the situation where μ has finite support, it yields that the sets $\phi(\mu, \cdot)$ are polyhedra (and, up to translation, ranges of atomless vector measures) outside a meager set of *measure zero*.

PROOF OF THEOREM 1: For each $a \in \mathcal{A}$ we apply Lemma 1 to the set $Q = \{p \in R^l_+ \mid pe(a) = 1\}$; this yields $\lambda_S \{p \in S \mid \dim \text{co } \varphi_p(a) > 1\} = 0$. According to Fubini's theorem, $(\lambda_S \otimes \mu) \{(p, a) \in S \times \mathcal{A} \mid \dim \text{co } \varphi_p(a) > 1\} = 0$, whence $\mu \{a \in \mathcal{A} \mid \dim \text{co } \varphi_p(a) > 1\} = 0$ for λ_S -almost every price in S . Thus, it follows, applying the previous lemma, that $\lambda_S(C_\mu) = 0$.

To show that C_μ is meager, we observe:

(i) With respect to the topology of the Hausdorff distance in R^l the set of translates of ranges of atomless measures is closed (consequence of Th. 5.3 in Bolker [3, p. 336]).

(ii) If the correspondence $\phi(\mu, \cdot): S^+ \rightarrow R^l$ is considered as a function into the space of nonempty, compact subsets of R^l with the Hausdorff distance topology,

then it is continuous if and only if it is continuous as a correspondence (see Hildenbrand [8, Prob. 4, p. 34]).

These observations suffice to show that a continuity point of $\phi(\mu, \cdot)$ cannot be in C_μ , since otherwise there would be an open subset of S^+ contained in C_μ , a contradiction to $\lambda_S(C_\mu) = 0$. Hence, C_μ is contained in the set of discontinuity points of $\phi(\mu, \cdot)$ which is meager. The latter follows from the fact that $\phi(\mu, \cdot)$ is compact-valued and u.h.c. (Hildenbrand [8, Th. 3, p. 117]), and thus is continuous on a set of second category, an immediate consequence of Fort's theorem (see Hildenbrand [8, Th. 4 and the Remark, p. 31]). Q.E.D.

5. MEAN EXCESS DEMAND FUNCTIONS

With the exception of Remark 3 we will not use in this section the concept of the integral of a correspondence.

If for $\mu \in \mathcal{E}$ and $p \in S$, $a \rightarrow \varphi_p(a)$ is a function μ -almost everywhere then we define the mean excess demand $\Phi(\mu, p) := \int \varphi_p d\mu - \int e d\mu$. If for every $p \in J \subset S$, $\Phi(\mu, p)$ is defined we say that μ has a mean excess demand function on J .

By a sequence $\mu_n \in \mathcal{E}$ approximating $\mu \in \mathcal{E}$ we shall mean that $\mu_n \rightarrow \mu$ weakly and $\int e d\mu_n \rightarrow \int e d\mu$. By abuse of language we refer to a single μ' as approximating μ if μ' is a generic element of an approximating sequence. The following lemma is well known.

LEMMA 3: *Let $\mu \in \mathcal{E}$; then μ can be approximated by a sequence $\mu_n \in \mathcal{E}$ such that every μ_n has finite support.*

PROOF: Let $\mu \in \mathcal{E}$ and $\varepsilon > 0$. Since \mathcal{A} is (topologically) complete, there is a compact set $K \subset \mathcal{A}$ such that $\mu(K) \geq 1 - \varepsilon$ and $|\int_K e d\mu - \int e d\mu| < \varepsilon$ (Hildenbrand [8, (33), p. 50]). Denoting by $\mu|_K$ the restriction of μ to K , it is obvious that if ε is small enough, then $(1/\mu(K))\mu|_K$ is arbitrarily close to μ with respect to the weak convergence. On the other hand, there is $\mu_n \in \mathcal{E}$ such that $\mu_n \rightarrow (1/\mu(K))\mu|_K$ weakly and μ_n has a finite support contained in K ; this implies $\int e d\mu_n \rightarrow (1/\mu(K)) \int_K e d\mu$. The combination of all these observations yields the lemma.

Q.E.D.

We now state the main result of this section.

THEOREM 2: *Every $\mu \in \mathcal{E}$ can be approximated by a $\mu' \in \mathcal{E}$ having a continuous mean excess demand function on S^+ and such that $p_n \rightarrow \partial S$, $p_n \in S^+$ implies $\|\Phi(\mu', p_n)\| \rightarrow \infty$.*

REMARK 3: The set $\widehat{\mathcal{E}} := \{\mu \in \mathcal{E} | \mu \text{ has a continuous mean demand function on } S^+ \text{ and } p_n \rightarrow \partial S \text{ implies } \|\Phi(\mu, p_n)\| \rightarrow \infty\}$ is not only dense in \mathcal{E} but also of the second category. Perhaps the most direct way to see this is with the help of the concept of the integral of a correspondence. As in Section 4, we let, for $p \in S^+$ and $\mu \in \mathcal{E}$, $\Phi(\mu, p) := \int \text{co } \varphi_p d\mu - \int e d\mu$. The correspondence Φ is u.h.c.

Note then: (i) for any compact $K \subset S^+$ the set $\{\mu \in \mathcal{E} | \text{diam } \Phi(\mu, p) < \varepsilon \text{ for all } p \in K\}$ is open (Hildenbrand [8, Th. 3, p. 117]) and, by the theorem, dense; (ii) if for any $\varepsilon > 0$ we let $\mathcal{E}^\varepsilon := \{\mu \in \mathcal{E} | \mu\{a | \succeq_a \text{ is strictly convex}\} \geq \varepsilon\}$, then the set $\bigcup_{\varepsilon > 0} \mathcal{E}^\varepsilon$ is dense in \mathcal{E} ; (iii) for any $r > 0$ the set $\{\mu \in \mathcal{E} | \liminf_{n \rightarrow \infty} \min \|\Phi(\mu, p_n)\| > r \text{ for every } p_n \rightarrow \partial S\}$ contains an open neighborhood of $\bigcup_{\varepsilon > 0} \mathcal{E}^\varepsilon$, hence an open, dense set (see Hildenbrand [8, Prop. 6, p. 119]). Combining (i) and (iii) we conclude that $\widehat{\mathcal{E}}$ contains a countable intersection of open, dense sets; since \mathcal{E} is a Baire space, $\widehat{\mathcal{E}}$ is second category.

We remark that the same could be proved using continuity properties of *individual* demand only.

PROOF OF THEOREM 2: Let $\mu \in \mathcal{E}$ be fixed.

By Lemma 3 we can as well assume that μ is a Dirac measure, i.e., $\mu = \delta_{\bar{a}}$ where $\bar{a} = (\succeq, e)$.

Lebesgue measure in R^m (or, in general, in an m -dimensional subspace of R^n) is denoted λ^m . From Lemma 1 we have $\lambda^l\{p \in R^l | \#\varphi(\bar{a}, p, w) > 1\} = 0$ for all $w \neq 0$. Hence, by Fubini's theorem, $\lambda^{l+1}\{(p, w) \in R^{l+1} | \#\varphi(\bar{a}, p, w) > 1\} = 0$. Therefore, by altering e slightly, we can assume (apply Fubini's again) that $\lambda^l\{p \in R^l | \#\varphi(\bar{a}, p, pe) > 1\} = 0$.

Now we construct an atomless $\nu \in \mathcal{E}$, having the desired aggregation property and with support close to \bar{a} . So, ν itself will approximate $\delta_{\bar{a}}$.

Consider the set of $l \times l$ diagonal matrices with positive diagonal entries. For such a matrix B we specify an agent $\bar{a}_B \in \mathcal{A}$ having the endowment vector Be and the preferences \succeq_B defined by $y \succeq_B x$ if and only if $B^{-1}y \succeq_{\bar{a}} B^{-1}x$ (if u is a utility function for $\succeq_{\bar{a}}$, the agent \bar{a}_B has the utility function $u \circ B^{-1}$). The map $B \rightarrow \bar{a}_B$ is clearly continuous and if \mathcal{U} is a sufficiently small open neighborhood of the identity map, then any ν with $\nu\{\bar{a}_B | B \in \mathcal{U}\} = 1$ is (arbitrarily) close to $\delta_{\bar{a}}$.

Let $\hat{\nu}$ be an arbitrary (Borel) probability measure on \mathcal{U} which is absolutely continuous with respect to the natural l -Lebesgue measure on \mathcal{U} . Define then $\nu \in \mathcal{E}$ by $\nu(C) := \hat{\nu}\{B \in \mathcal{U} | \bar{a}_B \in C\}$.

We show now that ν has a demand function on S^+ . It is easily checked that the following equation holds for all $p \in S: B\varphi_{pB}(\bar{a}) = B\varphi(\bar{a}, pB, pBe) = \varphi(\bar{a}_B, p, pBe) = \varphi_p(\bar{a}_B)$. Hence, if, for each fixed $p \in S^+$, we can prove $\lambda^l\{B \in \mathcal{U} | \#\varphi_{pB}(\bar{a}) > 1\} = 0$ we are done.

For a fixed $p \in S^+$ consider the linear map $f: \mathcal{U} \rightarrow R^l$ given by $f(B) = pB$. Since $p \gg 0$, f is one-to-one and since $\lambda^l\{q \in R^l | \#\varphi(\bar{a}, q, qe) > 1\} = 0$ we have $\lambda^l\{B \in \mathcal{U} | \#\varphi(\bar{a}, pB, pBe) > 1\} = 0$ as was to be proved.

We show now that $\Phi(\nu, \cdot)$ is continuous on S^+ . Let $p_n \rightarrow p \in S^+$. The correspondence $(a, p) \mapsto \varphi_p(a)$ is uniformly bounded and u.h.c. on $\text{supp}(\nu) \times K$ for $K := \bigcup_n \{p_n\} \cup \{p\}$. Since $\varphi_{p_n}(a), \varphi_p(a)$ are singletons ν -a.e. we have $\varphi_{p_n}(a) \rightarrow \varphi_p(a)$ ν -a.e. and therefore $\int \varphi_{p_n} d\nu \rightarrow \int \varphi_p d\nu$.

We observe finally that by transferring an arbitrarily small amount of mass from $\text{supp}(\nu)$ to an agent a with $e(a) \gg 0$ and \succeq_a represented by $u(x) := \prod_{i=1}^l x^i$ we obtain a new μ' which has a demand function on S^+ , does still approximate μ , and satisfies, in addition, the boundary condition. Q.E.D.

6. AN APPLICATION: EXISTENCE OF EQUILIBRIUM DISTRIBUTIONS

Let $\mathcal{B}(\mathcal{A} \times R^l)$ be the Borel σ -field on $\mathcal{A} \times R^l$. Given a probability measure τ on $(\mathcal{A} \times R^l, \mathcal{B}(\mathcal{A} \times R^l))$, $\tau^{\mathcal{A}}, \tau^{R^l}$ denote the corresponding marginal distributions.

For every $p \in S$ let $E_p := \{(a, x) \in \mathcal{A} \times R^l \mid x \in \varphi_p(a)\}$; the correspondence $p \rightarrow E_p$ has a closed graph on S (see Section 2).

Hart, Hildenbrand, and Kohlberg [7] have introduced the following notion of an equilibrium distribution on $(\mathcal{A} \times R^l, \mathcal{B}(\mathcal{A} \times R^l))$. Given $\mu \in \mathcal{E}$, a probability measure τ on $(\mathcal{A} \times R^l, \mathcal{B}(\mathcal{A} \times R^l))$ is an *equilibrium distribution* for μ if there is $p \in S$ such that: (i) $\tau^{\mathcal{A}} = \mu$; (ii) $\int x d\tau^{R^l}(x) \leq \int e d\mu$; (iii) $\tau(E_p) = 1$.

The concept of an equilibrium distribution is devised to put emphasis on the fact that, provided there are many traders (actually, a continuum), the notion of competitive (Walrasian) equilibrium depends only on the distribution of traders' characteristics.

As an application of Theorem 2, and to underline the usefulness of the Hart-Hildenbrand-Kohlberg notion, we shall give a new proof of the existence of equilibrium distributions which does not make use of the theory of the integral of a correspondence and does not rely on "convexifying" theorems such as Lyapunov's or Shapley-Folkman's. The use of Fatou's lemma in several dimensions is also avoided.

THEOREM 3: *Let $\mu \in \mathcal{E}$; then there is an equilibrium distribution τ for μ .*

REMARK 4: We should emphasize that all along we are assuming completeness of preferences, a hypothesis which is not required for the existence of equilibrium distributions but which is basic to our line of proof. We believe it possible, however, to obtain the more general result as, essentially, a corollary to Theorem 3.

PROOF: By Theorem 2, there is a sequence $\mu_n \in \mathcal{E}$ such that $\mu_n \rightarrow \mu$, $\int e d\mu_n \rightarrow \int e d\mu$ and $\Phi(\mu_n, \cdot)$ is a continuous function on S^+ .

It is a well-known consequence of Brouwer's fixed-point theorem (see, e.g., Dierker [5, Th. 8.3, p. 78]) that, under the above conditions, there is, for every n , a $p_n \in S^+$ for which $\Phi(\mu_n, p_n) = 0$. We can assume $p_n \rightarrow \bar{p} \in S$. For every n , we define a probability measure τ_n on $\mathcal{A} \times R^l$ by $\tau_n(C) := \mu_n\{a \in \mathcal{A} \mid (a, \varphi_{p_n}(a)) \in C\}$; τ_n is then an equilibrium distribution for μ_n .

We show that the sequence τ_n has a limit point. For this it suffices that the sequences μ_n and $\tau_n^{R^l}$ be tight. Since $\mu_n \rightarrow \mu$ the sequence μ_n is tight; the tightness of $\tau_n^{R^l}$ follows at once from the fact that $\tau_n^{R^l}(R^l_+) = 1$, $\int x d\tau_n^{R^l}(x) \leq \int e d\mu_n$ and $\int e d\mu_n \rightarrow \int e d\mu < \infty$. Hence the sequence τ_n is tight and we can assume $\tau_n \rightarrow \tau$. (For all of this see Hildenbrand [8, pp. 49–50].)

We show that, with prices \bar{p} , τ is an equilibrium distribution for μ : (i) Since $\mu_n \rightarrow \mu$, $\tau_n^{\mathcal{A}} = \mu_n$, and $\tau_n^{\mathcal{A}} \rightarrow \tau^{\mathcal{A}}$, we have $\tau^{\mathcal{A}} = \mu$. (ii) For every m and $x \in R^l$ define $x_m \in R^l$ by $x'_m := \min\{x^i, m\}$. By the monotone convergence theorem (for a statement see Hildenbrand [8, p. 46]), $\int x d\tau^{R^l}(x) = \lim_m \int x_m d\tau^{R^l}$. For every m

and $n, \int x_m d\tau_n^{R^i} \leq \int x d\tau_n^{R^i}(x) \leq \int e d\mu_n$ and, since $\tau_n^{R^i} \rightarrow \tau^{R^i}$ weakly, $\lim_n \int x_m d\tau_n^{R^i} = \int x_m d\tau^{R^i}$; hence $\int x_m d\tau_n^{R^i} \leq \lim_n \int e d\mu_n = \int e d\mu$. Therefore, $\int x d\tau_n^{R^i}(x) \leq \int e d\mu$.
 (iii) Since the graph of $p \mapsto E_p$ is closed on S , we have $\text{Li}(E_{p_n}) \subset \text{Ls}(E_{p_n}) \subset E_{\bar{p}}$. But $\text{supp}(\tau_n) \subset E_{p_n}$ (hence $\text{Li supp}(\tau_n) \subset \text{Li}(E_{p_n})$) and $\tau_n \rightarrow \tau$ implies $\text{supp}(\tau) \subset \text{Li supp}(\tau_n)$. Therefore, $\text{supp}(\tau) \subset E_{\bar{p}}$, i.e., $\tau(E_{\bar{p}}) = 1$. Q.E.D.

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¹ **A Class of Convex Bodies**

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