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A Multi-Objective Model for a Multi-Period Distribution Management Problem

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Abstract: The problems arising in commercial distribution are complex and involve several players and decision levels. One important decision is related with the design of routes to distribute the products, in an efficient and inexpensive way. This article deals with a complex vehicle routing problem that can be seen as a new extension of the basic and well-known vehicle routing problem. The proposed model is a multi-objective combinatorial optimization problem that considers three objectives and multiple periods, which models in a closer way real distribution problems than previous models. The first objective is the common cost minimization, the second is related with balancing work levels and the third is about assigning the same driver to the same client. An application of the model on a small example, with 5 clients and 3 days, is presented. The results of the model show the complexity of solving multi-objective combinatorial optimization problems and the contradiction between the several distribution management objectives.

Keywords: Multi-Objective, Vehicle Routing Problems, Distribution Problems

1 INTRODUCTION

The importance of good distribution strategy in today's competitive markets cannot be overstressed. The growing number of problems that firms are facing nowadays in relation with the distribution of their products and services, as the reduction of inventories and lead times, has lead logistics to be of primary concern to many industries. In many industries an important component of the distribution systems is the design of the routes of vehicles to serve their client's demand, since it is the element of the supply chain that is closer to the final customer.

New trends in the supply-chain management are, as pointed out by some industry leaders, better customer service, greater customer sophistication [Partyka, and Hall, 2000]. Customer service is becoming more important, clients demand more than a product; they demand a product arriving on time, an easy ordering system or a just-in-time distribution.

In this work, we will explore the decision-making problems in distribution management related with the delivery of products to customers, on a given period of time, using a fleet of vehicles. Decisions as how to assign customers to drivers, and to design the routes made by each vehicle constitute the Vehicle Routing Problem (VRP) .The motivation to work on this specific VRP arises by the distribution problems faced by the food and beverage industry.

Vehicle routing problems have been explored both in the management and operations research literature. The models found in the latter literature are often away from reality, since they do not consider issues present in real distribution, as for example multi-period planning. To get closer to real world problems and to reflect the multitude of concerns in distribution management we will extend the basic VRP. The final result of this extension will be a multi-objective model that takes into consideration three different objectives. These objectives are:

1- Cost objective
2- Human resource management objective
3- Marketing objective

The idea is to do a cross-functional planning in the supply-chain management by including in the model decisions that belong to different areas of the firm.

The first objective is the classical objective of VRP, which consists of minimizing total cost. However, this objective is often object of criticism by the users and planners, since it does not take into consideration other strategies of the company as, for example, customer service. The second objective expresses the need for balancing work levels. This objective is related with the human resources management and their remuneration, which is a growing source of competitiveness in today's firms. The third expansion of the model is the one that tries to implement a marketing policy. In a growing competitive environment many firms adopt strategies of tight relationships with their customers where loyalty and friendship play a key role. Therefore, when designing the route the firm will look forward to assign the same driver to the same customer.

The present article is organized in the following way: In the next section we will present a formulation of the basic model. Part 3 presents an approach for the multi-objective VRP. Then we will show the implementation of the model on a small example and the results obtained are analyzed. The difficulty to solve these problems on a real world case leads us to the need for a heuristic approach. Finally, the conclusions of the work are presented.

2 VEHICLE ROUTING PROBLEMS

Vehicle routing decisions are extremely important within a company to maintain its competitiveness an allow it to best exploit the available resources and to distribute its products at the lowest possible cost.

Significant amount of research efforts have been dedicated to VRP, see for example [Crainic and Laporte, 1998]. The most well known is a basic VRP which can be briefly defined as the following: given a set of customers with known demand and location, define a set of routes, starting and finishing at one depot, that visits all customers with minimal cost. A more detailed formulation will be presented next, since the basic VRP model is the starting point for any study in VRP. Despite the research efforts in academic research on VRP, there still exists a considerable gap in the application in practical problems of the models and techniques developed on the academic side.

2.1 The basic model for the VRP.

The basic model of the vehicle routing problem considers a set of nodes, representing retailers or customers, at a known location, that must be served by one depot. Each node has a known demand. A set of vehicles K, with equal capacity is available to serve the customers. The routes must start and finish at the depot. The objective is to define the set of routes to serve all customers with minimal cost.

For each pair of nodes, a fixed known *cost* is associated. Cost matrix can represent a real cost, distance or time. The main constraints of the problem are that all the demand must be satisfied and the vehicles capacity cannot be exceeded. There are several formulations of the classical VRP in the literature, for some of these formulations see [Fisher and Jaikumar, 1978], [Fisher and Jaikumar, 1981], [Kulkarni and Bhave, 1985] and [Gouveia, 1995]. The integer linear programming formulation described next was proposed by [Fisher and Jaikumar, 1978] and [Fisher and Jaikumar, 1981]. Consider the following data:

$I = 1,..., n$ set of nodes, that correspond to the different locations of the customers; node 1 corresponds to the depot.

$K = 1, ..., m$ set of vehicles;

Q capacity of each vehicle;

q_i demand of customer i, i=1,...,n;

c_{ij} cost of going from i to j, i=1,...,n ; j=1,...,n.

This formulation considers two types of variables:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ visits customer } j \text{ immediatly after customer } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \\ 0, & \text{otherwise} \end{cases}$$

The formulation of the problem is as follows:

Objective Function

$$Min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \sum_{k=1}^{m} x_{ijk}$$

Subject to

$$\sum_{k=1}^{m} y_{ik} = 1, \forall i = 2,...,n \qquad (1)$$

$$\sum_{k=1}^{m} y_{1k} = m \tag{2}$$

$$\sum_{k=2}^{n} q_i y_{ik} \le Q, \forall k = 1,...,m \tag{3}$$

$$\sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x_{jik} = y_{ik}, \forall i = 2,...,n; k = 1,...,m \tag{4}$$

$$\sum_{i,j \in S} x_{ijk} \le |S| - 1, \forall S \text{ non-empty subset of } \{2,...,n\}; k = 1,...,m \tag{5}$$

$$x_{ijk} \in \{0,1\}, y_{ik} \in \{0,1\}, \forall i = 1,...,n; k = 1,...,m \tag{6}$$

Constraint (1) ensures that each customer is visited by one vehicle only. Constraint (2) guarantees that all vehicles visit the depot. Constraint (3) represents the vehicle capacity constraint. The fourth constraint ensures that if a vehicle visits a client it also has to leave the client.

Constraint (5) is the sub-tour elimination constraint.

The last constraint defines the variables x and y as binary. The objective function is minimizing the total distance (or cost) of the routes.

The basic VRP is a generalization of the Traveling Salesman Problem, where more than one vehicle is available, for TSP references see for example [Lawer et al., 1985].

The TSP belongs to the class of NP-hard (non-deterministic polynomial time) problems, and so does the basic VRP and extensions. This means that the computational complexity of the problem grows exponentially with its size, i.e., it grows exponentially with the number of clients and vehicles.

For a more complete analysis of the VRP related research see the survey articles on VRP by [Laporte and Osman,1995], [Laporte,1992], [Desroisers et al, 1996] and [Fisher 1996] and the book of [Crainic and Laporte, 1998]. An extensive list of VRP research papers can be found on http://www.imm.dtu.dk/~orgroup/VRP_ref/.

3 THE MULTI-PERIOD DISTRIBUTION PROBLEM

3.1 Definition

The idea is to create a multi-objective VRP that models in a closer way the real distribution problems, since the basic model is rarely applied in real distribution problems due to its simplicity.

Here we will try to reflect these concerns of a firm when designing its distribution strategies. We assume that the firm is responsible for the distribution of its own products. Therefore, there are no questions of outsourcing to be handled. Another assumption is that these firms face the pressures of a competitive market making them concerned both on consumer satisfaction and internal efficiency.

As far as we know, the only multiple objective VRP model considered in the literature is the one by [Lee and Ueng, 1999]. These authors developed an integer linear model that searches for the shortest travel path and balances driver's load simultaneously. The work is measured in terms of traveling and loading time. The objective function is the weighted sum of the two objectives. This second objective minimizes the difference between the working time of each vehicle and the working time of the vehicle with the shortest working time. Our paper introduces a different measure of work and of balance, multiple periods and a third objective, related with marketing issues.

3.2 The multi-objective model

The classical VRP considers only one period and chooses the optimal routes for that period. Here we will introduce more periods by considering a week length of analysis. Each day we have a different set of clients to serve and quantities to deliver. The consideration of a multi-period model is essential to approach reality, since some industries plan their distributions weekly. In our model one of the objectives will be to minimize work-levels along several periods and not on a one period basis.

Other assumptions of the model are:

- All the demand is satisfied in the same day that it is required and not on any other day of the week.
- Only unload is done at each client.
- The number of vehicles is fixed and there are no fixed costs associated with the use of the vehicles. They all have the same capacity. Moreover, the number of vehicles available is enough to satisfy all the demand.
- Another assumption is that the cost matrix is known and fixed, independently of the day or the quantity loaded. The location of the clients is known.

- Each vehicle is assigned to a driver. For now we consider that they work every day in the period in question.
- One vehicle can only be used once a day and the time it takes to deliver the full capacity is less than a working day.
- Finally, we assume that each vehicle is driven always by the same driver. If we minimize differences in vehicles load we will balance employers work.

In the next sections the objectives and its formulations will be presented in detail. The following data is considered in the formulation:

i,I	Index and set of nodes, $I = 1,...,n$ where 1 is the depot and 2 to n are the clients locations;
k,K	Index and set of vehicles, $K = 1,...,m$;
t,T	Index and set of days which represent the period, $T = 1,...,p$;
T_i	Set of days where client i has a demand that is greater than zero, $i = 2,...,n$;
q_i^t	demand of customer i on day t with $i = 1,...,n$ and $t = 1,...,p$;
c_{ij}	the cost of going from i to j, this is a fixed matrix with $i = 1,...,n$ and $j = 1,...,n$;
Q	capacity of a vehicle.

The variables of the model are:

$$x_{ijk}^t = \begin{cases} 1, & \text{if vehicle } k \text{ visits customer } j \text{ immediatly after customer } i \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ik}^t = \begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

3.3 Objective A: Minimizing Cost

Cost reduction is one of the biggest concerns in transportation and distribution management, but not the only one, as we will see later. We want to find the route for each of the vehicles that will pass through the demand points in such a way as to satisfy all the demand with the smallest cost (or distance).

The formulation of this objective will be the same as the one used for the basic model but with a new parameter, t, representing the day of the week.

Objective function:

$$Min \sum_{t=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \sum_{k=1}^{m} x_{ijk}^t$$

Subject to

$$\sum_{k=1}^{m} y_{ik}^t = 1, \forall i = 2,...,n; t \in T_i \tag{1}$$

$$\sum_{k=1}^{m} y_{ik}^t = m, \forall t \in T_i; i = 1 \tag{2}$$

$$\sum_{k=2}^{n} q_i^t y_{ik}^t \le Q, \forall k = 1,...,m; t = 1,..., p \tag{3}$$

$$\sum_{j=1}^{n} x_{ijk}^t = \sum_{j=1}^{n} x_{jik}^t = y_{ik}^t, \forall i = 2,...,n; k = 1,...,m; t = 1,..., p \tag{4}$$

$$\sum_{i,j \in S} x_{ijk}^t \le |S| - 1, \forall S \text{ non-empty subset of } \{2,...,n\}; k = 1,...,m; t = 1,..., p \tag{5}$$

$$x_{ijk}^t \in \{0,1\}; y_{ik}^t \in \{0,1\}, \forall i = 1,...,n; k = 1,...,m; t = 1,..., p \tag{6}$$

Constraints (1) to (6) are similar to the ones in the basic model.

Constraint (1) ensures that in the days where the clients have a positive demand, that client is visited by only one vehicle. The second constraint forces that each day all vehicles go to the depot. Constraint (3) ensures that, the daily loading of a vehicle does not exceed its capacity. Constraint (4) guarantees that if the vehicle enters a node, on day t, it also has to leave that node, on the same day.

Finally constraint (5) avoids sub tours, but now not only for each vehicle but also for each day. The sub tour elimination constraint represents an exponential number of constraints. The problem with this constraint is that its number grows exponentially with n. Therefore, as it is usually done in the basic model, a weak form of the sub tour elimination constraint is introduced to be able to solve it by a commercial software.

[Desrochers and Laporte, 1991] presented a weak form of the sub tour elimination constraint for the TSP, this relaxation does not guarantee the elimination of sub tours for large problems. Here, we will transform this weak form so that it can be applied to the VRP. The last constraints (6) define all variables as binary.

3.4 Objective B: Balance Work-levels

The second objective function, seeks to balance the work between vehicles. The idea of making a multi-objective model that balances the work and minimizes cost has already been explored in the work of [Lee and Ueng, 1999]. In their work, the work-balance objective is defined as the minimization of the difference between the work of each vehicle and the work of the vehicle with the lowest work level, where the work was measured in terms of traveling and loading time.

In our model we measured work as the volume transported during all periods. This is particularly important in industries, like food and beverages, where everyday is more common to have a percentage of the remuneration related with the amount of sale and distributed products. Therefore, to measure the equilibrium of the routes we will consider a statistic, the standard deviation, since it is a well-known measure of deviations or spread around the mean.

The standard deviation of the work of each vehicle at the end of the period is minimized. The model allows a vehicle to work more than another on a given day as long as the total work of a vehicle at the end of the period is balanced. The problem of using the standard deviation as an objective function is that we no longer have a linear function , therefore, adds complexity to the model.

The total work of each vehicle will be represented by w_k , where:

$$w_k = \sum_{t=1}^{p} \sum_{i=1}^{n} q_i^t y_{ik}^t, \forall k = 1,...,m$$

and, the objective function will be:

$$Min \left\{ \sqrt{\frac{\sum_{k=1}^{m} w_k^2}{m} - \left(\frac{\sum_{k=1}^{m} w_k}{m} \right)^2} \right\}$$

Constraints (1) to (6) in this objective are the same as in the previous model and we add a new constraint:

$$\sum_{k=1}^{m} y_{ik}^t = 0, \forall i = 2,...,n; t \notin T_i \qquad (7)$$

Constraint (7) prohibits a vehicle to visit a client on a day where he has zero demand.

3.5 Objective C: Marketing Objective

In this model, each driver will try to serve always the same customer. This marketing policy is giving emphasis to the personal relationship between drivers and customers as a way to improve customer service. Since the policy to assign always the same driver to the same customer can violate other constraints as capacity and becoming expensive in terms of cost. Therefore, this requirement may have to be sacrificed but we will try to enforce this at least to the best clients., The idea is as follows: the better the client is the more interest exist in maintaining the same driver. This policy is becoming relevant in many industries, in particular the food and beverages, since the driver also performs commercial tasks.

In terms of mathematical formulation, the third objective works as follows: For each client we have a set of pairs of days with positive demand, T_i for each pair of days *(g,h)* in T_i (with $g \neq h$) we want to minimize the difference in the assignment to a vehicle k . The objective is to minimize $| y_{ik}^g - y_{ik}^h |$.

The importance is given by the total demand in the period; therefore a weight is introduced by the total amount ordered by each client. The objective function becomes:

$$Min \left\{ \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{\substack{g,h \in T_i \\ g<h}} \left[\left(\sum_{t=1}^{p} q_i \right) \times \left| y_{ik}^g - y_{ik}^h \right| \right] \right\}$$

The importance of a customer is measured in terms of sales. In some cases other measures could be used to classify the goodness of a client. For example, frequency of orders, credit history.

This objective function is again non linear. Constraints are the same as in section 3.3.

3.6 Summary of the complete model.

As mentioned before, when designing the distribution routes the three objectives play an important role. So, an appropriate model is a multi-objective one, which can be described as:

Objective Function

A $$Min \sum_{t=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \sum_{k=1}^{m} x_{ijk}^{t};$$

B $$Min\left\{\sqrt{\frac{\sum_{k=1}^{m} w_k^2}{m} - \left(\frac{\sum_{k=1}^{m} w_k}{m}\right)^2}\right\};$$

C $$Min\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{\substack{g,h\in T_i\\ g<h}}\left[\left(\sum_{t=1}^{p} q_i\right)\times\left|y_{ik}^{g} - y_{ik}^{h}\right|\right]\right\}$$

Subject to:

(1) to (7)

and $\sum_{t=1}^{p}\sum_{i=1}^{n} q_i^t y_{ik}^t = w_k, \forall k = 1,...,m$

To guarantee that the assumption on the number of the vehicles holds we can calculate the number of vehicles required.

$$m = \left\lceil \frac{Max\left(\sum_{i=1}^{n} q_i^1,..., \sum_{i=1}^{n} q_i^p\right)}{Q} \right\rceil$$

Ideally, we would like to find the solution that would be optimal for the three objectives at the same time. In multi-objective programming this solution point rarely exists. So, we would like to find solutions that are closer to this ideal point.

We will optimize the three objective functions to find non-dominated solutions. The choice among these non-dominated solutions is determined by the decision maker's preferences among the multiple objectives. In the next section these preferences are introduced as weights.

3.7 Weighting function approach

A usual approach to solve multi-objective problems is to consider all the objectives in the same objective function. The simplest method to do this is the *Objective Weighting*, where a weight is given to each of the objectives and the sum of them is optimized. In this method the optimal solution is controlled by the weight vector á. The preference of an objective can be changed by modifying the corresponding weight.

The use of this method may lead to Pareto-Optimal solutions but has some drawbacks. The solution is very sensitive to the weights that have been defined. The problem lies also on having objectives with different variables and scales. In our case, for example we are adding costs with standard deviations and quantities, forcing to use cost measures for the second and third objectives. This, in practical terms increases the complexity of the decision process.

To apply this method to our model we have to define objective B and C in terms of costs and in a similar scale. Let c_B be the cost per unit of standard deviation, this can be interpreted as the cost for a firm of having disequilibria in routes. And c_C is the unit cost to the company for not serving a client with the same salesman. This could also be interpreted as the penalty for not going to the same client. The weighted objective function with weights $á_1, á_2, á_3 \geq 0$, such that $\sum_{l=1}^{3} a_l = 1$ is as follows:

$$Min\, a_1 \sum_{t=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk}^{t} + a_2 c_B \left\{ \sqrt{\frac{\sum_{k=1}^{m} w_k^2}{m} - \left(\frac{\sum_{k=1}^{m} w_k}{m}\right)^2} \right\} + a_3 c_C \left\{ \sum_{i=1}^{n}\sum_{k=1}^{m} \sum_{\substack{g,h\in T_i \\ g<h}} \left[\left(\sum_{t=1}^{p} q_i\right)\times \left| y_{ik}^{g} - y_{ik}^{h}\right|\right]\right\}$$

Subject to :

(1) to (7)

and $\sum_{t=1}^{p}\sum_{i=1}^{n} q_i^t y_{ik}^t = w_k, \forall k = 1,\ldots,m$

4 SOLUTION APPROACH

In this section we will present solutions for the above problem using a small example. The Lingo 2.1 Industrial version was used to run the algorithms.

The example considers 5 clients, 2 vehicles and 3 days. The option to use an example with only three days is just to have a short computational time. And, on the other hand the number of days is large enough to allow the analysis of the results.

Figures 1 to 3 represent the solutions for 3 scenarios, corresponding to the optimization of the 3 objective functions (A, B and C) independently. In these pictures one can see the optimal routes and the total work of each vehicle. As can be observed all the routes are different depending on the objective we want to minimize, and, in general, the three objectives are contradictory.

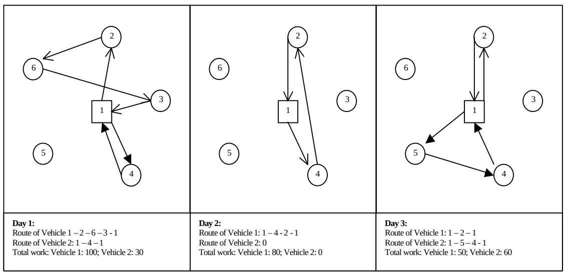

Figure 1: Optimal Solution-Objective A

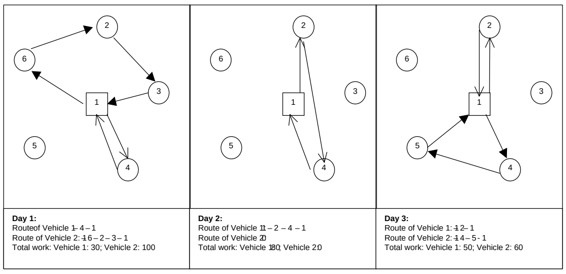

Figure 2: Optimal Solution-Objective B

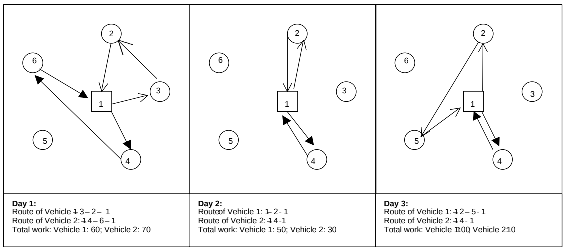

Figure 3: Optimal Solution-Objective C

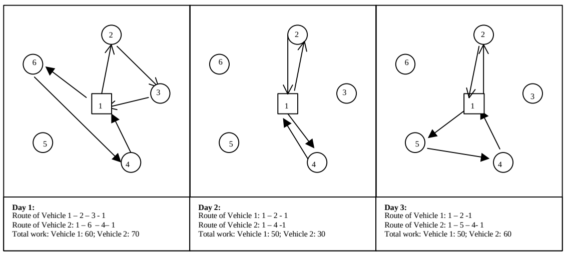

Figure 4: Optimal Solution-Objective M1

For example, when we minimize objective A, we obtain unbalanced routes. The total work of vehicle 1 is 230 and vehicle 2, 90. Or, when we balance the work we have that client 4 is served by different vehicles on the second and third days. Table 1 shows the results obtained for the three objective functions in the 3 cases. All the solutions are non-dominant. That is, in all the solutions there is at least one objective that has a better value on another solution.

Table 1

	A	B	C
Optimize A	13297	70	280
Optimize B	14428	0	880
Optimize C	15381	50	0

The next step will be to implement the weighting method approach to the same example, we will call it objective M. For this implementation we need to define the weights and the costs associated with objective B and C.

Let $c_B = 200$ and $c_C = 50$. Consider 2 examples: M1 and M2. The only difference between these examples is the weight given to each objective. In M1 all the objectives are equally important, all á are equal to 1/3 and in M2 the weights are $á_1 = 0{,}5$; $á_2 = á_3 = 0{,}25$. The solution of problem M1 (Figure 4) is different from the one found when considering the objectives separately. The solutions obtained when optimizing objective M1 and M2 dominate the one that results from optimizing objective C. It is an optimal solution for the objective C and produces better values for objectives A and B. For the non-linear objective functions we cannot guarantee that the solution is a global optimum.
If we try to increase the size of the example, by adding one more day, the computational complexity increases exponentially.

5 SUMMARY AND CONCLUSIONS

This work presents a new extension of the VRP, which tries to reflect closer the "real world" situations and business concerns. The model is a multi-objective combinatorial optimization problem with three objectives: minimizing cost, balancing work and improving customer service. We have implemented the model on a small example and found some non-dominated solutions. The results obtained matched our expectations: The three objectives are contradictory and, when optimizing one of them we obtain bad solutions for the other two. When using the weighted method as an approach to solve the multi-objective issue we face two problems: the first is that the costs associated with B and C are artificial and influence the results; and second, the definition of preferences for each objective also influence results. Just by analyzing such a small example, we were able to conclude the importance of the inclusion of multi-objectives in the multi-period model. This implies that we need to treat the problem as a multi objective and obtain the non-dominated solutions. However, since the idea is to approximate the model to reality we need to overcome the complexity of multi-objective models and also the time to find solutions. Therefore, as further work we will develop an heuristic approach, based on a technique known as iterated local search, that we hope that it will enable us to obtain good results in shorter running times. Only in this way we can apply it to real cases and help the user to take decisions.
For further research we also thought about extending this model to include stochastic demand in order to compete with the new trend of on-line delivery systems arising on the area of e-commerce.

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