

UNIVERSITAT POMPEU FABRA

APPLIED MACROECONOMICS

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Problem Set 2

Due: 25/10/04

1. Imagine a Ramsey model like the one we solved in class but assume that the utility function is $u(c_t) = \log(c_t)$ and the production function is $f(k_t) = k_t^\alpha$. There is no technical progress ($g=0$).

a) Solve for the central planner solution and show that it is the same as the competitive equilibrium.

b) Is there any important difference with respect to the results we show in class?

c) Discuss the evolution of the savings rate in the process of convergence.

d) Solve the same problem but assuming that there is labor augmenting technical change.

2. Consider a Ramsey model in discrete time

$$\max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(C_t)$$
$$Y_t \geq C_t + K_{t-1} - (1-\delta)K_t$$
$$N_t = N_0 n^t$$

where n is the growth rate of population. The utility function is the one we use in class and the production function is a Cobb-Douglas. There is no technical change.

a) Solve this model (central planner solution is fine).

b) Compare your results with the solution in continuous time.

3. Let H denote the stock of housing, I the rate of investment in new housing and p_H the real price of housing. Assume that the supply of new

houses can be characterized by $I(p_H)$, the rate of investment, where $I'(\cdot) > 0$. Assume the housing stock deteriorates at rate δ . The demand for housing services can be characterized by the function $R(H)$ where R is the rent and $R' < 0$. The real interest rate is constant and equal to r .

- a) Derive the equation relating $R(H), p_H, r$ and δ .
- b) Construct the phase diagram in the (H, p_H) space. Explain the locus of the $\dot{H} = 0$ and $\dot{p}_H = 0$. Describe the saddle path and the dynamics of the system.
- c) What is the steady state effect on H, p_H, I and R of an increase in the real interest rate, r . What is the effect of a temporary increase in the real interest rate?
- d) Imagine that there are taxes on income, τ . How does this change your model (arbitrage condition)? What is the effect of a permanent reduction of taxes?