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The Profit Motive in the Theory of Monopolistic Competition*

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1. Introduction

In the standard perfectly competitive general equilibrium model (Arrow and Hahn (1971), Debreu (1959)) the hypothesis that firms profit maximize plays a central role. Theoretically, profit maximization does not have the same primitive status as utility maximization but it can be justified in at least two grounds. In the prescriptive interpretation of the theory (i. e., welfare economics) profit maximization is the right objective for managers to follow if Pareto Optimality is to be attained (non-convexities should be ruled out). In the descriptive version of the theory profit maximization can be derived from utility maximization under the conditions that justify the perfect competition hypothesis itself (see Hart (1979)).

In monopolistically competitive theory (see Hart (1982b) and Mas-Colell (1982) for surveys) profit maximization is, again, at center stage but the foundations are now much less solid. It is well-known (see, e. g., Gabszewicz and Vial (1972)) that the profit maximizing decisions (and the eventual equilibria) need not be invariant to the choice of numeraire, which should be a very disquieting phenomenon. Also, even if well defined, there is in general no reason for the profit maximizing plan to convey the unanimous approval of owners. A notable exception where profit maximization is perfectly well grounded is Hart (1982 a). The present paper, let it be said once and for all, is in the line of Hart (1979) and (1982 a).

In spite of the remarks of the previous paragraph it is clear that, even if price-taking is not justified profit maximization makes intu-

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itive and descriptive sense. It is obviously this which accounts for its resilience as an ingredient of general equilibrium model building. It is therefore of interest to investigate the class of *general equilibrium* models in which price taking is not justified but, nevertheless, profit maximization is invariant to the choice of numeraire and is unanimously supported by firm's owners as an instruction to managers. In this note we shall not accomplish this task. We limit ourselves to describe a fully consistent particular limit case.

As in Hart (1979) the key aspect of our justification of profit maximization is the introduction of a strong asymmetry between ownership and consumption shares. The purpose is to make the income effects derived from ownership shares dominant relative to the consumption substitution effects.

It may be useful at this point to informally discuss an example.

Suppose there are two commidities, a generic input and a generic output. There is a single constant returns firm and a continuum of consumers (or, simply, many). Every consumer has the same preferences and owns an amount of input k. The model works à la Cournot: the manager of the firm sets a production z and the price p_z adapts so as to clear markets (see Fig. 1 for the case where profit shares are the same).

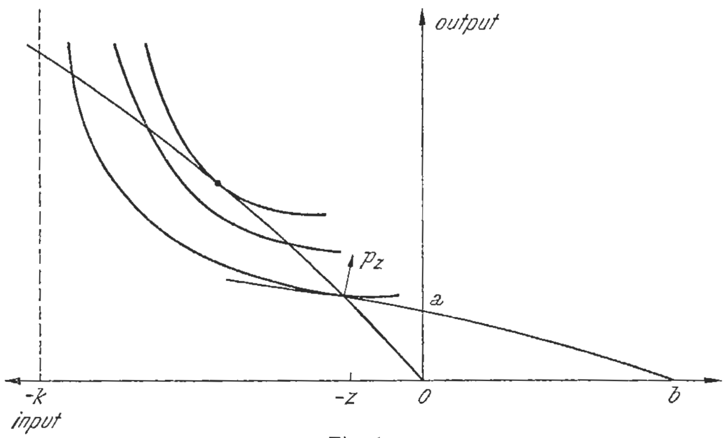

Fig. 1

The first problem is readily apparent from Fig. 1. There is no reason why profits in terms of output (measured by the segment 0a) and profits in terms of input (measured by 0b) should be maximized at the same z. As long as our model admits firms so large that its decisions affect all prices we will not be able to avoid this numeraire trouble. Therefore, in order to get rid of it we shall go to the extreme opposite and assume that we have many (in fact, a continuum) of firms.

Let's leave now aside the numeraire problem and convene that in the previous example profits are measured in terms of, say, input. Suppose, first, that every consumer has an equal claim on profits. Would the manager of the firm be instructed to maximize profits? Obviously not. In this example it is surplus (i. e. utility) not profit maximization the objective that conveys the unanimous approval of owners. But suppose now that the ownership of the firm is very concentrated, i. e., the profits are claimed by a small fraction of consumers (and distributed uniformly among them). It should then be expected that if this fraction is small enough profit maximization will be unanimously instructed to the managers. For any owner the loss as a consumer due to the increase in price is more than compensated by the increase in profit income.

We see, therefore, how profit maximization is tied to the concentration of the ownership of the firm. It should be emphasized that what matters is not that society be divided in owners and non-owners of firms. If there are many firms (as we will have in order to deal with the numeraire problem) the ownership of the "social stock of capital" can be very disperse. Every consumer may own comparable shares of firms. What is important is that every consumer's shares be concentrated in a few firms (while his purchases are dispersed among many).

We end this introduction by noting that, in contrast with the theory of perfect competition, profit maximization does not now have the status of an ideal welfare rule (although it has some desirable implications, e. g., cost minimization). Thus in the example it is when profit maximization does not make sense (i. e. when profit shares are disperse) that the welfare optimum is reached through a unanimous instruction to the manager. This observation seems to suggest that ceteribus paribus (i. e., keeping the same degree of monopoly power for every firm) a spread ownership of firms may be preferable to a concentrated one even on the grounds of the efficiency of the final allocation of resources. This is probably not, however, a promising line to follow. The fact of the matter is that if the ownerhsip of a firm is very disperse its decision matters very little to everyone. True, some decisions may be better than others but only by epsilon. Under these conditions the link between managers and shareholders is weak. To fill the vacuum a more managerial theory of the firm may be needed and any prediction on final allocation of resources will depend on which one is finally adopted.

2. An Illustrative Model

The model strives to be simple. Hence it is no more than an example. Every complication which we felt alien to the main point has been ruthlessly put aside.

2.1 Commodities

There will be a generic input (designated by z) and a continuum of products indexed by $s \in [0, 1] = I$. The unit interval only plays the role of providing a set of names. Those should not be interpreted as characteristics. The nearness of two indices s, s' is irrelevant to our analysis. In other words, it is the measure theoretic and not the metric properties of I that matter.

2.2 Firms and Production Sector

There is a firm for every product s. Hence, abusing notation slightly, we let s denote both the product s and the firm that produces s. Firm s has a production set $Y(s) \subset R_+ \times R_+$ with generic element $y(s) = (z(s), x(s))$ where $z(s)$ is the amount of input required to produce $x(s)$ of commodity s. We assume that the sets $Y(s)$ are uniformly bounded (i. e. $\bigcup_{s \in I} Y(s)$ is a bounded set) and that the correspondence $s \to Y(s)$ satisfies the technical requirement of measurability.

A function $y: I \to R^2$ is a feasible production if $y(s) \in Y(s)$ for a. e. $s \in I$. Given y the aggregate input use is denoted $\bar{z} = \int_0^1 z(s)\, ds$.

2.3 Consumption Sector: Preferences

Let L be the strictly positive orthant of the measurable and (essentially) bounded functions from $[0, 1]$ to R. We equip L with the induced L^1 norm, i. e. $x_n \to x$ if and only if $\int_0^1 |x_n(s) - x(s)|\, ds \to 0$.

Every consumer consumption set is $R_+ \times L$. This is not an innocent specification. It is tantamount to hypothesize that it is in the nature of things that consumers consume an infinitesimal amount of every commodity.

There is a continuum of consumers indexed by $t \in [0, T]$.

A consumer t is characterized by preferences, endowments of input and ownership shares of firms. We deal first with preferences.

We make the following two drastic hypotheses: (i) all consumers have the same preferences $\succsim$, and (ii) the preference relation $\succsim$ is homothetic (the reason for requiring this will be clear in 2.5). Given the convexity of $\succsim$ the following assumption, which we also make, is more in the nature of a regularity condition: for every $(z, x) \in R_{++} \times L$ there is a unique integrable function $p: I \to R_{++}$ such that $(z', x') > (z, x)$ implies $z' + \int_0^1 p(s)\, x'(s)\, ds > z + \int_0^1 p(s)\, x(s)\, ds$. Of course, with the price of the input normalized to be equal to one, p is just a price system supporting $\succsim$ at (z, x). So, the hypothesis is simply that p is unique and integrable.

Denoting by $p(z, x)$ the price system corresponding to (z, x) we also require that $(z, x) \to p(z, x)$ be uniformly continuous. The metric on p is given by the L^1 norm, i. e. $p_n \to p$ is defined to mean $\int_0^1 |p_n(s) - p(s)|\, ds \to 0$. It is important to our analysis that the continuity of $p(z, x)$ be respect to a weak topology on x. We use the L^1 norm. The sup norm will not do. The weak star topology would probably also work. Those are not merely technical remarks. It is well known that with infinitely many commodities the choice of topology has substantial economic implications.

As usual, the homotheticity of $\succsim$ implies $p(\lambda z, \lambda x) = p(z, x)$ for any $\lambda > 0$.

2.4 Consumption Sector: Ownership

The distribution of the ownership of firm s is described by a measure $\theta(s)$ on $[0, T]$. The function $s \to \theta(s)$ is assumed to be suitable measurable. (There is some intended vagueness here. At any rate, this is not an issue of substance).

We want to assume that the ownership of every s is concentrated in a few hands. In the limit, in a finite number of them. Hence, we assume that every $\theta(s)$ is finitely supported.

For every consumer t consider $g(t) = \sum_{s \in I} \theta(s)(\{t\})$, i. e. $g(t)$ is the sum of all ownership shares belonging to t. We assume there is a number $c > 0$ such that $g(t) \leq c$ for a. e. t. That is to say, there is an a-priori bound on the physical wealth of any consumer.

It is also convenient to assume that ownership shares, if positive, cannot be arbitrarily small. Hence, we suppose that $\theta(s)(\{t\}) > 0$ implies $\theta(s)(\{t\}) > \frac{1}{c}$. The description of the economy is completed by specifying the input endowment function $\omega: [0, T] \to R_{++}$. It is also assumed to be bounded. We let $\bar{\omega} = \int_0^T \omega(t)\, dt$.

2.5 The Inverse Demand Function

Let $y = (z, x)$ be a feasible production and p a price system. Then the wealth of consumer t is $w(t) = \omega(t) + \sum_{s \in I} \theta(s)(\{t\})(p(s)x(s) - z(s))$. We say that p is an equilibrium given y if there are $(z(t), x(t)) \in R_+ \times L$ such that, for a. e. t, $(z(t), x(t))$ maximizes preferences on $\beta(p, w(t)) = \{(z', x') : z' + p \cdot x' \leq w(t)\}$, and $\int_0^T (z(t), x(t))\, dt = (\overline{\omega} - \bar{z}, x)$. Remember that $\bar{z} = \int_0^1 z(s)\, ds$. The decisive advantage of assuming that preferences are identical and homothetic is that, provided y belongs to $J = \{y : \bar{z} < \overline{\omega}\}$, the set of equilibrium prices corresponding to $y = (z, x)$ is very easy to compute: it is simply $p(\overline{\omega} - \bar{z}, x)$. Indeed, consumers only differ on the wealth they have and with identical and homothetic preferences, demand is invariant to the distribution of wealth.

The continuous function $p(y) = p(\overline{\omega} - \bar{z}, x)$ defined on J is our general equilibrium analog of the inverse demand function.

We assume that ours is a quantity setting world, i. e. every firm has a manager that sets $y(s)$. Given a feasible production $y \in J$ the general equilibrium system is then completely solved via the prices $p(y)$. In particular the profits $\Pi_y(s)$ of a. e. $s \in I$ and the indirect utility $v_y(t)$ of a. e. $t \in [0, T]$ are well determined. They are simply:

$$\Pi_y(s) = p(y)(s) \cdot x(s) - z(s)$$

and

$$v_y(t) = u\left(\frac{w_y(t)}{W_y}(\overline{\omega} - \bar{z}, x)\right)$$

where u is a utility function for $\succsim$ (assumed to exist) and $\frac{w_y(t)}{W_y}$ the wealth distribution function given y and $p(y)$. Of course, $W_y = \int_0^1 w_y(t)\, dt$.

2.6 A Unanimity Result

The model has been devised so that its properties will be obvious. Hence, the following unanimity property should perhaps not be called a result.

Let the feasible production $y \in J$ be given and consider any $\varepsilon > 0$. Suppose that $y' \in J$ is such that the measure of $I(y, y') = \{s : y'(s) \neq y(s)\}$ is less than δ, where $\delta > 0$ is a very small number dependent on ε. We claim that if $\delta > 0$ is small enough and

$$\Pi_{y'}(s) \geq \Pi_y(s) + \varepsilon$$

for a. e. $s \in I(y, y')$ then

$$v_{y'}(t) > v_y(t)$$

for a. e. $t \in \{t: \theta(s)(\{t\}) > 0$ for some $s \in I(y, y')\}$. That is to say, if a very small fraction of firms change its production in a profit increasing direction then all the owners of any of those firms increase their utilities. The "very small fraction of firms" plays the role of the single player, a concept which in the present continuum context is meaningless.

The proof of the claim is clear. By the uniform continuity of $p(\cdot)$ we can make sure that if δ is small enough then, for a. e. $t \in [0, T]$,

$$\frac{w_y(t)}{W_y}\left(\int_0^1 (p(y')(s) - p(y)(s)) \cdot x(s)\, ds\right) < \frac{\varepsilon}{c}.$$

This means that for those t with $\theta(s)(\{t\}) > 0$ for some $s \in I(y, y')$ the increase in profit income $\left(\geq \frac{\varepsilon}{c}\right)$ more than compensates the variation in cost of the consumption at y. Hence, by revealed preference, the utility increases.

2.7 Comment on the Choice of Numeraire

The previous unanimity result is invariant to the choice of numeraire in the following sense. Suppose we renormalize prices by means of a linear price index. Then the argument of the previous subsection goes through unmodified. This roughly says that a manager of a firm will be in equilibrium if and only if it maximizes profits with respect to any numeraire, i. e. maximization with respect to a numeraire implies maximization with respect to any other.

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