Entry Patterns over the Product Life Cycle

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Abstract

We study a game-theoretic real options model of new market entry based on empirical evidence of demand for a new product growing over time and eventually falling. Yet, firms do not know ex ante when this will occur, which creates incentives to update information by delaying irreversible entry. By assuming that the construction of a new productive plant takes some time and is unobservable in the meantime, while operation in the market is not, we show that entry rates increase or decrease under certain conditions related to the rate at which flow profits decrease as more firms enter the industry.

Keywords: Bayesian Real Options, Investment Lags, Irreversibility, Strategic Competition, S-shaped Entry Paths.

JEL Classification: C73, D43, D92, L13, M30.

1 Introduction

Entry and exit processes are key determinants of market structure and firm profitability within a given industry, but also affect the evolution of significant variables such as input productivity, employment, or even immigration flows. Indeed, competition for markets stimulates economic growth as long as it results in new technologies and products being brought to the market, sometimes at a fast pace, as happens in the consumer electronics, semiconductor or pharmaceutical industries.

A robust empirical regularity is that new market entry typically proceeds in a gradual way, and, in fact, it is well documented that the path of entry in a new market is S-shaped, that is, entry rates first increase and then decrease (see e.g. Gort and Klepper 1982).[1] This empirical pattern clearly has relevant implications for economic phenomena such as the accumulation of human capital, job creation and destruction, or the allocation of financial resources within an economy.

There are several formal explanations of why the paths of entry in a new market are S-shaped.[2] Models of technological diffusion such as for example Jovanovic and Lach (1989) explain S-shaped entry paths by appealing to observational learning by late entrants (i.e., observing and learning from earlier entrants' experiences). Not all models that aim at explaining S-shaped entry paths focus on the supply side, though. A few handful of papers such as Vettas (1998, 2000) emphasize that demand

[1] In particular, Gort and Klepper (1982) study the S-shaped entry paths for 46 different products, among which one can find from electrocardiographs or electric shavers to outboard motors, shampoo, or televisions.

[2] Hoppe (2002) provides an extensive formal review of the industrial organization literature on new technology diffusion, and Geroski (2000)—somewhat more informally—surveys in great detail the reasons why the diffusion of new product innovations (the term used by Gort and Klepper 1982 for new market entry) adopts an S-shaped path. See Hall (2004) as well for a more recent survey with several real-world examples.

side aspects may also lead to entry rates that first increase and then decrease.[3] All of these models are based on theoretical models with (perfectly) competitive entry, and hence are not applicable to environments with strategic interaction, which is an important shortcoming insofar as most industries in the real world are oligopolistic.

The primary purpose of this paper is to show that S-shaped entry paths in oligopolistic industries may be explained by how (standardized) profit falls with the number of entrants. In our model, all firms are fully aware of the new technology/product and its characteristics, so technological diffusion plays no role. However, there exists uncertainty about future demand evolution, which, coupled with investment irreversibility, creates an incentive for firms to wait and see before entering the market and irreversibly committing resources.[4] Given this setting, we show that entry accelerates (decelerates) if the percentage reduction in profit due to an additional entrant decreases (increases) as more firms enter.

The main ingredients of our framework are the following: uncertainty about the nature of demand evolution over time, investment irreversibility and investment unobservability. Thus, we assume that the pattern of demand evolution is unknown to firms at the time they have to choose their entry strategies. There exists ample empirical evidence that shows that demand for some products—such as durable goods—grows over time until the market reaches its maturity, and gradually decays thereafter. The so-called theory of the (demand side) product life cycle (PLC) pro-

[3] For example, Vettas (1998) explains S-shaped entry patterns based on endogenous information diffusion and (Bayesian) learning both on the demand and supply sides. In turn, Vettas (2000) explains such patterns based on Bayesian learning about demand potential and the positive effect of past sales on current sales made by firms.

[4] Hence, investment opportunities are treated as real options that can be exercised at any instant of time. For an excellent survey on the theory of real options, see Dixit and Pindyck (1994).

vides a foundation to this pattern of consumer behavior,[5] based to a large extent on the Bass (1969) model.[6] In practice, uncertainty about the shape of the life cycle is critical for decision making insofar it is difficult to predict the various stages through which a product will go, as well as their duration.[7] Hence, entry decisions must account for the unknown evolution of demand.

The other crucial elements of our model that explain the S-shaped pattern of gradual entry in our strategic setup are the irreversibility and unobservability of investment processes. Time-consuming investments such as plant construction or technology/product development are largely irreversible and unobservable until completed, especially in the case of new markets. Investment irreversibility under uncertainty creates an incentive to delay entry, whereas the assumption that firms do not observe investment decisions but rather their actual outcome in the form of entry implies that they can only respond to rivals' observable actions with some long delay. Thus, we adopt an open-loop informational structure for the timing game we examine, which may be a more reasonable description of reality than a closed-loop formulation, especially if the number of potential entrants is not too small and/or investment lags are

[5]For instance, see Grant (1998, Chapter 10), Kotler (1999, Chapter 10), and Pisano and Wheelwright (1995) for a discussion of the existence of (demand side) product life cycles. For empirical evidence, see Bass (1969, 1980), Brockhoff (1967), Kwoka (1996), Polli and Cook (1969) and Tsurumi and Tsurumi (1980). As reported by Bass (1980), many leading companies such as IBM, Sears, Hewlett-Packard or Eastman Kodak have used the pattern of sales predicated by the PLC theory for forecasting purposes.

[6]In the industrial organization literature, the theory of the (supply side) PLC generally refers to a different phenomenon, in particular to the patterns of entry and exit of an industry based on intertemporal changes on supply side factors such as innovation capabilities or diffusion of technological knowledge among firms (Gort and Klepper 1982). Although our paper coincides with the research objective of this branch of the economic literature, our explanations of the same phenomena are significantly different, and our usage of the term "product life cycle" will be similar to that in the Bass model.

[7]For example, Tsurumi and Tsurumi (1980) use an econometric model in which the transition from one stage of the PLC to another is due to gradual shifts in key demand parameters (e.g., income or price elasticity) whose date of occurrence is unknown.

relatively long.[8]

Given this setup, we analyze the dynamics of irreversible entry into a market whose demand follows an unknown life cycle. The main contribution of our paper is to unveil a factor neglected in previous theoretical and empirical studies, but that turns out to be critical in determining the shape of entry paths: the rate at which flow profits decrease as more firms enter the market. At an intuitive level, investing in a productive plant or in technology/product development becomes relatively more (less) attractive if the standardized decrease in profits due to entry diminishes (increases) as more firms become active in the market, which accelerates (decelerates) entry by the remaining entrants. As a result, the discontinuous change from an increasing rate of entry to a decreasing one may follow from a discontinuous change in the nature of product market competition. This may be due for example to a critical switch from commercialization of a product that is basically homogenous to one that is highly differentiated, as usually happens at some stage in the evolution of numerous industries.

Despite the large number of papers on diffused entry in non-strategic settings with atomistic firms, the game-theoretic literature on diffusion to which we contribute is not very extensive. The classic papers are Reinganum (1981) as well as Fudenberg and Tirole (1985). Our work is closely related to the former, given its open-loop formulation, and may be considered an extension when demand is subject to a random dynamic and investment does not imply instantaneous entry. More importantly, Reinganum's (1981) paper does not provide conditions for S-shaped entry paths, unlike our work. In turn, the closed-loop formulation of Fudenberg and Tirole (1985)

[8] An open-loop formulation is particularly adequate if firms observe the strategic decisions made by rivals with a relatively long lag (see Athey and Schmutzler 2001, footnote 26).

is very fruitful for the analysis of preemptive actions, but is not particularly adequate when firms cannot respond in a short period of time to strategic moves by competitors, as happens in our framework with unobservable plant construction and relatively long investment lags. However, as recently shown by Ruiz-Aliseda and Zemsky (2007) based on Reinganum's (1981) model, open-loop and closed-loop equilibria coincide if productive plants (or technology development) take a long time-to-build and rivals cannot observe investment processes but rather their final outcome, namely operation in the market.[9]

In addition, there are a few previous papers that consider investment in a PLC setting, although their object of study is very different from ours. In the first place, Londregan (1990) examines the patterns of entry and exit of two firms with different capacities in a deterministic setting, and shows that a large firm can preempt its rival despite having higher entry costs. In the second place, Bollen (1999) considers a stochastic life cycle and shows that real options models that are based on a Geometric Brownian Motion overvalue (undervalue) the option to expand (contract) a project. In the third place, Lilien and Yoon (1990) perform an empirical analysis of the tension that exists between the risks of premature entry over the PLC and the problem of missed opportunities due to a delay in entry. Appendix B in their paper introduces a simple discrete-time dynamic programming model that focuses on the optimal entry time of a monopolist over the PLC, but they do not endogenize market structure as done in Amir and Lambson (2007) in a related setting that applies the powerful framework developed by Amir and Lambson (2003). In the last place, our paper

[9]Ruiz-Aliseda and Zemsky (2007) formalize this insight based on unobservable investment processes and endogenous time-to-build. Although we assume that time-to-build is a parameter, the main ideas remain if it is endogenized, as done by Gutiérrez and Ruiz-Aliseda (2007) in a single-firm setting with uncertainty similar to ours.

builds on the novel Bayesian real options framework introduced by Gutiérrez and Ruiz-Aliseda (2007), which is simpler than that of Bollen (1999). Its main feature is that demand grows over time until a random maturity date, after which the market gradually disappears. Hence, waiting to invest is valuable in that it allows firms to use Bayes' rule and update their beliefs about the maturity date of the market. This framework based on conditional probabilities greatly simplifies an analysis that otherwise would be very complicated in a setup with multiple firms, the focus of this paper.[10] In this sense, the present work is the oligopoly extension of the monopoly analysis in Gutiérrez and Ruiz-Aliseda (2007).

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 shows why entry rates may increase over the PLC. Section 4 briefly extends the basic model to highlight the underlying reasoning leading to accelerated or decelerated entry. Section 5 makes concluding remarks and points out directions for future research. The proofs of the results are simple but somewhat tedious, so they are relegated to an appendix.

2 Foundations of the theoretical model

Let time, denoted by t, be a continuous variable, with $t \in [0, \infty)$. Suppose that at date $t = 0$ a number of firms have to decide when to start building a plant so as to make a new product for which there already exists some latent demand. The number of firms is finite but arbitrary and is denoted by $n \geq 2$.[11] We also assume that such ex ante identical firms cannot perform technological improvements upon the product

[10] For instance, Bollen (1999) has to resort to numerical methods in a monopoly context.

[11] The finite number of firms seems an appropriate assumption, as suggested by Klepper and Graddy (1990, p. 36), because of expertise requirements.

or the production technology.[12]

All firms face uncertainty about the temporal evolution of demand for their products when making their investment decisions. Uncertainty is assumed to unravel partially over time, and demand evolves in the following manner. In a first stage, market size, which is positive at date $t = 0$, grows exponentially. This would represent the introduction and growth phases in the traditional PLC framework. However, the market reaches an ephemeral maturity at date τ, where τ is an exponentially distributed random variable with parameter $\lambda > 0$.[13] We will slightly abuse the notation and τ will also denote its realization, while the density function will be denoted by $f(\tau)$. Hence, in a second stage whose beginning is uncertain at date 0, market size decreases and converges to 0 as $t \to \infty$, because consumers perceive that new substitutes of the product can serve their needs better. This decaying behavior would represent the decline stage of the product, as well as its subsequent gradual disappearance. It is assumed that firms know the realized maturity date only once it arrives. For tractability, we make the following assumption.

Assumption 1 *Given a realization τ of the random maturity date, the flow of profits made by a firm if $i \in \{1, ..., n\}$ firms are active at date t evolves continuously over*

[12] Certainly, it would be reasonable to assume that imitators have to develop over time the necessary capabilities to serve the market. Indeed, according to some authors such as Gort and Klepper (1982, p. 651), it is precisely the diffusion rate of technological knowledge and the possibility to innovate that mainly explains the empirical patterns of entry into an industry. Given that the focus of our paper is on the demand side, we abstract away from these other aspects so as to provide a more transparent analysis of the forces into play.

[13] As shown empirically by Barbarino and Jovanovic (2004), it sometimes can be natural to assume that the demand hazard rate is non-increasing. Interestingly, at a theoretical level, this property follows if the true hazard rate is constant but unknown to firms, which can update their beliefs in a Bayesian fashion (see, e.g, Choi 1991), so that uncertainty would thus be two-folded. Our model can easily accommodate a random variable with a decreasing hazard rate, although a unique solution may fail to exist, especially for very long investment lags.

time as follows:

$$\Pi(i,t,\tau) = \begin{cases} \pi_i \exp(\alpha t) & \textit{if } t \leq \tau \\ \pi_i \exp\left[\alpha(2\tau - t)\right] & \textit{if } t > \tau \end{cases}.$$

We further assume that instantaneous profit has the following properties: it is monotone decreasing in n and $\pi_n > 0$. Note that π_i denotes a firm's initial profit if $i \in \{1, ..., n\}$ firms are active at $t = 0$, whereas $\alpha > 0$ denotes the growth rate of profit (if the market is in expansion; otherwise, it is its decay rate). Figure 1 graphically illustrates two possible realizations of this kind of PLC.

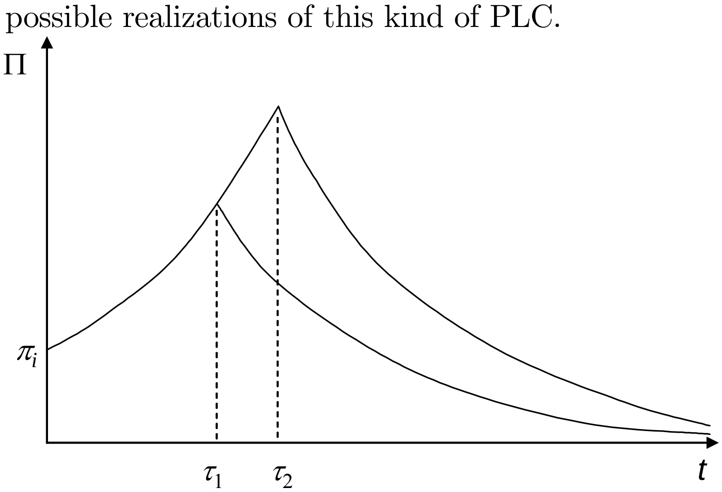

Figure 1: Two possible realizations of the proposed PLC

In addition, we assume that firms are risk-neutral, so they maximize expected payoffs, and discount cash-flows at the risk-free interest rate $r > 0$. It is also natural to assume that investments are completely irreversible, which makes risk-taking a potentially relevant factor in explaining the patterns of entry, which is exacerbated by the existence of investment lags.

Assumption 2 *Starting the construction of a plant entails paying a non-recoverable* *cost $K > 0$ and takes a fixed "time-to-build" denoted by $\delta \geq 0$.*[14]

Our last assumption ensures that the expected discounted value of one dollar that is capitalized at an instantaneous rate of α is finite no matter what the duration of the ascending phase of the PLC is, which bounds the value of the firms' investment opportunities.

Assumption 3 *$\lambda > \alpha - r$.*

3 The solution to the model

We employ the methodology of optimal stopping games in order to solve the dynamic game with uncertainty. Throughout this section, we use the solution concept of open-loop/pre-commitment equilibrium in pure strategies,[15] and thus, without any loss of generality, we let the firm's index denote the position of investment no matter whether the market is growing or not. In an open-loop equilibrium, no firm can observe the investment times of competitors, so using this solution concept is reasonable if firms observe rivals' actions with a sufficiently long lag. Note that there will be $n!$ identical equilibria of this kind, interchanging the indexes of all the firms.

The problem faced by firm $i \in \{1, ..., n\}$ at time $t = 0$ is to choose an investment rule $T_i(\cdot)$ that maximizes its expected discounted stream of cash flows conditional upon information available at the time of investment. More generally, we have $T_i : [0, \infty) \times \{0, 1\} \rightarrow \{In, Out\}$, that is, firm i's strategy assigns a decision of whether

[14] With an exponential random variable, the model with investment lags can be shown to yield exactly the same results as one in which firms that have not entered the market suffer from a (fixed) detection lag regarding the realized maturity date.

[15] Technically, the game we consider belongs to the category of piecewise deterministic differential games in which firms use open-loop strategies.

to invest so as to be in the market ("In") or not ("Out") to every instant of time t depending on whether τ has been revealed to the firm at that date or not (events that are denoted by "1" and "0", respectively). Note that the open-loop structure refers to the investment behavior of rival firms, not the random evolution of the market. As discussed in the introduction, such type of assumption is particularly adequate when investment lags are relatively long. In particular, the equilibrium outcome can be shown to coincide with that of a closed-loop assumption if plant construction is time-consuming and unobservable, but operation in the market is observed (Ruiz-Aliseda and Zemsky 2007).

When solving this type of timing game, one usually proceeds backwards and solves for firm i's optimal investment time t_i^e ($i \in \{1, ..., n\}$), subject to the constraint that $t_i^e \in [t_{i-1}^e, t_{i+1}^e]$ (where $t_0^e \equiv 0$ and $t_{n+1}^e \equiv \infty$). However, standard arguments (Reinganum 1981, pp. 620-622) show that it is enough to solve the game backwards with no such constraints for any firm. Before, though, let us proceed to characterize some of the common properties of the equilibrium strategies of all firms for the possible states of the system. Given our assumptions (no scrap value and positive profit-margin at any possible situation and date), one of the properties of these functions is that firm i stays in the market forever once it has paid the investment cost. This holds no matter if τ is known or not. In turn, Lemma 1 below describes the firms' behavior once the maturity of the market has been reached. According to this result, firms prefer to invest immediately once τ is revealed, but only if such date is sufficiently late (depending on the firm's index); otherwise, they prefer not to invest.

Lemma 1 *It is optimal for firm $i \in \{1, ..., n\}$ to invest immediately at the revealed maturity date τ for all $\tau \geq t_i^{\max} \equiv \max[0, \ln\left(K(\alpha + r)e^{(\alpha+r)\delta}/\pi_i\right)^{1/\alpha}]$, while invest-*

ment during the declining phase of the market is not profitable for all $\tau < t_i^{\max}$*.*

To characterize the equilibrium outcome fully, it only remains to focus on the optimal time t_i^e at which firm $i \in \{1, ..., n\}$ would invest were the cycle in its ascending phase (i.e., when τ is not known yet). We work backwards and derive first firm n's investment time when τ has not been yet realized, ignoring the constraint that $t_n^e \geq t_{n-1}^e$, as discussed above. Using Lemma 1, the value of firm n's investment opportunity at date $t = 0$ as a function of its investment time $t_n \geq 0$ is:

$$\begin{aligned} V_n(t_n) &= \int_0^{t_n} f(\tau) \max(0, \int_{\tau+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - K e^{-r\tau}) d\tau + \\ &\int_{t_n}^{t_n+\delta} f(\tau) (\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - K e^{-rt_n}) d\tau + \\ &\int_{t_n+\delta}^{\infty} f(\tau) (\int_{t_n+\delta}^{\tau} \pi_n e^{(\alpha-r)s} ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - K e^{-rt_n}) d\tau. \end{aligned}$$

If firm n chooses to wait to invest until t_n, then, for realizations smaller than t_n, it seizes the payoff to immediate investment at τ if and only if it is non-negative (by Lemma 1). In contrast, if firm n ends up investing at t_n while the product life cycle is growing, there are two cases to consider because of the delay in entry. If the maturity date occurs before entry takes place, then the firm will get a payoff during the decline of the market, since investments are irreversible once incurred. However, if entry occurs before the realization of the random variable, then it will also reap oligopoly profits for a period of time whose length is unknown at the time of investment.

Note that, by another application of Lemma 1, $V_n(t_n)$ can be rewritten as follows:

$$V_n(t_n) = \begin{cases} \begin{array}{c} \int_{t_n}^{t_n+\delta} f(\tau)(\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau + \\ \int_{t_n+\delta}^{\infty} f(\tau)(\int_{t_n+\delta}^{\tau} \pi_n e^{(\alpha-r)s} ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau \end{array} & \text{if } t_n \in [0, t_n^{\max}) \\ \begin{array}{c} \int_{t_n^{\max}}^{t_n+\delta} f(\tau)(\int_{\min(\tau,t_n)+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-r\min(\tau,t_n)}) d\tau + \\ \int_{t_n+\delta}^{\infty} f(\tau)(\int_{t_n+\delta}^{\tau} \pi_n e^{(\alpha-r)s} ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau \end{array} & \text{else} \end{cases}.$$

$V_n(t_n)$ can be easily shown to be continuously differentiable, so solving for firm n's optimal investment time amounts to maximizing $V_n(t_n)$ with respect to t_n using ordinary calculus (and subject to the constraint that the optimal entry time leads to a nonnegative payoff).

To avoid corner solutions, we assume that the date-0 monopoly profit flow is not too large, so that all firms find it optimal to wait to invest, including the first mover. Formally, $\pi_1 < K(r+\lambda)/C$, where

$$C \equiv e^{-(\lambda+r-\alpha)\delta} + \frac{\lambda e^{-(\alpha+r)\delta}}{\alpha+r} + \frac{\lambda(1-e^{-(\lambda-2\alpha)\delta})e^{-(\alpha+r)\delta}}{\lambda-2\alpha} > 0.$$

Then firm n's equilibrium strategy is as follows.

Lemma 2 *Firm n's optimal strategy is "invest at t_n^e if $t_n^e < \tau$; else, do not invest," where $t_n^e = \ln\left(K(r+\lambda)/C\pi_n\right)^{1/\alpha} > 0$.*

The proof of Lemma 2 shows that $t_n^e < t_n^{\max}$, so firm n will not invest in equilibrium while the PLC is decaying, as expected, although entry into the market may occur past the maturity date because of time-to-build. In addition, note that adding more firms (i.e., increasing n) would mean a later entry by the last mover. Finally, rewriting

t_n^e allows us to make the following observation.[16]

Remark 1 *The optimal investment time* t_n^e *uniquely solves the following equation:*

$$\left(\int_{t_n^e}^{t_n^e+\delta} \frac{f(s)}{\Pr(t_n^e < \tau)} \pi_n e^{2\alpha s} e^{-\alpha(t_n^e+\delta)} e^{-r\delta} ds + \int_{t_n^e+\delta}^{\infty} \frac{f(s)}{\Pr(t_n^e < \tau)} \pi_n e^{\alpha(t_n^e+\delta)} e^{-r\delta} ds\right) dt =$$
$$rKdt + \lambda dt \left(K - \int_{t_n^e+\delta}^{\infty} \pi_n e^{2\alpha t_n^e} e^{-\alpha s} e^{-r(s-t_n^e)} ds\right).$$

If the life cycle is still growing, then firm n decides to invest at the instant of time t_n^e such that the marginal value of waiting equals the marginal cost of postponing investment. The marginal cost is the expected discounted profit flow forgone by waiting a little bit more at date t_n^e, taking into account that the cycle may reach its peak before entry actually occurs, and given the informational situation at the investment date. In turn, the marginal value is the part of the sunk cost saved by delaying investment plus the marginal option value of waiting and avoiding an irreversible action. Thus, firm n believes that the demand of the product may suddenly decay right after time t_n^e, so waiting allows it to avoid making a negative payoff with probability λdt.[17]

After having examined firm n's optimal investment strategy when demand is still growing, let us solve the remainder of the game for all of its predecessors, taking into account that the latter correctly anticipate the investment threshold of subsequent entrants. As shown in the proof of Lemma 3, firm $n-1$'s marginal payoff does not depend on firm n's entry because firm $n-1$ perceives that firm n takes a *fixed* part of its rents with certain probability, so that investment by the latter is not relevant at the margin: firm n's entry time does not depend on firm $n-1$'s entry timing. As

[16] The proof of the remark follows from straightforward manipulations on the first-order condition.

[17] This follows from Lemma 1, which implies that the net present value of firm n's investment if it invested at date t is negative if demand suddenly decays for all $t < t_n^{\max}$.

a result, it is not very surprising that firm $n-1$'s optimal investment threshold is functionally equivalent to that of firm n.

Lemma 3 *Firm $n-1$'s optimal strategy is "invest at t_{n-1}^e if $t_{n-1}^e < \tau$; else, do not invest," where $t_{n-1}^e = \ln \left(K(r+\lambda)/C\pi_{n-1} \right)^{1/\alpha} > 0$.*

Finally, we use the same procedure to solve for firm i's optimal investment strategy, $i = 1, ..., n$. For these cases, recursiveness allows us to prove Proposition 1, which summarizes the optimal investment times for all firms.[18]

Proposition 1 *Firm i's optimal strategy is "invest at t_i^e if $t_i^e < \tau$; else, do not invest," where $t_i^e = \ln \left(K(r+\lambda)/C\pi_i \right)^{1/\alpha} > 0$.*

From Proposition 1, it is clear that $0 < t_1^e < ... < t_n^e < \infty$. That is, since early-movers find it more profitable to wait, subsequent entrants must wait even longer until the market is large enough to accommodate them, for the flow of profits is smaller the more firms invest. Hence, we have the following result, as in Reinganum (1981).

Corollary 1 *Open-loop equilibria are asymmetric, and the firms' equilibrium expected payoffs are decreasing in the position of entry.*

The first part of the corollary is particularly relevant for empirical work because it shows that entry may be dispersed over time even when firms have no differing capabilities and they are totally symmetric, which contradicts common wisdom (see, e.g., Klepper and Graddy 1990, p. 37). As for the effect of time-to-build on investment, it is easy to show that longer investment lags delay the firms' investment times, and

[18] Although we do not write the functional form of $V_i(t_i)$ on the region $[0, t_{i+1}^e)$ in order to avoid a messy expression that is constructed in the same way as $V_{n-1}(t_{n-1})$, it is straightforward to prove both Lemma 3 and Proposition 1 using the marginal interpretation that gives rise to Remark 1.

the impact is bigger the higher the firm's position of entry is.[19] On the other hand, the firms' equilibrium strategies lead to the following patterns of entry over the PLC.

Proposition 2 *Firm i enters at $t_i^e + \delta > 0$ for all $i = 1, ..., n$, provided $t_i^e < \tau$.*

Although firms follow a "wait-and-see" approach, none of them is willing to invest once the cycle has started to decay. However, mistakes may be possible because of time-to-build, and entry may be observed in industries that are declining because of investment lags and irreversibility. The proposition implies that one may never see entry into such a market, or, if demand grows sufficiently over time, entry may occur in a sequential fashion, with firms entering as the market gradually expands. Notwithstanding, Proposition 2 is silent about whether investment—and as a result entry—accelerates or decelerates as more firms enter. The following proposition identifies the determinant of increasing/decreasing entry rates in our setting: the rate at which (standardized) profits decrease as more firms enter the market.

Proposition 3 $(t_{i+1}^e + \delta) - (t_i^e + \delta) < (t_i^e + \delta) - (t_{i-1}^e + \delta)$ *if and only if* $\dfrac{\pi_i - \pi_{i+1}}{\pi_i} < \dfrac{\pi_{i-1} - \pi_i}{\pi_{i-1}}$.

The proposition implies that, as the number of firms grows large, the difference in the times of entry may be observed to converge to 0. More importantly, the assumptions of exponential growth and decay, and an exponentially distributed random variable for the length of the ascending phase of the PLC, are useful in identifying a factor that is critical for explaining why entry rates are accelerating or decelerating. In particular, such assumptions lead to a simple result by dropping time-dependence:

[19] This follows because $\dfrac{dC}{d\delta} = -\dfrac{e^{-(\alpha+r)\delta}(\lambda + r - \alpha)(\lambda - 2\alpha e^{-(\lambda-2\alpha)\delta})}{\lambda - 2\alpha} < 0$, as $\dfrac{\lambda - 2\alpha e^{-(\lambda-2\alpha)\delta}}{\lambda - 2\alpha} > 0$.

increasing entry rates in our setting are not due to the convex evolution of profits; rather, such patterns are driven by the percentage reduction in individual profit due to entry by an additional firm decreases with i. Indeed, entry slows down as more firms enter if the percentage reduction in profit due to entry is increasing in i. Therefore, our model suggests that one of the driving forces of S-shaped diffusion paths would be the existence of a unique i^* such that $(\pi_i - \pi_{i+1})/\pi_i < (\pi_{i-1} - \pi_i)/\pi_{i-1}$ for all $i \leq i^*$ and $(\pi_i - \pi_{i+1})/\pi_i > (\pi_{i-1} - \pi_i)/\pi_{i-1}$ for all $i > i^*$. This would definitely require some type of critical change in the nature of product market competition due to entry by some firm (denoted by the index i^*), such as for example switching from a collusive industry producing a homogenous product to one in which the product is more differentiated. Empirical work on this issue may result critical in validating or rejecting this conjecture that stems from our model.

We now provide an example. In particular, let us suppose that the realized maturity date is such that six firms choose to invest, so that firm i ends up entering at date $t_i^e(\delta) \equiv t_i^e + \delta$ for all $i = 1, ..., 6$ (provided $t_i^e < \tau$). Also, let us suppose that $i^* = 4$. Figure 2 shows how entry rates increase until the fourth firm enters, and decreases thereafter, given two realizations of the maturity date of the market. Note also that entry need neither accelerate nor decelerate during the market decline, since the latter arrives at a random date.

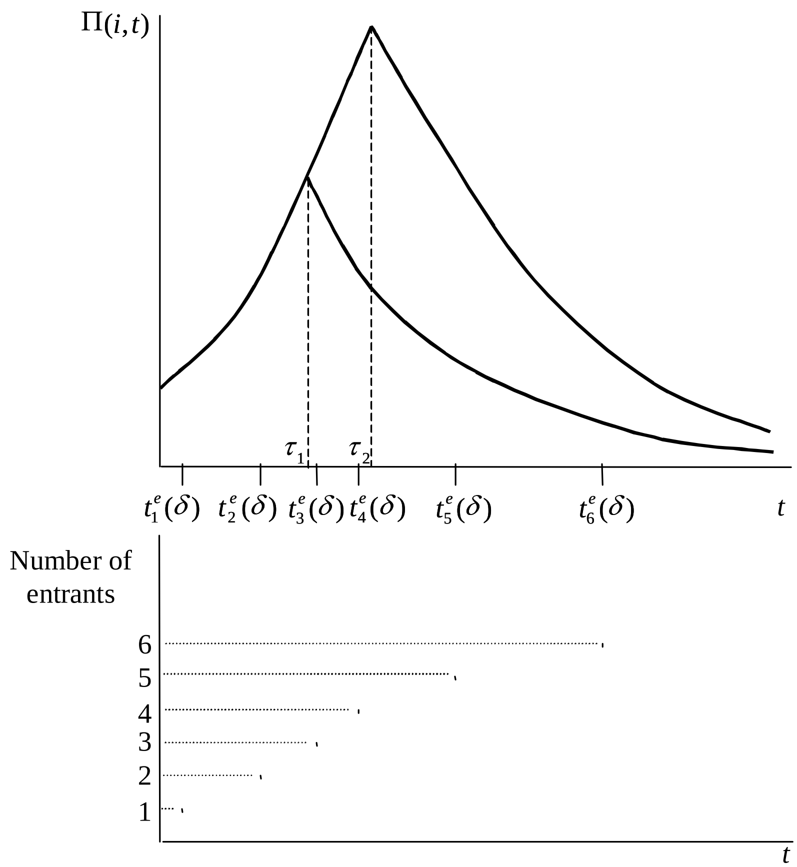

Figure 2: S-shaped entry paths

4 Conclusions and future research

The main purpose of this paper is to provide a theoretical framework that explains why entry patterns may by S-shaped. The driving factors of investment decisions are not supply side factors but rather strategic interaction, uncertainty about future demand evolution, investment lags and irreversibility. In this sense, we have examined conditions under which entry may accelerate or decelerate. The results strongly suggest that entry rates increase or decrease depending on whether the percentage reduction in individual profit due to entry by another firm decreases or increases with the number of remaining entrants. Whether or not demand factors, irreversibility and uncertainty about the PLC are relevant in explaining entry patterns is an empirical matter. Still, it seems interesting to test them versus inter-firm diffusion of technological knowledge and the rate of innovation within the industry.

The paper has some important limitations too, which would be worthwhile examining. We have assumed that the pattern of non-cumulative adoptions of the product by consumers is exogenous to the firms, but unknown. Specifically, the time of the peak is uncertain, as well as the adoption rate at the peak. The model can easily endogenize the distribution function governing the maturity date. This may be relevant if some investment activities undertaken by firms before entering the market can affect the properties of the PLC. For instance, let us consider a setting with symmetric firms and a hazard rate that is positively correlated to the pre-entry activities of each of the firms (e.g., due to a higher product awareness among the population of potential customers). Under these assumptions, the open-loop equilibria we have described would have to consider the incentives of late entrants to free-ride on the efforts of earlier entrants. In principle, early entrants would benefit more from exerting a higher effort in pre-entry activities because their expected payoff is higher (by Corollary 1). However, their incentives to invest in increasing the hazard rate would lessen because such investment would also benefit later entrants, so the latter would speed up entry.[20] As a result, pioneers may underinvest relative to later entrants, so that it may happen that it is early entrants that free ride on late entrants' pre-entry

[20]Effects of investment on own entry timing would be of second-order, while effects on later entrants' entry timing would be of first-order.

efforts.

Lastly, our main result may hold even if the hazard rate is not constant and the nature of competition in the market does not change as the number of active firms grows sufficiently large. If the hazard rate first decreased and then increased, it seems intuitive that the marginal option value of waiting of early-entrants (laggards) would decrease (increase), thus accelerating (decelerating) entry. This would result in an S-shaped entry path. Whether a constant or a non-monotonic hazard rate is a more reasonable assumption may constitute an interesting issue for empirical work on the topic of market entry under uncertainty.

Appendix

Proof of Lemma 1. Suppose that $0 \leq j < n$ firms had invested during the growth phase and suddenly τ were revealed. By relabeling the remaining firms (as well as the associated flow of profits), we can let $j = 0$ without loss of generality. Denoting the firms' optimal investment times in a declining market by t_i^d, let us solve for firm n's investment problem, knowing that it chooses not to invest before $t_{n-1}^d \geq \tau$. Since $NPV_n(t_n) = \int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}$ is a strictly quasiconvex function on $(-\infty, +\infty)$,[21] it follows that, for all $t_{n-1}^d < t_n' < t_n''$, $NPV_n(t_n') < \max(NPV_n(t_{n-1}^d), NPV_n(t_n''))$. Taking the limit as $t_n'' \to \infty$, $NPV_n(t_n') < \max(NPV_n(t_{n-1}^d), 0)$. Hence, if firm n finds it optimal to invest while the market is declining, then it must enter at t_{n-1}^d. Repeating this procedure, it is easy to show that $t_i^d = \tau$ for all $i \in \{1, ..., n\}$,[22] so noticing that $NPV_i(\tau) \geq 0$ if and only if

$$\tau \geq t_i^{\max} \equiv \max\left[0, \frac{1}{\alpha}\ln\left(\frac{K(\alpha+r)e^{(\alpha+r)\delta}}{\pi_i}\right)\right]$$

completes the proof. ■

Proof of Lemma 2. We shall show first that $V_n(t_n)$ is monotone decreasing on

[21] Formally, because $dNPV_n(t_n)/dt_n = 0$ implies that $d^2NPV_n(t_n)/dt_n^2 > 0$.

[22] Given such entry behavior by all firms, it is clear that our assumption that the entry order of firms is based on their index acts as a coordination device: if there is room only for i firms, then only the first i firms enter (not the last ones, for instance).

$[t_n^{\max}, \infty)$. In this case, the function becomes:

$$\begin{aligned} V_n(t_n) &= \int_0^{t_n} f(\tau)(\int_{\tau+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-r\tau}) d\tau + \\ & \int_{t_n}^{t_n+\delta} f(\tau)(\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau + \\ & \int_{t_n+\delta}^{\infty} f(\tau)(\int_{t_n+\delta}^{\tau} \pi_n e^{(\alpha-r)s} ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau. \end{aligned}$$

Differentiating $V_n(t_n)$ with respect to t_n, canceling terms and performing some algebraic manipulations taking into account that $\dfrac{f(t_n)}{\int_{t_n}^{\infty} f(\tau)d\tau} = \lambda$ yields:

$$V_n'(t_n) = e^{-(r+\lambda)t_n} \left[rK - \pi_n e^{-(\alpha+r)\delta} e^{\alpha t_n} \left(e^{-(\lambda-2\alpha)\delta} + \frac{\lambda(1 - e^{-(\lambda-2\alpha)\delta})}{\lambda - 2\alpha} \right) \right].$$

We claim that $V_n'(t_n) < 0$ for $t_n \geq 0$. Otherwise, we would reach a contradiction:

$$0 \leq e^{-(r+\lambda)t_n} \left[rK - \pi_n e^{-(\alpha+r)\delta} e^{\alpha t_n} \left(e^{-(\lambda-2\alpha)\delta} + \frac{\lambda(1 - e^{-(\lambda-2\alpha)\delta})}{\lambda - 2\alpha} \right) \right] \leq -\alpha K e^{-(r+\lambda)t_n} < 0,$$

since $\dfrac{\pi_n e^{\alpha t_n} e^{-(\alpha+r)\delta}}{\alpha + r} \geq \dfrac{\pi_n e^{\alpha t_n^{\max}} e^{-(\alpha+r)\delta}}{\alpha + r} \geq K$ for $t_n \geq t_n^{\max}$, and $e^{-(\lambda-2\alpha)\delta} + \dfrac{\lambda(1 - e^{-(\lambda-2\alpha)\delta})}{\lambda - 2\alpha} \geq 1$.

To conclude the proof, let us assume that $t_n^{\max} > 0$ (the last claim in this proof shows that this must hold) and note that the previous argument allows us to restrict our attention to the set $[0, t_n^{\max}]$. On this set, $V_n(t_n)$ can be rewritten as follows:

$$\begin{aligned} V_n(t_n) &= \int_{t_n}^{t_n+\delta} f(\tau)(\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau + \\ & \int_{t_n+\delta}^{\infty} f(\tau)(\int_{t_n+\delta}^{\tau} \pi_n e^{(\alpha-r)s} ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau} e^{-(\alpha+r)s} ds - Ke^{-rt_n}) d\tau. \end{aligned}$$

By Assumption 3, this function is bounded above. Differentiating it with respect to t_n, solving the integrals and rearranging yields:

$$\begin{aligned}
V_n'(t_n) &= f(t_n+\delta)\left(\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha(t_n+\delta)} e^{-(\alpha+r)s} ds - \int_{t_n}^{\infty} \pi_n e^{2\alpha(t_n+\delta)} e^{-(\alpha+r)s} ds\right) - \\
& \quad f(t_n)\left(\int_{t_n+\delta}^{\infty} \pi_n e^{2\alpha t_n} e^{-(\alpha+r)s} ds - K e^{-r t_n}\right) + rKe^{-rt_n}\int_{t_n}^{\infty} f(\tau)d\tau - \\
& \quad \int_{t_n+\delta}^{\infty} f(\tau)\pi_n e^{(\alpha-r)(t_n+\delta)} d\tau - \int_{t_n}^{t_n+\delta} f(\tau)\pi_n e^{2\alpha\tau} e^{-(\alpha+r)(t_n+\delta)} d\tau \\
&= e^{-(r+\lambda)t_n}\left[\begin{array}{c} -\frac{\lambda e^{-(\alpha+r)\delta}\pi_n e^{\alpha t_n}}{\alpha+r} + (r+\lambda)K - e^{-(\lambda+r-\alpha)\delta}\pi_n e^{\alpha t_n} \\ -\frac{\lambda e^{-(\alpha+r)\delta}(1-e^{-(\lambda-2\alpha)\delta})\pi_n e^{\alpha t_n}}{\lambda-2\alpha} \end{array}\right]
\end{aligned}$$

If an interior maximum t_n^e exists, it must be such that $V_n'(t_n^e) = 0$. Then the first-order condition yields:

$$t_n^e = \frac{1}{\alpha}\ln\left(\frac{K(r+\lambda)}{C\pi_n}\right) > 0. \tag{1}$$

Now let us check that $V_n''(t_n^e) < 0$, so that t_n^e is indeed a global maximizer. Because $V_n'(t_n) = e^{-(r+\lambda)t_n}\left((r+\lambda)K - C\pi_n e^{\alpha t_n}\right)$, (1) implies that $V_n''(t_n^e) = -\alpha C\pi_n e^{\alpha t_n^e} < 0$. It is also clear that $V_n(t_n^e) > 0$, since $\lim_{t_n\to\infty} V_n(t_n) = 0$ and $V_n(t_n)$ is single-peaked. Finally, we claim that $t_n^e < t_n^{\max}$, since otherwise the following would hold:

$$\begin{aligned}
t_n^e &\geq t_n^{\max} = \frac{1}{\alpha}\ln\left(\frac{K(r+\lambda)}{C\pi_n}\right) + \frac{1}{\alpha}\ln\left(\frac{C(\alpha+r)e^{(\alpha+r)\delta}}{r+\lambda}\right) = t_n^e + \frac{1}{\alpha}\ln\left(\frac{C(\alpha+r)e^{(\alpha+r)\delta}}{r+\lambda}\right) \\
& \implies 1 \geq \left(\frac{\alpha+r}{r+\lambda}\right)e^{-(\lambda-2\alpha)\delta} + \frac{\lambda}{r+\lambda} + \left(\frac{\alpha+r}{r+\lambda}\right)\frac{\lambda(1-e^{-(\lambda-2\alpha)\delta})}{\lambda-2\alpha} \\
& \implies \frac{-\alpha(\lambda+2r)}{(r+\lambda)(\lambda-2\alpha)} \geq \frac{-\alpha(2\alpha+2r)}{(r+\lambda)(\lambda-2\alpha)}e^{-(\lambda-2\alpha)\delta}.
\end{aligned}$$

This would lead to the following contradiction if $\lambda > 2\alpha$: $\lambda + 2r > 2(\alpha+r) \geq 2(\alpha+r)e^{-(\lambda-2\alpha)\delta} \geq \lambda + 2r$. Similarly, when $\lambda < 2\alpha$ holds, we would get another contradiction: $\lambda + 2r < 2(\alpha + r) \leq 2(\alpha + r)e^{-(\lambda - 2\alpha)\delta} < \lambda + 2r$. Hence, $t_n^{\max} > t_n^e > 0$.

■

Proof of Lemma 3. The only difference with Lemma 2 is the determination of t_{n-1}^e, so we next show how it is derived, which requires analyzing firm $n-1$'s optimization problem. $V_{n-1}(\cdot)$ can be shown to be continuously differentiable at $t_{n-1}^{\max}$, so it suffices to show that it is quasiconcave with its maximizer smaller than $t_{n-1}^{\max}$. Furthermore, we will only pay attention to the properties of $V_{n-1}(\cdot)$ on the region $[\max(0, t_n^e - \delta), t_n^e)$,[23] on which firm $n-1$'s payoff function is:

$$
\begin{aligned}
V_{n-1}(t_{n-1}) &= \int_{t_{n-1}}^{t_n^e} f(\tau)(\int_{t_{n-1}+\delta}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds)d\tau \\
&+ \int_{t_n^e}^{t_{n-1}+\delta} f(\tau)(\int_{t_{n-1}+\delta}^{t_n^e+\delta} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds + \int_{t_n^e+\delta}^{\infty} \pi_n e^{2\alpha\tau}e^{-(\alpha+r)s}ds)d\tau \\
&+ \int_{t_{n-1}+\delta}^{t_n^e+\delta} f(\tau)(\int_{t_{n-1}+\delta}^{\tau} \pi_{n-1}e^{(\alpha-r)s}ds + \int_{\tau}^{t_n^e+\delta} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds + \int_{t_n^e+\delta}^{\infty} \pi_n e^{2\alpha\tau}e^{-(\alpha+r)s}ds)d\tau \\
&+ \int_{t_n^e+\delta}^{\infty} f(\tau)(\int_{t_{n-1}+\delta}^{t_n^e+\delta} \pi_{n-1}e^{(\alpha-r)s}ds + \int_{t_n^e+\delta}^{\tau} \pi_n e^{(\alpha-r)s}ds + \int_{\tau}^{\infty} \pi_n e^{2\alpha\tau}e^{-(\alpha+r)s}ds)d\tau \\
&- \int_{t_{n-1}}^{\infty} f(\tau)Ke^{-rt_{n-1}}d\tau.
\end{aligned}
$$

If firm $n-1$ waits until t_{n-1} then, for realizations of τ larger than t_{n-1} but smaller than t_n^e, it would enter a declining industry and gain oligopolistic profits indefinitely (for $n-1$ firms). If the ascending phase of the cycle lasts enough so as to allow for firm n's investment, but not for firm $n-1$'s entry, then both firms would enter when the industry is declining and firm $n-1$ would gain oligopolistic profits for $n-1$ firms until $t_n^e + \delta$, and oligopolistic profits for n firms thereafter. If the growing stage of

[23] It can be easily checked that the first-order condition of firm $n-1$ does not vary for $0 \leq t_{n-1} < t_n^e - \delta$.

the cycle were sufficiently long so that firm $n-1$ could enter and firm n could invest, then firm $n-1$ would enjoy oligopoly profits for $n-1$ firms until n entered. From such moment onwards, it would enjoy oligopolistic profits indefinitely (for n firms).

Note that $V_{n-1}(t_{n-1})$ can rewritten as a function of $V_n(t_n^e)$ after some manipulations:

$$\begin{aligned}
V_{n-1}(t_{n-1}) &= \int_{t_{n-1}}^{t_{n-1}+\delta} f(\tau)(\int_{t_{n-1}+\delta}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_{n-1}})d\tau + \\
&\int_{t_{n-1}+\delta}^{\infty} f(\tau)(\int_{t_{n-1}+\delta}^{\tau} \pi_{n-1}e^{(\alpha-r)s}ds + \int_{\tau}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_{n-1}})d\tau - \\
&\int_{t_n^e}^{t_n^e+\delta} f(\tau)[\int_{t_n^e+\delta}^{\tau} (\pi_{n-1}-\pi_n)e^{2\alpha\tau}e^{-(\alpha+r)s}ds]d\tau - \\
&\int_{t_n^e+\delta}^{\infty} f(\tau)[\int_{t_n^e+\delta}^{\tau} (\pi_{n-1}-\pi_n)e^{(\alpha-r)s}ds + \int_{\tau}^{\infty} (\pi_{n-1}-\pi_n)e^{2\alpha\tau}e^{-(\alpha+r)s}ds]d\tau \\
&= \int_{t_{n-1}}^{t_{n-1}+\delta} f(\tau)(\int_{t_{n-1}+\delta}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_{n-1}})d\tau + \\
&\int_{t_{n-1}+\delta}^{\infty} f(\tau)(\int_{t_{n-1}+\delta}^{\tau} \pi_{n-1}e^{(\alpha-r)s}ds + \int_{\tau}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_{n-1}})d\tau + \\
&V_n(t_n^e) - \int_{t_n^e}^{t_n^e+\delta} f(\tau)(\int_{t_n^e+\delta}^{\tau} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_n^e})d\tau - \\
&\int_{t_n^e+\delta}^{\infty} f(\tau)(\int_{t_n^e+\delta}^{\tau} \pi_{n-1}e^{(\alpha-r)s}ds + \int_{\tau}^{\infty} \pi_{n-1}e^{2\alpha\tau}e^{-(\alpha+r)s}ds - Ke^{-rt_n^e})d\tau.
\end{aligned}$$

The value of firm $n-1$'s investment opportunity nests the value of the immediate follower (a constant from its viewpoint) so that its maximand is structurally similar to that of the follower plus a constant. Hence, its solution must be the same, *mutatis mutandis*, and thus $t_{n-1}^e = \dfrac{1}{\alpha}\ln\left(\dfrac{K(r+\lambda)}{C\pi_{n-1}}\right)$. ∎

Proof of Proposition 1. The proposition is true for firms n and $n-1$ by Lemmas 2 and 3, and the formal argument for the remaining firms parallels that of the proof of Lemma 3. ∎

Proof of Proposition 3. $(t^e_{i+1} + \delta) - (t^e_i + \delta) < (t^e_i + \delta) - (t^e_{i-1} + \delta)$ if and only if $t^e_{i-1} + t^e_{i+1} < 2t^e_i$, which holds if and only if

$$\frac{1}{\alpha} \ln \left[\left(\frac{1}{\pi_i} \right)^2 \left(\frac{K(r+\lambda)}{C} \right)^2 \right] > \frac{1}{\alpha} \ln \left[\frac{1}{\pi_{i-1}\pi_{i+1}} \left(\frac{K(r+\lambda)}{C} \right)^2 \right].$$

This inequality holds if and only if $\frac{\pi_{i+1}}{\pi_i} > \frac{\pi_i}{\pi_{i-1}}$, which is equivalent to $\frac{\pi_i - \pi_{i-1}}{\pi_{i-1}} < \frac{\pi_{i+1} - \pi_i}{\pi_i}$. Multiplying through by -1 completes the proof. ■

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