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Inventory-Routing Problem, for a multi-period problem with stochastic and deterministic demand

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Abstract-- The need for integration in the supply chain management leads us to consider the logistic planning functions of transportation planning and inventory control. The coordination of these activities can be an extremely important source of competitive advantage in the supply chain management. The battle for cost reduction can pass through the equilibrium of transportation costs versus inventory managing costs. In this inventory-routing problem we have two sets of customers, one with fixed demand that has to be fully satisfied on a specific day, and another set of customers with random demand. The objective is to determine the routes for a week period (5 periods) and the quantities delivered at the random demand points, minimizing total costs. We have to decide when to visit these customers and how much to deliver each time we visit them. In this paper we propose a meta-heuristic based on the Iterated Local Search to solve a new version the Inventory Routing Problem.
Keywords–Inventory-Routing Problem, Iterated Local Search.

I. INTRODUCTION

The need for integration in the supply chain management leads us to consider the logistic planning functions of transportation planning and inventory control. The coordination of these activities can be an extremely important source of competitive advantage in the supply chain management.

About the vehicle routing and inventory management problems, considered separately there is a vast amount of literature. However, when looking at the two problems together the amount of work found is much fewer. Many models have been proposed for inventory problems with no transportation costs considered and much literature exists on vehicle routing problems where no inventory management is mentioned.

We will now take a look at what has been done in these areas of the joint problem of routing and inventory, known as the inventory routing problems (IRP).

Federgruen and Simchi-Levi (1995) make a good summary on inventory-routing problems. These authors divided the IRP models into two variants: the single period model and the infinite horizon model.

For the single period inventory routing model, Federgruen and Zipkin (1984), addressed the problem of allocating a scarce resource, available at the central depot, among several locations, each with random demand. And, at the same time, planning the deliveries using a fleet of vehicles. At the beginning of the period the initial inventory is reported to the depot. This information is used to determine the allocation of the available products, for the next day, among the locations. At the same time the assignment of customers to vehicles and their routes are determined. After deliveries are made, the demands occur and inventory carrying and shortage costs are incurred at each location proportional to the end of the period inventory level. In this model it is possible to choose not to visit some of the location. The authors also present an interchange heuristic and an exact algorithm using generalized Benders' decomposition.

Dror and Ball (1987), Dror and Levy (1986), address the case where in each time interval, only customers who will reach their safety stock level during this interval are serviced. A single item has to be delivered from one depot to many customers, whose demand is different in each period and is deterministic. With stochastic demand see Trudeau and Dror (1992) and with deterministic and stochastic demand Dror and Trudeau (1996). They consider a fixed number of identical trucks. These authors developed different solution methods to solve the problem. Dror and Trudeau (1996), for example, focus the solution on the maximization of operational efficiency (average number of units delivered in one hour of operations) and the minimization of the number of stock outs in a period.

Bard, Huang, Jaillet and Dror (1998), present a decomposition scheme for solving inventory routing problems with satellite facilities, in which a central

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depot must restock a subset of customers on an intermittent basis. In this setting the customer demand is not known with certainty and routing decisions taken over the short run might conflict with the long run goal of minimizing annual operating costs. A unique aspect is the presence of satellite facilities, where vehicles can be reloaded and customer deliveries continue until the closing time is reached.

In the Infinite Horizon inventory-routing models, Anily and Federgruen (1990) developed a model of a one warehouse multi-retailer system with vehicle routing costs. The objective is to determine the feasible replenishment strategies minimizing long run average transportation and inventory costs. A replenishment strategy specifies a collection of regions covering all outlets, if an outlet belongs to several regions a specific fraction of its sales is assigned to each of these regions. Each time one of the outlets in a given region receives a delivery; this delivery is done by a vehicle that visits all other outlets in the region as well (in an efficient sequence or route). These authors allow regions to overlap. See Hall (1991) and Anily and Ferdergruen (1991) for more comments on this model.

Another work on the infinite horizon inventory-routing problem is the one by Barnes-Schuster and Bassok (1997). These authors studied the situation where retailers face random demands from known distributions, which are stationary over an infinite horizon and between retailers. Ordered goods arrive at depot and are allocated and delivered to the retailers, retailers see demands, report to the depot and are charged inventory holding and shortage costs; depot decides whether or not to place the order for retailer ι. Depot places the order for all retailers which are to receive goods. The question that the authors answer is when it will be effective for the depot to use direct shipping as its sole policy and myopic "order up to" rounded to full trucks as its sole ordering policy. Effectiveness is define as the ratio of the long run average cost per period of the policy at hand to a lower bound on the long run average cost over all possible policies.

II. THE MODEL

In this inventory-routing problem we have two sets of customers, one with fixed demand that has to be fully satisfied on a specific day, and another set of customers with random demand. The objective is to determine the routes for a week period (5 periods) and the quantities delivered at the random demand points, minimizing total costs. We have to decide when to visit these customers and how much to deliver each time we visit them. The cost includes travelling cost; inventory-managing costs associated with the random demand points. A fixed cost for using a vehicle.

The second set of customers has stochastic demand, the geographic location of these points is known but the quantities to deliver depend on the expected demand. At these points, there are inventory handling costs and a cost for stock out.

The motivation of this work is based on the need for coordination within the supply chain management. In this case, we are trying to coordinate decisions from the distribution management with decisions from the inventory management.

We consider the case of a company that has several types of customer, with different characteristics. However, we can divide them in two big sets. The first group are those customers that have a fixed demand, known in advance. Customers have posed orders, and we have to deliver to these customers, the quantity they order in the day the have specified, there are no inventory costs for the retailer, customer managed inventory case (VMI). The second group of customers are called the Vendor Managed Inventory (VMI) customers, where the demand is stochastic. For this group it is the distributor that decides how much to deliver and when. The distributor is responsible for managing the inventory at these points.

The distributor has two types of costs related to these points. The holding cost, the cost of having inventory at these points, cost per unit and per period (day). And a stock out cost, cost per unit not sold.

So, Based on this idea, the objective of this model will be to design the routes and the delivering quantities in such a way as to minimize total cost. The decisions will be based on the assumptions explained bellow.

A. Assumptions of the model

The model tries to define the best routes and the best delivering quantities. The first assumption is that we have 2 sets of customers and stochastic customers are visited at least once a week. Another important aspect is that we only know the initial inventory at the beginning of the week for the first period, and the decisions are taken for the all week, independently of what occurs during the week.

Other assumptions of the model are.

- Set of customers with well known geographical locations
- Week period delivery system is considered. (5 working days)
- Some of the customers have a demand that is deterministic, that is, at the beginning of the period we know the demand. And this exact amount has to be delivered at the desired day.
- There are no inventory costs to be managed at the customer with deterministic demand and no stock out penalties since the amount ordered is delivered.

- There are no handling stock costs at the depot, and we assume that there is enough amount of product at the depot (unlimited capacity).
- The second set of customers has stochastic demand. The geographical location of these points is known but the quantities to deliver depend on the expected demand.
- At the VMI points, there are inventory handling costs and a cost for stock out.
- The highest demand is always smaller than a vehicle capacity.
- The vehicles have fixed capacity.
- There is no fixed number of vehicles but a high fixed cost for the use of a vehicle
- Each location can only be visited once a day and is visited at least once a week.
- There is a fixed cost per vehicle used
- The holding and stock out costs only depend on quantities and not on customers.
- For the VMI customers, demand function varies by customer
- For now we will assume that demand is equal for each day of the week. Later on we can relax this assumption and allow for different functions on different days.

B. Model

The costs of the problem are:

- Transportation cost between locations;
- Stock out and inventory costs at the stochastic delivery points;
- Fixed cost per vehicle used.

The decisions to make are the following:

- Decide the routes for each day of the week for each vehicle;
- Decide the number of vehicles needed each day of the period;
- Decide for each day which of the stochastic points will be included and where in the tour;
- How much to deliver at these points on each day.

Objective function:

- Objective is to minimize the expected cost at the end of the week.
- Min weekly cost = transportation cost + inventory holding cost + stock out cost + fixed cost for the use of vehicles

Notation:

ν= number of locations, indexed from 1 to ν; index 0 denotes the central depot

Θ= capacity of\ a vehicle

Π= number of periods (in this case, 5 periods, from 0 to 4)

c_{ij} = cost of direct travel from location ι to φ

$F_i(.)$ = cumulative distribution function of the one period demand in location ι, which is assumed to be strictly increasing with $i \in B$.

η= inventory carrying cost per unit.

σ= shortage cost per unit.

b_{pi} = initial inventory at location ι on day π

A= set of locations where the demand is known.

B= set of locations where demand is unknown.

d_{ip} = demand of customer ι on day π with $i \in A$

T_i = set of days where ι has a positive demand, with $i \in A$.

X= fixed cost per vehicle used

Variables:

$$x_{ijk}^{p} = \begin{cases} 1, & \text{if vehicle } k \text{ visits client } j \\ & \text{immediatly after client } i \text{ on day } t; \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ik}^{p} = \begin{cases} 1, & \text{if client } i \text{ is visited} \\ & \text{by vehicle } k \text{ on day } t; \\ 0, & \text{otherwise.} \end{cases}$$

w_{ip} = amount delivered to location ι on day π

K_Π= number of vehicles needed on day π

Transportation costs:

$$\sum_{i,j,k} c_{ij} x_{ijk}^{p}$$

Inventory cost:

$$\sum_{i \in B} \sum_{p} \left[\begin{array}{l} h \sum_{0}^{b_{ip}+w_{ip}} (b_{ip} + w_{ip} - t_{ip}) f(t_{ip}) dt_{ip} \\ + s \sum_{b_{ip}+w_{ip}}^{\infty} (t_{ip} - b_{ip} - w_{ip}) f(t_{ip}) dt_{ip} \end{array} \right]$$

For each day, and for each VMI customer, the inventory cost associated is equal to the expected inventory holding cost if the initial stock plus the quantity delivered is less than the demand. And a stock out cost otherwise. This expression applies for the case where demand is a discrete variable.

Since the representation of the probability distribution is difficult to find, particularly when demand ranges over a large number of possible values, the discrete random variable is often approximated by a continuous random variable. Furthermore, when demand ranges from over a large number of possible values, this approximation will generally yield a nearly exact value of the optimal amount. In addition, when discrete demand is used, the resulting expression may become slightly more difficult to solve analytically.

So, from now on we will consider the demand as a continuous random variable:

Let $I_{ip}(w_{ip})$ represent the inventory managing cost of location ι on day π.

$$\sum_{i\in B}\sum_{p} I_{ip}(w_{ip}) =$$

$$\sum_{i\in B}\sum_{p}\left[\begin{array}{l} h\displaystyle\int_{0}^{b_{ip}+w_{ip}}(b_{ip}+w_{ip}-t_{ip})f(t_{ip})dt_{ip} \\ +s\displaystyle\int_{b_{ip}+w_{ip}}^{\infty}(t_{ip}-b_{ip}-w_{ip})f(t_{ip})dt_{ip}\end{array}\right]$$

The problem can be stated as follows:

$$Min\sum_{i,j,k}\sum_{p} c_{ij}x_{ijk}^{p}+\sum_{i\in B}\sum_{p} I_{ip}(w_{ip})+\sum_{p} K_p\times C$$

Subject to:

$$\sum_{k=1}^{K_p} y_{ik}^{p}=1, \forall i\in A; \forall p\in T_i \quad (1)$$

$$\sum_{k=1}^{K_p} y_{ik}^{p}=K_p, \forall i=0; \forall p=0,...,4 \quad (2)$$

$$\sum_{i\in A} d_i^p y_{ik}^{p}+\sum_{i\in B} w_{ip}\le Q, \quad \forall p=0,...,4; k=0,..., K_p \quad (3)$$

$$\sum_{j=1}^{n} x_{ijk}^{p}=\sum_{j=1}^{n} x_{jik}^{p}=y_{ik}^{p}, \quad \forall i=2,...,n; k=1,..., K_p; p=1,...,4 \quad (4)$$

$$\sum_{i,j\in S} x_{ijk}^{p}\le |S|-1, \quad \forall S \text{ non-empty subset of } \{2,..., n\}; \quad k=1,..., K_p; p=1,...,4 \quad (5)$$

$$1\le\sum_{k=0}^{K_p}\sum y_{ik}^{p}, \forall i\in B \quad (6)$$

$$\sum_{k=0}^{K_p} y_{ik}^{p}\le 1, \forall i\in B; p=0,...,4 \quad (7)$$

$$\sum_{k=0}^{K_p} y_{ik}^{p}\le w_{ip}\le\sum_{k=0}^{K_p} y_{ik}^{p}M, \forall i\in B; p=0,...,4 \quad (8)$$

$$\sum_{p=0}^{4} w_{ip}=\sum_{p=0}^{4} t_{ip} f(t_{ip})dt, \forall i\in B \quad (9)$$

$$w_{ip}\ge 0, \forall i\in B; p=0,...,4 \quad (10)$$

$$x_{ijk}^{p}\in\{0,1\}; y_{ik}^{p}\in\{0,1\}, \quad \forall i=1,...,n; k=1,..., K_p; p=1,...,4 \quad (11)$$

The meaning of the above constraints is:

(1) For the fixed demand locations, in days where a customer has a positive demand, that customer is visited by only one vehicle.

(2) The second constraint forces that each day all vehicles go to the depot.

(3) Constraint (3) ensures that, the daily loading of a vehicle does not exceed its capacity.

(4) Constraint (4) guarantees that if the vehicle enters a node, on day τ, it also has to leave that node, on the same day.

(5) Avoids sub tours, for each vehicle for each day. The sub tour elimination constraint represents an exponential number of constraints. The problem with this constraint is that its number grows exponentially with ν.

(6) The VMI customers are visited at least once a week.

(7) Each day, there can only be at most one vehicle visiting the VMI customers.

(8) If the demand delivered to a location with random demand on day π is positive than a vehicle has to visit that location on that day.

(9) The quantity delivered during the all period to a customer has to be equal to the expected demand of the period, to avoid the stock out option.

(10) The variable w_{ip}, representing the quantity delivered to a random demand customer on a given day is always greater or equal to 0.

(11) This constraint define the variables x_{ijk}^{p} and y_{ik}^{p} as binary.

The above inventory cost function only works if the initial inventory of each period is known in advance, or observed. Since we are planning for several periods in advance we do not know the demand on each period. The initial inventory is a random variable that depends on the demand of the previous periods and on quantities delivered during the previous periods.

We will assume that the only quantity we can observe is the initial inventory of each customer at the beginning of period 0.

At each period, if there is a stock out, a cost is incurred and the initial stock of the following period will be zero.

Then, the initial inventory for each period is:

$$b_{i0}=u_i>0$$

$$b_{i1}=Max\{0, u_i+w_{i0}-t_{i0}\}$$

$$b_{i2}=Max\{0, b_{i1}+w_{i1}-t_{i1}\}= Max\{0, Max\{0, u_i+w_{i0}-t_{i0}\}+w_{i1}-t_{i1}\}$$

$$b_{i3} = Max\{0, b_{i2} + w_{i2} - t_{i2}\} =$$

$$Max\left\{\begin{matrix} 0, Max\left\{\begin{matrix} 0, Max\{0, u_i + w_{i0} - t_{i0}\} \\ + w_{i1} - t_{i1} \end{matrix}\right\} \\ + w_{i2} - t_{i2} \end{matrix}\right\}$$

$$b_{i4} = ...$$

So, the initial stock of period π will be a function of the initial stock of period 0, of the quantities delivered and demand of all previous periods.

$$b_{ip}(u_i, w_{i0}, ..., w_{ip-1}, t_{i0}, ..., t_{ip-1})$$

Assume from now on that the demand follows an exponential distribution function with the form:

$$f(t) = a_i e^{-a_i t}$$

We need to calculate the inventory cost for each period. So, for each period we need to consider all the possible scenarios initial inventory for the previous periods. This is, for period 0, we know the initial inventory, for period 1 we need to consider two possibilities: stock out at the end of period zero (zero initial inventory at period 1) and positive stock at the end of period 0 (initial stock at period one positive). Then we extend this for the second, third and fourth periods for all possible combinations.

For simplicity, in the next sections we will derive the expressions for one customer only, then we will have to add for all customers. We will also transform the inventory cost expression in the following way:

$$h \int_0^{b_{ip}+w_{ip}} (b_{ip} + w_{ip} - t_{ip}) f(t_{ip}) dt_{ip}$$

$$+ s \int_{b_{ip}+w_{ip}}^{\infty} (t_{ip} - b_{ip} - w_{ip}) f(t_{ip}) dt_{ip}$$

Becomes:

$$1(t_p < b_p + w_p) = \begin{cases} 1 & if \quad t_p - b_p - w_p < 0 \\ 0 & if \quad otherwise \end{cases}$$

For one period, the inventory cost of a customer would be:

$$\int_0^{\infty} \begin{bmatrix} h1(u + w - t)(u + w - t) \\ - s1(-u - w + t)(u + w - t) \end{bmatrix} f(t) dt$$

Let us now calculate the inventory cost expression for the first two periods (for the other periods the same reasoning applies).

Period 0

$$\mathrm{I}_0 = \int_0^{\infty} \begin{bmatrix} h \times 1(u + w_0 - t_0)(u + w_0 - t_0) \\ - s \times 1(-u - w_0 + t_0)(u + w_0 - t_0) \end{bmatrix} f(t_0) dt_0$$

$$I_0 = \exp(-a(u + w_0))(s + h)/a + (u + w_0)h - 1/a \times h$$

I_0 is the inventory cost in period 0. It is equal to a holding cost of $h \times (u + w_0 - t_0)$ if $u + w_0 > t_0$ and a shortage cost of $s \times (t_0 - u - w_0)$ if $u + w_0 < t_0$

For period 1 we will have to consider the two possible outcomes of period 0.

Period 1

$$\mathrm{I}_1 = \int_0^{\infty}\int_0^{\infty} \begin{bmatrix} 1(u + w_0 - t_0)(h \times 1(u + w_0 + w_1 \\ - t_0 - t_1)(u + w_0 + w_1 - t_0 - t_1) \\ - s \times 1(-u - w_0 - w_1 + t_0 + t_1) \\ (u + w_0 + w_1 - t_0 - t_1)) \\ + 1(-u - w_0 + t_0)(h \times 1(w_1 - t_1)(w_1 - t_1) \\ - s \times 1(-w_1 + t_1)(w_1 - t_1)) \end{bmatrix}$$

$$f(t_0) f(t_1) dt_0 dt_1$$

$$I_1 = 1/a \times h \times \exp(-a(u + w_0)) + ((u + w_0)h$$
$$+ (u + w_0) \times s + (s + h)/a) \times \exp(-a(u + w_0 + w_1))$$
$$+ (u + w_0 + w_1) \times h - 2/a \times h$$

I_0 is the inventory cost of period 1. It is the inventory cost considering two possibilities:

Positive stock in period 0:

$$1(u + w_0 - t_0)(h \times 1(u + w_0 + w_1 - t_0 - t_1)(u + w_0$$
$$+ w_1 - t_0 - t_1) - s \times 1(-u - w_0 - w_1 + t_0$$
$$+ t_1)(u + w_0 + w_1 - t_0 - t_1))$$

Stock out in period 0:

$$1(-u - w_0 + t_0)(h \times 1(w_1 - t_1)(w_1 - t_1)$$
$$- s \times 1(-w_1 + t_1)(w_1 - t_1))$$

The total inventory cost will be the sum of the cost of each period: $\sum_{p=0}^{4} I_p$

III. Heuristics Development for the IRP with Deterministic and Stochastic Demand

The limitations of available computational techniques make it impractical to try to solve this problem directly for all but very small instances. The structure of the problem argues for some type of decomposition. As mentioned there are two sub problems embedded in the IRP. The first is to decide delivery days and the quantities and the second involves routing.

Our decomposition scheme for the IRP is outlined in the following steps:

Step 1: Obtain an initial solution where the stochastic points are delivered every day.

Step 2: For each day in the planning horizon, try to find a good feasible solution by solving a VRP.

Step 3: Reassign the delivery days, by delivery only certain days of the week.

Step 4: For each day in the planning horizon, try to find a good feasible solution by solving a VRP.

Step 5: Repeat steps 3 and 4 and a satisfied solutions is found.

Due to the complexity of these problems (NP-hard) we need to develop an heuristic. Our proposal is to use an heuristic algorithm that as proven to give quiet good results on other problems and is easy to implement and modify, adapting to different strategies.

A heuristic algorithm is a solution method that does not guarantee an optimal solution, but in general has a good level of performance in terms of solution quality and convergence. Heuristics may be constructive (producing a single solution) or local search (starting from one given random solutions and moving iteratively to other nearby solutions) or combination.

ILS is a simple and generally applicable meta-heuristic which iteratively applies local search to modifications of the current search point. At the start of the algorithm a local search is applied to some initial solution. Then, a main loop is repeated until a stopping criterion is satisfied. This main loop consists of a modification step ("perturbation"), which returns an intermediate solution corresponding to a modification of a previously found locally optimal solution.

Next, local search is applied to yielding a locally optimal solution. An "acceptance criterion" then decides from which solution the search is continued by applying the next "perturbation". Both, the perturbation step and the acceptance test may be influenced by the search history. ILS is expected to perform better than if we just restart local search from a new randomly generated solutions.

Here is architecture of the ILS

```
  Procedure ILS   s0= generate initial solution
        s*= local Search(s0)
        Repeat
        s'= perturbation(s*, history)
        s*'= local search (s')
        s*= acceptance criterion(s*, s*', history)
        Until termination condition met
  End
```

Next, we will explain in more detail each step of the heuristics for the new IRP.

Step 1: The Initial solution - The inventory problem

The previous inventory cost function only works if the initial inventory of each period is known in advance, or observed. Since we are planning for several periods in advance we do not know the demand on each period. Therefore, we cannot know the initial inventory for future periods.

We will assume that the only quantity we can observe is the initial inventory of each customer at the beginning of period 0 of the week.

At each period if there is a stock out, then a cost is incurred and the initial stock of the following period will be zero.

The inventory problem is different from other inventory problems that can be found in the literature. Since, in this case we have a set of days, 5 days, for which we need to plan the deliveries (and we do not known the demand, although we know its distribution function). And we do not known what is the initial stock at each day for each customer. We only know the initial sock at the beginning of the week. We have to decide how often and how much to deliver. The result is a complex expression. This inventory problem has a handling and a stock out cost, being that the stock-out cost has higher value than the handling cost.

This initial solution is obtained by solving the inventory model for each customer. The objective is to minimize inventory costs (holding + stock out costs). This expression is then minimized by applying the Gauss- Newton's method. The best solution is to deliver every day to the VMI customers, since there is no setup cost associated with the deliveries.

The initial solution is: $w_0^* > 0$, $w_1^* > 0$, $w_2^* > 0$, $w_3^* > 0$, $w_4^* > 0$.

In summary, in step 1 we calculate the optimal quantities to deliver on each day of the week to each customer with stochastic demand. We assume an initial solution for the first day and an exponential distribution function for the demand. All of these customers have an exponential demand function but, with a different parameter. For now we suppose that all customers have the same initial amount.

Step 2: The VRP

After step 1, we known the quantities and the days to delivery to each customer, stochastic and deterministic. Therefore, we have to solve a VRP each day to obtain the best routes. To solve the VRP each day, we consider an Iterated Local Search for all customers on that specific days and their respective quantities. Then, we calculate the total cost: The transportation cost and the inventory cost associated with the stochastic points.

In steps 1 and 2, we obtained an initial solution based on a sub-optimisation of the two existing problems: the routing and the inventory problem. Now, phases 3 and 4 consist in developing an heuristic method that improves this solution.

Step 3: Inventory and transportation - The setup cost approach

After steps 1 and 2, we deliver every day the stochastic points, based on the optimisation of the inventory costs. However the transportation costs can be quite high with this strategy, we need to balance the delivery costs with the inventory costs. So, may be it is not a good strategy to delivery everyday to the stochastic point especially if the transportation costs are high.

So, we consider next the transportation cost, this is: When we optimised considering only the inventory holding + stock out cost the solution is to deliver every day. But now consider the cost of delivering, i.e. the setup costs. This cost will depend on the cost associated with visiting customer ι on a specific day. So, what is the best day or days to delivery to the stochastic points? The calculation of the best day in the period will depend on the routes we have planned for that period. It is logical to think that if a customer is close to a group of customers that we will deliver on period t, than we have a lower cost of delivering that customer on that day. This is: the setup cost of customer ι will depend on the location of this customer with respect to all the other customer in the route.

So, for each stochastic demand customer we calculate the setup cost (related with the transportation cost) for each day.

This cost represents the cost associated with delivering to a customer on a day τ, and will only depend on distance. This setup cost is an approximation and will be calculated in the following way:

g_i^t = cost of going to customer ι on day τ.

Let φ and κ be the previous and successor customer in the route of customer ι on day τ. Since we are considering only the successor and preceded in the tour:

$$g_i^t = c_{ji} - c_{ik} + c_{jk}$$

The next steps of the algorithm are: to sort these costs by decreasing order; iterate starting with the highest costs; and set the delivery of this customer on this specific day to zero. And concentrate the quantities to be delivered in the remaining days of the week.

Step 4: For each day in the planning horizon, and using the new delivering quantities, try to find a good feasible solution by solving a VRP.

Step 5: Repeat steps 3 and 4 and a satisfied solutions is found.

We will present a computational experiment about the application of this decomposition method to the IRP with two types of customer.

IV. Computational Results

To analyse the model we propose a computational experiment with two heuristics methodologies: the first is the Clark and Wright methodology and the second is the Iterated Local search heuristic describe in the previous section.

For each of the examples we run both algorithms considering different changes at each step, this is, the number of customers that we pick up from the ordered list of cost varies. We consider 4 different quantities, 2%, 5%, 10% and 20% of the total number of customers and days. We start by assuming that all customers have the same initial stock and that the demand of a customer follows the same exponential distribution every day of the week.

A. The data

We start by generating some examples and then analysing the trade-off between the inventory cost and the transportation cost.

We have different sets of examples that vary according to:

The type of demand; the type of location (the coordinates associated with the location of the customers and the depot were taken from Solomon's instances on VRTW (www.ferminu-hagen.de/winf/touren/inhalte/probinst.htm). These instances are divided into 3 categories: the C-type, clustered customers; the R-type, uniformly distributed customers and RC-type, a mix of R and C type) and the percentage of customers with stochastic demand over the total number of customers (consider 25%, 50%, 75%, 90%).

B. Analysis of the results

From the experiment we have made we can conclude that clearly the tendency is an increase in inventory cost and a decrease in transportation cost, see Figure 1. The increase in the inventory is faster than the decrease in transportation cost as we concentrate demands. The total cost will depend on the relation between the costs involved, and the decision maker can decide how much to concentrate on each run.

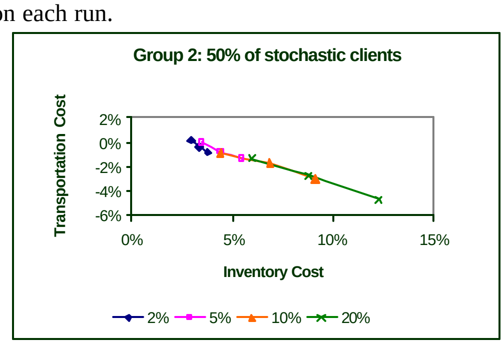

Figure 1

In Figure 2 we can also see the more we concentrate on each run, the higher is the decrease in transportation cost and the increase in inventory cost.

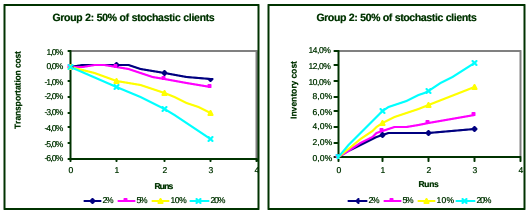

Figure 2

Considering the relation between the performance of the two algorithms, the Clarke and Wright (CW) and the ILS, see Figure 3, the ILS algorithm presents always a lower cost than the Clarke and Wright and a similar behaviour, however, the difference in the run time of Clarke and Wright algorithm is much smaller, 1 vs. 100 seconds when compared with the ILS. The important question here is to analyse if it is important this trade-off between quality and run - time, since we are trying to find a strategic solution for week problem.

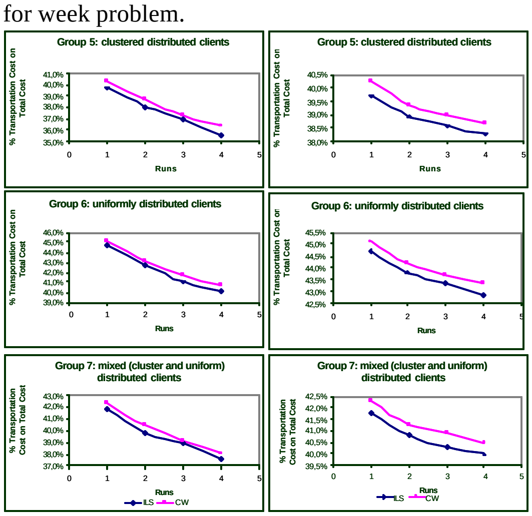

Figure 3

V. CONCLUSIONS

We have presented an inventory-routing problem and an heuristic approach to solve this problem. Considering the inventory and transportation management in an integrated mode can yield to a better performance. In our case, we have considered the situation where a distribution firm has two types of customers; the VMI and CMI customers and decisions have to be made in relation with the planning of the routing and deliveries for the complete set of customers. A week planning period was chosen and a heuristic, based on the iterated local search was constructed to solve this problem.

We have tested the algorithm on several types of examples, considering different networks of customers and different percentages of VMI customers. However, the effect of the cost is independent of the percentage of the VMI customers or network configuration.

Given the relationship between the inventory and transportation costs, the decision maker can decide how much of the deliveries to the VMI customers to concentrate.

VI. FURTHER RESEARCH

One area of research that could be interesting as the next step would be to build a model that decides the best days and quantities, instead of concentrating expected demands. We would also like to measure the setup costs in a dynamic way, this is, to measure of the cost effect of reassigning the deliveries of a VMI customer. In relations with the heuristics, we would also like to use Tabu Search on the IRP.

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