

UNIVERSITAT POMPEU FABRA

EMPIRICAL FINANCE

Final exam (14/12/99)

Good luck! Perfect score 110.

1. One basic fact on financial microstructure is the U shape of intraday volume. Give an explanation of this fact based on the intraday flow of information or on adverse selection.(10 points)

2. Consider the sequence of transaction prices $P(t_0), P(t_1), \dots, P(t_n)$ and denote by Z_k the corresponding price change which is supposed to be an integer multiple of a 'tick'. Let $Z_k^* = X_k' \beta + \epsilon_k$ where ϵ_k is a white noise and the 'desired' price change is unobservable. The observed price change, Z_k , is related with the unobservable Z_k^* in the following way

$$s_1 \quad \text{if} \quad Z_k^* \in (-\infty, \alpha_1] \quad (1)$$

$$s_2 \quad \text{if} \quad Z_k^* \in (\alpha_1, \alpha_2] \quad (2)$$

$$\dots \quad \dots \quad (3)$$

$$s_m \quad \text{if} \quad Z_k^* \in (\alpha_{m-1}, \infty) \quad (4)$$

Write down the density and the log-likelihood function of this process. Explain.(20 points)

3. With serial correlation in trade flows the conditional expectation of the trade indicator at time t-1 given the indicator at t-2, Q_{t-2} , is

$$E(Q_{t-1}|Q_{t-2}) = (1 - 2\pi)Q_{t-2} \quad (5)$$

where π is the probability that the trade at t is opposite in sign to the trade at t-1.

a) Show the above result.

b) Assuming that market participants know that conditional expectation show how to obtain the change in the fundamental value

$$\Delta V_t = \alpha S/2Q_{t-1} - \alpha S/2(1 - 2\pi)Q_{t-2} + \epsilon_t \quad (6)$$

c) Show how to obtain the final equation of the three way decomposition

$$\Delta P_t = S/2Q_t + (\alpha + \beta - 1)S/2Q_{t-1} - \alpha S/2(1 - 2\pi)Q_{t-2} + \epsilon_t \quad (7)$$

using the change in the fundamental, the definition of the midpoint bid-ask spread and the relationship between prices and midpoints

$$P_t = M_t + S/2Q_t + \eta_t \quad (8)$$

assuming that S is constant.

d) Write down the orthogonality conditions that would allow you to estimate the parameters using GMM. (40 points)

4. In the mixture distribution model at the time of the i th within-day equilibrium the desired position Q_{ij} of the j th trader is given by a linear relationship

$$Q_{ij} = \alpha(P_{ij}^* - P_i) \quad (9)$$

where α is a constant, P_{ij}^* is the j th trader's reservation price and P_i is the current market price. If change in the reservation price of agents is function of two components, public information and private information, in the following way

$$\Delta P_{ij}^* = \phi_i + \psi_{ij} \quad (10)$$

$$E(\phi_i) = E(\psi_i) = 0 \quad (11)$$

$$var(\phi_i) = \sigma_\phi^2 \quad (12)$$

$$var(\psi_i) = \sigma_\psi^2 \quad (13)$$

$$(14)$$

and ϕ and ψ are mutually independent across traders and through time.

a) Show what is the change in prices equal to in terms of the reservation prices and in terms of the two stochastic components.

b) Interpret public and private information in the context of this model.

c) Show that the distribution of de change in prices is

$$\Delta P_i \sim N(\mu_1, \sigma_1^2) \quad (15)$$

Derive the values of μ_1 and σ_1^2 in function of the moments of the two stochastic components.(30 points)

5. What are the advantages of the Weibull Autoregressive Conditional Duration model over the Exponential Autoregressive Duration Model? Show the relationship between ARCH-GARCH models and Autoregressive Conditional Duration models (ACD)? (10 points)