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MIRHA

Multi-start biased randomization of heuristics with adaptive local search for solving non-smooth routing problems

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1

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Outline of the Presentation

- Motivation and introduction
- VRP with non-smooth/non-convex objective functions
- MIRHA approach
- Computational results
- Conclusions and directions of future work.

2

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Motivation & Introduction

- Non-convex optimization problems (NCOPs) the objective function or even the feasible region are not convex.
- NCOPs result in a far more complex solution space than in the case of COPs.
- Many real applications that need a efficient answer…
- But not as many publications as for the COPs.

3

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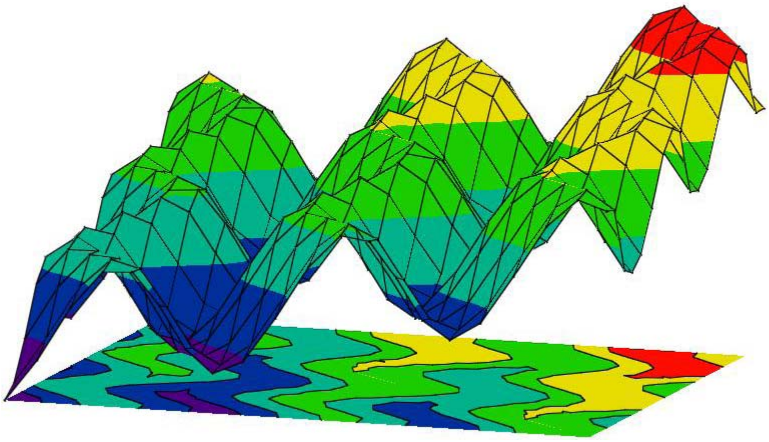
Motivation & Introduction

- We present a simple -almost parameter free- and efficient methodology, which can provide pseudo-optimal solutions to difficult problems in reasonable computing times.
- The method will be evaluated by its:
  - Accuracy (quality of results)
  - Simplicity of design and implementation
  - Efficiency
  - Robustness and Flexibility

4

Example of a non-convex function



VRP with non-smooth/non-convex objective functions

- Vehicle Routing Problems are well-known problems with many real applications.
- But more real problems tend to be more complex than the classical VRP.
- There is the need of an efficient and robust approach to deal with these real and complex applications of the VRP.

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VRP with non-smooth/non-convex objective functions

- Examples:
  - Minimization of fuel consumptions in surface transportation.
    - * costs related with roads slopes or types of asphalts, weather or temperature
  - Minimization of CO2 emissions related to road transportation.
  - Other costs, as penalty function for not complete cargo, time windows penalties, or drivers incentives. etc.
  - Penalties for soft constraints.

7

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VRP with non-smooth/non-convex objective functions

- Penalties for soft constraints
  - We considered soft constraints, which allow conditions to be violated, by incurring some penalty costs that must be added to the objective function.
  - Penalties for capacities and individual route costs associated with the cargo.
    - * For each route, if the route cost is less than C_{max}, this is the route cost, otherwise a penalty function is applied.
    - * non-linear and non-smooth function

8

VRP with non-smooth/non-convex objective functions

- Let ρ be a route in the solution
- The total cost of the route ρ is: $\gamma_\rho = \sum_{k=1}^{r} c_{\alpha_{k-1},\alpha_k}$
- The penalty function for one route:

$$C_\rho = \begin{cases} \gamma_\rho & \text{if } \gamma_\rho \leq C_{max} \\ \lambda(\gamma_\rho, C_{max}) & \text{otherwise} \end{cases}$$

- The objective function is:

$$\text{Minimize} \quad C_{total} = \sum_{all\ routes} C_\rho$$

MIRHA approach

- The basic aspects of our proposed algorithm is the use of classical greedy heuristics combined with a bias randomization and a local search.
- Multi-start approach
- Based on GRASP and HBSS
- Three main aspect:
  - Greedy classical heuristics
  - Bias random distribution combined with the greedy heuristic
  - Local search

MIRHA approach

- Get random solutions
  - Apply a Greedy Classical Heuristics randomized using a bias distribution:
    - The main idea of these heuristics is to select the next step from a list of available movements, usually according to a greedy criterion.
    - we consider non-uniform and nonsymmetric (biased) distributions, e.g.: the geometric distribution or the decreasing triangular distribution.
  - Local Search

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MIRHA approach

- procedure MIRHA(inputData, endConditions, prob.Dist., seed, heuristic)
  - initializeRandomGenerator(seed);
  - while endCondition[1]= false do
    - solution = getRandomSolution(inputData, heuristic, prob.Dist.);
    - solution = adaptiveLocalSearch(solution, endCondition[2]);
    - bestSolution = updateBestSolution(solution);
  - end while;
  - return bestSolution;
- end MIRHA

MIRHA approach

- Application to non-smooth/non-convex VRPs:
  - Get random solutions
    - * Apply the savings heuristic by Clarke and Wright, but a randomized version:
      - instead of having a single choice at every step, we will have multiple choices, each with a decreasing probability of being chosen.
  - Local Search
    - * The proposed method is an extension and improvement of the SR-GCWS-CS for the Capacitated Vehicle Routing Problem (CVRP).
    - * Juan A., Faulin J., Jorba J., Riera D., Masip D., and Barrios B., 2010

MIRHA approach

- Characteristics of the MIRHA
  - one important advantage of the proposed algorithm is its ***robustness and simplicity***
    - * MIRHA employs **very few or no parameters**, so there is no need to perform a complex fine-tuning process before using it.
  - the MIRHA is ***simple to design and implement***
    - * for most of the combinatorial optimization problem there exist a **classical greedy algorithm** and a **local search**.

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Computational results

- Classical CVRP benchmark instances from Christofides, Mingozzi and Toth
  - which feature the special constraints of the problem considered.
- The MIRHA algorithm has been implemented as a Java application.
- A standard personal computer, with an Intel R CoreTM2 Duo CPU processor at 2:4 GHz and a 2 GB RAM, was used to perform all tests.

15

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Computational results

- Penalty function used:

$$\lambda(\gamma_\rho, C_{max}) = \gamma_\rho + min\{\theta(\gamma_\rho, C_{max}), 20\}$$

where

$$\theta(\gamma_\rho, C_{max}) = 5 + 5000\left(\frac{\gamma_\rho - C_{max}}{\gamma_\rho}\right)^4$$

16

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Computational results

- Comparison with
  - BKS-H
    - * best-known solution when considering hard constraints (BKS-H) as published in Li, Lee, Ying, Lee (2009).
  - CWS-H
    - * CWS heuristic Clark and Wright (1964) when considering hard constraints (CWS-H)
  - MIRHA-H
  - GRASP-H
    - * Best solution obtained by GRASP using a restricted candidate list considering only *k* percent of edges

17

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Computational results

- Comparison among BKS-H, CWS-H, MIRHA-H and GRASP-H
  - Hard version

Instances	BKS-H	CWS-H	GAP	MIRHA-H	GAP	GRASP-H RCL=10%	GAP	GRASP-H RCL=50%	GAP
	1	2	(2-1)/1	3	(3-1)/1	4	(4-1)/1	5	(5-1)/1
vrpnc6	555.43	618.39	11.34%	555.43	0.00%	581.76	4.74%	557.49	0.37%
vrpnc7	909.68	975.46	7.23%	915.4	0.63%	927.26	1.93%	975.46	7.23%
vrpnc8	865.94	973.94	12.47%	867.58	0.19%	899.52	3.88%	973.94	12.47%
vrpnc9	1162.55	1287.64	10.76%	1188.63	2.24%	1189.85	2.35%	1287.64	10.76%
vrpnc10	1395.85	1538.66	10.23%	1435.79	2.86%	1457.7	4.43%	1538.66	10.23%
vrpnc13	1541.14	1592.26	3.32%	1547.79	0.43%	1592.26	3.32%	1592.26	3.32%
vrpnc14	866.37	875.75	1.08%	866.37	0.00%	866.37	0.00%	875.75	1.08%
Averages			**8.06%**		**0.91%**		**2.95%**		**6.49%**

18

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Computational results

- Comparison among BKS-H, CWS-S, MIRHA-S and GRASP-S
  - Soft version

Instances	BKS-H	CWS-S	GAP	MIRHA-S	GAP	GRASP-S RCL=10%	GAP	GRASP-S RCL=50%	GAP
	1	2	(2-1)/1	3	(3-1)/1	4	(4-1)/1	5	(5-1)/1
vrpnc6	555.43	629.88	13.40%	534.78	-3.72%	558.27	0.51%	534.8	-3.71%
vrpnc7	909.68	975.89	7.28%	874.84	-3.83%	892.7	-1.87%	975.89	7.28%
vrpnc8	865.94	941.17	8.69%	848.73	-1.99%	873.3	0.85%	941.17	8.69%
vrpnc9	1162.55	1252.59	7.75%	1112.38	-4.32%	1134.34	-2.43%	1252.59	7.75%
vrpnc10	1395.85	1475.57	5.71%	1389.32	-0.47%	1391.23	-0.33%	1475.57	5.71%
vrpnc13	1541.14	1194.52	-22.49%	1139.22	-26.08%	1145.58	-25.67%	1194.52	-22.49%
vrpnc14	866.37	868.68	0.27%	838.63	-3.20%	838.64	-3.20%	866.68	0.04%
Averages			**2.94%**		**-6.23%**		**-4.59%**		**0.46%**

* We can conclude the method is *efficient* and *accurate*.

19

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Conclusions

- The multi-start biased randomization of classical heuristics with adaptive local search (MIRHA) algorithm is proposed as a method for solving non-smooth/non-convex vehicle routing problems.
- The key idea in our approach is to employ non-uniform and bias probability distributions such as the geometric to add a random biased behavior to classical heuristics, e.g. the savings method.

20

Conclusions

- Our methodology has similarities with several antecedent methods (GRASP, HBSS) but, at the same time, it maintains significant differences that have already been discussed.
- Computational results show the efficiency of our approach when dealing with VRP with non-smooth objective functions.

Further Research

- Application of the MIRHA approach to other non-smooth / non-convex problems as for example scheduling problems.
- Improve the MIRHA approach by studying the most adequate bias random distributions.