Misinformative Advertising

Francisco Ruiz-Aliseda*

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Abstract

This paper analyzes how advertising can be used to mislead rivals in an oligopoly environment with demand uncertainty. In particular, we examine a two-period game in which two firms each sell a differentiated product whose attractiveness vis-à-vis the competitor's product is unknown. In each period, a firm sets prices for its product and exerts an advertising effort that is imperfectly observed by the rival later on. Advertising is persuasive in that it enhances willingness to pay, but it can also be used to manipulate rivals' beliefs about initially unobservable differences in consumers' quality perceptions. In equilibrium, each firm uses advertising to persuade consumers and to interfere with the rival's learning about this unknown dimension of demand. This can be done because the effect of imperfectly observed advertising cannot be separated out of the effect of the unknown quality differential, which creates a signal extraction problem for the competitor. There always exists a continuum of (symmetric) equilibria, but refining the equilibrium set selects out a unique one in which firms price in the first-period as in the static equilibrium, whereas the misinformative usage of advertising makes firms underadvertise if and only if the marginal cost of advertising is high enough.

Key words: Signal-Jamming, Imperfect Observability, Persuasive Advertising, Perfect Bayesian Equilibrium, Strategic Complementarity.

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*Universitat Pompeu Fabra and IESE SP-SP. E-mail: fran.ruiz@upf.edu. I am very grateful to Nicholas Argyres, Ramón Casadesús-Masanell, Sjaak Hurkens, Dmitri Kuksov, Gastón Llanes, David Myatt, Jackson Nickerson, Martin Peitz, Jennifer Reinganum, Xavier Vives, Govert Vroom, and participants at the IESE SP-SP Lunch Seminar and the 2009 CRES Foundations of Business Strategy Conference for valuable comments and suggestions. Special thanks are due to Ignatius Horstmann for a very insightful and thought-provoking discussion. Financial support from the IESE Public-Private Sector Research Center is gratefully acknowledged.

1 Introduction

Demand uncertainty is an element most firms have to cope with when making decisions ranging from investment in (in)tangible assets or technology development to product assortment and product design. Although some random elements affecting demand may have a transitory character, some other elements may be largely permanent, such as how substitutable consumers perceive products to be. As forcefully argued by Rothschild (1974), this creates an incentive for a firm to learn about the permanent features of the demand that it faces.[1] Learning about demand is not an easy process because signals are typically received with some noise (e.g., the random elements that are purely temporary), or because this learning process may depend on variables used by competitors to enhance the quantity demanded of their product, such as price or advertising effort. Indeed, in the presence of strategic interaction among rivals' learning processes, there is scope for manipulation in that a firm acknowledges that its activities can affect its rivals' learning about the random elements of demand, as analyzed by Keller and Rady (2003) in a price competition environment.

The main objective of this paper is to investigate how a firm learns about some unobservable drivers of demand in a setting in which competing firms set prices and also undertake promotional activities that are hard to monitor by rivals. In particular, we analyze how imperfectly observable advertising effort can be used to interfere with a competitor's learning of those demand drivers about which it is uncertain. Because we assume that advertising effort cannot be perfectly observed by a competitor, the most obvious cases to which our framework applies are those in which promotional activities are private. For instance, many (retailing) firms use direct-mail advertising to inform consumers about their product offerings (or about their existence). As another example, sales representatives in many industries make private visits to customers/retailers in order to persuade them about the goodness of a product. Although all these settings may come to mind more naturally, it is worth remarking that our theoretical insights also apply to public promotional activities as long as it is too costly to monitor all those done by a competitor, as is typically the case. In short, our model is not applicable whenever monitoring costs are so low that a competitor finds it optimal to monitor a firm's promotional activities so as to accurately quantify their effectiveness.

The theoretical contribution of this paper is not to explain why firms do advertising, but rather to examine whether there is an incentive to increase or decrease promotional intensity whenever there is demand uncertainty and advertising is not perfectly observed by competitors. Besides drawing implications for pricing and advertising strategies in these environments,

[1]See Keller and Rady (1999) for an analysis of this incentive that is richer than that in Rothschild (1974). See also Mazzola and McCardle (1996) for a similar idea in a context in which there is uncertainty about current production costs owing to an unknown learning curve.

we also aim at drawing empirical predictions for temporal patterns of prices and advertising expenditures. To accomplish these objectives, we analyze a two-period model in which two firms choose price and advertising at each period. Products are both horizontally and vertically differentiated, but firms are initially unsure of the degree of substitutability between the two products. In particular, we assume for simplicity that the extent of horizontal differentiation is known with certainty; however, firms cannot directly observe which product would be perceived to be most desirable to the eyes of consumers in the absence of horizontal differences.[2] Hence, there are some drivers of demand that are random and unobservable to firms. Besides setting prices, firms can also enhance their per-period sales by doing advertising, as in Dixit and Norman's (1978) oligopoly model of persuasive advertising.

If a firm's price and advertising effort were perfectly observed by the rival, then the latter could perfectly recover the quality differential when observing past sales. However, we focus on those situations in which, unlike the price charged, a firm's advertising effort cannot be perfectly observed by the competitor at the beginning of the second period. As a result, firms face a statistical identification problem when trying to infer the realized quality differential owing to the imperfect observability of the competitor's advertising effort. A firm's inferred quality differential will therefore depend on how much advertising the rival is believed to have done *and* on how much advertising the rival has actually done. Because the advertising actually done by the competitor is unobservable, it follows that the competitor can strategically manipulate the firm's inference about the realization of the quality differential.

Since second-period competition critically depends on the realized quality differential, each firm has an incentive to manipulate the rival's inference with the aim of softening future competitive interaction, as in any other signal-jamming oligopoly model (Riordan 1985). Although there exist infinitely many equilibria, a commonly used restriction on the off-the-equilibrium-path beliefs yields that the unique (symmetric) equilibrium exhibits a first-period price equal to that charged in the static equilibrium, and an advertising effort that is properly adjusted with the aim of softening future competition. In particular, we show that the advertising effort is lower than that exerted in the static equilibrium if and only if advertising costs are sufficiently high. Because firms perfectly learn their realized demand functions along the equilibrium path, it follows when the advertising technology is time-invariant that the expected time trend for advertising expenditures is positive if and only if advertising costs are high

[2] There might be several reasons for this uncertainty to arise: for example, the brand equity of firms to the eyes of consumers in the case of search goods, or unobservable past experiences and word-of-mouth effects in the case of experience goods. On the other hand, given that products have multidimensional characteristics, it is typically very hard to assess whether a particular combination of vertical attributes is preferred by consumers over another one (e.g., a recordable cassette tape vs. a more resistant, higher audio quality, but non-recordable CD). Because it is very difficult in practice to disentangle vertical differentiation from horizontal differentiation, this is another environment to which our setting may apply.

enough.

Our paper contributes to the literature on signal-jamming in imperfect competition settings. Several papers deal with oligopoly pricing behavior in this type of environments. The seminal paper by Riordan (1985) illustrates the signal-jamming role of unobservable prices or quantities: in the presence of demand uncertainty, his prediction is that the market price is expected to decrease (grow) over time if competitive interaction displays strategic complementarity (substitutability). In turn, the paper by Fudenberg and Tirole (1986) analyzes how an incumbent tries to predate a newly established firm by manipulating the entrant's inference about its fixed cost of production. Their main result is that that signal-jamming does not affect the incentive to exit, but it does discourage entry because of the incumbent's incentive to charge lower prices and thus interfere with the entrant's inference problem.

Other papers do not study how a firm tries to manipulate its competitors' learning process, but rather they focus on how to interfere with consumers' inferences, an aspect that is not examined in the current paper. For instance, in the setting considered by Caminal and Vives (1996), both consumers and firms are uncertain about the quality differential of two products, and consumers infer it based on noisily observed sales. Because firms observe each other's price and sales, they do not try to manipulate the competitor's inference process, but rather they signal-jam consumers' inferences about the quality differential by slashing prices below the static equilibrium level. Hence, the paper provides a rationale for aggressive battles for market share based on signal-jamming actions taken by firms. In the more recent paper by Iyer and Kuksov (2009), each of two firms chooses the intrinsic quality of its product together with some other variable that influences consumers' perception of the product quality (e.g., merchandising). These two costly choices are observed by the competing firm, but not by consumers, who make purchase decisions based on the joint effect of these two variables chosen by firms. Because consumers cannot disentangle intrinsic quality from these "atmospherics" when deciding whether or not to purchase a product, there arises an incentive to use atmospherics in order to signal-jam consumers' inferences about the intrinsic quality of a product. Hence, the paper provides a rationale for using merchandising grounded on its role as a signal-jamming device.

Both Riordan (1985) and Fudenberg and Tirole (1986) posit that the relevant strategic variable (price or quantity) is always unobservable to the competitor. In our paper, a firm does not signal-jam through price, which is observable, but through advertising effort, which cannot be perfectly observed by the competing firm. Hence, this parallels the literature on quality signaling in monopoly, which emphasizes that prices and/or advertising can be used as a signaling device.[3] In signaling models of advertising, a monopolist has private information

[3] Bagwell and Riordan (1991) emphasize the role of prices in signaling quality when the monopolist and a

about the quality of its product, and it signals quality through advertising effort (and price). By contrast, our oligopoly framework assumes that firms are symmetrically uninformed about the relative quality of their products (but consumers are not), and they learn about the realized quality differential over time, taking into account that each is able to manipulate the competitor's inference process through advertising. As a result, we highlight the signal-jamming role of advertising when advertising expands market size, whereas previous literature has highlighted the signaling role that advertising has about product quality even if advertising is purely dissipative. Our empirical predictions are also different from those in the monopoly signaling literature.

The paper is organized as follows. Section 2 introduces the game-theoretic model and describes the solution concept employed for solving it. Section 3 solves the game when second-period advertising costs are very high, and Section 4 pays particular attention to a certain type of symmetric equilibrium. Section 5 generalizes the lesson drawn from the previous two sections when advertising costs are not very high, whereas Section 6 concludes with directions for future research. An appendix to the paper presents qualitatively similar results derived from an alternative model in which advertising expands market size by informing consumers about the features of a product such as price.

2 The model

We examine a two-period game with two kinds of players.[4] On the one hand, there exists a unit mass of unit-demand consumers distributed along a Hotelling segment of unit length.[5] On the other hand, there are two ex ante identical firms, labeled 1 and 2, which are risk-neutral and discount future profits at a common rate $\delta \in [0, 1)$. Firm 1 is located at the left end of the segment and firm 2 is at the other extreme, and both produce goods at a constant marginal cost equal to zero (this is just a normalization without any loss of generality). Firm $i \in \{1, 2\}$ chooses in each period the price p_i at which to sell each unit of the product as well as advertising level a_i. Firm i's cost of exerting advertising effort a_i in period $\tau \in \{1, 2\}$ is

fraction of all consumers know the quality of a newly launched product. Milgrom and Roberts (1986) are the first to formally analyze how both price and advertising effort can be used for signaling quality in a repeat-purchase environment, whereas Hertzendorf (1993) extends their model to those situations in which consumers imperfectly observe advertising effort. See Bagwell (2007) for a thorough and deep survey of the economics and marketing literatures on advertising.

[4] See Vives (2009) for a general treatment of two-period games in which firms compete in pricing and advertising effort in the absence of signal-jamming.

[5] Any consumer is atomistic in that she cannot affect firms' second-period actions and beliefs just by herself. As a result, her first-period decision is purely static because she does not need to account for the outcome of her choices on subsequent play.

$C_\tau(a_i) = \dfrac{k_\tau a_i^2}{2}$, where k_1 and k_2 are positive constants. For simplicity, we let $k_2 = \infty$, so that second-period advertising effort is always zero for any of the firms. (We relax this assumption in Section 5.)

Given (p_i, a_i), a consumer located at distance $x \in [0,1]$ from firm $i \in \{1,2\}$ attains a (per-period) utility of

$$U_i^x(a_i, p_i) = V + \widetilde{v}_i + a_i - p_i - tx$$

when purchasing one unit of the good,[6] where V and t are positive scalars with the standard interpretation in a Hotelling setup, whereas $\widetilde{v}_1$ and $\widetilde{v}_2$ are positive continuous random variables with the same mean. The random variables are drawn just once, at the beginning of the game, and the values drawn are never observed by firms (although the realization of their difference can be inferred based on the first-period actions and outcomes, as we shall see). The random variable $\widetilde{v} \equiv \widetilde{v}_1 - \widetilde{v}_2$ represents the valuation differential in favor of firm 1, and it is assumed to have a positive continuous density $g(v)$ on the interval $[\underline{v}, \overline{v}]$, where $\underline{v} < 0 < \overline{v}$.[7] The standard deviation of the valuation differential $\widetilde{v}$ is denoted by σ. Notice that the assumption that $\int_{\underline{v}}^{\overline{v}} vg(v)dv = 0$ implies that no product is ex ante vertically differentiated, although $g(v)$ need not be symmetric about zero.

We now describe the information structure of the game regarding which actions are observable and which ones are not. In this sense, it is assumed that first-period prices are observed by the competitor at the beginning of the second period, but advertising levels are never observed by the rival. Unlike firms, consumers do observe the realization of $\widetilde{v} + a_1 - a_2$ when making their purchase decisions based on observed prices.[8] Whether the realized valuation differential can be separated out of the difference in advertising efforts is immaterial for the analysis and for the results, so we will make no particular assumption in this regard.

We follow the seminal work by Riordan (1985) in that our solution concept for this dynamic game with imperfect information will be that of pure-strategy Perfect Bayesian Equilibrium (PBE). In our context, equilibrium strategies must be sequentially rational given a system of beliefs and the system of beliefs must be consistent with the equilibrium strategies in a Bayesian way. We add a natural requirement to the PBE solution concept in order to rule out irrational belief formation off the equilibrium path. Thus, beliefs formed out of the equilibrium

[6] Note that advertising does not have a long-lasting impact (i.e., second-period utility does not depend on first-period advertising effort). However, the misinformative effect of advertising persists as long as a fraction of first-period advertising is short-lived, even if this fraction is negligible. On the other hand, it is worth remarking that qualitatitive results do not depend on all consumers being equally affected by advertising effort.

[7] The values of $\underline{v}$ and $\overline{v}$ are not too far away from zero so that solutions are always interior.

[8] Despite the assumption that consumers observe the value of $v + a_1 - a_2$, it is worth noting that they have no incentive to truthfully reveal it if asked by any of the firms, so our results are robust in this sense.

path about the advertising effort done by the rival and about the unknown quality differential should be consistent with all the information available to the firm. In particular, taking into account that a firm knows the advertising level it chose in the first period and the price that both charged, we require that beliefs about the advertising effort exerted by the competitor and about the random valuation differential be consistent with observed sales. Even with this extra rationality requirement, there will be a large number of outcomes that can be sustained in a PBE because Bayes' rule is not applicable out of the equilibrium path, so we will impose some reasonable restrictions on off-the-equilibrium-path beliefs after having characterized the set of PBE. This contrasts with Riordan's (1985) unique symmetric equilibrium in which (essentially) all first-period choices are shown to be on the equilibrium path because of their unobservability. In our setting, some first-period choices (prices) are observable, but others are not (advertising).[9] Therefore, there can be prices not expected on the equilibrium path with which we should associate some beliefs on the advertising effort that was exerted.[10]

Before characterizing the solution to the game, we would like to note that there are some assumptions that seem critical for obtaining our results, even though they are not. Aside from noting that advertising cannot be interpreted as secret price-cutting because it is a fixed cost that does not vary with output, we now discuss why two main ingredients of our model should not be taken at face value because they are not essential for the results to obtain.

On the one hand, we have assumed that the advertising done by a firm is unobservable to the competitor. Hence, advertising in our model may directly capture private activities such as direct-mail advertising, on-site promotions or visits to customers to explain the advantages of a good or service. However, it is important to remark that it may refer as well to public activities (such as media advertising intended to reinforce a firm's brand equity) insofar as they are imperfectly observed by the rival. Indeed, results are exactly the same if advertising effort is observed by the competitor with some (vanishingly small) noise: for example, if at the beginning of the second period firm i's rival observes $\widetilde{\gamma} a_i$, where a_i denotes firm i's first-period advertising effort and $\widetilde{\gamma}$ is an independent random variable with positive density on $(0, \infty)$. The key driver of our results is that firms face an *identification problem* when making inferences about the realized v based on first-period sales because they are unsure of how much

[9] See Dana (2001) for a setting in which prices are observed by consumers but inventories accumulated by firms are not, which also leads to a large number of equilibria depending on off-the-equilibrium-path beliefs held by consumers about the inventories carried by firms upon observing an unexpected price.

[10] There are are some (out-of-equilibrium) advertising levels that can result in a firm believing that the realized valuation differential is smaller or larger than it can possibly be (even if the price is kept at its equilibrium level). As explained in detail by Riordan (1985), this is not a problem if one modifies the framework and introduces an arbitrarily small prior probability that any nonnegative advertising level could be observed by the rival. In such a case in which Bayes' rule is applicable to all advertising levels, firms simply interpret an unexpected advertising effort as being entirely driven by the noise, and the PBE solution concept is equivalent to that of sequential equilibrium (Kreps and Wilson 1982).

advertising effort has been done by the rival.[11] The assumption of complete unobservability of advertising effort merely simplifies the analysis without affecting the underlying substance of the results: what is critical is that the rival's advertising effort cannot be fully disantangled from some unobservable drivers of demand, thus creating an incentive to engage in signal-jamming.

On the other hand, note that our setting is one of persuasive advertising because an increase in a_i directly leads to an increase in U_i^x. It is important to remark that advertising need not be persuasive in order for an incentive to use it as a signal-jamming instrument to arise, though. We have chosen this specification just because of its analytical simplicity. As should become clear, the insights stemming from this model are general and do not depend on whether advertising is done in order to persuade consumers or for other reasons such as informing them about some product features (e.g., the price at which a product is sold). Indeed, the appendix to this paper presents a model in which advertising is not persuasive and still has a misinformative role. In this alternative environment, advertising is done to increase awareness of a firm's product features, a setting that might be particularly appropriate whenever advertising activities are public but imperfectly observable because it is hard to believe that the rival can perfectly monitor how many consumers are being exposed to such promotional activities.

3 Resolution of the model

Throughout this section, it is assumed that $tk_1 > (9 + \delta)/36$ to ensure that payoff functions are strictly concave. Before solving the game, it is useful to examine how sales are generated in the first period whenever all consumers purchase one of the two goods for sale (i.e., the market is fully covered). Given $\{(p_i, a_i)\}_{i=1}^2$, we have that the consumer x indifferent between both firms is given by $U_1^x(a_1, p_1) = U_2^{1-x}(a_2, p_2)$. Denoting the realized quality differential by $v = v_1 - v_2$ and doing some manipulations yields that

$$x = \frac{t + a_1 - a_2 + p_2 - p_1 + v}{2t}.$$

Because all consumers to the left of x must purchase firm 1's product, it follows that firm 1's actual sales would be

$$q_1 = \frac{t + a_1 - a_2 + p_2 - p_1 + v}{2t},$$

[11] Whether or not a firm can observe the competitor's first-period sales is irrelevant, since such sales do not convey any information that is not already contained in the firm's own first-period sales.

whereas firm 2's actual sales would be

$$q_2 = \frac{t + a_2 - a_1 + p_1 - p_2 - v}{2t}.$$

We now proceed to solve the game backwards with the aid of these results. So suppose that the equilibrium strategies prescribe that firm $i \in \{1,2\}$ charges price $\overline{p}_i$ and exerts advertising effort $\overline{a}_i$ in the first period. Suppose without loss of generality that firm 2 has possibly deviated from this strategy and instead has charged price p_2 and exerted advertising effort a_2.[12] Let $\widehat{a}_2(p_2)$ be firm 1's belief about the advertising level done by its rival in the first period when it observes price p_2 being charged in the first period. (Of course, it holds that $\widehat{a}_2(\overline{p}_2) = \overline{a}_2$ even if $a_2 \neq \overline{a}_2$ because firm 1 believes that this is an on-equilibrium-path event given that price $\overline{p}_2$ was charged by firm 2 in the first period.) Then after observing sales of q_1 and firm 2's first-period price p_2, firm 1's inferred valuation differential $\widehat{v}$ should be given by the following equation because of the belief consistency requirement we imposed out of the equilibrium path:

$$\widehat{v} = 2tq_1 - t - \overline{a}_1 + \widehat{a}_2(p_2) - p_2 + \overline{p}_1.$$

Since it actually holds that $q_1 = (t + \overline{a}_1 - a_2 + p_2 - \overline{p}_1 + v)/2t$, we have that firm 1's estimate of the realized value of the random variable as a function of a_2, p_2 and v is as follows:

$$\widehat{v}(a_2, p_2 \,|v\,) = v + \widehat{a}_2(p_2) - a_2.$$

Unlike firm 1, firm 2 can perfectly identify the realized value of the valuation differential based upon its first-period sales. However, firm 1 believes that firm 2's estimate of the quality differential is $\widehat{v}$ (we will typically suppress the arguments of $\widehat{v}(\cdot)$ to save space). Therefore, given second-period prices (p'_1, p'_2), firm 1 believes that the location of the indifferent consumer is given by $\widehat{x} = (t - p'_1 + p'_2 + \widehat{v})/2t$, and hence it believes that firm 2 solves the following optimization programme in the second period:

$$\max_{p'_2}\{p'_2(t + p'_1 - p'_2 - \widehat{v})/2t\}.$$

The objective function is strictly concave, so it follows that firm 1 believes that firm 2 best responds by choosing $\mathring{p}_2 = (t + p'_1 - \widehat{v})/2$ when firm 1 chooses price p'_1. Firm 1 solves the

[12] Recall that checking for a Nash equilibrium only requires ruling out unilateral deviations. Because we shall focus on symmetric PBE, there is no loss of generality in considering unilateral deviations by one of the firms, 2 say.

following programme given its beliefs formed after observing the rival's first-period price p_2:

$$\max_{p_1'}\{p_1'(t + p_2' - p_1' + \widehat{v})/2t\}.$$

So solving the first-order conditions of this maximization programme taking into account that $p_2' = \mathring{p}_2 \equiv (t + p_1' - \widehat{v})/2$ yields that firm 1 charges a price of $p_1^*(\widehat{v}) = t + \dfrac{\widehat{v}}{3}$ when its estimate of the quality differential is $\widehat{v}$.

However, firm 2 knows the true realization of $\widetilde{v}$ and is also aware of firm 1 charging a price of $p_1^*(\widehat{v}) = t + \dfrac{\widehat{v}}{3}$, so it solves

$$\pi_2^*(\widehat{v}) \equiv \max_{p_2''}\{p_2''(t + p_1^*(\widehat{v}) - p_2'' - v)/2t\}.$$

As a result, firm 2 charges

$$p_2^*(\widehat{v}) = \frac{6t + \widehat{v} - 3v}{6}$$

and earns

$$\pi_2^*(\widehat{v}) = \frac{(6t + \widehat{v} - 3v)^2}{72t}.$$

Therefore, recalling that $\widehat{v}(a_2, p_2\,|v\,) = v + \widehat{a}_2(p_2) - a_2$, we have that

$$\frac{\partial \pi_2^*(\widehat{v})}{\partial a_2} = -\frac{6t - 2v + \widehat{a}_2(p_2) - a_2}{36t} \tag{1}$$

and

$$\frac{\partial \pi_2^*(\widehat{v})}{\partial p_2} = \left(\frac{6t - 2v + \widehat{a}_2(p_2) - a_2}{36t}\right)\frac{d\widehat{a}_2(p_2)}{dp_2} \tag{2}$$

at prices for which the belief function is differentiable.

We now turn to analyzing play in the first period. We restrict our attention to symmetric (Perfect Bayesian) equilibria in which firm $i \in \{1, 2\}$ charges price $\overline{p}_i = \overline{p}$ and exerts advertising effort $\overline{a}_i = \overline{a}$. For given firm 1's first-period actions, firm 2 should have no incentive to charge a different price from $\overline{p}$ and/or exert an advertising effort different from $\overline{a}$ (taking into consideration how subsequent play and beliefs would be affected if it deviates from one of these choices). As usual in this kind of games, there is a large number of symmetric equilibria because the solution concept does not impose any requirement on off-the-equilibrium-path beliefs. In the current section, we give a (partial) characterization of the full set of possible equilibria without any restriction other than symmetry. In the next section, though, we will restrict our attention to certain type of off-the-equilibrium-path beliefs that pin down a unique symmetric PBE.

Given that firm 1 charges $\overline{p}$ and exerts advertising effort $\overline{a}$, and taking into account that $\int_{\underline{v}}^{\overline{v}} vg(v)dv = 0$, we have that firm 2's expected stream of discounted profits as a function of its price p_2 and advertising a_2 is as follows:

$$\Pi_2(a_2, p_2 \,|\widehat{a}_2(p_2)\,) = \frac{(t + a_2 - \overline{a} + \overline{p} - p_2)p_2}{2t} + \delta \int_{\underline{v}}^{\overline{v}} \pi_2^*(\widehat{v})g(v)dv - \frac{k_1 a_2^2}{2}.$$

Differentiating with respect to a_2 with the aid of (1) yields that

$$\frac{\partial \Pi_2(a_2, p_2 \,|\widehat{a}_2(p_2)\,)}{\partial a_2} = \frac{p_2}{2t} - \delta \int_{\underline{v}}^{\overline{v}} (\frac{6t - 2v + \widehat{a}_2(p_2) - a_2}{36t})g(v)dv - k_1 a_2.$$

Using the assumption that $\int_{\underline{v}}^{\overline{v}} vg(v)dv = 0$, noting that $tk_1 > (9+\delta)/36$ ensures the strict concavity of $\Pi_2(a_2, p_2 \,|\widehat{a}_2(p_2)\,)$ with respect to its first argument, and equating $\partial \Pi_2(a_2, p_2 \,|\widehat{a}_2(p_2)\,)/\partial a_2$ to zero yields the optimal advertising effort that firm 2 must necessarily exert if it charges price p_2 (given the beliefs this price will induce):

$$a_2^*(p_2 \,|\widehat{a}_2(p_2)\,) = \frac{18p_2 - \delta(6t + \widehat{a}_2(p_2))}{36tk_1 - \delta}. \tag{3}$$

As a shorthand for $a_2^*(p_2 \,|\widehat{a}_2(p_2)\,)$, we will sometimes abuse notation and write $a_2^*(p_2)$ instead.

To characterize the entire set of possible symmetric PBE of the game, suppose that an equilibrium exhibits both firms choosing a price $\overline{p}$ and an advertising level $\overline{a} = \dfrac{18\overline{p} - 6t\delta}{36tk_1}$ (as follows from letting $a_2^*(\overline{p} \,|\widehat{a}_2(\overline{p})\,) = \overline{a}$ and $\widehat{a}_2(\overline{p}) = \overline{a}$ in (3) and then solving for $\overline{a}$). Easy manipulations using that $\int_{\underline{v}}^{\overline{v}} vg(v)dv = 0$ and $\int_{\underline{v}}^{\overline{v}} v^2 g(v)dv = \sigma^2$ show that a firm's expected stream of discounted profits equals

$$\Pi(\overline{p}) \equiv \Pi_2(\overline{a}, \overline{p} \,|\overline{a}\,) = \frac{36t^2k_1(\overline{p} + t\delta) - (3\overline{p} - t\delta)^2}{72t^2k_1} + \frac{\delta\sigma^2}{18t}. \tag{4}$$

Although the second term is always positive, we will require that the first term be non-negative as well so that results are not distorted by non-strategic effects.[13] We also need that $\overline{a} = \dfrac{18\overline{p} - 6t\delta}{36tk_1} \geq 0$, so putting together all the constraints on $\overline{p}$ yields that the following should hold:[14]

$$\frac{t\delta}{3} \leq \overline{p} \leq \frac{t(6tk_1 + \delta + \sqrt{12tk_1\,(3tk_1 + 4\delta)})}{3}. \tag{5}$$

[13] The second term arises because second-period profits are strictly convex in v along the equilibrium path (i.e., whenever $\widehat{v} = v$), so Jensen's inequality implies that $\mathcal{E}(\pi_2^*(\widehat{v})) > \pi_2^*(\mathcal{E}(\widehat{v}))$ despite firms are risk-neutral ($\mathcal{E}(\cdot)$ denotes the expectation operator).

[14] When deriving the bounds for the admissible values for $\overline{p}$, note that we are using the fact that $\delta \geq 0 > t(6tk_1 + \delta - \sqrt{12tk_1\,(3tk_1 + 4\delta)})/3$ for $\delta \in [0, 36tk_1)$.

This shows that it is not possible to have $\overline{p} < 0$ in equilibrium.

Given some belief function $\widehat{a}_2(\cdot)$, it should hold that no firm has an incentive to deviate if the rival is sticking to the equilibrium strategy, that is, we should have that

$$\Pi(\overline{p}) \geq \Pi_2(a_2^*(p_2), p_2 \, |\widehat{a}_2(p_2)) \text{ for all } p_2 \neq \overline{p}.$$

As mentioned earlier, the concept of PBE does not put restrictions on the belief function $\widehat{a}_2(\cdot)$ (except that $\widehat{a}_2(\overline{p}) = \overline{a}$). For this reason, we will look at the beliefs that are most favorable to sustaining a symmetric equilibrium.[15] For fixed $p_2 \neq \overline{p}$, minimizing $\Pi_2(a_2^*(p_2), p_2 \, |\widehat{a}_2(p_2))$ with respect to $\widehat{a}_2$ (subject to the constraint that $\widehat{a}_2$ cannot be negative) yields that $\widehat{a}_2^{\infty}(p_2) \equiv \max(0, \dfrac{p_2}{2tk_1} - 6t)$ gives the minimizer for each $p_2 \neq \overline{p}$. Henceforth, we will refer to $\widehat{a}_2^{\infty}(p_2) = \max(0, \dfrac{p_2}{2tk_1} - 6t)$ (with the understanding that $p_2 \neq \overline{p}$) as the "most favorable belief function." Working with this belief system, we have the following result.[16]

Proposition 1 *There is small enough $\varepsilon \geq 0$ such that there always exists a symmetric equilibrium in which $\overline{p} = t \pm \varepsilon$ and $\overline{a} = \dfrac{3-\delta}{6k_1} \pm \dfrac{\varepsilon}{2tk_1}$, with $\varepsilon > 0$ if and only if $\delta > 0$.*

Proof. We first study conditions under which it holds that $\Pi(\overline{p}) \geq \Pi_2(a_2^*(p_2), p_2 \, |\widehat{a}_2^{\infty}(p_2))$ for all $p_2 \neq \overline{p}$ (where $\widehat{a}_2^{\infty}(p_2) = \max(0, \dfrac{p_2}{2tk_1} - 6t)$ for $p_2 \neq \overline{p}$), and then we examine what happens in the specific case that $\overline{p} = t$.

To find the conditions for which $\Pi(\overline{p}) \geq \Pi_2(a_2^*(p_2), p_2 \, |\widehat{a}_2^{\infty}(p_2))$ for all $p_2 \neq \overline{p}$, we distinguish two cases, depending on whether or not $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$ holds (recall that $\overline{a} = \dfrac{18\overline{p} - 6t\delta}{36tk_1}$). When $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$, the fact that $tk_1 > 1/4$ implies that $\Pi_2(a_2^*(p_2), p_2 \, |\widehat{a}_2^{\infty}(p_2))$ is maximized at $\widehat{p}_2^+ \equiv \dfrac{2tk_1(t - \overline{a} + \overline{p})}{4tk_1 - 1} \geq 12t^2k_1$ (under the assumption that $tk_1 > 1/4$). Therefore, if $\overline{p}$ is such that $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$, then $\Pi(\overline{p})$ should not be lower than $\Pi^+(\overline{p}) \equiv \Pi_2(a_2^*(\widehat{p}_2^+), \widehat{p}_2^+ \, | \, \dfrac{\widehat{p}_2^+}{2tk_1} - 6t)$, where it can be shown that

$$\Pi^+(\overline{p}) = \frac{[6tk_1(t + \overline{p}) - (3\overline{p} - t\delta)]^2}{72t^2k_1(4tk_1 - 1)} + \frac{\delta\sigma^2}{18t}. \tag{6}$$

Using (4) and (6), several manipulations show that we have that $\Pi(\overline{p}) \geq \Pi^+(\overline{p})$ if and only if

[15] This is standard in dynamic games with private information (e.g., Mailath 1987). Imposing refinements that discard "unreasonable" off-the-equilibrium-path beliefs will typically reduce the equilibrium set, sometimes collapsing it into a singleton, as we show in the next section.

[16] Requiring that the indifferent consumer attains a positive utility in equilibrium can be accomplished by letting V be large enough.

$\lambda^+(\overline{p}) \leq 0$, where

$$\lambda^+(\overline{p}) \equiv 9k_1\overline{p}^2 - 3\overline{p}(6tk_1 + \delta) + 9t^2k_1 - t\delta(36tk_1 - 12 - \delta).$$

Therefore, if a price $\overline{p}$ such that $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$ satisfies $\lambda^+(\overline{p}) \leq 0$ (and condition (5)), then there exists a symmetric equilibrium in which firms set price $\overline{p}$ (and exert advertising effort $\overline{a} = \dfrac{18\overline{p} - 6t\delta}{36tk_1}$).

On the other hand, if it holds that $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$, then the assumption that $tk_1 > (9+\delta)/36$ implies that $\Pi_2(a_2^*(p_2), p_2 \,|\, \widehat{a}_2^{\infty}(p_2))$ is maximized at $\widehat{p_2} \equiv \dfrac{(t - \overline{a} + \overline{p})(36tk_1 - \delta) - 6t\delta}{2(36tk_1 - \delta) - 18} < 12t^2k_1$. Hence, if $\overline{p}$ is such that $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$, then $\Pi(\overline{p})$ should not be lower than $\Pi^-(\overline{p}) \equiv \Pi_2(a_2^*(\widehat{p_2}), \widehat{p_2} \,\big|\, 0)$, where it can be shown that

$$\Pi^-(\overline{p}) = \frac{[(t + \overline{p} - \dfrac{18\overline{p} - 6t\delta}{36tk_1})(36tk_1 - \delta) - 6t\delta]^2}{8t(36tk_1 - \delta)(36tk_1 - \delta - 9)} + \frac{36t^3k_1\delta}{2t(36tk_1 - \delta)} + \frac{\delta\sigma^2}{18t}. \tag{7}$$

Using (4) and (7), tedious manipulations show that we have that $\Pi(\overline{p}) \geq \Pi^-(\overline{p})$ if and only if $\lambda^-(\overline{p}) \leq 0$, where

$$\begin{aligned}\lambda^-(\overline{p}) \;\equiv\; & 9\overline{p}^2(4t^2k_1^2(36tk_1 - \delta) - \delta) - 6t\overline{p}(432t^3k_1^3 + 132t^2k_1^2\delta - 2tk_1\delta^2 - \delta^2 - 42tk_1\delta) - \\ & (4t^3k_1\delta^3 - 288t^4k_1^2\delta^2 - 36t^4k_1^2(36tk_1 - \delta) + 84t^3k_1\delta^2 + t^2\delta^3).\end{aligned}$$

Therefore, if a price $\overline{p}$ such that $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$ satisfies $\lambda^-(\overline{p}) \leq 0$ (and condition (5)), then there exists a symmetric equilibrium in which firms set price $\overline{p}$ (and exert advertising effort $\overline{a} = \dfrac{18\overline{p} - 6t\delta}{36tk_1}$).

To check whether $\overline{p} = t$ with associated advertising effort $\overline{a} = \dfrac{3 - \delta}{6k_1}$ can happen in equilibrium, we must verify whether it holds that $\lambda^+(\overline{p}) \leq 0$ if $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$ and $\lambda^-(\overline{p}) \leq 0$ if $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$. This is trivially satisfied for $\delta = 0$ (since $\lambda^+(t) = \lambda^-(t) = 0$), so let $\delta > 0$ from now on. Consider first the case in which the parameter space is such that $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$ for $\overline{p} = t$, which is true if the following two conditions hold at the same time: $1/4 < tk_1 < 1/3$ and $3 + 48tk_1(3tk_1 - 1) < \delta < 36tk_1 - 9$. Then $\lambda^+(t) = -t\delta(36tk_1 - 9 - \delta)$, so the assumption that $\delta < 36tk_1 - 9$, together with the fact that both t and δ are positive, implies that $\lambda^+(t) < 0$.

So we are left to consider the cases in which $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$. We must show in these cases that

$$\lambda^-(t) = -\delta t^2(9 + 864t^2k_1^2 - 6\delta - 252tk_1 + 4tk_1\delta^2 - 288t^2k_1^2\delta + 72tk_1\delta + \delta^2) \leq 0.$$

Equivalently, letting

$$\mu(tk_1, \delta) \equiv 9 + 864(tk_1)^2 - 6\delta - 252(tk_1) + 4(tk_1)\delta^2 - 288(tk_1)^2\delta + 72(tk_1)\delta + \delta^2,$$

we need to prove that $\mu(tk_1, \delta) \geq 0$ whenever it holds that $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$ for $\overline{p} = t$. There are two cases for which $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$. In both cases, it is sufficient to show that $\mu(tk_1, \delta) \geq 0$ for the largest admissible value for δ, since $\dfrac{\partial^2 \mu(tk_1, \delta)}{\partial \delta^2} = 8tk_1 + 2 > 0$ and $\left.\dfrac{\partial \mu(tk_1, \delta)}{\partial \delta}\right|_{\delta=1} = 80(tk_1) - 4 - 288(tk_1)^2 < 0$ (which holds because $tk_1 > 1/4$). The first case we deal with is that in which $1/4 < tk_1 < 1/3$ and $0 \leq \delta \leq 3 + 48tk_1(3tk_1 - 1)$. Because

$$\mu(tk_1, 3 + 48tk_1(3tk_1 - 1)) = (48tk_1)^2(36(tk_1)^3 - 33(tk_1)^2 + 10(tk_1) - 1),$$

it follows that $\mu(tk_1, 3 + 48tk_1(3tk_1 - 1)) > 0$ for $1/4 < tk_1 < 1/3$, as desired. The second and last case we need to study is that in which $tk_1 \geq 1/3$ (and hence $\delta < 1 < 36tk_1 - 9 \leq 3 + 48tk_1(3tk_1 - 1)$). Because

$$\mu(tk_1, 1) = 576(tk_1)^2 - 176(tk_1) + 4,$$

it follows that $\mu(tk_1, 1) > 0$ for $tk_1 \geq 1/3$, as desired.

Hence, we have just shown that there always exists a symmetric PBE in which $\overline{p} = t$ and $\overline{a} = \dfrac{3 - \delta}{6k_1}$ (that $\overline{p} = t$ satisfies condition (5) with strict inequalities is straightforward to show using the assumptions that $\delta < 1$ and $4tk_1 > 1$), whereas the facts that $\Pi(t) > 0$, $\lambda^+(t) < 0$, and $\lambda^-(t) < 0$ for $\delta > 0$, together with the continuity of $\Pi(\cdot)$, $\lambda^+(\cdot)$, and $\lambda^-(\cdot)$, imply that there must exist other equilibria in the neighborhood of this symmetric PBE. ■

Not only does this result prove that the equilibrium set is nonempty and infinitely large for $\delta > 0$,[17] but also it shows that the first-period price could be greater, equal or lower than the static price $\overline{p} = t$ charged when $\delta = 0$. By Proposition 1, we know that there always

[17] The nonemptiness of this set does not depend on the parameter values, but the measure of the set does. In fact, notice that despite we have just given a partial characterization of the equilibrium set, it is easy to fully characterize it. In particular, price $\overline{p}$ will be charged in an equilibrium if it satisfies (5), together with $\lambda^+(\overline{p}) \leq 0$ (if $t - \overline{a} + \overline{p} \geq 6t(4tk_1 - 1)$) or $\lambda^-(\overline{p}) \leq 0$ (if $t - \overline{a} + \overline{p} < 6t(4tk_1 - 1)$). It can be shown that the equilibrium set is compact and connected. However, it is difficult to perform comparative statics on the determinants of its size (at least analytically).

exists an equilibrium in which firms price as in the static equilibrium. As we will see in the next section, there are reasonable refinements on off-the-equilibrium-path beliefs that select out this symmetric equilibrium as the unique one, so it is useful to characterize some of its main properties.

Corollary 1 *In the unique symmetric equilibrium in which $\overline{p} = t$, it holds that $\overline{a} = \dfrac{3-\delta}{6k_1}$ is decreasing in δ, whereas $\Pi(\overline{p}) = \dfrac{36t^3k_1(1+\delta) - t^2(3-\delta)^2}{72t^2k_1} + \dfrac{\delta\sigma^2}{18t}$ is increasing in δ.*

Hence, we have for $\delta > 0$ that the static equilibrium in which $\overline{p} = t$ and $\overline{a} = \dfrac{1}{2k_1}$ vanishes because of the misinformative effect of advertising. In addition, the advertising level is distorted downwards in the equilibrium in which firms price as in the static equilibrium. That firms underadvertise relative to the static equilibrium captures the common wisdom of oligopoly models of signal-jamming. Thus, each firm advertises less than in the static equilibrium so as to try to fool the competitor in making it believe that the quality differential is greater than it actually is. Why does it make sense for a firm to try to induce its competitor to believe that the valuation differential is greater than it actually is? The key point is that doing so softens second-period price competition, which is in a firm's interest because of strategic complementarity. However, in equilibrium no firm is fooled and both act in the same way, so second-period price competition is not relaxed. But because advertising dissipates rents in a symmetric equilibrium, reducing advertising expenditures without vaying prices and sales is positive for firms, which explains why a firm's payoff grows with δ, and hence the misinformative effect of advertising results in greater payoffs for both firms.

To sum up, recall that in our model advertising has a dual nature: persuasive and misinformative. The persuasive nature of advertising—present even if $\delta = 0$—makes firms futilely engage in dissipative advertising, but this effect is partially offset if $\overline{p} = t$ due to the misinformative nature of advertising that arises when $\delta > 0$ (and $k_2 = \infty$).

4 Equilibrium refinements

Proposition 1 shows that there exists a continuum of symmetric PBE. That the set of equilibria is very big is a standard finding in dynamic games with private information because the belief function is largely unrestricted. For this reason, part of the game-theoretic literature has developed refinements that rule out "unreasonable" off-the-equilibrium-path beliefs. The purpose of this section is to sharpen our predictions by refining the set of symmetric equilibria.

Throughout this section, we make the technical assumption that belief functions are twice continuously differentiable in the (closed) neighborhood of the first-period equilibrium price $\overline{p}$;

in particular, we assume that they are differentiable at $p_2 \in B_r(\overline{p}) = \{p_2 \geq 0 : |p_2 - \overline{p}| \leq 1/r\}$, where $B_r(\overline{p})$ denotes the closed ball of radius $1/r > 0$ centered at price $\overline{p}$. (We will discuss later why this is a very weak assumption for large enough r.) Under this assumption, we can apply the envelope theorem when differentiating $\Pi_2(a_2^*(p_2), p_2 \,|\widehat{a}_2(p_2))$ with the aid of (2) so as to get

$$\frac{d\Pi_2(a_2^*(p_2), p_2 \,|\widehat{a}_2(p_2))}{dp_2} = \frac{t + a_2^*(p_2) - \overline{a} + \overline{p} - 2p_2}{2t} + \delta \left(\frac{6t + \widehat{a}_2(p_2) - a_2^*(p_2)}{36t} \right) \frac{d\widehat{a}_2(p_2)}{dp_2}$$

after using that $\int_{\underline{v}}^{\overline{v}} vg(v)dv = 0$.

In a symmetric equilibrium, it holds that $a_2^*(\overline{p}) = \overline{a}$ and $a_2^*(\overline{p}) = \widehat{a}_2(\overline{p})$, so if we equate $\left.\frac{d\Pi_2(a_2^*(p_2), p_2 \,|\widehat{a}_2(p_2))}{dp_2}\right|_{p_2 = \overline{p}}$ to 0,[18] and then rearrange the resulting expression, we arrive at the following:

$$\frac{\overline{p} - t}{2t} = \left(\frac{\delta}{6} \right) \left. \frac{d\widehat{a}_2(p_2)}{dp_2} \right|_{p_2 = \overline{p}}. \tag{8}$$

Expression (8) highlights the great multiplicity of outcomes that can be sustained because beliefs are unrestricted for p_2 different from $\overline{p}$ (despite our restriction on locally differentiable belief functions), so additional constraints on off-the-equilibrium-path beliefs are required to sharpen predictions.

Unfortunately, the refinements developed by the game-theoretic literature are not applicable to many imperfect information games. For this reason, we shall consider a weaker version of an assumption on off-the-equilibrium-path beliefs that is commonly used in the industrial organization literature dealing with unobservable actions (e.g., Hart and Tirole 1990, McAfee and Schwartz 1994, or Caminal and Vives 1996). In particular, this literature pays attention to "passive" out-equilibrium-path beliefs, which in our context means that $\widehat{a}_2(p_2) = \overline{a}$ for all p_2, something which may not be very reasonable if the unexpected price p_2 turns out to be very different from $\overline{p}$. However, this assumption is more plausible if $p_2 \in B_r(\overline{p})$ for very small r, since observing a price almost identical to $\overline{p}$ may be interpreted as a small but well-intended error when implementing prices. For this reason, we impose the restriction that $\left.\frac{d\widehat{a}_2(p_2)}{dp_2}\right|_{p_2 = \overline{p}} = 0$, which selects out the symmetric equilibrium in which $\overline{p} = t$ and $\overline{a} = \frac{3 - \delta}{6k_1}$ as the unique PBE. This equilibrium has some appealing properties that we already discussed in the light of Corollary 1, so our discussion now will focus on the system of beliefs that supports this equilibrium because there is an additional issue of robustness. To see this, let us consider the following family of belief functions that supports this equilibrium (the family

[18] Note that $tk_1 > (9 + \delta)/36$ implies that the payoff function is strictly quasi-concave.

is indexed by r):

$$\widehat{a}_2^r(p_2) = \begin{cases} \dfrac{3-\delta}{6k_1} \text{ if } p_2 \in B_r(t) \\ \widehat{a}_2^\infty(p_2) \text{ if } p_2 \notin B_r(t) \end{cases},$$

where $B_r(t) = \{p_2 \geq 0 : |p_2 - t| \leq 1/r\}$. This collection of functions can be made to be arbitrarily close to the most favorable belief function $\widehat{a}_2^\infty(p_2)$, since $B_r(t) \to \{t\}$ as $r \to \infty$ implies that the limit of this sequence of functions and $\widehat{a}_2^\infty(p_2)$ coincide almost everywhere.[19] As a result, we have that a slight but reasonable perturbation in the belief system considered in Section 3 selects out $\overline{p} = t$ and $\overline{a} = \dfrac{3-\delta}{6k_1}$ as the unique symmetric equilibrium of the game.[20]

Proposition 2 *Suppose that $\widehat{a}_2(p_2) = \widehat{a}_2^r(p_2)$. Then the unique symmetric PBE of the game exhibits price $\overline{p} = t$ and advertising effort $\overline{a} = \dfrac{3-\delta}{6k_1}$ for all $r > 0$.*

Proof. Small (local) price deviations have just been ruled out for $p_2 \in B_r(t)$ (see derivation of (8)), whereas large (global) price deviations are not profitable for $p_2 \notin B_r(t)$ by Proposition 1. ■

[19] This also shows that our local differentiability assumption is very weak, since it can be made to apply only to a set of arbitrarily small measure.

[20] Another plausible refinement that delivers uniqueness is based on a forward induction argument applied locally on the neighborhood of the equilibrium price. Thus, letting $a_2^*(p_2)$ be tangent to $\widehat{a}_2(p_2)$ at $p_2 = \overline{p}$ yields that $\left.\dfrac{d\widehat{a}_2(p_2)}{dp_2}\right|_{p_2=\overline{p}} = \dfrac{1}{2tk_1}$ (to show this, differentiate (3) with respect to p_2, and then impose the condition that $\left.\dfrac{da_2^{**}(p_2)}{dp_2}\right|_{p_2=\overline{p}} = \left.\dfrac{d\widehat{a}_2(p_2)}{dp_2}\right|_{p_2=\overline{p}}$). As a result, expression (8) implies that $\overline{p} = t + \dfrac{\delta}{6k_1}$, and hence $\overline{a} = \dfrac{1}{2k_1} - \dfrac{\delta(2tk_1 - 1)}{12tk_1^2}$. This refinement is much more sophisticated than the one based on "passive beliefs" in some sense. Thus, upon observing a price pretty close to the one expected in equilibrium, firm 1 asks itself whether there is some advertising level such that if it believed firm 2 did such advertising in the first period, then firm 2 could do better and hence firm 2 would actually exert such advertising effort (given that firm 1 would anticipate it). The (local) forward induction refinement considered discards these situations. Hence, for those belief functions whose derivative at the equilibrium price coincides with that of $a_2^*(p_2)$, it holds relative to the static equilibrium that price increases, whereas advertising done decreases if and only if $2tk_1 > 1$. The latter result arises because of two conflicting effects. On the one hand, there is always an incentive to signal-jam the competitor's inference by underadvertising relative to the static equilibrium. On the other, price and advertising are complements, so a firm that for some exogenous reason unrelated to t or k_1 prices higher than in the static equilibrium should advertise more. The latter effect is powerful relative to the former if and only if advertising is not too costly in the first period, which explains the result obtained under forward induction.

5 Overadvertising vs. underadvertising

This section deals with the cases in which $k_2 < \infty$, although it is assumed throughout that $tk_2 > 1/4$ to ensure that second-period profit functions are strictly concave in prices and advertising levels. Our objective is to check whether Proposition 1 and Corollary 1 extend to the cases in which firms find it optimal to exert some advertising effort in the second period.

To better understand the features of competition when firms choose price and advertising effort in a signal -jamming environment, let us first consider second-period competition if the quality differential were known to be equal to v. Understanding of this setting will shed a light on the more complicated setup examined later, in which firms use first-period advertising to manipulate the rival's perception of the realized valuation differential. As is standard in static oligopoly games, we will require second-period Nash equilibria to be stable.

If v were perfectly observed by firms at the beginning of the second period, firms 1 and 2 would respectively solve the following programs to maximize profits:

$$\max_{p_1',a_1'}\{\frac{p_1'(t+a_1'-a_2'+p_2'-p_1'+v)}{2t}-\frac{k_2(a_1')^2}{2}\} \tag{9}$$

and

$$\max_{p_2',a_2'}\{\frac{p_1'(t+a_2'-a_1'+p_1'-p_2'-v)}{2t}-\frac{k_2(a_2')^2}{2}\}. \tag{10}$$

Instead of working with complex bidimensional reaction functions, it is more instructive to work with a transformed game with a single strategic variable. Thus, note from (9) and (10) that firm i's optimal second-period advertising level depends only on p_i': $a_i' = \frac{p_i'}{2tk_2}$, $i = 1, 2$. When $2tk_2 < 1$, $da_i'/dp_i' > 1$ and advertising varies in the same direction as price (as happens in any model of persuasive advertising with increasing marginal cost of advertising), but in a larger amount. Substituting $a_i' = \frac{p_i'}{2tk_2}$ in (9) and (10) (this can be done because the optimal a_i' depends on neither p_{3-i}' nor a_{3-i}') allows us to transform the second-period game into one of price competition in which firms 1 and 2 respectively solve:

$$\max_{p_1'}\{\frac{p_1'(t+v+(p_1'-p_2')(\frac{1-2tk_2}{2tk_2}))}{2t}-\frac{(p_1')^2}{8t^2k_2}\}$$

and

$$\max_{p_2'}\{\frac{p_2'(t-v+(p_2'-p_1')(\frac{1-2tk_2}{2tk_2}))}{2t}-\frac{(p_2')^2}{8t^2k_2}\}.$$

This is a game in prices in which it is implicit how advertising varies when prices are changed. The (second-period) reaction functions of the transformed game, denoted by $R_i(\cdot)$ ($i = 1, 2$), are as follows:

$$R_1(p_2') = \frac{2tk_2(t+v)}{1+2(2tk_2-1)} + \frac{p_2'}{2+\frac{1}{(2tk_2-1)}}$$

and

$$R_2(p_1') = \frac{2tk_2(t-v)}{1+2(2tk_2-1)} + \frac{p_1'}{2+\frac{1}{(2tk_2-1)}}.$$

So we have that the equivalent game in which prices are the single strategic variable for firms displays strategic complementarity if and only if $2tk_2 > 1$. The surprising cases are hence those in which second-period competition exhibits strategic substitutability. In these cases in which $2tk_2 < 1$, advertising varies in the same direction with price, but by a larger amount (since $da_i'/dp_i' > 1$). As a result, an increase in price is accompanied by a larger increase in advertising effort, which decreases the elasticity of the competitor's residual demand, and hence makes it slightly lower its price and substantially decrease its advertising.

We pay attention to stable equilibria of the transformed game, that is, those in which $|dR_2(p_1')/dp_1'| < \left|dR_1^{-1}(p_1')/dp_1'\right|$. Given the linearity of reaction functions, stability simply boils down to requiring that it holds that $3tk_2 > 1$, since in this case we have that $1 > \dfrac{\partial R_1}{\partial p_2'}\dfrac{\partial R_2}{\partial p_1'}$. Let us call the (stable) Nash equilibrium point by (p_1^e, p_2^e) and let us parametrize it by v, so that we have that $p_1^e(v) = R_1(p_2^e(v), v)$ and $p_2^e(v) = R_2(p_1^e(v), v)$. As a result, it holds that $p_1^e(v) = R_1(R_2(p_1^e(v), v), v)$, so total differentiation with respect to v yields the following after rearranging:

$$\frac{dp_1^e(v)}{dv} = \frac{\dfrac{\partial R_1}{\partial v} + \dfrac{\partial R_1}{\partial p_2'}\dfrac{\partial R_2}{\partial v}}{1 - \dfrac{\partial R_1}{\partial p_2'}\dfrac{\partial R_2}{\partial p_1'}}.$$

Because $\dfrac{\partial R_1}{\partial v} > 0 > \dfrac{\partial R_2}{\partial v}$ and $\dfrac{\partial R_1}{\partial p_2'} < 0$, it follows that $\dfrac{dp_1^e(v)}{dv} > 0$ if and only if $1 > \dfrac{\partial R_1}{\partial p_2'}\dfrac{\partial R_2}{\partial p_1'}$. That is, we have that $\dfrac{dp_1^e(v)}{dv} > 0$ because are analyzing the part of the parameter space in which $3tk_2 > 1 > 2tk_2$. To better understand what happens when $3tk_2 > 1 > 2tk_2$, let us suppose that firm 1 is led to believe that the quality differential is greater that it actually is. This results in firm 1 perceiving that reaction functions shift as represented in Figure 1:

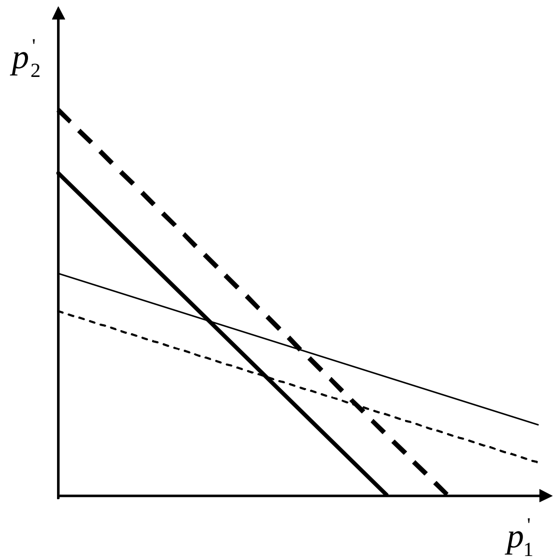

Figure 1: Shift in (perceived) reaction functions if $3tk_2 > 1 > 2tk_2$

The thick dashed line represents firm 1's reaction function when it is misled by firm 2 to believe that the quality differential is greater that it actually is. whereas the thick solid line represents firm 1's reaction function if it is not misled. The thin dashed line represents firm 1's perception of firm 2's reaction function if firm 1 is not misled, whereas the thin solid line represents firm 1's perception of firm 2's reaction function if firm 1 is not misled. As a result, inducing firm 1 to believe that the quality differential is greater than it really is leads to firm 1 charging a higher price that it would otherwise, since $dp_1^e(v)/dv > 0$.

It seems when $3tk_2 > 1 > 2tk_2$ that firm 2 would be better off by inducing firm 1 to believe that the quality differential is higher that it actually is, but this is not correct once we take into account that this is a transformation of the original game. In fact, if firm 1 were led to believe that the valuation differential is lower that it actually is, then its price would be lowered (since $dp_1^e(v)/dv > 0$), but advertising would lower even more (since $2tk_2 < 1$), which is in firm 2's interest. In short, when it holds that $3tk_2 > 1 > 2tk_2$, there arises an incentive for firms to *overadvertise* in the first period so as to lead the competitor to believe that the quality differential is lower that it really is, and thus soften second-period competition because the decrease in advertising effort is larger that the fall in price.

Based on the analysis for the two different cases that can possibly arise, we are now in the position of extending the results in Sections 3 and 4 to any finite k_2. Paralleling steps to those followed in Section 3, it is easy to show that firm 1 charges a price of $p_1^*(\widehat{v}) = t + \dfrac{tk_2\widehat{v}}{3tk_2 - 1}$ and exerts an advertising level of $a_1^*(\widehat{v}) = \dfrac{1}{2k_2} + \dfrac{\widehat{v}}{2(3tk_2 - 1)}$ when its estimate of the quality differential is $\widehat{v} = v + \widehat{a}_2(p_2) - a_2$. In turn, firm 2 charges

$$p_2^*(\widehat{v}) = \frac{t((2k_2(2t-v)-1)(3tk_2-1)+(2tk_2-1)k_2\widehat{v})}{(3tk_2-1)(4tk_2-1)}$$

and exerts advertising effort

$$a_2^*(\widehat{v}) = \frac{(2k_2(2t-v)-1)(3tk_2-1)+(2tk_2-1)k_2\widehat{v}}{2k_2(3tk_2-1)(4tk_2-1)}.$$

As a result, its second-period profit (net of advertising costs) is

$$\pi_2^*(\widehat{v}) = \frac{1}{8k_2(4tk_2-1)}\left(2k_2(2t-v)-1+\frac{k_2(2tk_2-1)\widehat{v}}{3tk_2-1}\right)^2.$$

Under the maintained assumption that $3tk_2 > 1$, $\pi_2^*(\widehat{v})$ is decreasing in $\widehat{v}$ if and only if it holds that $3tk_2 > 1 > 2tk_2$. As a result, there is an incentive to *underadvertise* in the first period whenever we have that $2tk_2 > 1$. Making the rival believe that the quality differential is higher than it actually is results in an increase in price and in a smaller increase in advertising effort, thus softening second-period competition. However, when $3tk_1 > 1 > 2tk_2$ there arises an incentive to *overadvertise* in the first period so as to mislead the competitor into thinking that the valuation differential is lower than it actually is. This softens second-period competition even though the price charged by the rival decreases, since its advertising effort decreases even more.

We will skip the steps, but it can be shown that the unique symmetric PBE selected out by the refinement used in Section 4 exhibits a first-period price $\overline{p} = t$ and a first-period advertising effort $\overline{a} = \frac{1}{2k_1} - \frac{\delta(2tk_2-1)}{4k_1(3tk_2-1)}$.[20] Hence, the general lesson one can draw from this PBE is that the strategic variable that is observable is kept at the static equilibrium level, and the strategic variable that is unobservable is adjusted so as to relax second-period price competition (as long as $\delta > 0$, of course). For $tk_2 \in (1/2, \infty)$, this means that firms will underadvertise relative to the static equilibrium, whereas the opposite happens for $tk_2 \in (1/3, 1/2)$.

Proposition 3 *In the symmetric PBE in which $\overline{p} = t$ and $\overline{a} = \frac{1}{2k_1} - \frac{\delta(2tk_2-1)}{4k_1(3tk_2-1)}$, it holds that $\frac{\partial \overline{a}}{\partial \delta} < 0$ if $k_2 \in (\frac{1}{2t}, \infty)$, whereas $\frac{\partial \overline{a}}{\partial \delta} > 0$ if $k_2 \in (\frac{1}{3t}, \frac{1}{2t})$.*

We can also draw predictions about the expected pattern of price and advertisement expenditure. Thus, we have in equilibrium that the expected price in the second period is

[20] When $3tk_2 > 1 > 2tk_2$, the nature of the most favorable belief function differs from the one derived in Section 3, though.

$\overline{p}' = t$, and hence price is not expected to vary over time. However, in the most likely case that $k_1 = k_2 \equiv k$, it holds that the expected advertising effort in the second period is equal to $\overline{a}' = \frac{1}{2k}$, the static equilibrium advertising level. Given that $3tk > 1$, it holds that $\overline{a} = \frac{1}{2k} - \frac{\delta(2tk-1)}{4k(3tk-1)} > \overline{a}'$ if and only if $tk \in (1/3, 1/2)$, so we have the following.

Proposition 4 *Price is expected to be constant over time. If $k_1 = k_2 \equiv k$, then advertising is expected to grow over time if $k \in (\frac{1}{2t}, \infty)$, whereas advertising is expected to decrease over time if $k \in (\frac{1}{3t}, \frac{1}{2t})$.*

Therefore, an increase in the marginal cost of advertising has a nonmonotonic effect on the sign of $\overline{a} - \overline{a}'$: for low k, $sign(\overline{a} - \overline{a}') > 0$, whereas for large k, $sign(\overline{a} - \overline{a}') < 0$.

6 Conclusion

This paper has studied the role of imperfectly observable advertising as a signal-jamming device in a two-period duopoly. In particular, the paper has showed that there exist many equilibria, all of which exhibit an attempt to strategically manipulate a competitor's inference about unobservable drivers of demand. Although in equilibrium no firm is able to fool its rival, the presence of these incentives to signal-jam learning about demand conditions may result in a higher or a lower price and in a higher or a lower advertising effort than those obtained in a static equilibrium.

We have also provided reasonable conditions under which the equilibrium outcome is unique. In this refined equilibrium, firms price as they would in a static equilibrium, and advertising is adjusted in an attempt to soften second-period competition, thus capturing the received wisdom. In a setting in which advertising is persuasive, we have found that a decrease in the marginal cost of advertising has a nonmonotonic effect on the sign of the difference between the equilibrium advertising effort and that of the static equilibrium. In particular, firms underadvertise (relative to the static equilibrium) if and only if the marginal cost of advertising is sufficiently high. Underadvertising results in less rent dissipation and an expected positive time trend for advertising expenditures, whereas overadvertising leads to more rents being dissipated owing to advertising and an expected negative time trend for advertising expenditures.

That advertising grows over time is found in the paper by Horstmann and MacDonald (2003) in their empirical study of the CD player industry. This study finds little support to the signaling role of advertising. Our paper may help to explain this evidence, although it cannot explain the decreasing time trend of the price path. However, it is well known that the manufacture of CD players was subject to learning curve effects. It would be interesting to extend our model by incorporating a learning curve, and it seems clear that such a model could simultaneously explain a positive time trend for advertising expenditures and a negative time trend for prices if there is a bit of cost reduction due to learning. (Whether this persists for substantial cost reductions is an open question.) Hence, perhaps firms were not signaling quality, but actually they were using advertising to try to signal-jam learning about the uncertain demand faced by each firm.[21]

There is one more aspect not dealt with in the paper that might be worthwhile exploring in detail. Thus, the paper has exclusively focused on the signal-jamming role that advertising can play, leaving its signaling role aside by assuming private information away. This is useful to isolate the effects that arise when firms can manipulate each other's inference processes through unobservable advertising. The real world is more complex, though, and it seems quite reasonable to assume that firms receive a private signal about the quality of its product (as perceived by consumers) before choosing price and advertising effort. In this case, the signaling role of prices (and perhaps advertising) would interact with the signal-jamming role that advertising would have. This would probably create a further incentive to soften second-period competition, which may lead to excessively low prices in the first-period. We leave the analysis of interaction between signaling and signal-jamming for future research.

[21] In fact, advertising data could only be observed with some noise by the rival firms because data were firm-specific (and not model-specific) and they were collected by a magazine issued monthly.

Appendix

In this appendix, we provide an analysis of the misinformative role advertising in a context in which advertising has no persuasive effects. Instead, we assume that firms do advertising in order to convey information about some features of their products (e.g., price). In this setting, firms acknowledge that imperfectly observed advertising can also be used to signal-jam the competitor's inference about some unobservable drivers of demand.

The model we use is a duopoly variant of that in Grossman's and Shapiro's (1984) seminal paper. We assume that there exists a unit mass of consumers who are identical and whose preferences over a *numéraire* good (denoted by 0) and the goods supplied by firms i and $3-i$ ($i \in \{1,2\}$) are as follows:

$$U(q_0, q_i, q_{3-i}) = q_0 + \widetilde{\alpha}(q_i + q_{3-i}) - (q_i^2 + q_{3-i}^2 + q_i q_{3-i})/3.$$

It is assumed that $\widetilde{\alpha}$ is a random variable with a positive continuous density $g(\alpha)$ defined on the interval $[\underline{\alpha}, \overline{\alpha}] \subset \Re_{++}$. The mean and the variance are respectively denoted by α_e and σ^2. As in the model dealing with persuasive advertising, we assume that the realized value of $\widetilde{\alpha}$ is always observed by consumers, but never by firms.

We consider a two-period game in which consumers initially do not know of the existence of firms (or their products). Besides setting the price p_i at which to sell its product, firm $i \in \{1,2\}$ can choose the advertising necessary to inform a fraction a_i of consumers about its existence and the features of its product, price included. The cost of doing advertising so as to reach $a_i \in [0,1]$ consumers is $C(a_i) = ka_i^2/2$. A consumer does not observe whether other consumers have received an ad from firm i, although this assumption is innocuous. To keep matters as simple as possible, we also assume that all consumers know of the existence of firms at the beginning of the second period, so firms simply compete in prices in that period. Given price competition with differentiated products, there should be an incentive to use imperfectly observed advertising as a signal-jamming device with the aim of softening second-period competition. In particular, a firm choosing its first-period advertising effort would like to lead its competitor to believe that the realized value of $\widetilde{\alpha}$ is greater than it really is so that the rival's second-period price increases. This creates an incentive for a firm to underadvertise relative to the static equilibrium so that the competitor receives higher sales than it would in the absence of the misinformative role of advertising. The analysis below simply confirms this intuition developed in the light of the model dealing with persuasive advertising.

Suppose that all consumers have a per-period income of $Y > 0$ to be spent in the *numéraire* and/or in the products sold by each of the firms. Conditional upon a consumer receiving an ad from firm i, we have that such consumer's demand if she does not receive an ad from firm $3-i$

is $q_i^m(p_i) \equiv 3(\widetilde{\alpha}-p_i)/2$. Conditional upon a consumer receiving an ad from firm i, we have that such consumer's demand if she receives an ad from firm $3-i$ is $q_i^d(p_i, p_{3-i}) \equiv \widetilde{\alpha} + p_{3-i} - 2p_i$. Therefore, given realization α and $\{(p_i, a_i)\}_{i=1}^2$, sales in the first period are formed as follows:

$$q_i = a_i[(1-a_{3-i})\frac{3(\alpha - p_i)}{2} + a_{3-i}(\alpha + p_{3-i} - 2p_i)].$$

To solve the game backwards, suppose that the equilibrium strategies prescribe that firm $i \in \{1,2\}$ charges price $\overline{p}_i$ and exerts advertising effort $\overline{a}_i$ in the first period. Suppose also that firm 2 has possibly deviated from this strategy and instead has charged price p_2 and exerted advertising effort a_2. Then after observing sales of q_1 and firm 2's first-period price p_2, firm 1's inference $\widehat{\alpha}$ about the realized value of $\widetilde{\alpha}$ would be given by the following equation:

$$q_1 = \overline{a}_1[\frac{3(1-\widehat{a}_2(p_2))(\widehat{\alpha} - \overline{p}_1) + 2\widehat{a}_2(p_2)(\widehat{\alpha} + p_2 - 2\overline{p}_1)}{2}],$$

where $\widehat{a}_2(p_2)$ denotes firm 1's belief about the advertising done by firm 2 upon observing a first-period price p_2. However, it actually holds that $q_1 = \overline{a}_1[\frac{3(1-a_2)(\alpha - \overline{p}_1) + 2a_2(\alpha + p_2 - 2\overline{p}_1)}{2}]$, so we have that firm 1's estimate of the realized value of the random variable as a function of a_2, p_2 and α is as follows:

$$\widehat{\alpha}(a_2, p_2\,|\,\alpha\,) = \frac{(3-a_2)\alpha + (a_2 - \widehat{a}_2(p_2))(2p_2 - \overline{p}_1)}{3 - \widehat{a}_2(p_2)}.$$

As usual, we shall suppress the arguments of $\widehat{\alpha}(\cdot)$ to ease exposition.

We now have all the information upon which each firm makes decisions in the second period. Recalling our assumption that all consumers are aware of the existence of firms (and of the price each charges) at the beginning of the second period, we have that firm 1 believes that firm 2 solves the following optimization programme in such period: $\max_{p_2'}\{p_2'(\widehat{\alpha}+p_1'-2p_2')\}$. Therefore, firm 1 solves the following programme (given its beliefs about the price charged by its rival, $\mathring{p}_2 = (\widehat{\alpha} + p_1')/4$): $\max_{p_1'}\{p_1'(\widehat{\alpha} + p_2' - 2p_1')\}$. Solving the first-order condition after taking into account that $\mathring{p}_2 = (\widehat{\alpha} + p_1')/4$ yields that $p_1^*(\widehat{\alpha}) = \widehat{\alpha}/3$.

Unlike firm 1, firm 2 observes the realized value of α and anticipates its rival's action, so it solves $\max_{p_2'}\{p_2'(\alpha + p_1^*(\widehat{\alpha}) - 2p_2')\}$, and hence it finds it optimal to charge a second-period price equal to $p_2^*(\widehat{\alpha}) = (3\alpha + \widehat{\alpha})/12$. Letting $\pi_2^*(\widehat{\alpha}) \equiv p_2^*(\widehat{\alpha})(\alpha + \dfrac{\widehat{\alpha}}{3} - 2p_2^*(\widehat{\alpha}))$, we have that $\pi_2^*(\widehat{\alpha}) = \dfrac{(3\alpha + \widehat{\alpha})^2}{72}$, so

$$\frac{\partial \pi_2^*(\widehat{\alpha})}{\partial a_2} = -\frac{(3\alpha + \widehat{\alpha})(\alpha + \overline{p}_1 - 2p_2)}{36(3 - \widehat{a}_2(p_2))}$$

and

$$\frac{\partial \pi_2^*(\widehat{\alpha})}{\partial p_2} = \frac{4(3\alpha+\widehat{\alpha})(a_2-\widehat{a}_2(p_2))(3-\widehat{a}_2(p_2)) + 2(3\alpha+\widehat{\alpha})(3-a_2)(\alpha-2p_2+\overline{p}_1)\dfrac{d\widehat{a}_2(p_2)}{dp_2}}{72(3-\widehat{a}_2(p_2))^2}$$

at prices at which the belief function is differentiable. Before proceeding to solve the first period, note that the critical driver of the results is that $\partial\pi_2^*(\widehat{\alpha})/\partial a_2 < 0$, so there is an incentive for firm 2 to signal-jam firm 1's inference about the realized value of $\widetilde{\alpha}$ by underadvertising. Doing so will lead firm 1 to believe that second-period demand is greater than it really is, and hence it will price higher than it would otherwise. This relaxes second-period competition and allows firm 2 to raise its second-period price.

Given that firm 1 charges $\overline{p}$ and exerts advertising effort $\overline{a}$, and taking into account that $\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha g(\alpha)d\alpha = \alpha_e$, we have that firm 2's expected stream of discounted profits as a function of its price p_2 and advertising a_2 is as follows:

$$\Pi_2(a_2,p_2\,|\widehat{a}_2(p_2)\,) = a_2p_2[\frac{3(1-\overline{a})(\alpha_e-p_2)+2\overline{a}(\alpha_e+\overline{p}-2p_2)}{2}] + \delta\int_{\underline{\alpha}}^{\overline{\alpha}}\pi_2^*(\widehat{\alpha})g(\alpha)d\alpha - \frac{ka_2^2}{2}.$$

Differentiating with respect to a_2 taking into account that $\dfrac{\partial\pi_2^*(\widehat{\alpha})}{\partial a_2} = -\dfrac{(3\alpha+\widehat{\alpha})(\alpha+\overline{p}_1-2p_2)}{36(3-\widehat{a}_2(p_2))}$ and $\int_{\underline{\alpha}}^{\overline{\alpha}}(\alpha-\alpha_e)^2 g(\alpha)d\alpha = \sigma^2$ yields that

$$\begin{aligned}\frac{\partial\Pi_2(a_2,p_2\,|\widehat{a}_2(p_2)\,)}{\partial a_2} &= p_2[\frac{3(1-\overline{a})(\alpha_e-p_2)+2\overline{a}(\alpha_e+\overline{p}-2p_2)}{2}] - \\ &\quad \frac{\delta(\alpha_e^2+\sigma^2+\alpha_e(\overline{p}-2p_2))(9-3\widehat{a}_2(p_2)+3-a_2)}{36(3-\widehat{a}_2(p_2))^2} - \\ &\quad \frac{\delta(a_2-\widehat{a}_2(p_2))(2p_2-\overline{p})(\alpha_e+\overline{p}-2p_2)}{36(3-\widehat{a}_2(p_2))^2} - ka_2.\end{aligned}$$

The second derivative can be made negative everywhere by assuming that $k \geq \left(\dfrac{\alpha_e+\overline{\alpha}}{12}\right)^2$.[21]

Under the assumption that $k \geq \left(\dfrac{\alpha_e+\overline{\alpha}}{12}\right)^2$, $a_2^*(p_2\,|\widehat{a}_2(p_2)\,)$ is uniquely defined and is (typically) given by the solution to $\partial\Pi_2(a_2,p_2\,|\widehat{a}_2(p_2)\,)/\partial a_2 = 0$.[22] Because it must hold that

[21] This follows because

$$\frac{\delta(\alpha_e+\overline{p}-2p_2)^2}{36(3-\widehat{a}_2(p_2))^2} - k \leq \frac{\delta(\alpha_e+\overline{p}-2p_2)^2}{144} - k \leq \frac{\delta(\alpha_e+\overline{\alpha})^2}{144} - k < \frac{(\alpha_e+\overline{\alpha})^2}{144} - k \leq 0,$$

where we have used the facts that $\widehat{a}_2(p_2) \leq 1$, $\delta < 1$, $p_2 \geq 0$ and $\overline{p} \leq \overline{\alpha}$.

[22] Of course, $a_2^*(p_2\,|\widehat{a}_2(p_2)\,) = 1$ for all p_2 for which it holds that $\partial\Pi_2(a_2,p_2\,|\widehat{a}_2(p_2)\,)/\partial a_2 > 0$ for all a_2,

$a_2^*(\overline{p}\,|\widehat{a}_2(\overline{p})\,) = \overline{a}$ and $\widehat{a}_2(\overline{p}) = \overline{a}$, solving for $\overline{a}$ in $\left.\dfrac{\partial \Pi_2(a_2, \overline{p}\,|\widehat{a}_2(\overline{p})\,)}{\partial a_2}\right|_{a_2=\overline{a}} = 0$ allows us to relate the equilibrium price and the equilibrium advertising efffort:

$$9(2k + (\alpha_e - \overline{p})\overline{p})\overline{a}^2 - 54(k + (\alpha_e - \overline{p})\overline{p})\overline{a} + 81(\alpha_e - \overline{p})\overline{p} - 2\delta(\alpha_e^2 + \sigma^2 - \alpha_e\overline{p}) = 0. \qquad (11)$$

As in the persuasive advertising case, we focus on the equilibrium in which $\left.\dfrac{d\widehat{a}_2(p_2)}{dp_2}\right|_{p_2=\overline{p}} = 0$, so using the shorthand $a_2^*(p_2)$ in lieu of $a_2^*(p_2\,|\widehat{a}_2(p_2)\,)$, one can easily show the following under the restriction that $\left.\dfrac{d\widehat{a}_2(p_2)}{dp_2}\right|_{p_2=\overline{p}} = 0$:

$$\begin{aligned}\frac{d\Pi_2(a_2^*(p_2), p_2\,|\widehat{a}_2(p_2)\,)}{dp_2} &= \frac{a_2^*(p_2)[3(1-\overline{a})(\alpha_e - p_2) + 2\overline{a}(\alpha_e + \overline{p} - 2p_2) - 3p_2 - p_2\overline{a}]}{2} + \\ &\quad \frac{4\delta\alpha_e(a_2^*(p_2) - \widehat{a}_2(p_2))(12 - 3\widehat{a}_2(p_2) - a_2^*(p_2))}{72(3 - \widehat{a}_2(p_2))^2} + \\ &\quad \frac{4\delta(a_2^*(p_2) - \widehat{a}_2(p_2))^2(2p_2 - \overline{p})}{72(3 - \widehat{a}_2(p_2))^2}\end{aligned}$$

Because $a_2^*(\overline{p}) = \widehat{a}_2(\overline{p}) = \overline{a}$, letting $\left.\dfrac{d\Pi_2(a_2^*(p_2), p_2\,|\widehat{a}_2(p_2)\,)}{dp_2}\right|_{p_2=\overline{p}} = 0$ yields

$$\overline{p} = \frac{(3 - \overline{a})\alpha_e}{6}. \qquad (12)$$

The solution to the system of equations formed by (11) and (12) delivers the equilibrium values (as long as the resulting $\overline{a}$ is neither smaller than 0 nor larger than 1). Note that the equilibrium payoff for each firm is

$$\Pi(\overline{p}) = \frac{\overline{ap}(3 - \overline{a})(\alpha_e - \overline{p})}{2} + \frac{2\delta(\alpha_e^2 + \sigma^2)}{9} - \frac{k\overline{a}^2}{2}.$$

We now provide a numerical illustration of how this setting leads to underadvertising relative to the static equilibrium (i.e., the one in which $\delta = 0$). In our numerical examples, we set $k = 0.25$ and $\sigma^2 = 1$, with $\alpha_e = 1$, which is without any loss of generality.[23] Figure 2 shows how the equilibrium advertising effort decreases with the discount factor, as was to be expected in the presence of the signal-jamming effect of advertising and the strategic complementarity displayed by second-period product market competition:

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whereas $a_2^*(p_2\,|\widehat{a}_2(p_2)\,) = 0$ for all p_2 for which it holds that $\partial\Pi_2(a_2, p_2\,|\widehat{a}_2(p_2)\,)/\partial a_2 < 0$ for all a_2.

[23] Setting $\alpha_e = 1$ is without loss of generality because it simply boils down to a standardization on $\overline{p}$, σ^2, k, $\overline{\alpha}$ and $\underline{\alpha}$. On the other hand, note that it holds that $k \geq \left(\dfrac{\alpha_e + \overline{\alpha}}{12}\right)^2$ as long as $\overline{\alpha} \leq 5$.

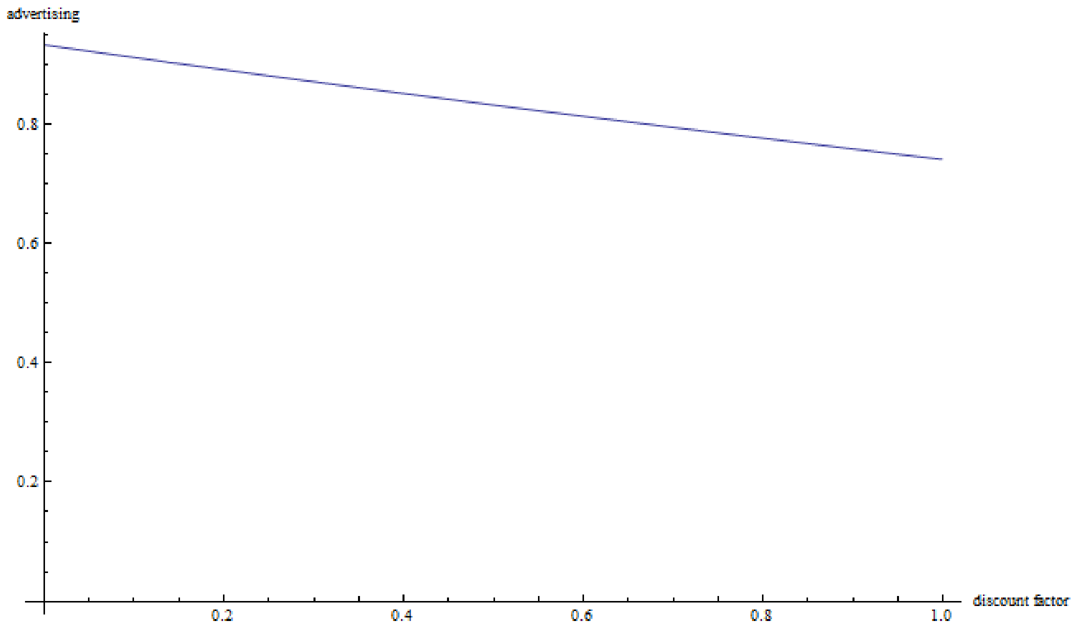

Figure 2: Equilibrium advertising effort as a function of the discount factor

In this particular example, payoffs are increasing in δ: underadvertising lowers costs and implies greater market power vis-à-vis those consumers who receive just one ad. Unlike the persuasive advertising case, though, this may not be a general result because lowering advertising expenditures has a negative effect in that the fraction of consumers who receive the firm's ad decreases.

References

[1] Bagwell, K. (2007), "The Economic Analysis of Advertising", *Handbook of Industrial Organization*, Vol. 3, North-Holland: 1701-1844.

[2] Bagwell, K. and Michael H. Riordan (1991), "High and Declining Prices Signal Product Quality", *American Economic Review*, 81(1): 224-239.

[3] Caminal, R. and Xavier Vives (1996), "Why Market Shares Matter: An Information-Based Theory", *RAND Journal of Economics*, 27(2): 221-239.

[4] Dana, J. D. (2001), "Competition in Price and Availability when Availability is Unobservable", *RAND Journal of Economics*, 32(4): 497-513.

[5] Dixit, A. K. and Victor Norman (1978), "Advertising and Welfare", *Bell Journal of Economics*, 9(1): 1-17.

[6] Fudenberg, D. and Jean Tirole (1986), "A "Signal-Jamming" Theory of Predation", *RAND Journal of Economics*, 17(3): 366-376.

[7] Grossman, G. M. and Carl Shapiro (1984), "Informative Advertising with Differentiated Products", *Review of Economic Studies*, 51(1): 63-81.

[8] Hart, O. D. and Jean Tirole (1990), "Vertical Integration and Market Forecluse", *Brookings Papers on Economic Activity*, special issue: 205-276.

[9] Hertzendorf, M. N. (1993), "I'm Not a High-Quality Firm-But I Play One on TV", *RAND Journal of Economics*, 24(2): 236-247.

[10] Horstmann, I. and Glenn MacDonald (2003), "Is Advertising a Signal of Product Quality? Evidence from the Compact Disc Player Market, 1983-1992", *International Journal of Industrial Organization*, 21(3): 317-345.

[11] Iyer, G. and Dmitri Kuksov (2009), "Consumer Feelings and Equilibrium Product Quality", *Journal of Economics and Management Strategy,* forthcoming.

[12] Keller, G. and Sven Rady (1999), "Optimal Experimentation in a Changing Environment", *Review of Economic Studies*, 66(3): 475-507.

[13] Keller, G. and Sven Rady (2003), "Price Dispersion and Learning in a Dynamic Differentiated-Goods Duopoly", *RAND Journal of Economics*, 34(1): 138-165.

[14] Kreps, D. M. and Robert Wilson (1982), "Sequential Equilibria", *Econometrica*, 50(4): 863-894.

[15] Mailath, G. J. (1987), "Incentive Compatibility in Signaling Games with a Continuum of Types", *Econometrica*, 55(6): 1349-1365.

[16] Mazzola, J. B. and Kevin F. McCardle (1996), "A Bayesian Approach to Managing Learning-Curve Uncertainty", *Management Science*, 42(5): 680-692.

[17] McAfee, R. P. and Marius Schwartz (1994), "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity", *American Economic Review*, 84(1): 210-230.

[18] Milgrom, P. and John Roberts (1982), "Price and Advertising Signals of Product Quality", *Journal of Political Economy*, 94(4): 796-821.

[19] Riordan, M. H. (1985), "Imperfect Information and Dynamic Conjectural Variations", *RAND Journal of Economics*, 16(1): 41-50.

[20] Rothschild, M. (1974), "A Two-Armed Bandit Theory of Market Pricing", *Journal of Economic Theory*, 9(2): 185-202.

[21] Vives, X. (2009), "Strategic Complementarity in Multi-Stage Games", *Economic Theory*, forthcoming.