# *Matheuristics*
## *Optimized Search Heuristics*

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## Search…

> **Levantou-se, e, fiel à regra de que em todas as operações de busca o melhor é começar sempre por uma ponta e avançar com método e disciplina, atacou o trabalho pelo extremo de uma fileira de estantes, resolvido a não deixar papel sobre papel sem verificar se, entre o de baixo e o de cima, outro papel não estaria escondido.**

*Todos os nomes*, José Saramago, Editora, 1999

> "He stands up and, following the law that in all search operations the best thing is always to start from one point and advance methodically and with discipline, he attacks the job from one end of the bookshelf, resolved not to leave any page unturned without checking whether, between the lower and upper one, there is another paper hidden."
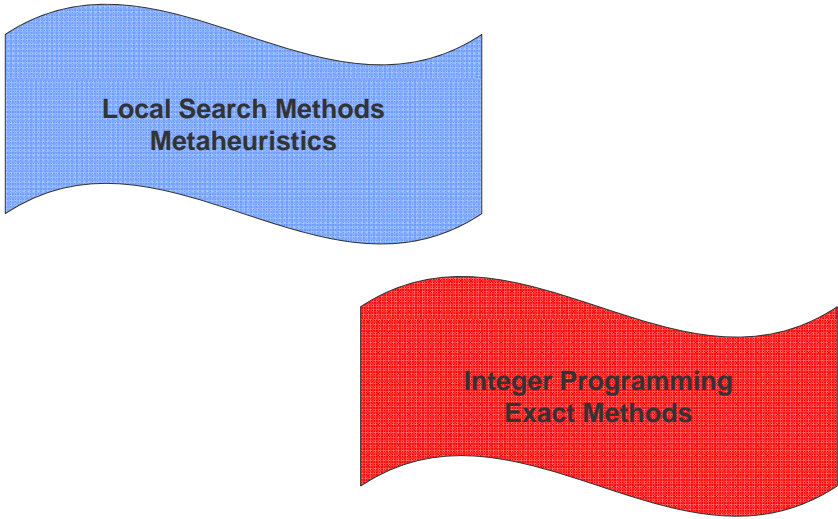
All the names, José Saramago

## Outline of the Presentation

- Background and Motivation
- Example of applications
- Classification
- Literature review
  - By problem
  - By approach
- Conclusions

## Background and Motivation

Local Search Methods
Metaheuristics

Integer Programming
Exact Methods

## Background and Motivation
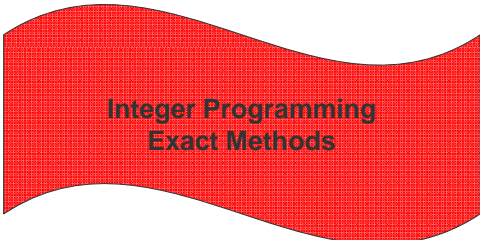
**Local Search Methods
Metaheuristics**

- Local Improvement
- Tabu Search
- Iterated Local Search
- Simulated annealing
- Genetic Algorithms
- Evolutionary algorithms
- Ant Colony Optimization
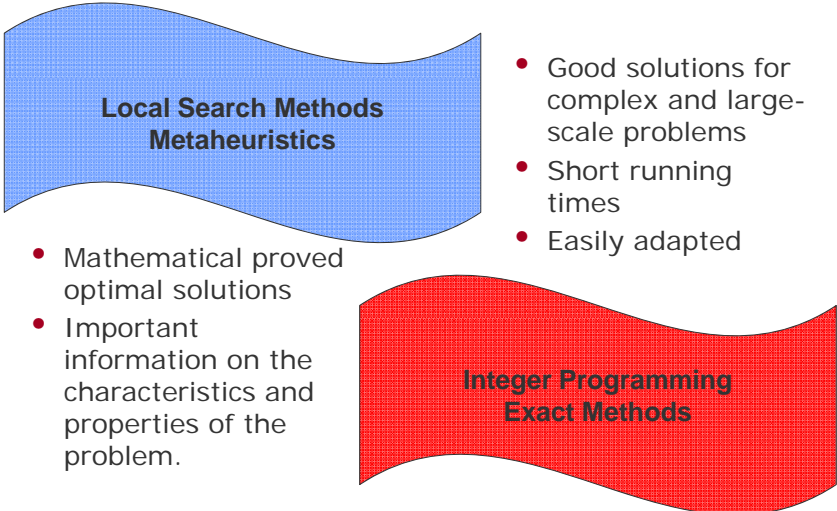- Scatter Search
- Memetic Algorithms
- Etc....

## Background and Motivation

- Branch-and-bound
- Branch-and-cut
- Column generation
- Cutting and price
- Dynamic programming
- Lagrangian relaxation
- Linear relaxation
- Surrogate relaxation
- Lower bounds
- Etc...

**Integer Programming
Exact Methods**

## Background and Motivation

**Local Search Methods Metaheuristics**
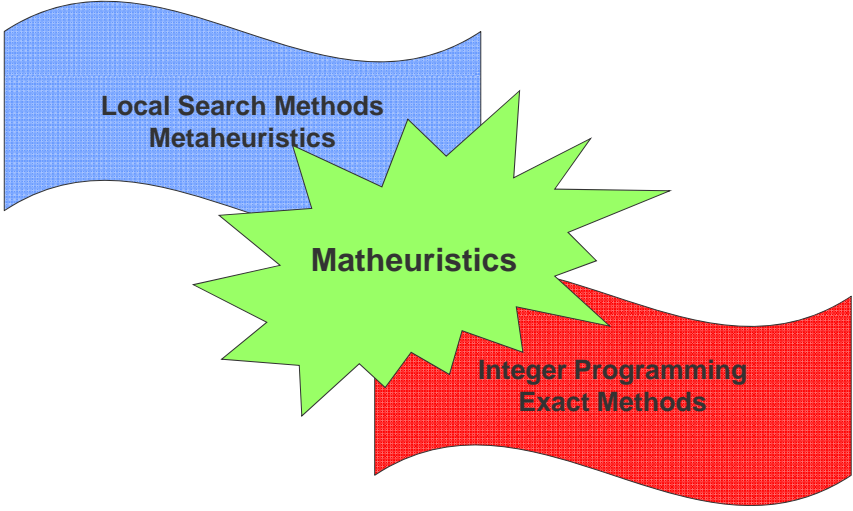
- Good solutions for complex and large-scale problems
- Short running times
- Easily adapted

- Mathematical proved optimal solutions
- Important information on the characteristics and properties of the problem.

**Integer Programming Exact Methods**

## Background and Motivation

**Local Search Methods Metaheuristics**

**Matheuristics**

**Integer Programming Exact Methods**

## Matheuristics

- Refers to …
  - on exploiting mathematical programming (MP) techniques in a (meta)heuristic framework or
  - on granting to mathematical programming approaches the cross-problem robustness and constrained-CPU-time effectiveness which characterize metaheuristics.
    - Copied from **Matheuristics 2010 Conference webpage.**

## Example of Applications

- Maybe the first application…
- Use an exact algorithm to solve a sub problem within a **Iterated Local Search** heuristic for the **Job-Shop Scheduling Problem**
  - Solving to optimality the one-machine scheduling problem with due dates and delivery times using the Carlier Algorithm (1982)
  - Lourenço (1993)
  - Named **Optimized Search Heuristics**

## Example of Applications

- More applications to the Job-Shop Scheduling Problem
  - Balas & Vazacoupolos (1998)
    - Include time-lags
    - work with the shifting bottleneck heuristic and design a guided local search, over a tree search structure, that reconstructs partially destroyed solutions
  - Fernandes & Lourenço (2007)
    - Combination with GRASP

## Example of Applications

- Set Covering / Partitioning Problem
- Crew scheduling problem
  - Crossover operator considering the columns in the parents and solving to optimality the reduced set covering problem.
    - Perfect Child /offspring
  - Aggarwal, Orlin & Tai (1997)
  - Portugal, Lourenço & Paixão (2002)

## Example of Applications

- Mixed Integer Programming
  - Construction of promising neighborhood using information contained in a continuous relaxation of the MIP model.
    - Relaxation Induced Neighborhood Search
  - Danna, Rothberg and Le Pape (2005)
    - Network design and multicommodity routing
    - Job-shop scheduling with earliness and tardiness
    - Local branching

## Example of Applications

- Vehicle Routing Problem
  - Iterated Local Search to assign customer to route and optimize the sequencing of the customers.
    - Solve a TSP using Concorde algorithm,
  - Dynamic programming is applied to determine the arriving time at each customer.
    - Ibaraki, Kubo, Masuda, Uno & Yagiura (2001)
  - Exploring a large scale neighborhood using dynamic programming in a VRP problem.
    - Thompson & Psaraftis(1993)

## Example of Applications

- Real Applications
  - Maybe the best set of problems to apply Matheuristics/OSH methods…
  - Why?
    - Complex problems with a large number of constraints.
    - Sometimes difficult to model…
    - But, a simplification of the problem is frequently a well-studied optimization problem.
  - Apply metaheuristics for the real general problem, and exact methods for the well-known problem.

## Classification

- Exact algorithms to explore large neighborhoods within local search.
- Information of high quality solutions found in several runs of local search is used to define smaller problems solvable by exact algorithms.
- Exploit lower bounds in constructive heuristics.
- Local search guided by information from integer programming relaxations.
- Use exact algorithms for specific procedures within metaheuristics.
  - Dumitrescu, I. and T. Stutzle (2003). Combinations of local search and exact algorithms. In G. R. Raidl ed. Applications of Evolutionary Computation. vol 2611 of LNCS, pp. 211-223. Springer.

## Classification

- Collaborative Combinations
  - Sequential Combinations
  - Parallel Execution
- Integrative Combinations
  - Incorporating exact methods in metaheuritics
  - Incorporating metaheuristics in exact methods

  - Puchinger, J. and G. R. Raidl (2005). "Combining Metaheuristics and Exact Algorithms in Combinatorial Optimization: A Survey and Classification." Lecture Notes in Computer Science, vol. 3562.

## Literature review

- Optimization problems
  - Graph Theory
  - Network design
  - Storage and Retrieval
  - Sequencing and Scheduling
  - Mathematical Programming
  - Other…
    - http://www.nada.kth.se/~viggo/problemlist/

# Literature review

| | 1.1 Sequential execution | 2.1 Incorporating exact algorithms in metaheuristics | 2.2 Incorporating metaheuristics in exact methods |
|---|---|---|---|
| Mixed Integer | (Pedroso 2004) **LS, LR** | (Pedroso 2004) **TS, BB** | (French et al. 2001) **BB, GA** (Kostikas et al. 2004) **BB, GP** (Danna et al. 2005) **BC, LS** (Fischetti et al. 2003) **BB, LS** |
| Graph Colouring | | (Marino et al. 1999) **GA, LP** | (Filho et al. 2000) **CG, GA** |
| Frequency Assignment | | (Maniezzo et al. 2000) **ACO, LR, D, BB** | |
| Partitioning | | (Ahuja et al. 2000) **LS, DP** (Ahuja et al. 2002) **LS, DP** (Yagiura et al. 1996) **GA, DP** | |
| Maximum Independent Set | | (Aggarwal et al. 1997) **GA, IP** | |

# Literature review

| | 1.1 Sequential execution | 2.1 Incorporating exact algorithms in metaheuristics | 2.2 Incorporating metaheuristics in exact methods |
|---|---|---|---|
| Network Design | | (Büdenbender et al. 2000) **LS, IP** | (Danna et al. 2005) **BC, LS** |
| P-Median | (Rosing et al. 1997) (Rosing 2000) **LS, BB** | | (Della-Croce et al. 2004) **BS, LS** |
| Quadratic Assignment | | (Mautor et al. 1997) (Mautor 2002) **LS, IP** (Maniezzo 1999) **ACO, LR, D, BB** | |
| TSP | (Applegate et al. 1999) **ILK, BC** (Cook et al. 2003) **ILK, DP** | (Pesant et al. 1996) **LS,CP** (Congram 2000) **ILS, DP** (Voudouris et al. 1999) **GLS, DP** **LS, VNS, DP** (Cowling et al. 2005) **ILS, DP** | |
| Vehicle Routing | (Ibaraki et al. 2001) **ILS, DP** | (Thompson et al. 1993) **VNS, DP** | (Danna et al. 2005) **BC, LS** |

## OSH Web page

- http://www.econ.upf.edu/~ramalhin/OSHwebpage/index.html

- Work in progress... need some help on it!

Search
Optimized
Heuristics

## Conclusions

- Optimized Search Heuristics (OSH)
  - Combining metaheuristics with exact methods.
  - The best of both worlds.
  - Best features from both solution techniques.
- Promising area of research
  - The classification helps to structure the different approaches in the literature.
  - The literature review helps to identify the potential areas of future applications.
- Working on a theoretical general framework for the Matheuristics /OSH methods.

## Matheuristics conferences

- Matheuristics 2010: third international workshop on model-based metaheuristics
  - June 28–30, 2010 :: Vienna, Austria
    Kardinal König Haus
  - http://homepage.univie.ac.at/matheuristics2010/
- Matheuristics 2008
  - June 16 - 18, 2008, Bertinoro (Forlì-Cesena), Italy
- Matheuristics 2006
  - 27-30 August 2006, Bertinoro (Forlì - Cesena),Italy