UNIVERSITAT POMPEU FABRA

APPLIED MACROECONOMICS

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Fall 2004

Problem Set 4
Due: 8/11/04

1. Consider a simple Ramsey model in discrete time

$$
\begin{aligned}
& \max \sum \beta^t u(C_t) \\
Y_t &= K_t^{\alpha}(A_t L_t)^{(1-\alpha)} \\
u(C_t) &= \frac{C_t^{1-\theta}-1}{1-\theta} \\
K_{t+1}-K_t &= Y_t - C_t - \delta K_t \\
A_t &= \overline{A} e^{gt} z_t \\
L_t &= \overline{L} \\
K_t &\geq 0
\end{aligned}
$$

We discussed in class that fluctuations in the RBC model are caused by technology fluctuations. What happen in the Ramsey model if we introduce a permanent shock in technology? In this model z_t represents fluctuations in technology around the trend growth rate. Imagine that technology is fixed at A. Then the production function in efficient units is $\widehat{y}_t = z_t^{1-\alpha}\widehat{k}_t^{\alpha}$. Imagine a multiplicative shock in technology ($\Delta z_t = B$).

a) Show the effect of this change on the locus $\Delta k = 0$ and $\Delta c = 0$. What happen with the steady state of consumption and capital in efficient units?

b) Discuss the transitional dynamics of consumption and capital over time from the old steady state to the new one.

2. Consider a very simple RBC model

$$\max E_t \sum \left(\frac{1}{1+\beta}\right)^t u(C_t) \qquad \rho > 0$$

$$\begin{aligned}
Y_t &= \beta K_t + \varepsilon_t \\
u(C_t) &= C_t - \theta C_t^2 \qquad\qquad \theta > 0 \\
K_{t+1} - K_t &= Y_t - C_t \quad\; so\ \delta = 0 \\
\varepsilon_t &= \rho\varepsilon_{t-1} + u_t \qquad -1 < \rho < 1 \\
u_t &\sim iid \quad E(u_t) = 0
\end{aligned}$$

Notice that in this simple case the production function is linear, the interest rate is β (why?) and technology shocks (ε) are additive.

a) Find the first order condition relating C$_t$ with the expectation of C$_{t+1}$ (Euler equation).

b) Solve for K_{t+1} as a function of K_t and ε_t. You can use the guess $C_t = \alpha_1 + \alpha_2 K_t + \alpha_3 \varepsilon_t$.

c) Solve for the undetermined coefficients α_1, α_2, and α_3. For this obtain from the Euler equation the values of the coefficients that satisfy the equation for all values of K_t and ε_t.

d) Imagime that we have a productivity shock (one time shock in u). Describe the paths of Y, K and C.

3. Consider the Keynesian model we discuss in class.

a) Obtain the balanced budget multiplier (increase in government expenditure equal to the increase in taxes).

b) Imagine that the demand for money depends on disposable income instead of income. Obtain the balanced budget multiplier. Comment the differences with respect to a).

4. (Not compulsory) Consider the RBC model we discuss in class but, to simplify, assume that $n = g = \bar{A} = \bar{N} = 0$. In class we solved the competitive equilibrium from a decentralized perspective. In this problem you have to solve the social optimum. For this problem the Bellman equation is

$$V(K_t, A_t) = \max_{C,l}([\ln C_t + b\ln(1 - l_t) + e^{-\rho} E_t[V(K_{t+1}, A_{t+1})])$$

a) Explain in a simple way this Bellman equation.

b) Since the model is log-linear, let's guess that V(.,.) is $V(K_t, A_t) = \alpha_1 + \alpha_2 \ln K_t + \alpha_3 \ln A_t$ where, as before, the $\alpha' s$ are the undetermined coefficients. (Hint: substitute the conjecture and use the expressions of $K_{t+1} = Y_t - C_t$ and $E_t(\ln A_{t+1}) = \rho \ln A_t$.

c) Find the first-order condition for C_t. Show that it implies that C_t/Y_t does not depend on K or A.

d) Find the first-order condition for l_t. Use this condition and the one obtain in c) to show that l_t does not depend on K or A.