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Hybrids combining Local Search Heuristics with Exact Algorithm

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Abstract-- Recently hybrid metaheuristics have been design to find solutions for combinatorial optimization problems. We focus on hybrid procedures that combine local search based metaheuristics with exact algorithms of the operations research field. We present a mapping that outlines the metaheuristic and exact procedures used, the way they are related and the problems they have been applied to.

Keywords— hybrid metaheuristics, exact algorithms, combinatorial optimization algorithms.

I. INTRODUCTION

In this research we are particularly interested in hybrid procedures that combine local search based metaheuristics with exact algorithms of the operations research field. We name them Optimized Search Heuristics (OSH). We present how this kind of procedures has been applied to combinatorial optimization problems. We start by comparing and examining the correspondences of two existent classifications of such procedures and transform the more general one adding a new item and renaming other. To stress the distribution of these applications over the different problems of combinatorial optimization, we group the problems following a classification of NP optimization problems and indicate the work where OSH methods were applied to these problems. We also present a large number of references that use Optimized search Heuristics.

II. CLASSIFICATION OF OSH PROCEDURES

In the literature we can find two classifications of Optimized Search Heuristics. The first one, done by (Dumitrescu and Stützle 2003), presents a classification of solution methods that combines local search with exact algorithms. In particular they consider that the main framework is based on local search and the subproblems are approached by exact methods. These authors consider the following categories:

- 1) Exact algorithms to explore large neighbourhoods within local search.
- 2) Information of high quality solutions found in several runs of local search is used to define smaller problems solvable by exact algorithms.
- 3) Exploit bounds in constructive heuristics.

- 4) Local search guided by information from integer programming relaxations.
- 5) Use exact algorithms for specific procedures within metaheuristics.

The next classification done by (Puchinger and Raidl 2005) considers a combination of exact methods and metaheuristics and includes the following categories:

1. Collaborative algorithms exchange information but are not part of each other. The authors consider two subcategories: one, sequential the other parallel and intertwined. In sequential execution, one technique does a preprocessing before the other or the second one is a post processing of the solution(s) generated by the first. Sometimes both techniques have equal importance and we cannot speak of pre or post processing. In parallel execution, several processors perform simultaneous tasks acting as teams and interchanging information. In intertwined execution a single processor executes some steps of one procedure, then some steps of another.
2. Integrative combinations. One technique is a subordinate embedded component of the other technique. The authors consider the following
 - 2.1. Incorporating exact algorithms in metaheuristics by solving exactly relaxed problems - (as solutions to relaxations heuristically guide neighbourhood search, recombination, mutation, repair and/or local improvement); or by searching exactly large neighborhoods (exact algorithms are used to search neighborhoods in local search based metaheuristics). Merging solutions - Exact algorithms are used to solve sub problems generating partial solutions. Merging these partial solutions is iteratively applied within a metaheuristics. In evolutionary algorithms where solutions are incompletely represented in the chromosome, exact algorithms are used as decoders to find the correspondent best solution.
 - 2.2. Incorporating metaheuristics in exact algorithms. Metaheuristics can be used to obtain incumbent solutions and bounds. In branch-and-cut and branch-and-price

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algorithms, metaheuristics are used to dynamically separate cutting-planes and pricing columns, respectively. Metaheuristics can also be used for strategic guidance of exact algorithms, or to determine the branching strategy in branch-and-bound techniques.

III. CONNECTING THE CLASSIFICATION OF DUMITRESCU AND STÜTZLE WITH THE ONE OF PUCHINGER AND RAIDL

Almost all items in the classification of Dumitrescu and Stützle correspond to sub items of item 2.1 (incorporating exact algorithms in metaheuristics), of the classification of Puchinger and Raidl. The exceptions are procedures classified by Dumitrescu and Stützle in item 2. (information of high quality solutions found in several runs of local search is used to define smaller problems solvable by exact algorithms), that have a sequential nature, running a local search based heuristic several times before an exact algorithm; and also the work of (Umetani et al. 2003), allocated to item 4 of Dumitrescu et al., that sequentially executes tabu search after solving the integer programming relaxation.

TABLE I
CORRESPONDENCE BETWEEN THE DUMITRESCU AND STÜTZLE (2003) AND PUCHINGER AND RAIDL (2005) CLASSIFICATIONS

Classification of Dumitrescu and Stützle	Classification of Puchinger and Raidl
1. Exact algorithms to explore large neighborhoods within local search	2.1 Exactly searching large neighborhoods. Merging solutions – exactly solving sub problems
2. Information of high quality solutions found in several runs of local search is used to define smaller problems solvable by exact algorithms	1. Sequential execution
3. Exploit bounds in constructive heuristics.	2.1 Exact algorithms for strategic guidance of metaheuristics
4. Local search guided by information from integer programming relaxations	1. Sequential execution
5. Use exact algorithms for specific procedures within metaheuristics	2.1 Merging solutions

Some works included in item 1 of the classification of Dumitrescu et al., exactly searching large neighbourhoods, can be viewed as a merging solutions kind of procedure.

We introduce a new item in the classification of Puchinger and Raidl, 2.1. related with exact algorithms for strategic guidance of metaheuristics.

Here, we include all works of item 3 of the classification of Dumitrescu and Stützle.

We believe item 2.1. related with merging solutions should be generalized and renamed exactly solving sub problems.

We can say that the classification of Dumitrescu and Stützle is more specific and the one of Puchinger and Raidl is more general.

IV. APPLICATION TO COMBINATORIAL OPTIMIZATION PROBLEMS

Using the classification of NP optimization problems proposed by Crescenci and Kann in <http://www.nada.kth.se/~viggo/problemlist/>, we show the distribution of the OSH heuristics application to the different combinatorial optimization problems. In table II we present the problem type along with the reference of the work where the OSH method is presented.

We can see that a lot of the research of procedures that combine metaheuristics with exact algorithms has been dedicated to the job shop scheduling problem and to routing problems. Packing problems and the multiple constraint knapsack problem have also received some considerable attention, as well as the more general class of mixed integer programming problems. We believe this can be viewed as a measurement of both the difficulty and the practical relevance of these problems. Practitioners are still not satisfied with the results achieved by traditional applications from stand-alone fields of knowledge.

When looking at the type of combination implemented, we see that the most popular are sequential execution, exactly searching large neighbourhoods (and here dynamic programming is the most used exact algorithm) and exactly solving subproblems. Genetic algorithms have been the metaheuristics procedures more frequently used in combination with exact algorithms, maybe because of its low performance on their one.

The most common exact algorithms in this OSH procedures are, aside from dynamic programming, linear relaxations and branch-and-bound.

We believe using exact algorithms for strategic guidance of metaheuristics to be a very promising line of research. This way we can profit from the fast search of the space of solutions of the metaheuristics without getting lost in a “wandering” path, because of the guidance given by the exact algorithms. We find that another very interesting idea is the one of “applying the spirit of metaheuristics” when designing exact algorithms.

TABLE II
CLASSIFICATION OF ARTICLES PER TYPE OF PROBLEM

Problems	Classified articles	Problems	Classified articles
Mixed Integer	<p>1.1 Sequential execution (Pedroso 2004)</p> <p>2.1.3 Exactly solving sub problems (Pedroso 2004)</p> <p>2.2.3 Metaheuristics for strategic guidance of exact algorithms (French et al. 2001) (Kostikas et al. 2004)</p> <p>2.2.4 Applying the spirit of metaheuristics (Danna et al. 2005) (Fischetti et al. 2003)</p>	Vehicle Routing	<p>1.1 Sequential execution (Ibaraki et al. 2001)</p> <p>2.1.2 Exactly searching large neighbourhoods (Thompson et al. 1989) (Thompson et al. 1993)</p> <p>2.1.3 Exactly solving sub problems (Shaw 1998)</p> <p>2.2.4 Applying the spirit of metaheuristics (Danna et al. 2005)</p>
Graph Colouring	<p>2.1.3 Exactly solving sub problems (Marino et al. 1999)</p> <p>2.2.2 Metaheuristics for column and cut generation (Filho et al. 2000)</p>	Packing	<p>2.1.4 Exact algorithms as decoders (Puchinger et al. 2004) (Imahori et al. 2003)</p> <p>2.1.5 Exact algorithms for strategic guidance of metaheuristics (Dowland et al. 2004)</p> <p>2.2.1 Metaheuristics for obtaining incumbent solutions and bounds (Alvim et al. 2003)</p> <p>2.2.2 Metaheuristics for column and cut generation (Puchinger et al. 2004) (Puchinger 2006)</p>
Frequency Assignment	<p>2.1.5 Exact algorithms for strategic guidance of metaheuristics (Maniezzo et al. 2000)</p>	Cutting Stock	<p>1.1 Sequential execution (Umetani et al. 2003) (Bennell et al. 2001)</p>
Partitioning	<p>2.1.2 Exactly searching large neighbourhoods (Ahuja et al. 2000) (Ahuja et al. 2002)</p> <p>2.1.3 Exactly solving sub problems (Yagiura et al. 1996)</p>	Lot-sizing	<p>2.1.4 Exact algorithms as decoders (Staggemeier et al. 2002)</p> <p>2.2.1 Metaheuristics for obtaining incumbent solutions and bounds (Ozdamar et al. 2000)</p>
Maximum Independent Set	<p>2.1.3 Exactly solving sub problems (Aggarwal et al. 1997)</p>	Flow-Shop Scheduling	<p>1.1 Sequential execution (Nagar et al. 1995)</p> <p>2.2.3 Metaheuristics for strategic guidance of exact algorithms (Della-Croce et al. 2004)</p>
Maximum Clique	<p>2.1.3 Exactly solving sub problems (Balas et al. 1998)</p>	Job-Shop Scheduling	<p>1.2 Parallel or intertwined execution (Chen et al. 1993) (Denzinger et al. 1999)</p> <p>2.1.1 Exactly solving relaxed problems (Tamura et al. 1994)</p> <p>2.1.3 Exactly solving sub problems (Caseau et al. 1995) (Applegate et al. 1991) (Adams et al. 1988) (Balas et al. 1998)</p> <p>2.1.5 Exact algorithms for strategic guidance of metaheuristics (Lourenço 1995) (Lourenço et al. 1996)</p> <p>2.2.1 Metaheuristics for obtaining incumbent solutions and bounds (Schaal et al. 1999)</p> <p>2.2.4 Applying the spirit of metaheuristics (Danna et al. 2005)</p>
Network Design	<p>2.1.3 Exactly solving sub problems (Büdenbender et al. 2000)</p> <p>2.2.4 Applying the spirit of metaheuristics (Danna et al. 2005)</p>	One Machine Scheduling	<p>2.1.2 Exactly searching large neighbourhoods (Congram et al. 2002) (Lourenço et al. 2002)</p> <p>2.1.3 Exactly solving sub problems (Yagiura et al. 1996)</p>
p-Median	<p>1.1 Sequential execution (Rosing et al. 1997) (Rosing et al. 1998) (Rosing 2000)</p> <p>2.2.3 Metaheuristics for strategic guidance of exact algorithms (Della-Croce et al. 2004)</p>	Parallel Machine Scheduling	<p>1.1 Sequential execution (Clements et al. 1997)</p> <p>2.2.3 Metaheuristics for strategic guidance of exact algorithms (Ghirardi et al. 2005)</p>
Quadratic Assignment	<p>2.1.3 Exactly solving sub problems (Mautor et al. 1997) (Mautor et al. 2001) (Mautor 2002)</p> <p>2.1.5 Exact algorithms for strategic guidance of metaheuristics (Maniezzo 1999)</p>		
Steiner Tree	<p>1.1 Sequential execution (Klau et al. 2004)</p> <p>2.1.2 Exactly searching large neighbourhoods (Klau et al. 2004)</p>		
Travelling Salesman	<p>1.1 Sequential execution (Applegate et al. 1999) (Cook et al. 2003)</p> <p>1.2 Parallel or intertwined execution (Talukdar et al. 1998)</p> <p>2.1.2 Exactly searching large neighbourhoods (Cowling et al. 2005) (Burke et al. 2001) (Pesant et al. 1996) (Pesant et al. 1999) (Congram 2000) (Voudouris et al. 1999)</p> <p>2.1.3 Exactly solving sub problems (Yagiura et al. 1996)</p>		

Problems	Classified articles
Knapsack	1.1 Sequential execution (Vasquez et al. 2001) (Plateau et al. 2002) 2.1.1 Exactly solving relaxed problems (Chu et al. 1998) (Raidl 1998) 2.1.5 Exact algorithms for strategic guidance of metaheuristics (Puchinger 2006)
Markov Decision Processes	1.1 Sequential execution (Lin et al. 2004)
Generalized Schwefel Function	2.1.3 Exactly solving sub problems (Cotta et al. 2003)
Optimisation of continuous problems	2.1.5 Exact algorithms for strategic guidance of metaheuristics (Hedar et al. 2004)

V. CONCLUSIONS

Recently hybrid metaheuristics have been design to find solutions for combinatorial optimization problems. This work focus on hybrid procedures that combine local search based metaheuristics with exact algorithms of the operations research field. We present two classifications proposed for these solution methods and we also develop a mapping that outlines the metaheuristic and exact procedures used, the way they are related and the problems they have been applied to. The main conclusion of this work is that there are many research opportunities to develop Optimized Search heuristics and a large opportunity to apply them to difficult problems. The OSH method can extract the best features of the Metaheuristics and Exact method and provide an integrated solution method that as proved already by several authors can lead to excellent results.

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