

UNIVERSITAT POMPEU FABRA

APPLIED MACROECONOMICS

José García Montalvo

Office 20.218

Fall 2004

Problem Set 5

Due: 22/11/04

1. Suppose that consumers make their consumption decisions at the first of each month and then hold constant their level of consumption until the first of next month. Suppose that these consumers obey to the permanent income hypothesis (Hall's version) and, therefore, their monthly consumption follows a random walk. Denote C_t as the measure of consumption in quarter t (this is the highest periodicity of consumption in National Accounting data). Let $c_{3t-2}, c_{3t-1}, c_{3t}$ denote consumption in the three months of quarter t . Check if quarterly consumption should follow a random walk under the following definitions. The easiest way to check this it to look at the autocorrelation of $C_t - C_{t-1}$.

a) $C_t = c_{3t}$

b) $C_t = (c_{3t-2} + c_{3t-1} + c_{3t})/3$

2. The government faces a present value budget constraint

$$\sum_{s=0}^{\infty} (1+r)^{-s} (T_{t+s} - G_{t+s}) = B_t$$

where T is real (tax) revenue, G is real expenditure, B is real debt outstanding and r is the real interest rate (assume that it is constant).

Imagine that you are able to fix taxes but expenditures, G , are given (exogenous). Expenditures may change from time to time depending on the decisions of Parliament (legislative body) and other exogenous events (like wars). As a tax authority you have to fix the level of taxes such that the budget constraint is satisfied.

The taxes in this economy are not lump-sum and, therefore, there is a deadweight loss, which is equal to $f(T) = kT^2$ and k is a positive constant. Your objective is to choose T in each period such that you minimize the expected deadweight loss.

$$\min E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+r} \right)^s f(T_{t+s}) \quad k > 0$$

subject to the intertemporal budget constraint.

a) Show that the optimal T follows a random walk.

b) Derive the optimal T as a function of current and expected future expenditure and the level of debt.

c) How does the level of taxation, T , and the deficit ($T - G$) respond to a permanent increase in G ? Explain.

d) In future decades a lot of workers will reach their age of retirement in many developed countries and they will start collecting social security benefits. What does the model implies for optimal fiscal policy over the next decades?

3. Suppose that consumption follows the permanent income hypothesis. What happen to savings (income less consumption) in response to a positive shock to income ($\epsilon > 0$) if income follows the stochastic processes below?

a) $Y_t = \alpha + \rho Y_{t-1} + \epsilon_t$

b) $Y_t = \alpha + Y_{t-1} + \epsilon_t$

c) $Y_t - Y_{t-1} = \alpha + \rho(Y_{t-1} - Y_{t-2}) + \epsilon_t$

4. (Not compulsory) Consider a consumer who gets utility from both his consumption of a nondurable good, C , and from his use of his stock of durable goods, denoted by K . His intertemporal utility function is

$$E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s [U(C_{t+s}) + V(K_{t+s})]$$

The intertemporal budget constraint is

$$\sum_{s=0}^{\infty} (1+r)^{-s} (C_{t+s} + X_{t+s} - Y_{t+s}) = A_t$$

where X is the expenditure in the durable, Y is labor income, A is non-human wealth and r is the real interest rate. The relative price of the durable

and the non-durable is fixed at unity. The relation between the stock and the flow of the durable is

$$K_{t+1} - K_t = X_{t+1} - \delta K_t$$

Assume that $U(\cdot)$ and $V(\cdot)$ are quadratic, and $r = \rho$.

a) Is it still true that consumption follows a random walk? If not, what process does it follow? Explain.

b) Show that

$$\frac{(r + \delta)}{(1 + r)} U'(C_t) = V'(K_t)$$

Explain.

c) What is the correlation between $C_{t+1} - C_t$ and $K_{t+1} - K_t$?

d) What are the autocorrelations of $X_t - X_{t-1}$?

e) In U.S. quarterly data all the autocorrelations of $X_t - X_{t-1}$ are about

0. Is there any plausible interpretation for this finding?