Assessing brand image through communalities and asymmetries in brand-to-attribute and attribute-to-brand associations

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Received 13 October 2005; accepted 15 February 2008

Abstract

Brand image is a key component of customer-based brand equity, and refers to the associations a consumer holds in memory. Such associations are often directional; one should distinguish between brand-to-attribute and attribute-to-brand associations. Information on these associations arise from two ways of collecting data, respectively: brand-by-brand evaluations of all attributes and attribute-by-attributes evaluations of all brands. In this paper, the authors present a methodological approach, namely correspondence analysis of matched matrices, to assess the communalities as well as asymmetries between brand-to-attribute and attribute-to-brand associations. This allows studying whether or not there is match in a brand’s positioning (brand-to-attribute associations) and relative advantage (attribute-to-brand associations). The methodology results in perceptual maps visualizing brand image. The approach is illustrated in an empirical market research project in which two samples of consumers evaluated ten brands of deodorants and eleven attributes. The stability of the solution is examined using bootstrap resampling procedures.

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Keywords: Marketing; Brand image; Brand-to-attribute and attribute-to-brand associations; Multivariate statistics; Perceptual mapping; Correspondence analysis; Bootstrap

1. Introduction

The value of a brand lies in what consumers have experienced and learned about the brand. The resulting brand associations held in a consumer’s memory constitute the brand image, and affect their behavior. Brand associations are thereby important building blocks of customer-based brand equity (Keller, 1993, 2003; Krishnan, 1996), and marketers should aim to optimize the attributes and benefits that the brand is associated with by the consumers, satisfying their core needs and wants (Keller, 2003; Park et al., 1986). Such strongly held, favorably evaluated associations that are unique to the brand and imply superiority over other brands will be critical for a brand success (Broniarczyk and Alba, 1994; Thelen and Woodside, 1997). Hence, brand associations will have implications for many marketing mix actions, such as (re-)positioning and (re-)design of a brand (Kaul and Rao, 1995), and extending a brand to other product categories (Czellar, 2003).

Associations between brands and attributes are often directional (Anderson, 1983; Holden and Lutz, 1992; Farquhar and Herr, 1992; Krishnan, 1996): the association is from the brand to the attribute and/or the other way around. For example, the brand equity of BMW is affected by the extent to which positive features like safety and sportiness are evoked by that car brand. On the other hand, whether or not certain cues or attributes enhance brand recall in a purchase or consumption setting contributes to the equity of the brand. Insights in the communalities and asymmetries of these bi-directional associations can direct towards recommendations for brand managers.
Holden and Lutz (1992) stated that when measuring advertising effectiveness, one has to assess effects on attributes evoked by the brand as well as on attributes that are likely to evoke the brand. Farquhar and Herr (1992) showed that the dual nature of brand associations is an essential part of determining the limits of a brand’s stretch. Hence, when assessing brand image, one should consider both brand-to-attribute and attribute-to-brand associations.

Previous conceptual and empirical studies related to the description and assessment of brand image largely ignored the bi-directional nature of brand associations. Exceptions are Farquhar and Herr (1992), Holden and Lutz (1992), and Krishnan (1996), which provided conceptual foundations for studying such associations. However, existing literature does not present methodological tools adapted to the bi-directional nature of the association data.

A variety of methodologies have been proposed to assess and visualize brand images spatially on the basis of brand ratings or associations regarding a set of attributes, so-called perceptual mapping methods (see for example Dillon et al., 1985 or Shocker and Srinivasan, 1979). In this stream of literature, several studies (Jaffe and Nebenzahl, 1984; Olsen and Olsson, 2002; Teas and Wong, 1992; Wong and Teas, 2001) have demonstrated important differences for multi-attribute ratings collected through brand-by-brand judgment of all attributes versus attribute-by-attribute judgment of all brands. However, this stream of literature is specifically dealing with multi-attribute rating judgments, instead of binary associations which are widely used in positioning studies (i.e. Hoffman and Franke, 1986; Dillon et al., 2001).

Furthermore, again no methodological tools are presented that account for the directional nature of the data.

Here, we aim to contribute to this stream of publications by providing a perceptual mapping procedure to assess brand image based on bi-directional associations. In particular, we present a methodological approach, correspondence analysis of matched matrices (Greenacre, 2003; Greenacre and Clavel, 2002), which provides insightful spatial representations of the communalities and asymmetries between the brand-to-attribute and attribute-to-brand associations. Further more, to examine the stability of the results obtained we modify and apply bootstrap resampling procedures as proposed by Lebart (2006) and Greenacre (2007, Chapter 25).

In this paper, we first discuss the background on brand associations within the customer-based brand equity model. Next, we discuss the potential communalities and asymmetries between attribute-to-brand and brand-to-attribute associations. We present a methodology for assessing the communalities and asymmetries and apply it in a study of brand image for deodorants. The application illustrates the insights obtained from the methodology and implications that can be derived. The bootstrap resampling is used to examine the stability of the associations previously described. Finally, we discuss general implications for research on brand image and provide directions for future research.

2. Customer-based brand equity

Customer-based brand equity occurs when consumers are familiar with the brand and hold favorable, strong, and unique brand associations in memory (Keller, 1993, 2003). Memory for a concept, such as a brand, consists of a network of nodes and linkages among these nodes (Anderson, 1983). The nodes represent concepts and linkages represent the relationship between the concepts. The strength of the association linking two nodes reflects the likelihood that activation of one node will activate the other (Higgins and King, 1981). A brand node can have a variety of associations linked to that node, like attributes or benefits. Customer-based brand equity implies a certain amount of brand knowledge causing differential consumer responses to marketing of the brand. Brand knowledge has two components (Keller, 1993, 2003): brand awareness and brand image.

Brand awareness is related to the strength of the brand as reflected by consumers’ ability to identify the brand under different conditions (Alba and Chattopadhyay, 1985). Brand awareness is often measured by means of brand recall, which refers to the number of consumers that retrieve the brand when no cue at all or a cue like the product category or an attribute is given. Mature brands often score higher on brand recall compared to new brands (Kent and Allen, 1994), which can be attributed to longer history of media support, purchases, and consumption occasions.

Brand image can be defined as consumer perceptions about a brand as reflected by brand associations held in memory. Brand associations are informational nodes linked to the brand node in memory and contain the meaning of the brand for consumers. The favorable, strength, and uniqueness of brand associations are the dimensions of brand knowledge that play an important role in determining the differential response that makes up brand equity (Keller, 1993). The links in memory are often conceptualized as directional (Anderson, 1983), and may start or end at the brand node. Farquhar and Herr (1992) further elaborated on the dual nature of brand association and show that failure to account for the directionality and possible asymmetries can lead to incorrect conclusions.

One of the dimensions of brand image within the customer-based brand equity model is the strength of the associations between a brand and other concepts, such as attributes. The strength of an association is labelled as connectivity by Nelson et al. (1993). As our research deals with bi-directional associations, we adopt terminology by among others Ashcraft (1978), Farquhar and Herr (1992), and Loftus (1973), who used the term dominance, which combines direction and strength of an association. In particular, we use “attribute dominance” to refer to the strength of the directional association from a brand to an attribute, and “brand dominance” as the strength of the directional association from an attribute to a brand. Attribute dominance is operationalized by the number of...
people who give the attribute in response to the brand and, in a similar way, brand dominance by the number of people who give the brand in response to the attribute, with appropriate adjustments for total frequencies in order to normalize the measures. Theoretically and empirically, brand dominance (attribute-to-brand associations) and attribute dominance (brand-to-attribute associations) will not be the same. Communalities between both directional associations will refer to overall brand strength. Independent of the direction of questioning, the link between a brand and an attribute is strong. Asymmetries between both sets relate to differences between positioning (collected by the brand-by-attribute table) and competitive advantage of brands (collected by the attribute-by-brand table). For example, Keller (1993) describes a situation where asymmetries appear. Certain attributes or benefits may be considered “prototypical” and essential to all brands in the category, but a specific brand may be considered an “exemplar” that is most representative of the product category.

3. Empirical data

To explain and illustrate the methodology for studying bi-directional associations, we present an actual marketing research project on brand images in the deodorant product category. Interviews were conducted in London, in the year 2000. Within this project, brand association data has been collected from two samples of 198 and 203 subjects, respectively. The data has been collected by a market research agency and the samples were considered to be drawn at random and to be representative for the UK population. Respondents, in both samples, were asked a series of questions about the deodorant brands, such as “most often used brand”, “a brand one would switch to”, “a brand one would not use again”, and “brands used nowadays”. Any mentioning of a brand would include that brand among a respondent’s personal list of evoked brands (Howard and Sheth, 1969). This so-called “free-choice” questioning indirectly measures brand familiarity, the first component of brand equity. It results in different evoked brands across subjects. This procedure is considered a better option compared to “forced-choice”, where respondents have to evaluate all brands. The carry-over effect may be fairly large even after a considerable amount of time. In particular, when a within-subject design had been used in our study, subjects would have learned about the stimuli and the strategies to be used in the association task and would probably have strived to be consistent across the two tasks, which may considerably bias the second set of associations (Poulton, 1982; Keren and Raaijmakers, 1988). Furthermore, a large amount of time between measures may result in a change of the actual brand associations.

The final format of the tables is the same for both samples: frequencies representing the number of people who made a positive association between a brand and an attribute. The rows are 10 brands of deodorants and the column 11 attributes (Table 1).

The set of attributes was obtained using an unrestricted attribute-elicitation method (Steenkamp et al., 1994): the direct elicitation method. The subjects were asked directly what attributes he or she would consider when evaluating different brands of the same category (Olson and Muderrisoglu, 1979). It allows consumers to describe the brands in familiar with the brand and holds brand associations in memory (Keller, 1993).

Information on the directional associations were collected in two ways, namely: brand-by-brand evaluations of all attributes and attribute-by-attributes evaluations of all brands. The first sample of 198 subjects provided brand-to-attribute associations. For each one of the evoked brands, the subjects had to indicate which of the listed attributes is strongly associated with it. The question made during the interview was: “Which of these attributes particularly apply to this brand?”. The second sample of 203 subjects provided attribute-to-brand associations. For each attribute, they stated whether it applied to each of the brands they had evoked. The question asked during the interview was: “To which of these brands does this particularly apply?”. Collecting data in two different samples increases the differences between the brand-to-attribute and the attribute-to-brand associations, by introducing sample variability, which might underestimate the communality and overestimate the asymmetry. However, a within-subject design is not feasible in this case, as the first measurement will have a carry-over effect on the second measurement. The carry-over effect may be fairly large even after a considerable amount of time. In particular, a within-subject design had been used in our study, subjects would have learned about the stimuli and the strategies to be used in the association task and would probably have strived to be consistent across the two tasks, which may considerably bias the second set of associations (Poulton, 1982; Keren and Raaijmakers, 1988). Furthermore, a large amount of time between measures may result in a change of the actual brand associations.

The negative counterpart is that this type of data includes responses unfamiliar with a brand, the “forced-choice” procedure may give rise to response strategies such as guessing or ya-saying (Greenleaf, 1992). This more refined structure allows consumers with higher product experience to exercise greater discernment when recalling and/or evaluating brand-to-attribute associations (Mason et al., 2001).

The negative counterpart is that this type of data includes the usage effect, which implies that brands with larger market shares score higher on most attributes (Barnard and Ehrenberg, 1990). Finally, limiting the association task to evoked brands matches with the observation that customer-based brand equity occurs when a consumer is

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terms of their own terminology. This is a valid and common applied methodology to elicit relevant product attributes (Alpert, 1971; Breivik and Supphellen, 2003). In line with Keller (1993, 2003), we distinguish between category-specific attributes and non-specific, more general attributes (A9–A11).

4. Preliminary description of dominance, communalities, and asymmetries

To describe dominance relations, Loftus (1973), Ashcraft (1978) and Farquhar and Herr (1992) measured the strength of association simply by the frequency with which the item was mentioned, with normalizations that do not include information related to the number of evoked brands. For example, for the attribute A3 from the attribute-to-brand association table, the raw frequencies are:

\[ 59, 33, 40, 28, 32, 43, 79, 31, 29, 10 \]

Since the highest value corresponds to B7: Vaseline Intensive Care, following Ashcraft (1978), we take this brand as reference point and transform the data as percentages relative to it:

\[ [74.7, 41.8, 50.7, 35.4, 40.5, 54.4, 100, 39.2, 36.7, 12.7]. \]

Traditionally, dominance has been discretized into high and low dominance using somewhat arbitrary thresholds, e.g. at 50% by Ashcraft (1978). For attribute A3 we observe the high dominant property for brands B1: Dove, B3: Natrel Plus, B6: Sure, and B7: Vaseline Intensive Care.

This definition of dominance does not take into account that more well-known brands tend to receive higher values and so will establish more dominant relations. We improve the definition of dominance by analyzing percentages calculated from the frequencies relative to the number of times each brand is evoked. This results in tables that measure brand dominance independently from brand familiarity, which allows comparison of dominance and other evaluations across brands.

Table 2 shows the obtained brand-to-attribute association data: frequency of associations relative to the number of brand evocations. For example, the cell (B4, A1) tells us that 44.4% of the subjects who evoked Right Guard, associated it with attribute “prevents body odour all day”. The attribute-to-brand association table (Table 3), similarly expresses frequencies of attribute-to-brand associations relative to the number of times the brand is evoked. For example, the cell (B4, A1) tells us that 44.4% of the subjects who evoked Right Guard named it in response to “prevents body odour all day”.

Now, for attribute A3 (does not irritate my skin) there exists brand dominance in the cases for the following brands (again using 50% as cut-off point): B1: Dove, B7: Vaseline Intensive Care, B8: Secret and B9: Impulse. Notice that brand dominance changes when the brand familiarity effect is eliminated. Brands like B3: Natrel Plus and B6: Sure are highly evoked brands and have high dominance only when associations are not expressed relative to the number of evoked brands. On the other hand, B8: Secret, and B9: Impulse are not highly evoked brands but have high dominance when values are expressed relative to the frequency the brand is evoked.

We follow our exposition with the introduction of the idea of communalities and asymmetries in dominance relationships. For the cell (B1, A3) in the attribute-to-brand association table (Table 3), we find a value of 60.2%, indicating that a high brand dominance relation between Dove and “does not irritate my skin” exists. For the same pair (B1, A3) in the brand-to-attribute analysis (Table 2) the percentage is very similar (60.4%), so in this case associations in both directions share a common high dominance and asymmetry does not appear. The brand displays a strong brand image with respect to this attribute independent of the direction in which the associations are measured. But if we take another pair such as (A1, B10), in the brand-to-attribute case, the proportion of people who made the association is equal to 55.0%, which means that a high attribute dominance relationship exist between Body Shop and “prevents body odour all day” is given. However, the attribute-to-brand association takes a value of 22.7%, concluding that high brand dominance is not present. These percentages are quite different indicating that asymmetry between the directional associations exist. In the rest of this paper we will introduce and illustrate a methodology to explore the communalities and asymmetries of the bi-directional associations.

Table 2
Brand-to-attribute associations: percentages relative to the number of evoked brands

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>A8</th>
<th>A9</th>
<th>A10</th>
<th>A11</th>
<th>Times evoked</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>53</td>
<td>49</td>
<td>60</td>
<td>40</td>
<td>49</td>
<td>38</td>
<td>47</td>
<td>27</td>
<td>36</td>
<td>41</td>
<td>11</td>
<td>(101)</td>
</tr>
<tr>
<td>B2</td>
<td>35</td>
<td>19</td>
<td>33</td>
<td>59</td>
<td>69</td>
<td>35</td>
<td>39</td>
<td>60</td>
<td>47</td>
<td>45</td>
<td>13</td>
<td>(104)</td>
</tr>
<tr>
<td>B3</td>
<td>36</td>
<td>28</td>
<td>38</td>
<td>18</td>
<td>40</td>
<td>21</td>
<td>27</td>
<td>11</td>
<td>39</td>
<td>50</td>
<td>27</td>
<td>(103)</td>
</tr>
<tr>
<td>B4</td>
<td>45</td>
<td>38</td>
<td>43</td>
<td>22</td>
<td>41</td>
<td>24</td>
<td>24</td>
<td>20</td>
<td>17</td>
<td>47</td>
<td>9</td>
<td>(76)</td>
</tr>
<tr>
<td>B5</td>
<td>46</td>
<td>43</td>
<td>38</td>
<td>25</td>
<td>30</td>
<td>40</td>
<td>31</td>
<td>15</td>
<td>19</td>
<td>48</td>
<td>19</td>
<td>(93)</td>
</tr>
<tr>
<td>B6</td>
<td>45</td>
<td>35</td>
<td>48</td>
<td>34</td>
<td>54</td>
<td>40</td>
<td>35</td>
<td>24</td>
<td>27</td>
<td>47</td>
<td>18</td>
<td>(116)</td>
</tr>
<tr>
<td>B7</td>
<td>62</td>
<td>62</td>
<td>52</td>
<td>37</td>
<td>52</td>
<td>48</td>
<td>37</td>
<td>23</td>
<td>33</td>
<td>52</td>
<td>17</td>
<td>(155)</td>
</tr>
<tr>
<td>B8</td>
<td>38</td>
<td>46</td>
<td>47</td>
<td>18</td>
<td>44</td>
<td>27</td>
<td>29</td>
<td>22</td>
<td>24</td>
<td>44</td>
<td>6</td>
<td>(55)</td>
</tr>
<tr>
<td>B9</td>
<td>43</td>
<td>63</td>
<td>52</td>
<td>38</td>
<td>53</td>
<td>35</td>
<td>35</td>
<td>13</td>
<td>40</td>
<td>53</td>
<td>15</td>
<td>(60)</td>
</tr>
<tr>
<td>B10</td>
<td>55</td>
<td>25</td>
<td>70</td>
<td>35</td>
<td>55</td>
<td>25</td>
<td>35</td>
<td>25</td>
<td>50</td>
<td>30</td>
<td>5</td>
<td>(20)</td>
</tr>
</tbody>
</table>

5. Correspondence analysis of matched matrices

As the number of brands and/or attributes that define brand image increase, the total number of associations become very large. At some point, it becomes difficult to inspect the communalities and asymmetries of the relative associations from frequencies. Therefore, the application of correspondence analysis has been proposed for visualizing brand images in a perceptual map (e.g. Greenacre, 1984; Hoffman and Franke, 1986). This yields independent factors, which define orthogonal dimensions of a perceptual map, where brand and attributes are points projected on the map. The center of the map, or centroid, can be interpreted as an “average brand” characterized as having an average degree of association with each attribute. Then, dominance relationships between a brand and an attribute will be established relative to that average brand. Then, one advantage is that, this sets us free from fixing a percentage threshold for high dominance (e.g. 50%). As a consequence, we can describe high dominance relationships also for attributes with low averages, while with the previous definitions, these might not be recognized as having any high dominance relations. Furthermore, it makes brands directly comparable in terms of the attributes, and so, the results describe the degree of uniqueness in associations between a brand and an attribute in a particular sample.

Separate correspondence analyses of the brand-to-attribute and attribute-to-brand associations will display each of the set of bi-directional associations separately in a perceptual map. In empirical studies, these two maps will only be the same to some degree. However, determining the differences and communalities will be hard, if not impossible, by simply inspecting the two maps. Therefore, we introduce correspondence analysis of matched matrices (Greenacre, 2003; Greenacre and Clavel, 2002) to the customer-based brand equity field to capture communalities and asymmetries in brand image due to the directionality of the associations. In particular, the communality will capture those associations which are independent of the direction of making evaluations (brand-to-attribute or attribute-to-brand), and the asymmetry will capture those associations which change due to the direction of making evaluations as well as the source of asymmetry (attributes or brands). Applying correspondence analysis of matched matrices to the tables of frequencies relative to the number of times the brand is evoked yields a visualization of the communalities and asymmetries between the two sets of associations.

The methodology we present consists of applying correspondence analysis to the two matrices, attribute-to-brand and brand-to-attribute associations, combined in a particular block format. This leads to an analysis of the centered sum and the uncentered difference components, which are desirable properties: (a) recovering the centered sum still enables describing strength of the associations relative to a “mean brand” correspondence analysis to the sum and (b) the displayed asymmetries are uncentered, which allows interpretation of locations in the map as positive or negative differences.

Greenacre (2003) shows that, if $B$ and $A$ are two $n \times m$ matrices, where $B$ represents the brand-to-attribute association table measuring attribute dominance and $A$ the attribute-to-brand association table measuring brand dominance, the data matrix to apply correspondence analysis takes the form:

$$
\begin{bmatrix}
B & A \\
A & B
\end{bmatrix}
$$

The sum component $B + A$ will capture communalities: associations which are high dominant relations (compared to a “mean brand”) in both tables, that means independently of the direction of making evaluations. The difference component $B - A$ will capture asymmetries between the two association matrices. Note that the communalities can also be assessed with a simple correspondence analysis of the sum of the two matrices ($B + A$), which is not the case for the difference component.

If $A$ and $B$ are two tables coming from different subsamples, the total or marginal frequencies may differ, like in our case, where we have different “total evoked” for each brand. The asymmetry analysis would reflect differences due to different totals between both samples and not due to the directionality. However, by analyzing observed percentages relative to the number of times the brand is evoked, this issue is solved in the approach presented here.

We are not going to describe correspondence analysis in detail since it is not the aim of this paper. We expose the
correspondence analysis concepts that a marketing researcher needs, to interpret the correspondence analysis output: the maps, once CA is applied to the block matrix presented as (1). Some geometrical aspects are included as an Appendix. For more details of this technique we refer to Greenacre (1984).

The results of correspondence analysis can be visualized in a map, based on the decomposition of the inertia or variance, from the data matrix (Greenacre, 1984; Blasius and Greenacre, 1994). In our case, some dimensions represent the asymmetries. The inertia of each one of these dimensions can be decomposed into components in the following way:

\[
\begin{bmatrix}
D_{xx} & 0 \\
0 & D_{yy}
\end{bmatrix} = F^T D F = G^T D G.
\]  

(4)

where \(F\) is the matrix collecting principal coordinates for rows (brands), \(G\) is the matrix collecting principal coordinates for columns (attributes), \(D_{xx}\) and \(D_{yy}\) are the diagonal matrices with the principal inertias of sum and difference components respectively in their main diagonal. Greenacre (1984) shows that singular values are the square roots of the principal inertias (\(D = D_{1}/D_{2}\) and \(D_{c} = D_{3}/D_{4}\)). Finally, \(D_{x}\) and \(D_{y}\) are diagonal matrices with the row masses \(r_{i}\) (row sums divided by the grand total) and column masses \(c_{j}\) (column sums divided by the grand total) in their main diagonal, respectively.

The contribution of inertia shows to what extent the geometric interpretation of an axis is determined by the variable categories (Blasius and Greenacre, 1994). In our particular application this can be used to assess which attributes have contributed most to the principal axis and thereby to give meaning to the obtained dimensions. The contribution of the columns (attributes) to the principal inertia are defined as the inertia components relative to their total. For the communality analysis, taking the elements of matrices appearing on (4), for each particular attribute the values is \((c_{i}g_{i}^{2}/\lambda_{i})\) and for the asymmetry analysis, it becomes \((c_{i}g_{i}^{2}/\lambda_{i+})\). Given the configuration of the data matrix (see (1)), the sum of contributions for each point to the principal axis do not add up 1 but 0.5. Then, the highest values, or the ones closer to 0.5, will correspond to attributes that contribute most in the meaning of that axis. The decomposition of the inertia gives these values in absolute terms. We will examine correlation values for rows (brands) and columns (attributes) to complement the interpretation of dimensions as well as to see which brands are well represented by them.

From matrices on (4), the values corresponding to the squared correlations of the rows/columns with the principal axes are the inertia components \(r_{i}f_{i}^{2}(c_{i}g_{i}^{2})\) expressed relative to the row (brand)/column (attribute) inertia, which is \(\sum r_{i}f_{i}^{2}(\sum c_{i}g_{i}^{2})\). The higher value for each item can be 1. Then highest values, or values closer to 1 will correspond to brands/attributes more correlated to that particular axis.

The graphical interpretation of communalities is made in the usual way, such that, in a symmetric map (Greenacre, 1984) closer relative positions between points (brands and attributes) will be understood as a higher association with respect to a centroid value. The interpretation of the asymmetry map needs some clarifications. To justify this graphical interpretation we should go to the geometry of correspondence analysis (Greenacre, 1984). Correspondence analysis can be understood as to apply SVD to a data matrix which is centered and standardized in the following way:

\[
D_{x}^{-1/2}(P - rc^{T})D_{y}^{-1/2} = M\Gamma N^{T},
\]

where \(P\) is the original data set, where each cell collects the frequencies describing the association between rows and columns category levels but, divided by the grand total; \(M\) and \(N\) correspond to the singular vectors of the rows and columns respectively; and \(\Gamma\) is a diagonal matrix with the singular values in its main diagonal. Once the terms arranged, we can get the following expression:

\[
(P - rc^{T}) = FL^{2}D_{c},
\]

(5)

where \(L (= D_{x}^{-1/2}N)\) represents de standard coordinates for columns (attributes). Greenacre (2003) shows that the total sum-of-squares in the block matrix (1), once centered, can be decomposed into two components, one component due to the matrix sum (the communalities) and one component due to the matrix difference (the asymmetries) in the following way:

\[
\sum a_{ij}^2 + \sum b_{ij}^2 = \sum (a_{ij} - c_{j} + b_{ij} - c_{j})^2 + \sum (a_{ij} - b_{ij})^2
\]

so that the component corresponding to the matrix difference is clearly uncentered. Then the expression (5) is simplified since the centroid becomes zero and the data matrix collects differences. For each cell of the correspondence analysis matrix \(P\), the resulting expression is the following: \(p_{ij} = \frac{1}{n}(b_{ij} - a_{ij})\), where \(b_{ij}\) and \(a_{ij}\) are the cells corresponding to the combination of brand \(i\) and attribute \(j\) of tables B (attribute-by-brand) and A (brand-by-attribute) respectively. If \(p_{ij}\) is positive, it is due to the same sign in both coordinates, \(f_{ik}\) and \(l_{ik}\). If the sign of \(p_{ij}\) is negative, it is due to differences of the signs of the coordinates. These signs determine the positions in the maps as we describe in the next section.

6. Empirical results

Part of the total inertia (0.064) of the analysis corresponds to the communality between both tables (\(\sum \lambda_{i} = 0.042\), while the rest is due to the asymmetries between the brand images described in the studies (\(\sum \lambda_{i} = 0.021\)). The two principal axes with higher inertia corresponding to the communality are the first one (with inertia of 0.024) and the third one (with inertia of 0.008), which represent 57.14 % and 19.05% respectively of the inertia.

of communality. Hence, together these two dimensions explain 76.19% of the communality variance. The two principal axes with highest inertia corresponding to the differences between both analyses are the second one (with inertia of 0.01) and the forth one (with inertia of 0.006). The first value represents 47.62% of the total inertia of asymmetries and the second one 28.57%. Then, both together represent 76.19% of the variance of the asymmetries. From the relative values of the inertia, we can see that the asymmetries are less important than the communalias in this particular application, but some differences between bi-directional associations of some brands can still be captured and interpreted. Next, we describe the communality between both types of associations, followed by the asymmetries.

6.1. Communalities

For interpretation of the maps, we display the coordinates ($k$) and correlation values (CORR) for brands with respect to the first and the second principal axes of communalias. The meaning of principal axes displaying commumalities is determined by the contribution (CTR) and correlation values of the attributes with respect to these axes. This information is collected in Table 4.

Since the inertia for the sum and the difference are not very high, we display symmetric maps (Greenacre, 1984), in other words, a map with principal coordinates for attributes and another map with principal coordinates for brands. Since points for brands and points for all attributes are in different spaces, only relative positions can be interpreted and not distances between points of different category variables. The symmetric maps are represented in Figs. 1 and 2.

Table 4

<table>
<thead>
<tr>
<th>k = 1</th>
<th>CORR</th>
<th>CTR</th>
<th>Inertia</th>
<th>k = 3</th>
<th>CORR</th>
<th>CTR</th>
<th>Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>-0.0078</td>
<td>0.0079</td>
<td>-</td>
<td>-</td>
<td>-0.0214</td>
<td>0.0939</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>0.3957</td>
<td>0.9701</td>
<td>-</td>
<td>-</td>
<td>-0.0612</td>
<td>0.0232</td>
<td>-</td>
</tr>
<tr>
<td>B3</td>
<td>-0.0264</td>
<td>0.0118</td>
<td>-</td>
<td>-</td>
<td>0.1898</td>
<td>0.6100</td>
<td>-</td>
</tr>
<tr>
<td>B4</td>
<td>-0.0748</td>
<td>0.2628</td>
<td>-</td>
<td>-</td>
<td>-0.0838</td>
<td>0.3300</td>
<td>-</td>
</tr>
<tr>
<td>B5</td>
<td>-0.1463</td>
<td>0.5362</td>
<td>-</td>
<td>-</td>
<td>-0.0981</td>
<td>0.2412</td>
<td>-</td>
</tr>
<tr>
<td>B6</td>
<td>0.0257</td>
<td>0.0520</td>
<td>-</td>
<td>-</td>
<td>0.0224</td>
<td>0.0395</td>
<td>-</td>
</tr>
<tr>
<td>B7</td>
<td>-0.1124</td>
<td>0.5412</td>
<td>-</td>
<td>-</td>
<td>-0.0825</td>
<td>0.2921</td>
<td>-</td>
</tr>
<tr>
<td>B8</td>
<td>-0.0561</td>
<td>0.3034</td>
<td>-</td>
<td>-</td>
<td>-0.0164</td>
<td>0.0258</td>
<td>-</td>
</tr>
<tr>
<td>B9</td>
<td>-0.1088</td>
<td>0.3898</td>
<td>-</td>
<td>-</td>
<td>0.0411</td>
<td>0.0555</td>
<td>-</td>
</tr>
<tr>
<td>B10</td>
<td>0.0460</td>
<td>0.0470</td>
<td>-</td>
<td>-</td>
<td>0.1534</td>
<td>0.5241</td>
<td>-</td>
</tr>
<tr>
<td>A1</td>
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<td>0.7018</td>
<td>0.0041</td>
<td>0.0000984</td>
<td>-0.0455</td>
<td>0.0752</td>
<td>0.0013</td>
</tr>
<tr>
<td>A2</td>
<td>-0.2265</td>
<td>0.7749</td>
<td>0.0104</td>
<td>0.0002496</td>
<td>-0.0933</td>
<td>0.1314</td>
<td>0.048</td>
</tr>
<tr>
<td>A3</td>
<td>-0.0961</td>
<td>0.3085</td>
<td>0.0023</td>
<td>0.0000552</td>
<td>0.0780</td>
<td>0.2032</td>
<td>0.040</td>
</tr>
<tr>
<td>A4</td>
<td>0.1919</td>
<td>0.7713</td>
<td>0.006</td>
<td>0.000144</td>
<td>-0.0756</td>
<td>0.1196</td>
<td>0.028</td>
</tr>
<tr>
<td>A5</td>
<td>0.1007</td>
<td>0.6304</td>
<td>0.0023</td>
<td>0.0000552</td>
<td>0.0442</td>
<td>0.1215</td>
<td>0.011</td>
</tr>
<tr>
<td>A6</td>
<td>-0.0816</td>
<td>0.2807</td>
<td>0.0012</td>
<td>0.0000288</td>
<td>-0.1097</td>
<td>0.5068</td>
<td>0.060</td>
</tr>
<tr>
<td>A7</td>
<td>0.0119</td>
<td>0.0260</td>
<td>0</td>
<td>0</td>
<td>0.0437</td>
<td>0.3499</td>
<td>0.009</td>
</tr>
<tr>
<td>A8</td>
<td>0.4003</td>
<td>0.8902</td>
<td>0.0223</td>
<td>0.0000552</td>
<td>-0.1169</td>
<td>0.0759</td>
<td>0.055</td>
</tr>
<tr>
<td>A9</td>
<td>0.0637</td>
<td>0.1166</td>
<td>0.009</td>
<td>0.000216</td>
<td>0.1566</td>
<td>0.7054</td>
<td>0.154</td>
</tr>
<tr>
<td>A10</td>
<td>-0.0500</td>
<td>0.1827</td>
<td>0.006</td>
<td>0.000144</td>
<td>-0.0312</td>
<td>0.0714</td>
<td>0.006</td>
</tr>
<tr>
<td>A11</td>
<td>-0.0062</td>
<td>0.0003</td>
<td>0</td>
<td>0</td>
<td>0.1742</td>
<td>0.2491</td>
<td>0.076</td>
</tr>
</tbody>
</table>

The attribute which contributes most to the first principal axis is A8: can use all over the body. The contributions for the second principal axis of communalias are quite small. Then, we check the attributes correlated with such axes to complement the map interpretation. For the first principal axis, on the right hand side, besides A8, it is located A4: has a long lasting fragrance and A5: has a pleasant fragrance. On the left hand side, A2: keeps me dry all day and A1: prevents body odour all day. For the second principal axis, a high correlation is encountered by attribute A6: leaves me feeling confident, located close to durability attributes. A high correlation is also given by A9: is portable. Hence, the first principal axis differentiates between deodorants that can be use all over the body, more related with fragrance benefits, and, attributes related with benefits of durability and confidence. On the other hand, attributes implying benefits not related with fragrance and durability, like not irritate the skin, leaves no marks on clothes, or being portable, display association with the second principal axis of communalias. From the map we appreciate that attributes related with the first principal axes of communalias, the one capturing higher level of inertia, are related to benefits specific for the deodorants product category.

As an example of the interpretation, consider Brand B2: Mum. It is positioned on the right hand side of Fig. 1 and displays a strong association with A8: can be used all over the body and A4: has a long lasting fragrance. So, Mum is related with “fragrance” consistently for both directions of associations. The map also informs about a strong association between the brands B5: Soft&Gentle and B7: Vaseline Intensive Care and the attribute A2: keeps me dry all day. Considering both directions of associations, these brands display a brand image related to “durability.”
Finally, one brand in the sample, B10: Body Shop, displays a stronger association with A9: being portable, independently of the situation. This is the brand with lower level of involvement in our sample.

6.2. Asymmetries

For the asymmetry component, the coordinates, the correlation values for brands, and the contribution and correlation values for attributes appear in Table 5. The symmetric maps are presented in Figs. 3 and 4. A close relative position between a brand, from Fig. 4, and an attribute, from Fig. 3, means that the association was stronger in the attribute-to-brand association table (attribute dominance) than in the brand-to-attribute association table (brand dominance).

The main explanatory attributes in the map of asymmetries are not category-specific attributes. A9: to be portable and A10: quick to apply have important correlations with the first principal axis of differences. A minor following value is given by A5: has a pleasant fragrance. A1: prevents body odour all day, a category-specific attribute, displays the highest correlation with respect to the second principal axis of asymmetries.

Brands situated further from the centroid (0,0) are the ones displaying higher differences between both directional associations. In our case, this holds in particular for B8 (with respect to the first principal axis of difference) and B10 (with respect to second principal axis of difference), which are the brands lowest in familiarity.

The deodorant brand B8: Secret is situated just opposite to A9. It means that Secret is strongly associated with the attribute of “being a portable deodorant” when we ask attribute-to-brand but not when we ask brand-to-attribute. In managerial terms we can conclude that Secret is not positioned as a portable brand, but it nevertheless has a stronger association with respect to this attribute compared to other brands of the sample. Actually, a huge set of brands: B8, B1, B4, B3, B5 and B9, are situated in the opposite position (counter signs for first and second principal axes of asymmetry) with respect to A9: to be portable. This means that these brands receive a higher percentage of association with respect to that attribute in the attribute-to-brand condition compared to the brand-to-attribute condition, which indicates that A9 is a general attribute and not specifically linked with one of the brands.

The other brand displaying higher asymmetries in the map is B10: Body Shop. Its close relative position with respect to A1: prevents body odour all day, indicates that Body Shop is positioned (brand-to-attribute associations) with an image of durability, a benefit which is not perceived as a competitive advantage (attribute-to-brand associations) of this brand in the set of the analyzed brands. However, this brand is more associated with the attributes A10: is quick to apply in the attribute-to-brand compared to the brand-to-attribute associations, as B10 is situated opposite to A10.

From the application we conclude that the researcher should consider both attribute-to-brand and brand-to-attribute associations to describe brand images. Consumers may consider a limited set of brands and establish their perceptions and purchase behavior based on the uniqueness and strength of brand-to-attribute associations. In other cases, consumers may have in mind some attribute and the set of competing brands is established depending on the strength of the links that start in that particular attribute. For example, Thelen and Woodside (1997) showed the importance of strong attribute-to-brand associations for choices among alternatives (stores or brands). Non-product-specific attributes turn out to be important sources
of asymmetries and they display a minor role in the analysis of communalities, where more well-known brands are specially represented. If the asymmetries are not involved in dominant relationships, the source of asymmetry can be due to the higher number of answers in the attribute-to-brand association tasks for the non-specific attributes. An explanation could be that these are secondary attributes in the positioning of the brands.

### Table 5
Asymmetry results

<table>
<thead>
<tr>
<th></th>
<th>k = 2</th>
<th></th>
<th></th>
<th></th>
<th>k = 4</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.0966</td>
<td>0.6321</td>
<td>–</td>
<td>–</td>
<td>0.0318</td>
<td>0.0684</td>
<td>–</td>
</tr>
<tr>
<td>B2</td>
<td>0.0818</td>
<td>0.6673</td>
<td>–</td>
<td>–</td>
<td>-0.0136</td>
<td>0.0184</td>
<td>–</td>
</tr>
<tr>
<td>B3</td>
<td>0.0667</td>
<td>0.3152</td>
<td>–</td>
<td>–</td>
<td>0.0680</td>
<td>0.3279</td>
<td>–</td>
</tr>
<tr>
<td>B4</td>
<td>0.0842</td>
<td>0.4418</td>
<td>–</td>
<td>–</td>
<td>0.0765</td>
<td>0.3648</td>
<td>–</td>
</tr>
<tr>
<td>B5</td>
<td>0.0790</td>
<td>0.3233</td>
<td>–</td>
<td>–</td>
<td>0.0354</td>
<td>0.0650</td>
<td>–</td>
</tr>
<tr>
<td>B6</td>
<td>0.1198</td>
<td>0.7110</td>
<td>–</td>
<td>–</td>
<td>-0.0530</td>
<td>0.1393</td>
<td>–</td>
</tr>
<tr>
<td>B7</td>
<td>0.0756</td>
<td>0.8277</td>
<td>–</td>
<td>–</td>
<td>-0.0101</td>
<td>0.0149</td>
<td>–</td>
</tr>
<tr>
<td>B8</td>
<td>0.1638</td>
<td>0.6336</td>
<td>–</td>
<td>–</td>
<td>0.0669</td>
<td>0.1058</td>
<td>–</td>
</tr>
<tr>
<td>B9</td>
<td>0.0785</td>
<td>0.2529</td>
<td>–</td>
<td>–</td>
<td>0.0179</td>
<td>0.0131</td>
<td>–</td>
</tr>
<tr>
<td>B10</td>
<td>0.1030</td>
<td>0.1952</td>
<td>–</td>
<td>–</td>
<td>-0.2054</td>
<td>0.7766</td>
<td>–</td>
</tr>
<tr>
<td>A1</td>
<td>0.0833</td>
<td>0.3185</td>
<td>0.038</td>
<td>–</td>
<td>0.00038</td>
<td>0.5741</td>
<td>0.111</td>
</tr>
<tr>
<td>A2</td>
<td>0.0689</td>
<td>0.2728</td>
<td>0.023</td>
<td>–</td>
<td>0.00023</td>
<td>0.0065</td>
<td>0.0025</td>
</tr>
<tr>
<td>A3</td>
<td>0.0207</td>
<td>0.0552</td>
<td>0.002</td>
<td>–</td>
<td>0.00002</td>
<td>0.7361</td>
<td>0.057</td>
</tr>
<tr>
<td>A4</td>
<td>0.0148</td>
<td>0.0116</td>
<td>0.001</td>
<td>–</td>
<td>0.00001</td>
<td>0.5991</td>
<td>0.078</td>
</tr>
<tr>
<td>A5</td>
<td>0.0922</td>
<td>0.5928</td>
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<td>–</td>
<td>0.00049</td>
<td>0.2218</td>
<td>0.032</td>
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<td>A6</td>
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<td>–</td>
<td>0.00001</td>
<td>0.4969</td>
<td>0.046</td>
</tr>
<tr>
<td>A7</td>
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<td>0.1747</td>
<td>0.005</td>
<td>–</td>
<td>0.00005</td>
<td>0.1039</td>
<td>0.005</td>
</tr>
<tr>
<td>A8</td>
<td>-0.1034</td>
<td>0.6054</td>
<td>0.037</td>
<td>–</td>
<td>0.00037</td>
<td>0.1963</td>
<td>0.017</td>
</tr>
<tr>
<td>A9</td>
<td>-0.2115</td>
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<td>0.1917</td>
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</tr>
<tr>
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<td>0.7341</td>
<td>0.073</td>
<td>0.00073</td>
<td>0.0216</td>
<td>0.0299</td>
<td>0.007</td>
</tr>
<tr>
<td>A11</td>
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<td>0.024</td>
<td>0.00024</td>
<td>0.1241</td>
<td>0.1846</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Principal coordinates (k), correlation values (CORR), contribution values (CTR) and inertia decomposition for attributes.

### 7. Stability of the solutions

To examine the stability of the maps obtained through correspondence analysis of matched matrices, we modify and apply bootstrap resampling procedures as proposed by Lebart (2006) and Greenacre (2007, Chapter 25). The idea of bootstrap is to treat the observed data as the population, and then create new data sets by resampling from
the data. As the raw data is not available in this study, we follow the approach of resampling from the frequency distribution of the original data set. In particular, the procedure we apply consists of the following steps:

1. Binomial sampling to obtain the number of times each brand is evoked, using the original sample sizes (198 and 203) and the original proportions of respondents who evoked each brand (Tables 2 and 3).
2. Binomial sampling to obtain the number of brand-to-attribute and attribute-to-brand associations, using the numbers obtained in step 1 and the original proportions for each association (Tables 2 and 3).
3. Correspondence Analysis of Matched Matrices of the association matrices obtained in step 2.
4. Procrustes rotation of the new results obtained in step 3 to the solution based on the original data set.
5. Repeat steps 1–4 a substantial number of times. Here, we use 100 replications.
6. For each brand and attribute, computation of the convex hull containing a certain percentage of the points. Here, we restrict the convex hull to the 90% points that are closest to the centroid.

The results of the bootstrap procedure are presented in Figs. 5 and 6, for the communality analysis and Figs. 7 and 8 for the asymmetry one. The confidence regions differ considerably in size across brands and attributes. Whereas for most points the confidence region is small to moderate, for a few points the region is fairly large. If the observed number of respondents who know the brand is low, the binomial sampling in step 2 of our bootstrap procedure will be based on a small number of draws. As a result, the proportion for the associations with this brand will fluctuate substantially. The brand point will become unstable and the convex hull becomes relatively large. This is what we can observe for brand 10: The Body Shop. Similarly, the number of associations with attribute 11 (costs a little less than most others) is relatively small, which leads to large convex hull regions for that attribute. Thus, bootstrap results point out that one has to be cautious when considering associations based on small sample sizes, which can be due to brands that are unfamiliar to most respondents or attributes that are not relevant for most respondents.

The convex hulls of many brands (and attributes) show some overlap. For example, in the lower left part of Fig. 5,
the confidence region of B5 overlaps with that of B4 and B7, and B9. The association communalities of these three brands are not significantly different from those of B5. This indicates a similar positioning by the brands, competition between the brands and suggests potential brand confusion. On the other hand, the convex hull of brand B2 is rather small and does not overlap with a convex hull of another brand, which suggests a clear, distinctive positioning of this brand.

Next to the total bootstrap procedure described above, we applied the partial bootstrap method suggested by Lebart (2006) and Greenacre (2007, Chapter 25). The partial bootstrap procedure does not include a correspondence analyses of matched matrices on the new data set (steps 3 and 4), but projects the rows and columns of the new data set using the original correspondence analysis of matched matrices solution. In this empirical study, the partial bootstrap procedure yields very small regions for the convex hulls, as shown in Fig. 9 for the symmetric map of the brands. We consider the total bootstrap procedure to be more correct, though also more conservative. The total bootstrap procedure correctly mirrors the situation that a researcher would have to perform correspondence analysis of matched matrices when another (bootstrap) sample is available, whereas the partial bootstrap procedure makes the less realistic assumption that the correspondence analysis of matched matrices projections are somehow available.

8. Discussion, conclusions and future research

We have addressed the problem of communalities and asymmetries in brand-to-attribute and attribute-to-brand associations. This topic has been addressed before, but basically focused on category-brand associations and tended to be conceptual (e.g. Farquhar and Herr, 1992). We show the relevance of taking into consideration bi-directional associations when assessing and visualizing brand image. We introduced a particular way of correspondence analysis, named correspondence analysis of matched matrices (Greenacre, 2003; Greenacre and Clavel, 2002), as a tool for studying communalities and potential asymmetries between brand-to-attribute and attribute-to-brand association tables. It identifies the brands and attributes which are critical sources of asymmetries.

We can corroborate that when the association between brands and attributes is measured asking brand-to-attribute associations, which is a non-comparative format, the stronger links from the brands to the attributes dominate the associations. On the other hand, if a researcher measures brand image asking attribute-to-brand associations (a comparative format), stronger links from the attributes to the brands will determine the perceptions of the consumers. Non-product-related attributes, which are less core attributes related with the product category, have received more associations in the attribute-to-brand (brand dominance) tables. Thus, we suggest that both directions of associations should be considered when brand image is assessed to make managerial recommendations.

It is largely unclear to what extent certain types of products, brands, and/or attributes will display large communalities or asymmetries between the two directions of associations. One might argue that brands which are positioned (captured by the brand-to-attribute associations table) with respect to attributes that differ from their competitive advantage (captured by the attribute-to-brand association table) should display substantial asymmetries. In addition, leading brands or more mature brands (more associated with unique attributes or, if shared ones with the category product, related to present stronger associations with respect to other brands) may display the same brand associations in both directions, while secondary
brands in a particular market may display large asymmetries. Finally, the nature of the attributes may affect the degree of asymmetry between both tables. While category-specific attributes could show high communality, non-category-specific attributes could show high asymmetry. In our empirical study, the more general attributes “portable”, “quick to apply”, and “price” indeed show relatively large asymmetries. These attributes are apparently not the first ones people consider to describe a brand image. However, they do display high attribute-to-brand associations for many brands, and may therefore be important in specific purchase situations such as for traveling or for going to the gym. Further research is required to examine the drivers and consequences of communalities and asymmetries between brand-to-attribute and attribute-to-brand associations.

To examine the stability of the CAMM solution, we modified and applied bootstrap resampling procedures. In particular, we performed binomial sampling for each cell of both matrices (Tables 2 and 3), which reflects the data generating process. Next, on each set of new data matrices we performed CAMM and rotated the new solution to the original one. The stability of most brand and attribute points is satisfactory. However, points based on small sample sizes, for example brands evoked by just a few respondents, have relatively large confidence regions. The brand with the smallest sample size (B10: 20 and 22) has a large confidence region, whereas all other brands (with a minimum sample size of 55 and 61) have considerably smaller confidence regions. This underlines the need for a sufficient number of observations. We also performed a partial bootstrap, in which CAMM is not applied to the new data matrices, but instead the new columns and rows are simply exposed, are the fact that the points have different weights projected into the original solution. The confidence regions obtained in this way are extremely small and much smaller than obtained with the full bootstrap procedure. An interesting area for future research would be to study the differences between these and other bootstrap procedures for correspondence analysis.

It could also be interesting to study the time dynamics of the communalities and asymmetries of the associations. If asymmetries are not desired (i.e. signal of secondary brands or due to divergences between competitive advantage and positioning), the analysis of matched matrices applied to different periods could describe the change of such asymmetries due to managerial decisions. This would require the data to be combined as follows (Greenacre and Clavel, 2002):

\[
\begin{bmatrix}
A_1 & B_1 & A_2 & B_2 \\
B_1 & A_1 & B_2 & A_2 \\
A_2 & B_2 & A_1 & B_1 \\
B_2 & A_2 & B_1 & A_1
\end{bmatrix},
\]

where \(A_1\) and \(B_1\) represent the attribute-to-brand and the brand-to-attribute association tables at period 1 and the \(A_2\) and \(B_2\) collects the same tables at period 2. Once we apply correspondence analysis to the previous expression, we are able to recover asymmetries as well as communalities, and to visualize the changes over time.

Acknowledgements

The authors thank Alain de Beuckelaer for the data and information used in the empirical application, and Erjen van Nierop and Frederieke van Perlo – Ten Kleij for comments on previous versions. Anna Torres acknowledge the MEC (SEJ 2004-00672/ECON and SEJ2006-14098) for research funding support.

Appendix. Let the singular value decomposition (SVD) (Eckart and Young, 1936) of \(B + A\) and \(B - A\) be respectively:

\[
B + A = UD_AV^T, \quad B - A = XD_BY^T,
\]

where \(U, V, X\) and \(Y\) are singular vectors of the sum and differences, respectively, and \(D_A\) and \(D_B\) are diagonal matrices with the singular values of the communalities and asymmetries, respectively, in their main diagonal. Then the singular value decomposition of the \(2n \times 2m\) block matrix is (Greenacre, 2003):

\[
\begin{bmatrix}
B & A \\
A & B
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
U & X \\
U & -X
\end{bmatrix} \begin{bmatrix}
D_2 & 0 \\
0 & D_2
\end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix}
V & Y \\
V & -Y
\end{bmatrix}^T,
\]

where

\[
\frac{1}{\sqrt{2}} \begin{bmatrix}
U^T \\
U
\end{bmatrix}^T \frac{1}{\sqrt{2}} \begin{bmatrix}
U \\
U
\end{bmatrix} = \frac{1}{2} U^T U + \frac{1}{2} U^T U = I.
\]

Thus the solutions corresponding to the difference component appear as repeated vectors with a change in sign in the singular vectors. The differences with respect to just apply a singular value decomposition to the data matrix previously exposed, are the fact that the points have different weights in correspondence analysis and that there is a standardization in the form of the chi-square metric (Greenacre, 1984).

References


