Breaking the law when others do: A model of law enforcement with neighborhood externalities

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Abstract

A standard assumption in the economics of law enforcement is that the probability of a violator being punished depends only on the resources devoted to enforcement. However, it is often true that the productivity of enforcement resources decreases with the number of violators. In this paper, an individual who violates the law provides a positive externality for other offenders because the probability of being punished decreases with the number of individuals violating the law. This externality explains the existence of correlation between individuals’ decisions to break a law. The model evaluates the implications when determining the socially optimal enforcement expenditure, focusing specifically on the case of neighborhood crime. In particular, using a parametrized functional form, I show that neighborhood externalities will enhance or impede enforcement, depending on the crime rate.

1 Introduction

Socioeconomic conditions of poverty, inequality, education and unemployment do not fully explain the differences in crime rates across locations. Such differences in crime rates remain an open question in the law enforcement literature. This paper studies an alternative explanation that regards the interdependence of individuals’ decisions to break the law as an important source of the variance in rates of compliance. In contrast to previous work that focuses on behavioral

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assumptions,\textsuperscript{1} this paper demonstrates that such interdependence can also arise from conventional assumptions of rational utility maximizing behavior.\textsuperscript{2}

Standard theories of the economics of law enforcement assume that the likelihood that a violator is punished depends only on the level of resources that are devoted to enforcement. However, it is often true that the productivity of enforcement resources depends upon the number of people that engage in the illegal activity.\textsuperscript{3} This paper considers two positive externalities among offenders that may explain neighborhood differentiation. By affecting the productivity of enforcement resources, these externalities create interdependence between individuals’ decisions to violate the law.

As shown below, these externalities must be accounted for in evaluating individual payoffs from violating the law because they have a considerable effect on optimal enforcement policy. One externality is caused by congestion in enforcement. It creates a positive externality among offenders because (for a fixed level of enforcement resources) an increase in the number of violators leads to a lower amount of enforcement resources per violator, yielding a lower likelihood that a violator is punished. This case arises when enforcement resources are needed for punishment and detection activities, rather than for detection alone. For example, if there is only one tow-truck in the neighborhood, an increase in the number of cars that are illegally-parked reduces the probability that a given car is towed since there are fewer enforcement resources (tow-trucks) per violator. In the model the number of violators is determined in equilibrium; hence, the magnitude of this externality is generated endogenously.

The second externality is caused by the community’s degree of involvement in enforcement activities. The role of citizens in the enforcement process is important since they may alert authorities, provide evidence, and denounce offenders.\textsuperscript{4} Thus, the productivity of enforcement resources increases in the community’s degree of involvement. First, I consider the degree of involvement as an exogenous characteristic of the neighborhood; this allows me to discuss the effect of policies that may change this degree of involvement. Second, Section 4.4 extends the results to the case in which neighborhood involvement is a decreasing function of the non-compliance rate (and thus, being determined endogenously), as is the case when non-compliers may retaliate against neighbors who provide information to police.

The probability of punishing a violator is determined endogenously depending on the enforcement resources and on individuals’ decisions in equilibrium. A functional form for this probability permits me to evaluate the effects of the externalities. The results show that the externalities have

\textsuperscript{1}See for instance Glaeser et al. (1996) and Sah (1991), which are discussed below.

\textsuperscript{2}This interdependence results in multiple equilibria. I will use a refinement to select among them; this is discussed in Section 4.2.

\textsuperscript{3}The outcome of the enforcement process is the likelihood that a violator is punished. Thus, the enforcement resources’ productivity is measured in terms of that likelihood, or consequently, in terms of the resulting crime rate.

\textsuperscript{4}For instance, Akerlof and Yellen (1994) argue that "the major deterrent to crime is not an active police presence but rather the presence of knowledgeable civilians, prepared to report crimes and cooperate in police investigations." In a model of gang behavior, Akerlof and Yellen (1994) study the optimal level of cooperation of a community. However, there are no externalities across criminals because their behavior is modeled by a representative gang that chooses the intensity of criminal activity.
crucial effects on optimal law enforcement policy. First, they create multiple equilibria; thus, more than one compliance level may result for a given amount of enforcement resources. To find the optimal enforcement policy, the enforcement agency must be able to identify which of the equilibria will be selected. As I argue, risk dominance seems the most suitable selection criterion in this framework. After one equilibrium has been selected, the effects of the externalities remain. In particular, they may cause enforcement to be too costly, which helps explain how some neighborhoods become “no-go” zones for police.

In such cases, an alternative is to enforce the law through community policing. The paper formally models how differences in the involvement in enforcement activities between two otherwise identical neighborhoods may create a divergence in crime rates. Other alternatives are policies that make apprehension and punishment depend less on the number of violators. Examples of these types of policies for traffic violations are demerit point systems and electronic citation programs.

The model is presented in Section 2. As a benchmark, Section 3 provides the results when there are no externalities. Section 4 solves the model with externalities. Section 5 studies other possible neighborhood externalities and discusses the case where congestion is the only externality. Section 6 extends the model to a framework with heterogeneous individuals. Finally, Section 7 concludes.

Related literature

Glaeser et al. (1996) show that the variance of crime rates clearly exceeds what one would predict considering observable socioeconomic characteristics. Furthermore, they show that, with the exception of murders and rapes, such variance can be explained by the correlation of agents’ decisions. To interpret such correlation the authors use a behavioral model where a fraction of the population simply imitates their neighbors. In an earlier behavioral model Sah (1991) also studies how individuals’ decisions to violate the law may be interdependent. In his model, individuals respond to the "perception" of the likelihood of the punishment rather than to the true likelihood which implies that each individual chooses whether to be a criminal depending on her own and her acquaintances’ past experiences.

In contrast with these behavioral models, I show how individuals’ decisions may be interdependent (which generates multiple equilibria) in a framework where all individuals are utility maximizers. As discussed above, such interdependence arises when the crime rate is a "negative input" on criminal apprehension system. In an empirical study on crime, Ehrlich (1973) showed that the probability of apprehending and convicting felons is not only positively related to the level of current police resources, but also negatively related to the crime rate. He argued that the productivity of the resources "is likely to be lower at higher levels of criminal activity because more offenders must then be apprehended, charged and tried in court in order to achieve a given level of P [probability of the sanction]." However, much of the literature on the economics of law

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5 Unemployment rate, high school dropout rate, property taxes per capita, police per capita, regional dummy variable, persons over age of 25, etc.
enforcement does not allow for this kind of externality among criminals.

One notable exception is Freeman, Grogger and Sonstelie (1996). In a model with two neighborhoods, they study the tradeoffs between two externalities across criminals: when the number of thieves increases, the probability of being arrested decreases while the returns of crime decrease because there is less to steal. They find multiple equilibria; in particular, one possible equilibrium is that crime may concentrate in one neighborhood instead of spreading to the other. My work differs from theirs in several ways. First, rather than taking enforcement resources as exogenous, I take these enforcement resources as strategically determined by the enforcement agency, which is an active agent of the model. Second, my model considers also the externality that arises through the involvement of the neighborhood in enforcement activities, and assumes that returns from illegal behavior do not decrease in the number of criminals. Third, I study further effects of the externalities by introducing criteria of equilibrium selection. After adopting a criterion of equilibrium selection, I show that it matters how sensitive the enforcement technology is to the externalities; in particular, it is crucial in determining the optimal enforcement policy. This result is particularly relevant because Glaeser et al. (1996) argue that, although crime models with multiple equilibria generate a higher variance in the crime rate than do other models, the existence of multiple equilibria is not enough to explain the high variance in crime rates. In their data, they show that differences in crime rate across communities (once they control for socioeconomic conditions) cannot be explained by crime rates clustering around a few possible equilibria.

After Ehrlich (1973), the empirical literature on crime has continued studying the relationship between the likelihood of the punishment and the crime rate. In general, these studies find a significant negative relationship between the crime rate and the arrest rate (the usual proxy for the likelihood of the punishment). As discussed in Levitt (1998) and Ehrlich (1996) there are several empirical difficulties in these studies that may make the empirical analysis overly complex. In particular, two of the empirical problems are related to the externalities discussed in this paper. First, regressing crime rates on arrest rates may suggest a (spurious) correlation when the arrest rate is also affected by the crime rate. Second, there may be a measurement error when using the arrest rates to proxy the likelihood of the punishment because it might not provide enough

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6 Another exception is Bar-Gill and Harel (2001) which discusses in detail several ways in which the crime rate might feed back into the expected sanction. They argue that the expected sanction could be a decreasing function of the crime rate, either because of resource congestion or due to learning from fellow criminals; this is also the approach taken in this paper. They also note the possibility of multiple equilibria, but they do not characterize the full set of equilibria nor select among them. In addition to characterizing and selecting among equilibria, my analysis also differs from Bar-Gill and Harel in that I use a parametrized functional form for the enforcement technology that allows me to vary the sensitivity of the technology to the externalities.

7 The literature on illegal behavior has studied other causes of multiple equilibria. In Schrag and Scotchmer (1997), when the crime rate is high, individuals’ likelihood to be punished is almost the same regardless of being innocent or guilty, thus it is actually rational for an individual to commit a crime only when the crime rate is high. Because of the multiple equilibria, the authors conclude that the crime rate cannot be predicted from the enforcement policy and do not undertake an analysis of optimal law enforcement. In Rasmusen (1996) employers have incomplete information about workers’ criminal activity. Multiple equilibria arise because the stigma of being convicted (reduction in the wage employers are willing to pay someone with a criminal record) decreases with the crime rate. Similarly, Silverman (2004) finds multiple equilibria in a model where committing a crime is beneficial in terms of "street reputation."
information about the ratio of criminals that are effectively punished. More specifically, using the arrest rates leaves out the possibility of congestion during the investigation and conviction process, which may have important implications as shown in this paper. Although Glaeser et al. (1996) focus on showing the importance of the interactions across individuals and “not the form of that interaction or the mechanisms that aid that interaction,” they believe congestion is not the form of interaction because they do not find a correlation between arrest rates and crime rates in New York City precincts. However, as just mentioned, a spurious correlation problem may arise when studying the relationship between these two variables.

With this paper I intend to provide further insights on how enforcement resources, the crime rate, and other factors determine the likelihood of the punishment. Ehrlich (1996) considers this “production function” as an essential part of the simultaneous equations econometric structure needed to study illegal behavior and law enforcement. Moreover, my analysis points out two additional challenges in the empirical analysis. First, there may exist threshold levels of enforcement resources that make the individuals’ behavior vary drastically. In such cases, increases in enforcement resources below that threshold level might not result in meaningful changes in the crime rate, which might make the estimation more difficult. Second, when the level of enforcement resources is a strategic decision of the enforcement agency, a separate regression equation might be needed to model this decision. For instance, it would be interesting to study how the harm caused by the criminal activities (measured through victimization costs) affects the choice of level of enforcement resources.

In what follows I will assume that the fine is fixed exogenously, so that the level of enforcement resources is the sole decision variable of the enforcement agency. This is reasonable because the fine is set by a legislative body (or perhaps by judicial precedent) with broad jurisdiction, while the level of enforcement resources is chosen at a more local level and through a shorter-term process. Notice that although the total amount of enforcement resources might be decided at a supralocal level (e.g., decided by the state), urban and local authorities may decide how those resources are distributed across neighborhoods or communities within their area.

2 The Model

The basic structure of the model is similar to Polinsky and Shavell (1979). There is an enforcement agency that aims at maximizing social welfare and a continuum of risk neutral utility-maximizing individuals.

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8 According to Becker’s (1968) seminal study on crime and punishment (see also Stigler, 1970), optimal enforcement involves the highest possible fine and the lowest possible apprehension probability that are consistent with the desired expected sanction. Others have argued that less-than-maximal fines may be optimal when more complicated incentives are involved. Stigler (1970) and Mookherjee (1994) invoke the need for marginal deterrence; Polinsky and Shavell (1979) and Block and Sidak (1980) include costs associated with risk-bearing; and Malik (1990) includes avoidance and/or collection costs that increase with the magnitude of the fine.
2.1 The individuals

There is a continuum of risk-neutral individuals of measure 1, which represents the population of potential offenders. Individuals are assumed to be homogenous in the main model; this assumption is relaxed in Section 6. Each of the individuals may either comply with the law, denoted as \{C\}, which yields zero payoff, or not comply with the law, denoted as \{NC\}, which implies a benefit, \( b \), but also a possible fine, \( f > 0 \). The fine is imposed with a probability \( P \) that is determined endogenously as explained below. The benefit is net of any cost (excluding the fine) associated with not complying (e.g., moral cost). Both \( b \) and \( f \) are exogenous to the model. I assume that \( b < f \), that is, there is always a high enough probability, \( P \geq b/f \), that deters individuals from violating the law.

The choice of a single individual has a negligible impact on the crime rate; therefore it has no effect on \( P \). Thus, an individual commits an offense if \( P < b/f \), but not if \( P > b/f \), and will be indifferent if \( P = b/f \). In order to simplify the exposition, I maintain the following assumption\(^{10}\): ASSUMPTION 1: Individuals comply with the law in case of indifference.

Therefore, the proportion of individuals not complying is given by a function of the probability of the sanction, \( \mu = R(P) \) where:

\[
R(P) = \begin{cases} 
1 & \text{if } P < b/f \\
0 & \text{if } P \geq b/f
\end{cases}
\]  

2.2 The enforcement agency

The enforcement agency maximizes social welfare by choosing the amount of enforcement resources \( c \geq 0 \). The sanction associated with non-compliance, \( f > 0 \), is exogenous for reasons explained in the Introduction. Therefore the decision variable of the agency is the level of enforcement resources, \( c \), which includes all the needed expenses in detecting, prosecuting, and fining, so that the fine is actually imposed. Hence, \( c \) describes the public resources that are used for enforcement activities.

Social welfare is measured by considering that first, non-compliance should be deterred because each individual not complying with the law generates a harm, \( h \), to the community, and that second, (for a fixed level of aggregate harm) the lower the expenditure on enforcement the better off society is. For a given non-compliance rate, denoted above as \( \mu \), the harm generated is \( h \cdot \mu \). In addition, the fines are assumed to be mere transfers of money and hence the revenue obtained from them does not affect the choice of the agency. Therefore, the enforcement policy that maximizes social welfare is given by:

\[
c_{\text{opt}} = \arg \max_c \ SW(c) = \arg \min_c \ h \cdot \mu^*(c) + c,
\]  

\(^9\) Although every individual could be considered as a potential criminal, some individuals are deterred by very small levels of enforcement resources due, for instance, to moral costs. Thus, I am excluding them from the analysis.

\(^{10}\) I thank an anonymous referee for suggesting this assumption. For a model that involves mixed strategies, see Ferrer (2008).
where $\mu^*(c)$ is the equilibrium non-compliance rate among individuals who are contemplating crime.

The timing of the decisions of the agency and of the individuals is:

- **Stage 1**: The agency decides how much to spend on enforcement, $c$.
- **Stage 2**: Individuals decide whether or not to comply with the law.

The agency anticipates the behavior of the individuals since it has perfect information about the individuals’ payoffs.

### 2.3 The enforcement technology

The enforcement technology consists of the process that determines the probability of a law-violator being sanctioned, $P$. Therefore, it assembles all the activities related to detection, apprehension, and punishment.

The probability of the sanction, $P$, is not a (direct) decision variable of the agency and it will be determined endogenously. Given the enforcement resources, $c$, a non-compliance rate, $\mu$, and a measure of neighborhood involvement, denoted $\eta$, the probability of being sanctioned is given by $P = p(c, \mu, \eta)$. The triple $(c, \mu, \eta) \in \mathbb{R}^+ \times [0, 1] \times [0, 1]$ and the function $p$ is increasing in $c$ and $\eta$ and decreasing in $\mu$.

Because of the positive externality among offenders, the enforcement technology is such that the higher the non-compliance rate, the lower the probability of the sanction (i.e., $p_\mu < 0$).\(^{11}\) This positive externality among offenders arises due to congestion in enforcement resources. Also, it could arise when offenders share information or techniques on how to avoid detection and punishment. Since the non-compliance rate is determined in equilibrium, the magnitude of this externality is endogenous in the model.

In addition, the involvement of a neighborhood in enforcement is measured by $\eta$. The information that members of the community have plays an important role in the enforcement process since they may alert authorities, provide evidence, and denounce offenders. That is, since such information has an effect on the productivity of enforcement: the higher is the involvement of neighbors in enforcement, the higher the probability of the sanction (i.e., $p_\eta > 0$).\(^{12}\) Diverse factors and policies may affect the value of this parameter. For instance, language differences between the police and the neighbors may decrease the level of neighbors’ involvement. First I consider $\eta$ as exogenously given for each neighborhood. Section 4.4 extends the results to the case where $\eta$ is decreasing in $\mu$; that is, to the case where the neighbors’ involvement is decreasing in the non-compliance rate.

In order to have closed-form solutions for the optimal level of enforcement resources, a particular functional form for $p$ is employed. A second advantage of assuming a specific functional form is that

\(^{11}\)In this paper, the externality among offenders is always positive. In contrast, Calvó-Armengol and Zenou (2004) consider a model of social networks in which there is a negative externality among delinquents because they compete in criminal activities. Competition in criminal activities commonly arises in environments of organized crime, however, in other illegal activities there is usually no (significant) booty to fight for. Bar-Gill and Harel (2001) also discuss the possibility of a negative externality among offenders. While this negative externality is certainly possible, it would predict a negative correlation in criminal behavior, which seems to be at odds with the available evidence.

\(^{12}\)Sampson (2004) finds evidence that "exposes the centrality of citizens as the engine of crime control."
it allows me to measure the results in terms of the sensitivity of the technology to the externalities. A specific functional form is a restrictive assumption; however, the form assumed represents a large family of functions and satisfies desirable properties.

The probability of the sanction is given by the following function defined over the enforcement resources and the non-compliance rate:

$$P = p(c, \mu, \eta) = \frac{kc^\alpha}{(1 + \mu - \eta)^\chi}.$$  \hspace{1cm} (3)

The parameters of this production function are $k > 0$, which expresses additional factors that may affect the enforcement technology such as specific characteristics of the type of illegal behavior; and $\alpha \in (0, 1)$, which implies that there are decreasing returns with respect to the level of enforcement resources (i.e., $p_{c \infty} < 0$). Also, $1 + \mu - \eta$ measures the overall neighborhood effect. Finally, $\chi \in (0, 1)$ measures how sensitive the technology is to the externalities. Notice that $-\chi$ is the elasticity of $p$ with respect to the overall neighborhood effect $1 + \mu - \eta$. In the limiting case of $\chi = 0$, then $\mu$ and $\eta$ have no effect on the probability of the sanction, and $P$ depends only on the level of enforcement resources. This case will be used as a benchmark. Furthermore, if the rate of community involvement is higher than the crime rate (i.e., $\eta > \mu$) then the net effect of the externalities is positive for enforcement since then $p_\chi > 0$. Alternatively, the externalities have a negative net effect when the rate of community involvement is lower than the crime rate (i.e., $\eta < \mu$) because then $p_\chi < 0$. In Section 5, I study the case in which congestion is the only externality; that is, the case where $\eta = 0$.

This specific functional form aggregates the two distinct neighborhood effects ($\mu$ and $\eta$) into an overall neighborhood effect; let $N \equiv \mu - \eta$ denote this overall effect. By employing this specific functional form, I impose assumptions on the signs of the second cross-partial derivatives of the enforcement technology. Thus, $p_{cN} < 0$; that is, a higher overall neighborhood effect (due to either an increase in the crime rate or a decrease in community involvement) reduces the marginal productivity of expenditures on enforcement. The effect of the externalities on the marginal productivity of enforcement expenditures is given by $p_{c\chi} > 0$ if $\mu < \eta$ and $p_{c\chi} < 0$ if $\mu > \eta$. That is, if the crime rate is lower than the rate of community involvement, then the marginal productivity of enforcement expenditures is decreased by the presence of the externalities, while if the crime rate is higher than the rate of community involvement, then the marginal productivity of enforcement expenditures is decreased by the presence of the externalities. Finally, $p_{\chi N} < 0$; that is, the overall neighborhood effect is stronger when the enforcement technology is more sensitive to the externalities. While these implications about the signs of cross-partial derivatives seem plausible, they are not crucial for the main results of the paper.\footnote{I will make the necessary parametric assumptions in order to ensure that $P \in [0, 1]$. In particular, I impose that $P = 1$ when the level of enforcement resources is $c > ((1 + \mu - \eta)^\chi/k)^{1/\alpha}$. Footnote (18) discusses the implications when modeling the equilibrium selection.}

\footnote{In particular, the existence of multiple equilibria, and a unique risk-dominant equilibrium, also hold when $P$ is determined by a generic differentiable function such that for any $\mu$ and $\eta$, $p(c, \mu, \eta)$ is strictly increasing, surjective,}
2.4 Equilibrium condition for the individuals’ behavior

Because of the positive externality among offenders, each individual cares about the rest of the individuals’ decisions with respect to compliance. Therefore an individuals’ equilibrium is only reached when, given the non-compliance rate, no individual is willing to change her decision as to whether to comply or not.

**Definition 1**: Given the enforcement resources, $c$, the non-compliance rate $\mu^* \in [0, 1]$ is an equilibrium for the individuals’ behavior if it satisfies the following condition: $\mu^* = R(p(c, \mu^*, \eta))$. That is, the non-compliance rate $\mu^*$ is consistent with the probability of the sanction resulting from $c$ enforcement resources and a non-compliance rate $\mu^*$.

Given $\eta$, the equilibrium condition for individuals’ behavior can be rewritten as a function of the enforcement resources, $c$, through the following function $\mu^*: [0, 1] \to [0, 1]$:

$$\mu^*(c; \eta) = \begin{cases} 0 & \text{if } p(c, 0, \eta) \geq b/f \\ 1 & \text{if } p(c, 1, \eta) < b/f \end{cases}.$$  \hfill (4)

3 The benchmark: optimal policy in the absence of externalities

The analysis excluding externalities (i.e., when imposing $\chi = 0$) provides the results obtained in the standard law enforcement literature. For this reason, the results of this section are used as a benchmark. Notice that when $\chi = 0$, $p$ becomes a one-to-one, increasing and concave function of the enforcement resources, $c$, alone. For any given $c$, $p(c)$ is uniquely determined, independent of the rate of non-compliance:

$$P = p(c) = kc^\alpha \quad \text{for all } c \geq 0.$$  \hfill (5)

Whenever $\chi = 0$, each individual complies or not with the law depending only on the enforcement resources, since other individuals’ choices have no effect on her payoff function. Notice that there is a level of enforcement resources that constitutes a threshold for the individuals.

**Definition 2** Let $\tilde{c} \geq 0$ be such that $p(\tilde{c}) = \frac{b}{f}$ is satisfied. I refer to $\tilde{c}$ as the threshold level of enforcement resources in the absence of the externalities.

Considering the functional form of $p$, notice that $\tilde{c} = (b/fk)^{1/\alpha}$ and $p(\tilde{c}) \leq 1$. Thus, the individuals’ non-compliance rate in equilibrium can be rewritten as a function of $c$. In order to find the optimal enforcement policy let me first find the equilibrium of the second stage. For any $c \geq 0$, the equilibrium of the individuals’ behavior is given by:

$$\mu^*(c) = \begin{cases} 1 & \text{if } c < \tilde{c} \\ 0 & \text{if } c \geq \tilde{c} \end{cases}.$$  \hfill (6)

and strictly concave in $c$. 

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Therefore, the equilibrium of the individuals’ behavior is unique for any given $c$. The agency anticipates the behavior of the individuals and chooses the optimal policy according to it. Therefore, the optimal policy of the enforcement agency, $c_{opt}$, is obtained by backwards induction:

$$c_{opt} = \arg \min_c \ h \cdot \mu^*(c) + c.$$  \hspace{1cm} (7)

From the second stage, it is clear that the agency will either invest $c = 0$ so that $\mu^* = 1$ or $c = \tilde{c}$ so that $\mu^* = 0$. This decision will be based on how large is $h$, the harm imposed to the community. If the agency chooses $c = 0$, then the welfare is equal to $h \cdot \mu^*(c) + c = h$ while if it choose $c = \tilde{c}$, the $h \cdot \mu^*(c) + c = \tilde{c}$. Thus, we have the following result:

**Proposition 1** In the absence of the externalities, equilibrium enforcement and compliance can be characterized as follows:

i) If $\tilde{c} > h$ the agency will spend $c_{opt} = 0$, which yields a no-compliance equilibrium, $\mu^*_{opt} = 1$.

ii) If $\tilde{c} < h$ the agency will spend $c_{opt} = \tilde{c}$, which yields a full-compliance equilibrium, $\mu^*_{opt} = 0$.

iii) If $\tilde{c} = h$ then the agency is indifferent between spending $c_{opt} = \tilde{c}$, which induces a full-compliance equilibrium, $\mu^*_{opt} = 0$, and spending $c_{opt} = 0$, which induces a no-compliance equilibrium, $\mu^*_{opt} = 1$.

Considering zero enforcement resources as optimal (as is the case in the model for certain parameter values) or having equilibrium rates with either full or no compliance may seem unusual in real life. However, let me emphasize that $\mu$ measures the compliance rate among potential criminals. There may be many members of the community that do not behave illegally even when enforcement resources are very low (for instance because illegal behavior has strong moral costs for them) and thus are outside of my analysis. In any case, Section 6 extends the model to a framework where the potential offenders are heterogeneous.

4 Law enforcement under the externalities

As already discussed in subsection 2.3, the enforcement technology depends on its sensitivity to the externalities, $\chi$, and on their net effect, $\mu - \eta$. Recall that $p_\chi < 0$ when $\mu > \eta$ and $p_\chi > 0$ when $\mu < \eta$. That is, the presence of externalities decreases the probability of sanction if the noncompliance rate exceeds the rate of community involvement, and increases the probability of sanction if the rate of community involvement exceeds the non-compliance rate.

4.1 The individuals’ behavior: multiple equilibria

In contrast with the benchmark, whenever $\chi > 0$ the probability of the sanction depends also on the non-compliance rate. Then the decision of an individual with respect to compliance depends on other individuals’ choices. When the individual decides not to comply, her utility is given by
$b - p(c, \mu, \eta) \cdot f$. Hence, an individual may decide to comply with the law when the non-compliance rate is low (which yields a larger value of $p(c, \mu, \eta)$) and not to comply when the non-compliance rate is high (because it yields a smaller value of $p(c, \mu, \eta)$).

**Definition 3** Let $c_0 \geq 0$ be such that $p(c_0, 0, \eta) = \frac{b}{f}$. I refer to $c_0$ as the minimal enforcement resources needed to reach $P = b/f$.

That is, $c_0$ is the level of enforcement resources needed to make individuals indifferent between compliance and non-compliance when the rate of non-compliance is zero. For the functional form specified for $p$, $c_0 = (b(1 - \eta)^\alpha/fk)^1/\alpha$.

**Definition 4** Let $\bar{c} \geq 0$ be such that $p(\bar{c}, 1, \eta) = \frac{b}{f}$. I refer to $\bar{c}$ as the maximal enforcement resources needed to reach $P = b/f$.

That is, $\bar{c}$ is the level of enforcement resources needed to make individuals indifferent between compliance and non-compliance when the rate of non-compliance is 1. For the functional form specified for $p$, $\bar{c} = (b(2 - \eta)^\alpha/fk)^1/\alpha$. Thus, for $\eta \in [0, 1)$:

$$c_0 \leq \bar{c} \leq \bar{c}.$$  \hspace{1cm} (8)

Notice that if the agency is not able to benefit from the information of the neighbors (i.e., $\eta = 0$) then $c_0$ coincides with $\bar{c}$. The equilibria for the individuals’ behavior can be characterized in terms of $c_0$ and $\bar{c}$ as shown in Figure 1. Notice that for $c < c_0$ and for $c \geq \bar{c}$ the individuals’ equilibria coincides with those of the benchmark case. First, for $c < c_0$, no compliance ($\mu^v = 1$) is the unique equilibrium possible, since by definition of $c_0$, if $c < c_0$ then $p(c, \mu, \eta) < \frac{b}{f}$ for all $\mu$; hence it is optimal for the individuals to violate the law. Second, for $c > \bar{c}$, full compliance ($\mu^v = 0$) is the unique equilibrium possible since by definition of $\bar{c}$, if $c > \bar{c}$ then $p(c, \mu, \eta) > \frac{b}{f}$ for all $\mu$; hence it is optimal for all individuals to comply. Finally, full compliance is also the unique equilibria for $c = \bar{c}$ since I have assumed that individuals comply with the law in case of indifference.

![Equilibria of the individuals](image)
However, because of the externalities there is an interval $[c, \tilde{c})$ of enforcement resources for which individuals’ equilibria differ from the benchmark. In particular, as shown in Figure 1, for this interval of resources there are multiple equilibria.

**Proposition 2** For any level of enforcement resources $c \in [c, \tilde{c})$:

i) There exists a no-compliance equilibrium, $\mu^* = 1$.

ii) There exists a full-compliance equilibrium, $\mu^* = 0$.

Furthermore, the length of the interval $[c, \tilde{c})$ is increasing in $\chi$.

**Proof.** See the Appendix

Due to the presence of externalities, more than one equilibrium arises for a given amount of enforcement resources $c \in [c, \tilde{c})$. Intuitively, because the net effect of the externalities on enforcement may be positive, some of the new equilibria that arise are good equilibria from the perspective of the enforcement agency. In particular, full-compliance equilibria can now be sustained for levels of enforcement resources below the threshold $\tilde{c}$.

Furthermore, since a larger $\chi$ results in a smaller $c$ and a larger $\tilde{c}$, there is more scope for multiple equilibria the more sensitive the technology is to the externalities. In other words, a higher elasticity of the enforcement technology with respect to the overall neighborhood effect implies a larger range of enforcement resources for which there are multiple equilibria.

### 4.2 Equilibrium selection

As just shown, multiple equilibria arise for any given $c$ in the range $[c, \tilde{c})$. The full-compliance equilibrium is the socially desirable one; however, it may be that it is not the equilibrium selected by the individuals. In this subsection I discuss possible selection criteria and characterize the risk-dominant equilibrium, as it seems to be the most compelling criterion. The concept of risk dominance (Harsanyi and Selten, 1988) consists of individuals choosing the less risky equilibrium action, incorporating each individual’s uncertainty about the strategy that the rest will end up choosing in equilibrium.\(^{15}\) Given $c \in [c, \tilde{c})$, notice that choosing not to comply is a risky strategy for the individuals since the rest of individuals might choose to comply. In particular, the higher is $c$, the higher is this strategic risk.

Alternative equilibrium selection methods are the global games’ approach (introduced by Carlson and van Damme, 1993), Young (1993)’s dynamic evolutionary process, or assuming payoff dominance as a focal point. As shown in Angeletos, Hellwig, and Pavan (2006), policy analysis in global games is quite complex. In a global game, a unique equilibrium is selected when agents have asymmetric uncertainty about the payoff structure. However, Angeletos et al. (2006) show that there is no equilibrium selection when the policy choice affects that uncertainty by signaling some

\(^{15}\)For a recent application of risk dominance in a law and economics setting, see Spier (2002).
Alternatively, the dynamic evolutionary model assumes that agents are boundedly rational in the sense that they have “an incomplete knowledge of recent precedents” (Young, 1993, page 75). Thus, I find that risk dominance is a more compelling equilibrium selection criterion. Also, it is important to note that both the global games approach and Young (1993)’s dynamic evolutionary model select the risk-dominant equilibrium in 2 x 2 coordination games.

Finally, in comparison with payoff dominance, risk dominance seems to be a more adequate criterion for this framework. Recent experimental findings (van Huyck et al, 1990; Straub, 1995; and Schmidt et al., 2003) have shown the difficulty players have in coordinating to reach the payoff-dominant equilibrium, and also the important role of risk dominance in explaining individuals’ behavior in coordination games. Moreover, there is not a clear payoff-dominant equilibrium in the model when the incidence of harm is taken into consideration. At the no-compliance equilibrium, each individual attains a payoff:

\[ b - p(c, \mu, \eta = 1) \cdot f > 0 \text{ for } c < \bar{c}. \tag{9} \]

Thus, it appears that the no-compliance equilibrium payoff dominates the full-compliance equilibrium since the latter yields a payoff of zero. However, such a comparison does not consider that the harm caused by non-compliance, \( \mu \cdot h \), may affect the payoffs of the individuals even though it does not affect their decision-making processes. Each individual takes the risk of harm as given and including it does not alter the optimal responses of the individuals; however, it may affect their payoffs. For instance, living in a neighborhood where illegal behavior is the rule might make individuals worse off, but it will not keep them from breaking the law when it is optimal for them individually.

This problem is not new in game theory. Harsanyi and Selten (1988) argue that, for rational individuals, transformations of the game that do not affect their best-response correspondences should not affect which equilibrium is considered as focal. Payoff dominance does not satisfy this requirement. In contrast, as explained by Harsanyi and Selten (1988) and by Myerson (1991), risk dominance is a solution concept that is invariant to changes in the agents’ payoffs that do not affect their best-response correspondences.

The risk-dominance selection concept is typically applied in the context of two player games, while in this context there is (formally) a continuum of players. However, the setup of the model allows us to easily interpret individuals’ behavior as a game with two players and two strategies per player. Consider the decision process of an individual \( i \) by interpreting the model as a 2 x 2 game where all other players are represented by a "representative agent." Thus, individual \( i \) decides whether to comply or not, and his payoff depends on the strategy chosen by a representative agent.

---

\[ ^{16} \text{In my framework, if agents had uncertainty about } k, \text{ then asymmetric uncertainty among the individuals would lead to the selection of a unique equilibrium. However, multiple equilibria would arise again when the choice of enforcement resources conveys information about } k \text{ (i.e., when the choice of } c \text{ signals the type of } k \text{ to the individuals).} \]

\[ ^{17} \text{Section 5 studies the case where the individuals’ decisions are affected by the harm from criminal activities because it affects non-criminals more than criminals.} \]
that reflects the choice of the rest of the potential offenders. Because there is a continuum of individuals the contribution of individual $i$’s choice to the payoff of the representative agent is negligible. As a consequence, the non-compliance rate is 0 whenever the representative individual decides to comply and 1 when she decides not to comply.

<table>
<thead>
<tr>
<th>PAYOFFS FOR INDIVIDUAL $i$</th>
<th>Representative individual</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMPLY</td>
<td>NOT COMPLY</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b - \frac{kfc^a}{(1-\eta)^x})</td>
<td>(b - \frac{kfc^a}{(2-\eta)^x})</td>
</tr>
</tbody>
</table>

Table 1: Payoffs for individual $i$ when $\eta$ is exogenous

Payoffs for individual $i$ are shown in Table 1. Under risk dominance each individual’s strategy consists of the best response when assigning a positive probability to the possibility of other individuals choosing the non-equilibrium strategy (i.e., there is a risk that the rest of the individuals will choose to comply when the individual has chosen not to comply or vice versa). Given the enforcement resources, individuals choose the risk-dominant strategy. As a consequence, the following equilibrium selection takes place.

**Proposition 3** There exists a level of enforcement resources $c^* \in (\bar{c}, \tilde{c})$, such that:

i) For any enforcement policy $c < c^*$ the no-compliance equilibrium is risk dominant.

ii) For any enforcement policy $c > c^*$ the full-compliance equilibrium is risk dominant.

iii) For an enforcement policy $c = c^*$, there is not a risk dominant equilibrium.

Furthermore, $c^*$ is decreasing in $\eta$ and is invariant to including the harm caused by non-compliance in the individuals’ payoffs.

**Proof.** See the Appendix □

For each level of enforcement resources, risk dominance makes one of the equilibria focal, but which one depends on resources being above or below a level of enforcement resources $c^*$.$^{18}$ In particular, $c^*$ is increasing in $\chi$, $b$ and $\alpha$, and decreasing in $k$ and $f$.

Thus, whenever $\tilde{c} < c^*$ the externalities imply that more resources are needed to enforce the law. In contrast, whenever $\tilde{c} > c^*$ the externalities allow enforcement of the law with fewer resources.

---

$^{18}$ In order to ensure that $p(c^*, \mu, \eta) \leq 1$ for all $\mu$, then I impose $b/f \leq \frac{1}{2} + \frac{(1-\eta)^x}{(2-\eta)^x}$. This condition guarantees that $c^* \leq ((1 + \mu - \eta)^x/k)$ which, as discussed in footnote (13), is the level of resources such that $p(c, \mu, \eta) = 1$. 

14
4.3 Policy implications

The problem of the enforcement agency can be solved by maximizing social welfare.

**Proposition 4** In the presence of the externalities, equilibrium enforcement and compliance can be characterized as follows:

i) If \( c^* > h \) the optimal enforcement resources are \( c_{opt} = 0 \), which yields a no-compliance equilibrium, \( \mu_{opt}^* = 1 \).

ii) If \( c^* < h \) the optimal enforcement resources are \( c_{opt} = c^* \), which yields a full-compliance equilibrium, \( \mu_{opt}^* = 0 \).

iii) If \( c^* = h \) then the agency is indifferent between spending \( c_{opt} = c^* \), which induces a full-compliance equilibrium, \( \mu_{opt}^* = 0 \), and spending \( c_{opt} = 0 \), which induces a no-compliance equilibrium, \( \mu_{opt}^* = 1 \).

**Proof.** The proof is straightforward once the results from the previous proposition are inserted into the social welfare function. Note that full compliance must follow an expenditure of \( c^* \) in order in order for there to be an equilibrium at \( c^* \). Therefore, there are values of \( h \) for which the externalities have relevant policy implications. Comparing this result with Proposition 1 illustrates the impact of the externalities on the optimal enforcement policy. First, for the case where \( \tilde{c} < c^* \) and whenever \( \tilde{c} < h < c^* \), it is optimal to enforce the law only when there are no externalities. The externalities increase the amount of resources needed to enforce the law to a level at which it is no longer socially optimal. This result illustrates situations that may happen in high crime neighborhoods; when the situation is considered to be "hopeless," some laws are no longer enforced. The model explains how the positive externality among criminals may be such that the law is too costly to be enforced.

Second, if \( \tilde{c} < c^* < h \), the law is enforced both in the benchmark and when there are externalities, although more enforcement resources are needed in the latter case. Third, for the case where \( \tilde{c} > c^* \) and whenever \( c^* < h < \tilde{c} \), it is optimal to enforce the law only under the externalities. Finally, if \( \tilde{c} > c^* \) and \( c^* < \tilde{c} < h \), the law is enforced in both the benchmark and in the presence of the externalities, but now more enforcement resources are needed in the former case.\(^{19}\)

**Corollary 1** The net effect of the externalities on enforcement is determined in equilibrium:

i) When \( \mu_{opt}^* = 1 \), in equilibrium the net effect is negative \((p_\lambda < 0)\).

ii) When \( \mu_{opt}^* = 0 \), in equilibrium the net effect is positive \((p_\lambda > 0)\).

\(^{19}\)This result is in contrast to the claim by Bar-Gill and Harel (2001) that when a higher crime rate reduces the likelihood of the sanction, then the optimal investment in enforcement is always lower in the benchmark than in the model that incorporates the crime rate as a determinant of the expected sanction. They come to this conclusion because they fail to account for the fact that the probability of sanction function is different when there is an externality than when no externality exists. Essentially, the function has another argument that reflects the intensity of the externality, and they do not take account of this argument’s independent influence on the probability of sanction function.
Intuitively, whether the net effect of the externalities is positive or negative in equilibrium depends on the compliance rate resulting from the optimal enforcement resources. If $\mu_{opt}^* < \eta$, then the equilibrium compliance is such that the net effect of the externality on the enforcement technology is positive. Alternatively, if $\mu_{opt}^* > \eta$ then in equilibrium the net effect of the externality on enforcement is negative. Hence, in equilibrium, two communities that differ only in their values of $\eta$ may end up with different non-compliance rates. However, also note that there are ranges of $\eta$ which would generate the same effect, so that empirical analysis will not find a simple monotonicity between a measurement of $\eta$ and one of $\mu$.

This result is particularly relevant since there exist policies that may affect the value of $\eta$. Considering a technology with sensitivity to the externalities $\chi$, the following proposition summarizes what happens when the enforcement agency may influence the involvement of the community, $\eta$.

**Proposition 5** For any $h > 0$ there exists a large enough $\tilde{\eta} < 1$, above which enforcing the law becomes optimal for the agency. As a consequence, laws that were unenforced in the benchmark (because $h < \tilde{c}$), may be enforced in the presence of the externalities. The critical value $\tilde{\eta}$ might be decreasing or increasing in the sensitivity of the enforcement technology to the externalities, $\chi$.

**Proof.** See the Appendix

Given the value of harm generated by non-compliance, $h$, and a technology with sensitivity to the externalities $\chi$, it may not be optimal for the agency to enforce the law. However, the agency may reduce the necessary level of resources to enforce the law, $c^*$, by increasing $\eta$ to $\tilde{\eta}$. In particular, the value of $\tilde{\eta}$ provides an index to measure the objective that community policing must accomplish.

The Neighborhood Watch Program created in 1972 is an example of the type of policies that promote communication between neighbors and the police in the United States. The purpose of this program is to reduce residential crime by involving citizens and private organizations in law enforcement activities. As the Neighborhood Watch Manual (elaborated by the United States’ National Sheriffs’ Association\(^{20}\)) argues, "the impact of law enforcement alone is minimal when compared with the power of private citizens working with law enforcement." Efforts on community policing were encouraged through the US Violent Crime Control and Law Enforcement Act of 1994 (the Crime Act). The results obtained in this section provide a rationale for how these programs may have substantial effects if they succeed in increasing the communication between the police and the public.

In Europe, several countries have established community policing programs, for instance the *police de proximité* in France or the *Komunale Kriminalprävention* in Germany. However, as observed in Brogden and Nijhar (2005), "practice and understanding of the problem seem a long way" from the Anglo-American experience. Nevertheless, rising recorded crime rates and riots by

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ethnic minorities in France have prompted calls for a determined implementation of community policing.\textsuperscript{21}

4.4 Endogenous neighborhood involvement in enforcement

If a larger number of offenders in a community leads to a lower neighbors’ involvement in enforcement, then the involvement will depend on $\mu$. For instance, this is the case if violators can retaliate against those who provide information to the enforcement authority or if witnesses are intimidated. In some urban (generally high crime) communities of the United States, campaigns known as "Stop Snitchin’" attempt to deter collaboration between neighbors and the police. To model this kind of situation, the involvement of a neighborhood must be endogenously determined.

Until now I have assumed that the involvement of a neighborhood in enforcement is exogenous, and measured by the parameter $\eta$. Consider now that the degree of involvement is a monotonically decreasing function $n$ of $\mu$ such that $n(\mu) \in [0, 1)$ for all $\mu$.\textsuperscript{22} Then the probability of the sanction is given by:

$$P = p(c, \mu) = kc^\alpha / (1 + \mu - n(\mu))^\chi,$$

where $p_\mu < 0$ since $n'(\cdot) \leq 0$.

Notice that introducing the function $n$ does not alter any of the definitions. The equilibrium condition for the individuals’ behavior still implies that:

$$\mu^*(c) = \begin{cases} 0 & \text{if } p(c, 0) \geq b/f, \\ 1 & \text{if } p(c, 1) < b/f, \end{cases}, \quad (11)$$

where now $\mu^*$ is determined solely by $c$.

Using the definitions for minimal and maximal resources needed to reach $P = b/f$ then, under an endogenous involvement of the neighborhood, they are $c = (b(1 - n(0))^\chi / fk)^{1/\alpha}$ and $\hat{c} = (b(2 - n(1))^\chi / fk)^{1/\alpha}$, respectively. Notice that $c \leq \hat{c} < \hat{c}$ still holds. Therefore, as with the exogenous neighborhood involvement, multiple equilibria arise for enforcement resources in the


\textsuperscript{22}In contrast, Huck and Kosfeld (2007) consider a model where new members’ recruitment for a neighborhood watch program is easier when there is a "crime crisis." In such a framework, a higher number of burglaries makes it more likely for neighbors to be enrolled in the neighborhood watch program, which leads to a higher probability of catching a burglar. Their model differs from mine in several aspects; in particular, it evaluates the optimal magnitude of the sanction rather than the optimal level of enforcement resources, and it does not allow for congestion. Nevertheless, I can adjust my model to study a framework analogous to theirs. Assuming that neighbors’ involvement is increasing in the crime rate (and excluding the congestion effect) I would have that:

$$p(c, \mu) = kc^\alpha / (1 + n(\mu))^\chi$$

where $n(\mu)$ would be increasing in $\mu$ rather than decreasing; hence, there would be a negative externality among offenders. While this negative externality is certainly possible, it would predict a negative correlation in criminal behavior (as in footnote (11)’s case), which seems to be at odds with the available evidence.
interval $c \in [\underline{c}, \overline{c}]$, as shown in the following proposition.\(^{23}\)

**Proposition 6** When the neighborhood involvement is endogenous, Proposition 2 still holds. Furthermore, the interval $[\underline{c}, \overline{c})$ is increasing in the spread between $n(0)$ and $n(1)$, for $n(0)$ or $n(1)$ held fixed.

**Proof.** See the Appendix. ■

Notice that $n(0) - n(1)$ measures the change in the neighbors’ involvement when the compliance rate switches from no compliance to full compliance. Thus, a larger change in the neighborhood involvement leads to a larger range of enforcement resources for which there are multiple equilibria.

The payoffs for individual $i$ are shown in Table 2. Following the same steps as in Section 4.2, I find the risk-dominant equilibrium for each level of enforcement resources in the interval $[\underline{c}, \overline{c})$.

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<td>COMPLY</td>
<td>NOT COMPLY</td>
</tr>
<tr>
<td>COMPLY</td>
<td>0</td>
</tr>
<tr>
<td>NOT COMPLY</td>
<td>$b - \frac{kfc^a}{(1-n(0))^z}$</td>
</tr>
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</table>

Table 2: Payoffs for individual $i$ when the involvement is endogenous, $\eta = n(\mu)$

\(^{24}\)As with $c^*$, in order to ensure that $p(c^*_{\text{endog}}, \mu) \leq 1$ for all $\mu$, then I impose $b/f \leq \frac{1}{2} + \frac{(1-n(0))^z}{(2-n(1))^z}$.

**Proposition 7** For an endogenous neighborhood involvement described by the function $n$, there exists a level of enforcement, $c^*_{\text{endog}}$, such that:

i) For any enforcement policy $c < c^*_{\text{endog}}$ the no-compliance equilibrium is risk dominant.

ii) For any enforcement policy $c > c^*_{\text{endog}}$ the full-compliance equilibrium is risk dominant.

iii) For an enforcement policy $c = c^*_{\text{endog}}$ there is not a risk dominant equilibrium.

Furthermore, $c^*_{\text{endog}}$ is decreasing in $n(1)$ and in $n(0)$.

**Proof.** As shown in the Appendix, the proof is almost identical to the proof in Proposition 3. ■

As in Proposition 3, for each level of enforcement resources, risk dominance makes one of the equilibria focal. However, the threshold level of resources $c^*_{\text{endog}}$ depends now on $n(1)$ and $n(0)$.\(^{24}\) In particular, a lower level of neighborhood's involvement under no compliance, $n(1)$, results in a higher $c^*_{\text{endog}}$ needed to enforce the law. Likewise, a lower level of neighborhood’s involvement under full-compliance, $n(0)$, results in a higher $c^*_{\text{endog}}$.

Having a unique equilibrium per level of enforcement resources, I can solve the problem of the enforcement agency as in Proposition 4. Specifically, a small enough $n(1)$ may result in $c^*_{\text{endog}} > h$, which implies that the optimal resources are zero. Therefore, as the model illustrates, campaigns

\(^{23}\)Notice that $\underline{c} < \overline{c}$ since $1 - n(0) < 2 - n(1)$. Also, notice that $\underline{c} \leq \overline{c} < \overline{c}$ still holds.

\(^{24}\)As with $c^*$, in order to ensure that $p(c^*_{\text{endog}}, \mu) \leq 1$ for all $\mu$, then I impose $b/f \leq \frac{1}{2} + \frac{(1-n(0))^z}{(2-n(1))^z}$. \(18\)
like "Stop Snitchin'" can clearly cause an increase in the resources needed for enforcement. Furthermore, enforcement may become non-optimal because of a decrease in \( n(1) \). The Baltimore police have launched the counter-campaign "Keep Talking" to prevent the negative consequences of a deterioration in the communication between police and neighbors.\(^{25}\)

5 Further discussion

5.1 Other possible neighborhood externalities

Until now I have assumed that becoming a criminal does not affect the harm perceived from others’ criminal activities. That is, becoming a criminal does not make individuals less likely to be victims of crime. However, perhaps individuals can avoid being victims of a crime by becoming criminals themselves.\(^{26}\) In this case, a third externality arises because this additional benefit from becoming a criminal is increasing in the crime rate (since a higher crime rate is associated with a higher degree of the harm). In this subsection I study the consequences of adding this additional externality into the model.

Let \( d(\mu) \) be the difference between the degree of harm faced by compliers and the degree of harm faced by violators given a non-compliance rate \( \mu \). Then, normalizing to zero the payoff of complying with the law, the payoff from not complying is \( b + d(\mu) - Pf \), where \( d(\mu) > 0 \) represents the gain from avoiding part of the harm by becoming a violator.\(^{27}\) In addition, I assume that \( d(0) = 0 \), that is, when the rest of individuals are complying with the law, then becoming a criminal does not imply any gain in terms of avoiding harm. Therefore the equilibria of the individuals is given by:

\[
\mu^*(c) = \begin{cases} 
0 & \text{if } p(c, 0, \eta) \geq (b + d(0))/f \\
1 & \text{if } p(c, 1, \eta) < (b + d(1))/f 
\end{cases}.
\] (12)

Using the definition for minimal enforcement resources, then \( c = ((b(1 - \eta) + d(0))/kf)^{1/\alpha} = (b(1 - \eta)^{\alpha}/fk)^{1/\alpha} \). That is, this additional externality does not affect \( c \). In contrast, the maximal level of enforcement resources is now \( \tilde{c} = ((2 - \eta)^{\alpha}(b + d(1))/kf)^{1/\alpha} \) which is greater than the initial \( (b(2 - \eta)^{\alpha}/kf)^{1/\alpha} \). Then, again multiple equilibria arise for any level of enforcement resources \( c \in [c, \tilde{c}] \). Furthermore, this additional externality implies a larger range of enforcement resources for which there are multiple equilibria.

The payoffs for individual \( i \) are shown in Table 3. As in the previous sections an equilibrium is reached when all of the individuals choose the same strategy. The following proposition summarizes these results and studies the implications for the risk-dominant equilibrium.


\(^{26}\)I thank an anonymous referee for suggesting the analysis of this case.

\(^{27}\)Recall that when the harm from criminal activities affects criminals and non-criminals the same, as assumed in previous sections, such harm is irrelevant for the decision of whether to comply with the law or not.
Table 3: Payoffs for individual $i$ when compliers perceive a higher degree of the harm

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</tr>
<tr>
<td>Individual $i$</td>
<td></td>
</tr>
<tr>
<td>COMPLY</td>
<td>0</td>
</tr>
<tr>
<td>NOT COMPLY</td>
<td>$b - \frac{kfc^a}{(1-\eta)\chi} + d(l)$</td>
</tr>
</tbody>
</table>

Table 3: Payoffs for individual $i$ when compliers perceive a higher degree of the harm

**Proposition 8** When compliers perceive a higher degree of the harm than violators, Propositions 2 and 3 hold. Furthermore, $c$ and $c^*$ are larger than when there are no differences in how individuals perceive the harm.

Therefore, this additional externality increases the cost of enforcement further. Moreover, following Proposition 4, such an increase in $c^*$ may imply that enforcing the law in that neighborhood is no longer socially optimal. Finally, notice that this same analysis serves to describe reputational benefits from becoming a criminal that are increasing in $\mu$, such as the ones modeled in Silverman (2004). The results here complement those in Silverman (2004) since I am able to select among the multiple equilibria.

### 5.2 Isolating the effect of congestion

In this subsection I impose $\eta = 0$ to focus on the externality caused by congestion. Notice that congestion at the neighborhood level is not the only form of congestion that may arise in enforcement activities (e.g., there may be congestion in the administrative or court procedures that ensure punishment that depend on the “technology” determined at the city or state level).

Since $\eta = 0$, $P$ is given by:

$$P = p(c, \mu) = kc^a/(1 + \mu)^\chi;$$

(13)

hence, $p_\chi < 0$ for all $\mu$. The benchmark’s results in Section 3 still hold for the case with no externality, $\chi = 0$. Figure 2 compares the equilibria for both the benchmark and the case of congestion. Since congestion has a negative effect on enforcement, all the new equilibria under the externality for $c \in [c, \bar{c})$ are no-compliance equilibria.
Using the equilibrium selection results, there is a $c^* \in (\bar{c}, \tilde{c})$ such that for any $c > c^*$, the full compliance equilibrium is selected. The main difference of this specific case with respect to the general model is that now $\partial c^*/\partial \chi > 0$ (recall that for $\eta > 0$ the sign of this partial derivative was ambiguous). As a consequence, the more sensitive is the enforcement technology to congestion, the higher the amount of resources that are needed to induce the full-compliance equilibrium. Thus, identical locations differing only in how sensitive their technology is to the externality, require different amounts of resources to induce compliance since $c^*$ is increasing in $\chi$.

Moreover, a high enough $\chi$ implies that the enforcement agency optimally chooses not to enforce the law because $c^* > h$. Thus, if congestion exists and it is not accounted for, the optimal policy will not be correctly specified. Also, and more importantly, if it is possible to decrease the sensitivity of the technology to this externality, then the amount of resources needed for enforcement will be lower. Decreasing the sensitivity of the technology to congestion is possible by decreasing the amount of enforcement resources that are specifically needed for activities related to punishment.\footnote{Recall that, as explained in Section 2.3, in the enforcement process resources are used for activities related to detection, apprehension and punishment.}

For example, if punishment could take place instantaneously at the moment of detection, then resources would only be needed for detecting violators (e.g., there would be no need for tow trucks), which reduces considerably the possibility of congestion.

Examples of policies aimed at reducing the resources needed specifically for punishing activities are: incentives for voluntary payment of fines; demerit point systems for traffic violations,\footnote{Demerit point systems associated with traffic regulation are becoming a popular policy. Punishment is more immediate than with traditional fines. Since the agency administers the number of points of each driver, the punishment becomes effective by simply reducing the number of points of the violator.} or allowing for plea bargaining in criminal cases. More recently, the use of information and computer technology may become an effective way of reducing the need of resources for punishing activities. Electronic citation programs and other forms of electronic processing technology programs are being adopted throughout the United States (Department of Transportation (2003)). The International Association of Chiefs of Police (2003) recommends the use of electronic citations because “the physical process of writing and issuing traffic citations demands a significant amount of time and
effort” from the patrol officer, the offices’ personnel and the court office staff. In Europe, the European Commission launched in 2005 the project Fully Automatic Integrated Road Control to promote the use of this type of technology. However, the adoption of these techniques into traffic management is proceeding slowly in most European countries.

6 Heterogeneous individuals

In this section, I assume that the benefit from violating the law follows a uniform distribution on the interval $[0,1]$. This extension allows me to examine how the effects of the externalities remain when the individuals are not homogenous. Since $b$ follows a uniform distribution, the non-compliance rate is given by:

$$
\mu(c) = \int_{p(c,\mu,\eta)}^{1} db = 1 - f \cdot p(c, \mu, \eta) = 1 - \frac{f k c^\alpha}{(1 + \mu - \eta)^\chi}.
$$

Let $c(\mu; \chi, \eta)$ be the level of enforcement resources that induce a non-compliance rate of $\mu$ given a technology with a sensitivity to the externalities, $\chi$, and a neighborhood with involvement, $\eta$. Then:

$$
c(\mu; \chi, \eta) = \left(\frac{(1 - \mu)(1 + \mu - \eta)^\chi}{f k}\right)^{1/\alpha}.
$$

If the technology is not sensitive to the externalities (i.e., $\chi = 0$) then $c(\mu; 0, \eta) = \left(1 - \mu / f k\right)^{1/\alpha}$ which is a one-to-one function of $\mu$. The externalities introduce distortions in the level of enforcement resources needed to reach a specific non-compliance rate. Also, multiple equilibria arise when $c(\mu; \chi, \eta)$ is not a one-to-one function of $\mu$. This is because if $c(\mu; \chi, \eta)$ is not monotone in $\mu$, the same level of enforcement resources may induce more than one non-compliance rate.

**Proposition 9** For $\mu > \eta$ ($\mu < \eta$) enforcement is more (less) costly with the externalities (i.e., $\chi > 0$) than without them (i.e., $\chi = 0$). Furthermore, for $\chi > 1 - \eta$ the externalities lead to multiple equilibria.

**Proof.** See the Appendix

Figure 3 shows the equilibria of the individuals for the case where multiple equilibria arise (i.e., for $\chi > 1 - \eta$). I denote as $[\underline{C}, \overline{C}]$ the interval of enforcement resources for which multiple equilibria arise. Since individuals differ now in their benefit from violating the law, $b$, definitions 3 and 4 do not apply here. Instead, $\underline{C}$ is the minimal enforcement resources needed to reach $\mu^* = 0$ as an equilibrium. That is, $\underline{C} = c(0; \chi, \eta)$ which implies $\underline{C} = \left(1 - \eta / f k\right)^{1/\alpha}$. Then, for any level of enforcement resources $c > \underline{C}$, $\mu^* = 0$, is a possible equilibrium, although there may be others. Finally, $\overline{C}$ is the maximal level of enforcement resources for which a compliance rate $\mu^* > 0$ is an equilibrium, in other words $\overline{C}$ is the maximum of $c(\mu; \chi, \eta)$. Thus, for $c > \overline{C}$, $\mu^* = 0$ is the unique equilibrium.
7 Conclusion

This paper studies externalities that affect the productivity of enforcement resources. The first externality is due to congestion of enforcement resources, which creates a positive externality among offenders by decreasing the probability of the punishment. The second externality is determined by the community’s involvement in enforcement activities. Neighborhoods with a higher degree of involvement lead to a higher productivity of enforcement resources. When the involvement of the neighborhood is decreasing in the number of offenders, an additional positive externality among offenders arises.

These externalities explain the interdependence of individuals’ decisions to break the law, and generate neighborhood effects. Multiple equilibria arise for a given level of enforcement resources. Using risk dominance to select among the equilibria, I show how the externalities affect the optimal compliance rate and the optimal level of enforcement resources. When the net neighborhood effect is negative and strong enough, it may be too costly to enforce the law in that neighborhood.

While a significant number of empirical studies have established the importance of neighborhood effects on crime, the issue has been largely neglected in theoretical models on enforcement. This paper provides a theoretical framework that explains how neighborhood effects may be related to the productivity of the enforcement technology. In relation to particular residential policies, the model allows for a better understanding of community policing and its consequences.

The results are extended to a framework where individuals are heterogeneous in the benefit from breaking the law which follows a uniform distribution. Alternative distribution functions are left for further research; however, multiple equilibria and similar conclusions are expected. Future progress in game theory is needed to find an equilibrium selection concept that can be applied to the framework with heterogeneous individuals.

\footnote{For a survey, see Sampson et al. (2002).}
Further research could also measure the impact of the externalities. However, important methodological problems arise when trying to study neighborhood effects (such as selection bias or how to determine the boundaries of local communities).\footnote{Again, see Sampson et al. (2002).} More importantly, differences in the technology’s sensitivity to congestion and in the community’s involvement in enforcement activities are hard to observe and measure. Nevertheless, this paper provides a rational explanation for the interdependence of individuals’ decisions to break the law, which is a stylized fact that has already been documented.

References


\footnote{Again, see Sampson et al. (2002).}


APPENDIX

Proof of Proposition 2:

i) For $c < \bar{c}$ and $\chi > 0$, then $\lambda b - \frac{\lambda f}{\lambda} > 0$ by definition of $\bar{c}$. Thus, it is optimal for each individual to break the law if all others do, and $\mu^* = 1$ is an equilibrium.

ii) For $c \geq \bar{c}$ and $\chi > 0$, then $\lambda p(c,0,\eta) = b/(1-\eta)^\chi \geq b/f$ by definition of $\bar{c}$. Thus, it is optimal for each individual to comply if all others do, and $\mu^* = 0$ is an equilibrium.

Finally, $\bar{c} - \bar{c} = \lambda b/(2f)^{\chi+\alpha}/(2-\eta)^\chi - (1-\eta)^\chi$, which is increasing in $\chi$.

Proof of Proposition 3:

As shown in Table 1, individual $i$ and the representative individual play a 2 x 2 coordination game. Therefore, an equilibrium is reached when both players choose the same strategy.

When being at the no-compliance equilibrium, let $\lambda$ be the probability that the representative individual chooses the compliance equilibrium; then, individual $i$ faces a deviation loss of $\lambda(b - \lambda f/c)/(1-\eta)^\chi)$. Since individual $i$ obtains a payoff of zero in case of deviating to comply, then she chooses not to comply as long as:

$$\lambda b - \frac{\lambda f}{\lambda} + (1-\lambda) b - \frac{\lambda f}{(2-\eta)^\chi} \geq 0.$$ 

That is, as long as:

$$\lambda \leq \frac{(b(2-\eta)^\chi - \lambda f/c)(1-\eta)^\chi}{\lambda f/c((2-\eta)^\chi - (1-\eta)^\chi)}.$$ 

I denote as $\bar{\lambda}$ the highest probability for which this condition holds (i.e., the highest for which $i$ chooses to not comply).

Similarly, when being at the full compliance equilibrium, I denote as $\gamma$ to the probability that the representative individual deviates to not comply. Then individual $i$ chooses to comply only as long as the payoff from deviating is lower than the zero payoff from maintaining non-compliance. That is, as long as:

$$(1-\gamma) b - \frac{\gamma f/c}{(1-\eta)^\chi} + \gamma b - \frac{\gamma f/c}{(2-\eta)^\chi} \leq 0.$$
Then:
\[
\gamma \leq \frac{(fke^\alpha - b(1-\eta)^\chi)(2-\eta)^\chi}{fke^\alpha((2-\eta)^\chi - (1-\eta)^\chi)}.
\]
I denote as \( \tilde{\gamma} \) the highest probability for which this condition holds. Then, for individual \( i \) to comply risk dominates not to comply whenever \( \tilde{\gamma} > \tilde{\lambda} \). Meanwhile, not to comply risk dominates to comply whenever \( \tilde{\lambda} > \tilde{\gamma} \). Meanwhile, none of the equilibria risk dominates the other when \( \tilde{\lambda} = \tilde{\gamma} \).

Let \( c^* \) be the threshold amount of enforcement resources that satisfies this equality, then:
\[
c^* = \left( \frac{2b(1-\eta)^\chi}{(2-\eta)^\chi} \right)^{1/\alpha}.
\]
Thus, for any \( c < c^* \) the risk-dominant strategy for player \( i \) is not to comply. Since every player faces the same setup and the same payoff function, for any \( c < c^* \) the equilibrium selected is the no-compliance equilibrium. Similarly, for \( c > c^* \) the equilibrium selected is the full-compliance equilibrium.

Notice that for all \( \chi \in (0, 1] \) it is the case that \( c^* \in (\underline{c}, \bar{c}) \). More precisely:
\[
c^* = \underline{c} \cdot \left( \frac{2(2-\eta)^\chi}{(2-\eta)^\chi + (1-\eta)^\chi} \right)^{1/\alpha},
\]
where \( \frac{2(2-\eta)^\chi}{(2-\eta)^\chi + (1-\eta)^\chi} > 1 \) for all \( \chi > 0 \). Also:
\[
c^* = \bar{c} \cdot \left( \frac{2(1-\eta)^\chi}{(2-\eta)^\chi + (1-\eta)^\chi} \right)^{1/\alpha},
\]
where \( \frac{2(1-\eta)^\chi}{(2-\eta)^\chi + (1-\eta)^\chi} < 1 \) for all \( \chi > 0 \). Also, notice that \( \frac{\partial c^*}{\partial \eta} < 0 \).

Finally, \( c^* \) is invariant to including the harm caused by non-compliance into the individuals’ payoffs. Notice that if non-compliance causes a harm \( v_i(h\mu) \) to individual \( i \), then the condition for \( i \) not to deviate from the non-compliance strategy is given by:
\[
\lambda \left( b - \frac{fke^\alpha}{(1-\eta)^\chi} \right) + (1-\lambda) \left( b - \frac{fke^\alpha}{(2-\eta)^\chi} - v_i(h\mu) \right) \geq -(1-\lambda)v_i(h\mu),
\]
which is equivalent to the condition imposed previously. It can be shown analogously for \( \gamma \).

**Proof of Proposition 5:**

From proposition 4 we know that it is optimal to enforce a law for any \( h > 0 \) if and only if \( h > c^* \). Let \( \tilde{\eta} \) be the value such that \( h = c^* \). The existence of \( \tilde{\eta} \) in the interval \([0, 1)\) is ensured because as shown in the proof of Proposition 4, \( c^* = \underline{c} (2(2-\eta)^\chi /((2-\eta)^\chi + (1-\eta)^\chi))^{1/\alpha} \); and thus:
\[
\lim_{\eta \to 1} c^* = 0 \quad \text{since} \quad \lim_{\eta \to 1} c = 0.
\]
Moreover, $\tilde{\eta}$ is unique since $\partial c^*/\partial \eta < 0$. Therefore, for any $\eta \in (\tilde{\eta}, 1)$ it holds that $h > c^*$ (i.e., it is optimal to enforce the law).

Finally, the sign of $\frac{\partial \eta}{\partial \chi}$ can be obtained locally to $\eta = \tilde{\eta}(\chi)$ by applying the implicit function theorem:

$$\frac{\partial \eta}{\partial \chi} = -\frac{\partial c^*/\partial \chi}{\partial c^*/\partial \eta},$$

where $\frac{\partial c^*/\partial \eta} > 0$ as shown in the proof of proposition 4. Therefore, $\frac{\partial \eta}{\partial \chi} < 0$ if and only if $\partial c^*/\partial \chi > 0$ which is only true for an interval of values of $\eta$.

**Proof of Proposition 6:**

In the proof of Proposition 2, the results are shown for $\eta \in [0, 1)$. Thus, introducing $n(\mu) \in [0, 1)$ instead of $\eta$ does not affect the results. In particular, for part i) notice that for $c < \bar{c}$ (using the new value obtained for $\bar{c}$) and $\chi > 0$, then $b - p(c, 1):f > 0$. Therefore, it is optimal for each individual to break the law if all others do; hence, $\mu^* = 1$. Similarly in part ii), for $c > \bar{c}$ (using the new value obtained for $\bar{c}$), then $b - p(c, 0):f < 0$. Therefore, it is optimal for each individual to comply if all others do; hence, $\mu^* = 0$ is an equilibrium.

Finally, $\bar{c} - \bar{c} = \frac{b}{f(2 + n(1))}((2 - n(1))^{\chi/\alpha} - (1 - n(0))^{\chi/\alpha})$, which is increasing in $\chi$ as in the exogenous case. In addition, denoting the difference $n(0) - n(1) > 0$ as $D$, I can rewrite $\bar{c} - \bar{c} = \frac{b}{f(2 + D - n(0))}((2 + D - n(0))^{\chi/\alpha} - (1 - n(0))^{\chi/\alpha})$. Then, $\bar{c} - \bar{c}$ is increasing in $D$ when holding $n(0)$ fixed. Similarly, substituting $n(0)$ with $D + n(1)$, I find that $\bar{c} - \bar{c}$ is increasing in $D$ when holding $n(1)$ fixed.

**Proof of Proposition 7:**

Using the payoffs in Table 2, and using the same procedure as in Proposition 3, let $\lambda_{\text{endog}}$ be the probability that the representative individual deviates from the non-compliance equilibrium by choosing to comply. Then individual $i$ chooses not to comply only as long as:

$$\lambda_{\text{endog}} \leq \frac{(b(2 - n(1))^{\chi} - fke^\alpha)(1 - n(0))^{\chi}}{fke^\alpha((2 - n(1))^{\chi} - (1 - n(0))^{\chi})}.$$

Similarly, when being at the full compliance equilibrium, I denote as $\gamma_{\text{endog}}$ to the probability that the representative individual deviates to not comply. Then individual $i$ chooses to comply only as long as:

$$\gamma_{\text{endog}} \leq \frac{(fke^\alpha - b(1 - n(0))^{\chi})(2 - n(1))^{\chi}}{fke^\alpha((2 - n(1))^{\chi} - (1 - n(0))^{\chi})}.$$

Therefore, following the same steps as in the proof of Proposition 3, I find a threshold level of enforcement resources $c^*_{\text{endog}}$ such that for $c < c^*_{\text{endog}}$ the full-compliance equilibrium is risk dominant, and for $c > c^*_{\text{endog}}$ the no-compliance equilibrium is risk dominant, where $c^*_{\text{endog}}$ is given by:

$$c^*_{\text{endog}} = \left(\frac{2b(1 - n(0))^{\chi}(2 - n(1))^{\chi}}{((2 - n(1))^{\chi} + (1 - n(0))^{\chi})f(k)}\right)^{1/\alpha}.$$
Notice that $\frac{\partial c^*_{endog}}{\partial (1)} < 0$ and $\frac{\partial c^*_{endog}}{\partial (0)} < 0$. Also, as in Proposition 3, $c^*_{endog} \in (c, \bar{c})$ and $c^*_{endog}$ is invariant to including the harm caused by non-compliance into the individuals’ payoffs.

**Proof of Proposition 8:**

This proof is as the proof of Proposition 3 except that now if both individual $i$ and the representative individual choose to not comply then the payoff of individual $i$ is $\left( b - \frac{fk\alpha}{(2-\eta)x} + d(1) \right)$ rather than $\left( b - \frac{fk\alpha}{(2-\eta)x} \right)$. Then individual $i$ chooses not to comply as long as:

$$\lambda\left( b - \frac{fk\alpha}{(1-\eta)x} \right) + (1 - \lambda) \left( b - \frac{fk\alpha}{(2-\eta)x} + d(1) \right) \geq 0.$$ 

That is, as long as:

$$\lambda \leq \frac{(b(2-\eta)x - fkc\alpha + d(1) \cdot (2-\eta)^x)(1-\eta)^x}{fk\alpha((2-\eta)x - (1-\eta)x) + d(1) \cdot (2-\eta)^x(1-\eta)^x}.$$ 

I denote as $\bar{\lambda}$ the highest probability for which this condition holds.

Similarly, when being at the full compliance equilibrium, individual $i$ chooses to comply only as long as:

$$(1 - \gamma) \left( b - \frac{fk\alpha}{(1-\eta)x} \right) + \gamma \left( b - \frac{fk\alpha}{(2-\eta)x} + d(1) \right) \leq 0.$$ 

Then:

$$\gamma \leq \frac{(fk\alpha - b(1-\eta)x)(2-\eta)^x}{fk\alpha((2-\eta)x - (1-\eta)x) + d(1) \cdot (2-\eta)^x(1-\eta)^x}.$$ 

I denote as $\bar{\gamma}$ the highest probability for which this condition holds (i.e., the highest for which $i$ chooses to comply). Then for individual $i$ to comply risk dominates not to comply whenever $\bar{\gamma} > \bar{\lambda}$. Meanwhile, not to comply risk dominates to comply whenever $\bar{\lambda} > \bar{\gamma}$. Let $c^*$ be the threshold amount of enforcement resources such that $\bar{\gamma} = \bar{\lambda}$, then:

$$c^* = \left( \frac{(2b + d(1))(1-\eta)x(2-\eta)x}{((2-\eta)x + (1-\eta)x)f\kappa} \right)^{1/\alpha} > \left( \frac{(b(1-\eta)x(2-\eta)x}{((2-\eta)x + (1-\eta)x)f\kappa} \right)^{1/\alpha}.$$ 

Thus, under this third possible externality, the law enforcement agency has to choose a higher level of enforcement resources than in Proposition 3 to induce individuals to select compliance.

**Proof of Proposition 9:**

Comparing $c(\mu; \chi > 0, \eta)$ and $c(\mu; 0, \eta)$, we see that if $\mu > \eta$ a larger amount of resources are needed because of the externalities since $c(\mu; \chi > 0, \eta) > c(\mu; 0, \eta)$. Meanwhile, if $\mu > \eta$ less resources because $c(\mu; \chi > 0, \eta) > c(\mu; 0, \eta)$.

Also, whenever $\chi > 1 - \eta$ then $c(\mu; \chi > 0, \eta)$ is not a one-to-one function. Notice that $c(\mu; \chi, \eta)$ is a one-to-one function only if $\partial c(\mu; \chi, \eta)/\partial \mu < 0$ for all $\mu$. However, there are values of $\chi$ and $\eta$
for which this condition does not hold since:

\[
\frac{\partial c(\mu; \chi, \eta)}{\partial \mu} = \frac{1}{\alpha} \left( \frac{1}{f k} \right)^{1/\alpha} (1 - \mu)^{1/\alpha} (1 + \mu - \eta)^{\chi/\alpha} \left( \frac{\chi}{1 + \mu - \eta} - \frac{1}{1 - \mu} \right),
\]

where all the elements are non-negative, except the last term in parenthesis that might be positive or negative. In particular,

\[
\frac{\partial c(\mu; \chi, \eta)}{\partial \mu} \begin{cases} 
> 0 & \text{if } \mu < \frac{\eta + \chi - 1}{1 + \chi} \\
< 0 & \text{if } \mu > \frac{\eta + \chi - 1}{1 + \chi}
\end{cases}.
\]

Thus, \(\frac{\partial c(\mu; \chi, \eta)}{\partial \mu} < 0\) for all \(\mu\) only if \(\chi < 1 - \eta\). Whenever this condition holds, each level of enforcement resources \(c\) induces a unique non-compliance rate, \(\mu\). However, for \(\eta > 1 - \chi\) then \(c(\mu; \chi, \eta)\) is not a one to one function.